**Course on Momentum Transfer in Process Engineering**
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**Lecture 3**
**Module 1**
**Equation of continuity in cartesian coordinates**

Good evening actually we had to finish previous class if you remember that it should be that equation of continuity, right? If you remember that we had told now we had given the different different ways of representation of equation of continuity and we had also given what is the meaning of or significance of those expressions we had also given. So we also told that in this class we will try to show you how you can develop those equation of continuity for different coordinate systems.

Obviously there are three coordinate systems as you know Cartesian coordinate or cylindrical coordinate or normally that we try to (())(1:24) because the other one that is spherical coordinate is normally not so much used in normal processing that is why that we will not go to develop and there is even more complicated unnecessarily time will be taken too much, so we will avoid that giving you the result final result of the spherical coordinate.

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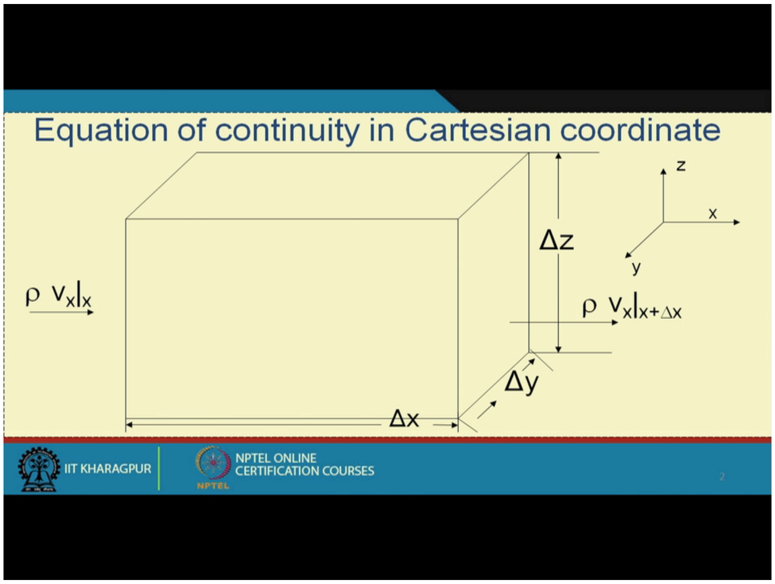

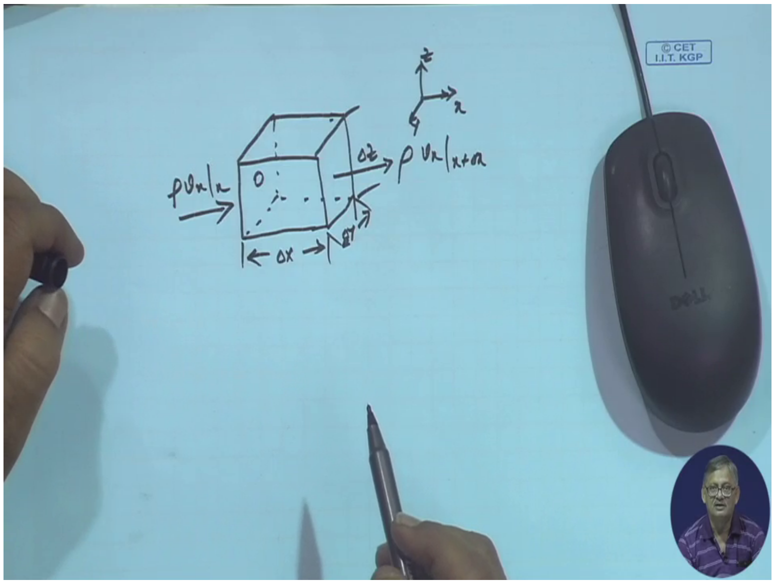

So if you look at so we can go to equation of continuity in the Cartesian coordinate system, right? We said in the beginning that we will take a volume element as small as we can think of infinite is (())(2:18) small as small as we can think of. And in that case as you see from here we have taken a small rectangular or Cartesian coordinate system, right? And we have defined them delta x delta y and delta z, right? So these three we had said and we assume that the mass which is coming in at the surface this surface the mass which is coming in is rho vx at the phase x and the mass which is going out is rho vx at the phase x plus delta x, right? And this is the Cartesian coordinate system, right?

We have given this are to be x this to be z and this to be y, right? These three coordinate systems we have given, right? So this small volume element we have taken and its volume is del x del y del z and we will do the analysis on this volume element, right?

