

Course on Momentum Transfer in Process Engineering
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Lecture 30
Module 6
Nozzle flow-problems and solutions

So we had finished in the previous class that sonic I mean not sonic velocity we had done with the nozzle flow, right? Nozzle flow, right? And we also has seen that what is the what is the velocity when you are nozzle flow is normal and what is the maximum velocity and what is the discharge corresponding to the maximum discharge or corresponding to the velocity maximum the discharge maximum that also we have seen, right?

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NOZZLE FLOW.

$$v_0 = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho} \left[1 - \left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} \right]}$$

$$v_{cr} = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho} \left[1 - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma-1}{\gamma}} \right]}$$

$$W = C_D A_0 \sqrt{\frac{2\gamma p p_0}{(\gamma-1)\rho} \left[\left(\frac{p_0}{p}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{p_0}{p}\right)^{\frac{\gamma+1}{\gamma-1}} \right]}$$

$$W_{cr} = C_D A_0 \sqrt{\frac{2\gamma p p_0}{(\gamma-1)\rho} \left[\left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma}{\gamma-1}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma+1}{\gamma-1}} \right]}$$

Now that if you remember we had given that v_0 was equals to v_0 was equals to, yeah under root $2 \gamma p$ divided by γ minus 1 into ρ into $1 - p_0$ by p to the power γ minus 1 by γ , right? This was our normal velocity and velocity under maximum which we call it to be critical velocity and this was under $2 \gamma p$, right? By γ minus 1 into ρ into $1 - p_0$ by p to the power γ minus 1 by γ under critical, right?

So if we remember that it was like this critical was that, okay p_0 by p yes W was this was $2 \gamma p$ no this was not this this was velocity, okay and W was given like this, W discharge was given as $C_D A_0$ under root $2 \gamma p$ by γ minus 1, right? And this with ρ and this was

p_0 by p to the power 2 by γ and this was p_0 by p to the power γ plus 1 by γ
 this was normal discharge and discharge under critical condition was $C_d A_0$, right? 2 by γ minus 1 and this was p_0 by p to the power 2 by γ under critical condition by p_0 by p to the power γ plus 1 by γ , right? So this was our critical velocity under nozzle flow and critical discharge under nozzle flow or flow through the nozzle, right?

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Prob. Air flows through a nozzle of diameter 1.3 mm having a discharge coefficient of 0.95, from a pressure of 5 atm to a pressure of 1 atm at 25 °C. What is the maximum velocity and mass flow rate?

Ans. We know that, $\rho = \frac{pM}{RT} = \frac{5 \times 101325 \times 29}{8314 \times 298} = 5.93 \text{ kg m}^{-3}$

Now, upstream pressure is 5 times greater than that of down stream pressure. Hence pressure ratio is at critical condition

$$v_0 = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 5 \times 101325}{(1.4-1) \times 5.93} \left[1 - (0.528)^{\frac{1.4-1}{1.4}} \right]}$$

$$= 315.83 \text{ ms}^{-1}$$




And we had done also your on solved the problem, so now if we see another problem let it be here if we see another problem which tells that (())(4:20) this problem we had done, so this is another problem that it says, Air flows through a nozzle of diameter 1.3 millimeter having a discharge coefficient of 0.95, from a pressure of 5 atmosphere to a pressure of 1 atmosphere at 25 degree centigrade, what is the maximum velocity and mass flow rate? Right?

I repeat, Air flows through a nozzle of diameter 1.3 millimeter having a discharge coefficient of 0.95, from a pressure of 5 atmosphere to a pressure of 1 atmosphere at 25 degree centigrade. What is the maximum velocity and mass flow rate? Right?

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NOZZLE FLOW. ✓

$$V_0 = \sqrt{\frac{2\gamma P}{(\gamma-1)\rho} \left[1 - \left(\frac{P_1}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$
$$V_{0cr} = \sqrt{\frac{2\gamma P}{(\gamma-1)\rho} \left[1 - \left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma-1}{\gamma}}\right]}$$
$$W = C_D A_0 \sqrt{\frac{2\gamma P P}{(\gamma-1)\rho} \left[\left(\frac{P_1}{P}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{P_1}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$
$$W_{cr} = C_D A_0 \sqrt{\frac{2\gamma P P}{(\gamma-1)\rho} \left[\left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma}{\gamma-1}} - \left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma+1}{\gamma}}\right]}$$

So if we go back to that previous which we have already written here that in the nozzle flow the velocity is like this and velocity under critical condition is like this discharge was like this and discharge under critical condition was like this. Now what was that critical condition?

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NOZZLE FLOW. ✓

$$V_0 = \sqrt{\frac{2\gamma P}{(\gamma-1)\rho} \left[1 - \left(\frac{P_1}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$
$$V_{0cr} = \sqrt{\frac{2\gamma P}{(\gamma-1)\rho} \left[1 - \left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma-1}{\gamma}}\right]}$$
$$W = C_D A_0 \sqrt{\frac{2\gamma P P}{(\gamma-1)\rho} \left[\left(\frac{P_1}{P}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{P_1}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$
$$W_{cr} = C_D A_0 \sqrt{\frac{2\gamma P P}{(\gamma-1)\rho} \left[\left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma}{\gamma-1}} - \left(\frac{P_1}{P}\right)_{cr}^{\frac{\gamma+1}{\gamma}}\right]}$$

$\left(\frac{P_0}{P}\right)_{cr} = 0.528$ critical

$\left(\frac{P}{P_0}\right)_{cr} = 1.893$

If we just remember a little that we said if the ratio of p_0 over p if this is equal to 0.528, then we call that this is under critical pressure ratio, right? Or the converse is also true that if the pressure ratio of p by p_0 is equals to 1.893, then also it is under critical pressure ratio that is called p_0 by p under critical this is also p by p_0 under critical, right? So if we apply that, right?

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$p = 5 \text{ atm}$
 $p_0 = 1 \text{ atm}$
 $D = 1.3 \text{ mm} = 1.3 \times 10^{-3} \text{ m}$
 $C_D = 0.95$
 $T = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$
 $\gamma = 1.4$
 $M = 29$

$\rho = \frac{PM}{RT} = \frac{5 \times 101325 \times 29}{8314 \times 298} = 5.93 \text{ kg/m}^3$

$v_0 = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma}{\gamma-1}} \right]}$

$= \sqrt{\frac{2 \times 1.4 \times 5 \times 101325}{(1.4-1) \times 5.93} \left[1 - \left(0.528 \right)^{\frac{1.4-1}{1.4}} \right]}$

$= 315.83 \text{ m/s}$

Maximum velocity = 315.83 m/s \Rightarrow critical pressure ratio.

And here in this problem we have been given that the pressure p is equals to 5 atmosphere and pressure p_0 is equals to 1 atmosphere we are also given a diameter we are also given a diameter of D is equals to 1.3 millimeter is equals to 1.3 into 10 to the power minus 3 meter, right? And we are also given that the coefficient of discharge that is C_d is equals to 0.95 and temperature T is 25 degree centigrade is equals to 25 plus 273 is equals to 298 kelvin, right?

So these are all given, obviously from the relation which we had given that velocity v_0 is equals to under root $\frac{2\gamma p}{(\gamma-1)\rho} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma}{\gamma-1}} \right]$ to know that we know p we know p_0 we know we do not know we know γ , γ we cannot given so we can assume it to be say 1.4 like the previous one.

So γ is equals to 1.4 if we assume, right? And ρ we have we have it has not been given, so ρ we can write equals to $\frac{PM}{RT}$ is equals to p is 5 atmosphere 5 into 101325 into air so we can say molecular weight of air is 29 so it would be 29 divided by R 8314 into 298 T . So this comes to equal to let us look into that calculator, let us look into that calculator 5 into 101325 into 29 is equal to this divided by 8314 divided by 298 is equal to 5.93 is equal to 5.93 kg per meter cube, right?

So if it is 5.93 kg per meter cube, then v_0 from this we can write to be equal to under root $\frac{2 \times 1.4 \times 5 \times 101325}{(1.4-1) \times 5.93} \left[1 - \left(0.528 \right)^{\frac{1.4-1}{1.4}} \right]$

minus p_0 by p critical, so we can write 0.528 to the power 1.4 minus 1 divided by 1.4, right? So if we start from here, so we can calculate it like this, this is 1 minus one bracket 0.528 to the power γ to the power 1.4 minus 1 that is 0.4 divided by 1.4, right? This is equals to that is equals to this is equal to 0.166, right?

So this into 101325 into 5 into 1.4 into 2 this is equal to that divided by 1.4 minus 1 that is 0.4 into 5.93. So this is that is equal to 99749, right? So hope we have done somewhere something wrong, hope we have done somewhere something wrong, so let us redo that is 0.528 to the power 1.4 minus 1 that is 0.4 divided by 1.4 is this to the power is that, right? So this is plus minus this so plus 1 is equal to 0.166 fine, into 101325 into 5 into 1.4 into 2 is equal to this, right? Divided by 0.4 into 5.93 is equal to so is equal to this, okay this is under root so if we take square root yes it is 315.83 so equal to 315.83 meter per second this is the velocity under critical or maximum velocity. So maximum velocity is 315.83 meter per second that is the velocity under critical pressure ratio, right? So this we have done, Now we like to see what is the discharge know.