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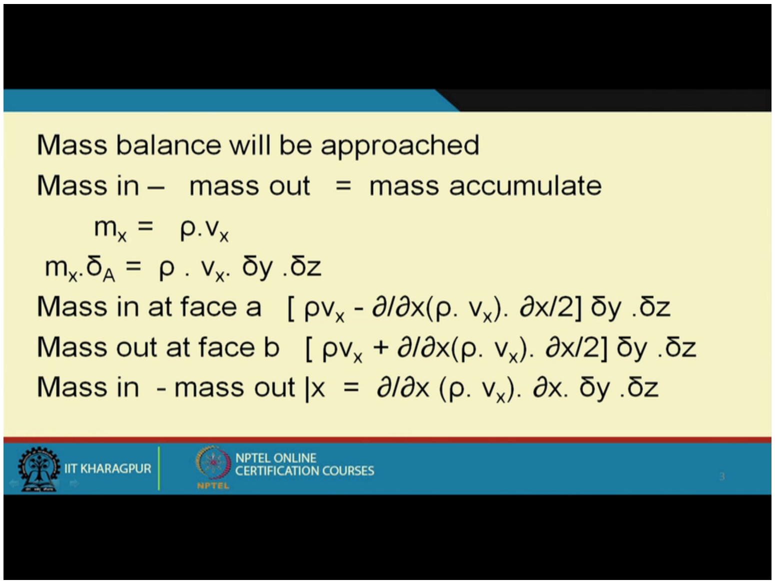


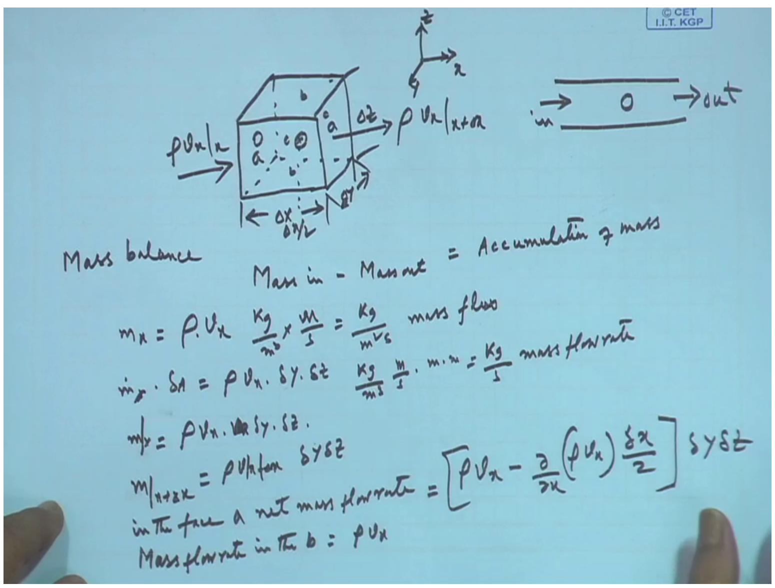

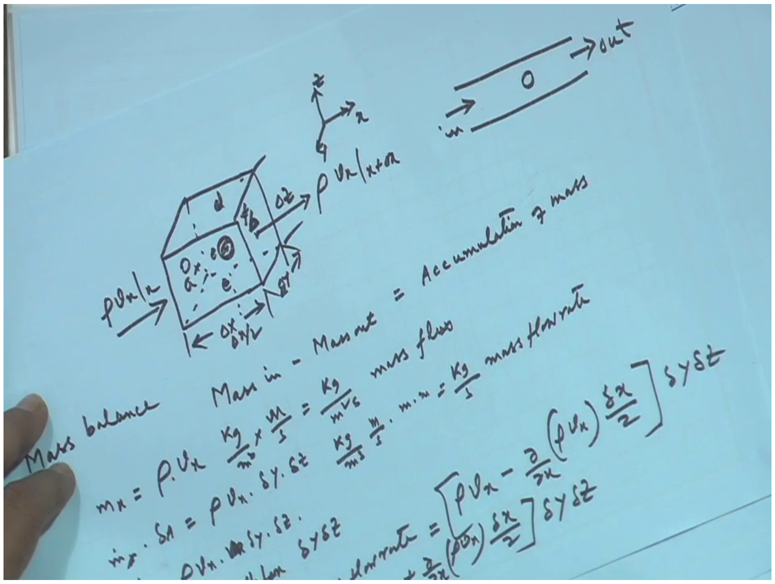


So if we look at this the basic equation is based on mass balance is that mass in minus mass out should be mass accumulation you remember there we had given you this kind of that in a pipe if we had taken this as the volume element and if the in is like this and out is like this, then we can say that if on the basis of mass balance we can say that mass in minus mass out that should be the accumulation of mass, right? Accumulation of mass.

So this we have already said, now how much mass is getting in at the x direction? It is if we tell it to be mx that is equals to rho times vx, right? rho is the density and vx is the x component velocity, right? x component velocity is vx and rho is the density. So rho vx is the mx and if we tell you see this unit is coming kg per meter cube into meter per second, so it becomes kg per meter square second that is we already said this is nothing but mass flux, right? So the total mass which is coming in at the phase x we can write mx is into the area del a is rho vx times del y into del z the other two that is del y del z this is the del y and this is the del z, so this area this area is del y del z.

So whatever is coming on this phase where we have put this one is that rho vx to del y to del z, what is the unit of this? Kg per meter cube into meter per second into meter into meter equal to kg per second that is the mass flow rate, right? So if this is true, then we can say that mx is equals to nothing but rho vx into vx rho vx into sorry del y del z, right? Similarly this is at the phase x similarly at the phase x plus delta x we can say this is again nothing but rho vx into at the phase that area is same rho into v at x plus delta x, right?

So that should be x plus delta x and the area remaining same that is del y del z. So we can say that at in the phase a in the phase say a if this to be a, right? So whatever is coming in and out the net in and out is equals to net in and out net mass flow rate this can be said equals to rho vx, right? Now if we assume that our center is here, right? Our center is here so the balance if we do on this point so whatever is coming here some will be accumulated and some will be going out, right? So in that case we can say this is rho vx minus del del x of rho vx into del x by 2 times the area that is del y del z, why we are saying you see this is at the x by the mid-point, so it is x by 2 or delta x by 2, right? If it is (del) delta x by 2, then the rate of change of rho vx with respect to ax in the or at the distance of del x by 2 is this and this was the rho vx at the phase where it is a.

So the net will be rho vx minus del lx of rho vx times del x by 2 and this into del y to del z, right? Similarly, we can also write in the same way mass flow rate in the phase say if it be b, so if it is b b is what we can say this to be b or this to be b and the third one is that bottom so this to be c opposite to that to be c, right? So in that case at the phase b what we can say in that case this will area will be this that is del y del z, right? In the in the phase this will be del y del x that b and c will be in this phase that is del y del z, right?

So if we write that in the phase b we can say this will be rho vx or this will be a for x component, right? So this will be rho vx in and the okay let us look into let us look into this again that we had we had here in this center so this was a and this is b sorry, this is b so this phase is a and this phase is b and similarly the other phases could be this phase could be c, this phase could be d or the other two phases this could be e and this phase could be f but we are both first for the x direction we are bothered for this how much is coming in this phase and how much is going out from this phase, right?

So in this phase rho vx was coming minus this was not coming that del del x of rho vx into del x by 2. So in the other phase what we are getting this rho vx plus this which is going there that is del del x of rho vx del del of del del x of rho vx into del x by 2 the area that is del y del z, right?

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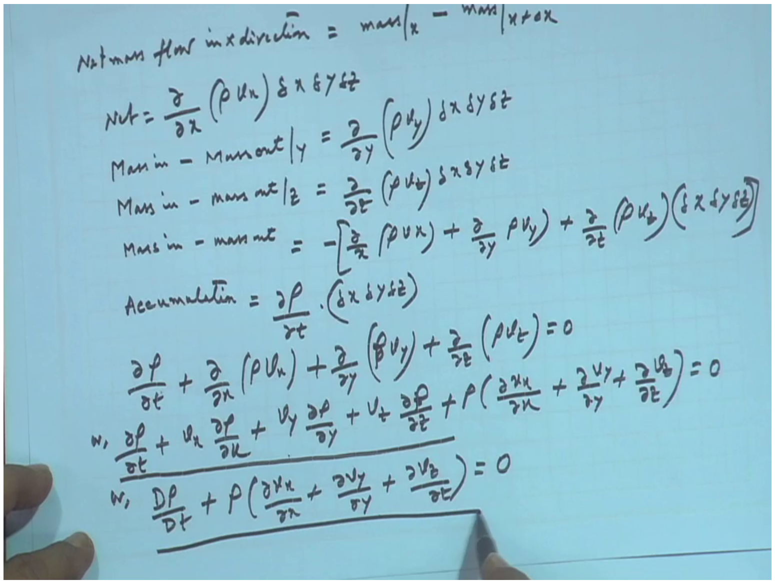

So we can write the net mass flow in x direction that we can write that is equals to the mass which is coming in at x minus mass which is going out at x plus delta x, right? So this is mass in minus mass out if we write that then we can write from there whatever we have written earlier here that this minus this, right?