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$$W_{cr} = C_d A_0 \sqrt{\frac{2\gamma p}{(\gamma-1)} \left[\left(\frac{p_0}{p_{cr}}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{p_0}{p_{cr}}\right)^{\frac{\gamma}{\gamma-1}} \right]}$$

$$A_0 = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1.3 \times 10^{-3})^2 = 1.327 \times 10^{-6} \text{ m}^2$$

$$W_{cr} = 0.95 \times 1.327 \times 10^{-6} \sqrt{\frac{2 \times 1.4 \times (5 \times 10^5 \times 101325 \times 5.93)}{(1.4-1)} \left[(0.528)^{\frac{1.4}{1.4}} - (0.528)^{\frac{1.4}{1.4}} \right]}$$

$$= 0.001496 = 1.4 \times 10^{-3} \text{ kg/s}$$

$$= 5.386 \text{ kg/k}$$

And for that discharge under critical pressure ratio we have seen W_{cr} is $C_d A_0$ under root 2 gamma p rho by gamma minus 1 into p_0 by p to the power 2 by gamma under critical pressure ratio minus p_0 by p to the power gamma plus 1 by gamma under critical pressure ratio, right? So if we know see that what is the A_0 , right? What is the value of A_0 , A_0 because C_d is given, so

what is the value of A_0 if we look at then we can see that A_0 is π by 4 into D square that is π by 4 into D was D was given, yeah D was given 1.3×10 to the power minus 3 meter, right?

So 1.3×10 to the power minus 3 square, so this becomes equals to cancel π by 4 into 1.3×10 to the power minus 3, right? So this becomes that this square of this now square of this is this, right? So this becomes equals to 1.327×10 to the power minus 6 meter square, right? So if that is true, then C_d is given W_{cr} critical condition C_d is given C_d was given 0.95, right? So C_d was given 0.95 A 1.327×10 to the power minus 6 and this is under root 2 γ is 1.4 p is 5 into 101325 ρ we got previously yeah ρ we got 5.93, right?

So we got 5.93 by 1 minus γ or 1.4 minus 1 into this times p_0 by p critical, so 0.528 to the power 2 by γ 1.4 minus 0.528 to the power γ plus 1 1.4 plus 1 divided by 1.4, right? So this is that, so if we look at what is the value so then let us start from the other end that is 0.528 x to the power 2 by γ that is 1.4, right? So this is equal to that minus how much 0.528 so 0.528 to the power 1.4 plus 1 that is 2.4 divided by 1.4 is this, so is this so is equal to this, right? So this multiplied by 2 multiplied by 1.4 multiplied by 5 multiplied by 101325 multiplied by 5.93, right?

So this is equal to that divided by 1.4 minus 1 that is 0.4, right? So much, so the whole is under root so this becomes 2905, right? This become 2905 so that means again we have done a mistake, so let us redo and first we check whether we have written everything we have not done mistake my goodness, so after square root we had to add this, so this we have not given unnecessarily we are redoing, so 0.528 to the power 2 by γ is this equal to this minus 0.528 to the power (1.) 2.4 divided by 1.4, right?

So this becomes that, right? And then the second one then is equals to 0.06 earlier also hopefully it was like that into 2 into 1.4 into 5 into 101325 into 5.93 is equals to so much divided by 1.4 minus 1 that is 0.4 is equal to so much under root of this is this much into which we missed last time 0.95 is equal to this into 1.327×10 to the power minus 6 so this is equal to 0.00149, right? So 0.00149 so it is then 0.001496 that means 1, 2, 3 so if 1, 2, 3, 4, 5, 6 so it is it is 1, 2, 3 1.4 is equals to 1.4×10 to the power minus 3 kg per second, right?

So this can be made into kg per hour by multiplying 3600 is equal to 5.386, okay kg per hour, right? Now the difference is that hopefully here it is differing but 0.528 to the power 2 by 1.4 we

have taken, right? 0.528 to the power 1.4 plus 1 by 1.4 we have done, right? 5.93 is the density we have taken correctly 101325 we had taken for that, but perhaps 5 is missing here, right? So from this value whatever we have gotten there that is $(2.)$ no, 6.69 10 to the power minus 4 so we know our value this is 5.386 so if we do 6.69 into 10 to the power minus 4 is equal right into 5 under root is this so it comes like that.

So we missed 0.01496 so we missed here somehow this term p that is 5 into 101325 that should be there, right? So correct it and then it is okay, right? So correct in the slide that 5 this figure was not there in the p so that should have been so we have made it to 2 Pascal that is true but somehow missed there so correct one is this 5.386 kg per hour or 0.001496 or 1.4 10 to the power minus 3 kg per second, right? So we have found out the critical velocity and critical mass flow rate, right? Thank you.