So if we make that this del del x of rho vx into del x del y del z, right? And this is the mass in minus mass out, right? So that becomes del del x of rho vx del x del y del z, why del x? It is because this del x by 2 and this del x by 2, right? So this two put together is this minus this so it will be 1 plus, right? So out minus in is that so this is del del x of rho vx that is net which is flowing. Then if we look at similarly on the other two coordinates that y and z, then mass in minus mass out at the phase or in the direction y we also in the similar say can write del del y of rho vy into del x del y del z, right?

Similarly, mass in minus mass out in the z direction that also we can write del del z of rho vz into del x del y del z, right? So if we have taken care of all the three directions del x in the x direction, in the y direction and in the z direction if we have taken all whatever mass is coming in and whatever mass is going out that is the net mass flow in all three x, y and z directions then we can say that the total mass which has flown there. Now we have to look into from the basic equation if you remember we said net mass in and out or net mass flow must be equal to accumulation mass accumulation.

So accumulation we can say in that case or adding all these three, then we can that mass in minus mass out in all the three directions that can be written as minus of del del x of rho vx plus del del y of rho vy plus del del z of rho vz into area del x del y del z, right? So this is mass in minus mass out, so if we put like this, then we can write like that, okay, then accumulation that can be written, what is the accumulation? With respect to time we said that with respect to time if there is accumulation that is it is not steady it is an unsteady with time there will be some accumulation.

So what is the change in mass with time in the entire volume is the accumulation, so if we write that thing mathematically then we can say del del t del del or del rho del t, right? del del t of rho times del x del y del z this is the volume element, right? Kg per meter cube per unit time into meter cube so kg per second that is the accumulation, right? So this is the accumulation, so if we now add this two or if we put it in the original equation which was our mass balance equation that is mass in minus mass out is the accumulation if we look into that, then we can say that del rho del t plus del del x of rho vx plus del del y of rho vy plus del del z of rho vz.

So this must be equal to 0 because when we are equating this with this then the volume term this volume term is cancelling out, so in that case we get del rho del t plus this side when it is going del del x of rho vx plus del del y of rho vy plus del del z of rho vz is equal to 0, right? So if that be true, then we can expand this rho vx, no? This is nothing but uv product so we can write del rho del t, right? plus vx into del rho del x plus vy into del rho del y plus vz into del z del rho sorry del rho del z, right?

So this plus with when rho is constant uv so when rho is constant, then we can write del del x of vx plus del del y of vy plus del del z of vz, right? So this must be equals to 0. So we can now if you remember in the morning we had said that this del rho del t plus vx del rho del x plus vy del rho del y plus vz del rho del z, this can be replaced by the operand that was capital D if you remember we are said in the previous class that operand capital D so which we can substitute here as D rho Dt plus rho del vx del x plus del vy del y plus del vz del z is equals to 0, right?

So these we can we have substituted so this we had said that this is nothing but substantial time derivative if you remember, right? That same example that you were going through a canoe and this can canoe is flowing (alo) across not across along the flow of the stream. So when you are flowing like that so that situation was the substantial time derivative and we have taken that into consideration and there we showed that del rho del t plus vx del rho del x plus vy del rho del y plus vz del rho del z was the substantial time derivative, right?

So if now we replace that with D rho Dt we get this form, right?

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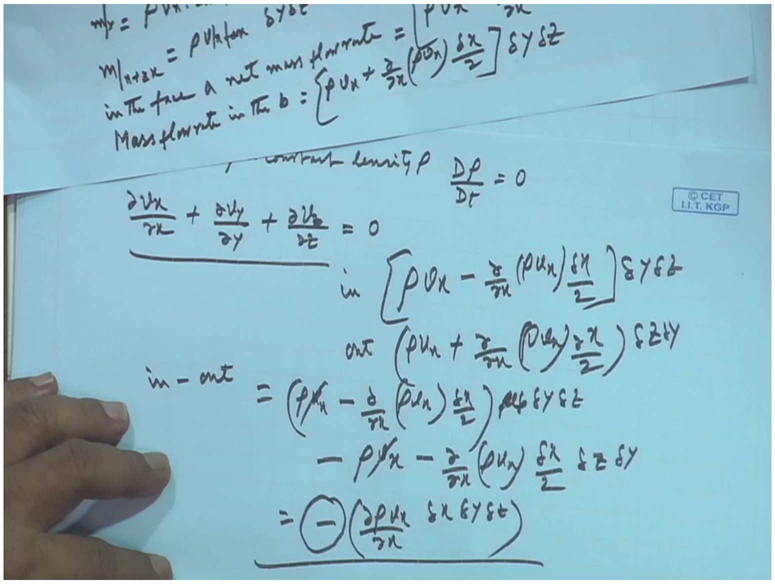


So when we are getting this if now we assume that the fluid is having a constant density that is rho is independent of time and position, right? If rho is independent of time and position, then we can say that rho is constant and in under that situation we can write this substantial time derivative D rho Dt is equal to 0, right? This we said earlier also that substantial time derivative if the density is constant and then we can say this D rho Dt to be equal to 0.

So we can write del vx del x plus del vy del y plus del vz del z this to be equal to 0, right? So this if you remember in the morning in earlier class we had said that this type of continuity equation we had develop or we had shown and its implementation or its its significance that also we had said in the previous class we said that now we are showing you this forms or different forms of continuity of equation or equation of continuity, but we will be deriving them in the rights situation or in the subsequent classes. So this is what we are now we have shown that time (deriva) rather in the Cartesian coordinate what is the equation of continuity and how it can be derived that we have shown, right?

And in that if you remember that we had taken a volume element, right? Of del x del y and del z this volume element we have taken we have said that are the phase a this rho vx at phase x is coming and at the phase b at x plus delta x distance from x this mass flow rate is rho vx at the phase x plus delta x, right? And we said we will do the mass balance that is mass in minus mass out and that should be equal to mass accumulation if you remember, and we said that what is mass? That is mx rho into vx, right?

So that is kg per meter square into second that is mass flux if it is multiplied with area, then it becomes the mass flow rate, right? This we said and we had shown so (m) this is why that mx or normally when it is rate it is represented with a dot. So dot x into del a is rho vx into area, which is the area? Perpendicular to the flow, so this is flowing on this phase so perpendicular to this flow is this so this is del y and del z, so del y del z came so rho vx this is the mass flow rate.

Now and that we have shown this is in kg per meter second, right? So rho vx del y del z is the mass flow rate. Now, similarly we can also write what is that with the respect to at the phase x plus delta x we said that was also rho vx at x plus delta x. Now how much is the quantity in that case? Now if we said that if we take the the balance equation or balancing point that this entire so whatever is coming here and whatever is going out from here that, so what we are getting that yes rho vx which has come but what had did not come is del del x of rho vx del x by 2 del y del z and at the phase b, right?

So this was rho vx which has come at the phase a but this del del x of rho vx del x by 2 did not come so that is why it is minus and at the phase b it is rho vx has come plus this much has come from there due to the distance. So del del x of rho vx del x by 2 del y del z, right? So if we added them so we added them, then the mass in minus mass out that we had written it was del del x of rho vx how much so it was like this, okay so mass in was this and mass out was this.

So here what I said earlier if you remember that we will do some mistake which you if you can identify here hopefully by this time we have seen suddenly this negative have has come, how the negative has come? If you write it clearly then you see it was mass in rho vx minus del del x of rho vx del x by 2, right? Into the area, area was del y del z, right? This was in and out was rho vx plus del del x of rho vx into del x by 2 into del y del z, right?

Then, if we make mass in minus out, then it should be rho vx minus del del x of rho vx del x by 2 into rho v into del y del sorry into del y del z, right? Minus because minus mass in minus mass out minus so it becomes rho vx minus del del x of rho vx to del x by 2 into del z del y or del y del z, right? So in that case this rho vx this rho vx goes out, so this becomes 2 rho 2 negative becomes 1 and then this becomes rho vx so this is del x by 2 del x by 2 so it becomes del x del y del z, right? So del del x of rho vx into del x del y del z that is the in the x direction that is how the negative term has come up.

So this I am telling you that sometime maybe purposefully we will do it I shall do it so that you can identify if there is any fault and if you really can identify and let us know then we will definitely let you know back the real thing or what was the what was the wrong in that. So you identify here I identified for you and showed you that how in future we will do. so please keep in mind that any such thing you will be able to find out if you can able to find, then definitely you can say that you are following the procedure or you are following the derivation or you are following the subject so that is one indication how you can identify if there be any mistake, right? So with this let us stop here with the Cartesian coordinate, next we will go for the spherical not spherical that is for the cylindrical coordinate, next class we go for the cylindrical coordinate because this takes time and since this takes time it is not possible that all the three coordinates if we go on doing and deriving and it, then it may not be justified by the time management, right?

So we will do with the Cartesian and then show you the polar coordinate how much that is your r theta phi, right? Thank you.