Course on Momentum Transfer in Process Engineering
By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur
Lecture 33
Module 7
Variable fluid flow

Hello, we were with sonic velocity, right?

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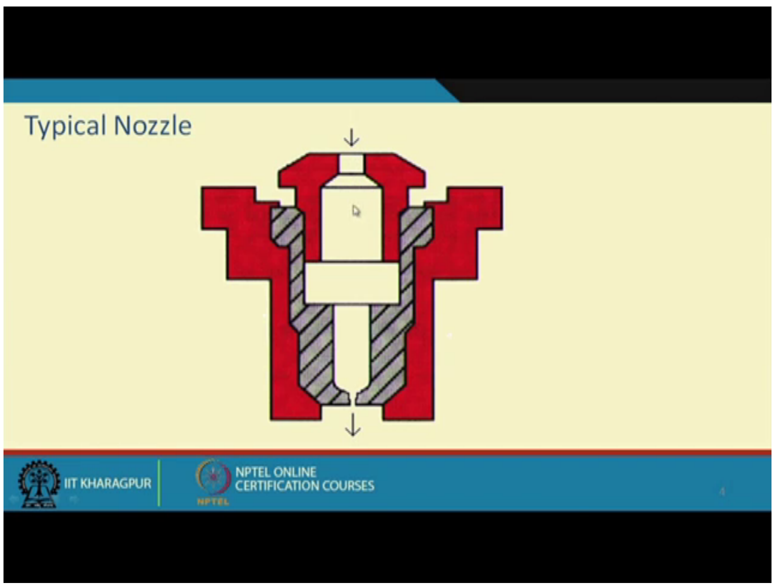

And we said that there are many examples we would like to show you also here that this is one such typical nozzle, right? So it is coming from this end so this is the incoming and this is the outgoing and this is called tip, right? So this tip velocity which we developed earlier so that is v0, right? Or vt whatever you call it to be so that or outlet v outlet. So from high pressure at with a lower velocity to low pressure with very high velocity this will come out and we also showed that the velocity corresponding to this at the tip is the velocity of the sound if that pressure ratio is under critical condition that is 0.528 this we said and that is what it is also being shown this is in a nozzle.

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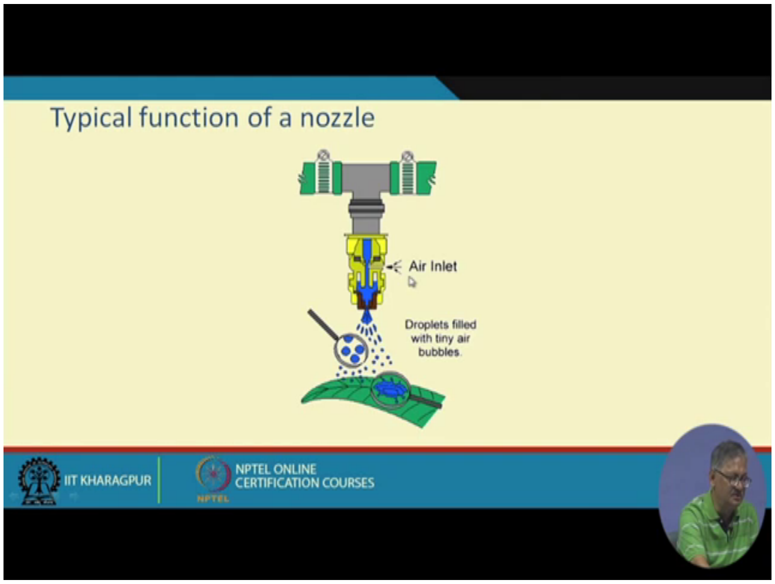

Typical flow throw through nozzle like this this diagram we had shown this is another typical function of nozzle where air is going in and you see that many nozzles are used particularly in spray drying if you see that spray dryer where it is there. So nozzles are there which are atomizing the inlet of the fluid and they are also similar behaviors you can expect, right?

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So this things are there and this is a typical siren I referred to earlier that typical siren when it is in the industrial area you get that siren sound or we all I also said that during war which you have not seen but we have and we have seen that during that period to warn people or alert people this siren was ringing and it was really a huge sound from anywhere you can hear it, right? So this is another example of this nozzle flow, okay.

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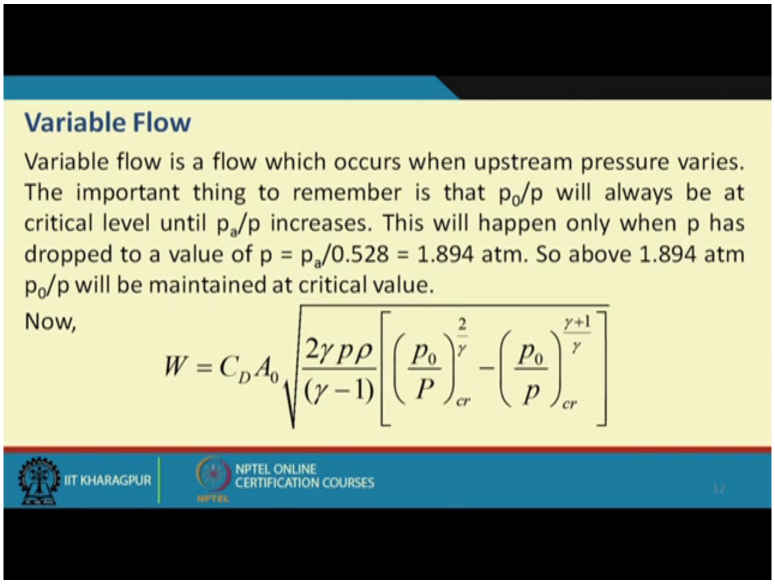

Now we come back to another which is called variable flow, right? I give an example of it very very now and then you come across, nowadays there is a go for celebrating birthday or some other kind of days, right? So and there you have seen that many balloons are there which is decorative and while you I do not know how many of you have seen that while you are pumping balloon with your mouth, so as sometimes you are pumping it is blowing up and again when you are stopping that time it is again going down.

So this process of pumping and getting off from the balloon these two or together and ultimately you come with a balloon. Now with that balloon if you had ever seen that by chance if it slips from your hand it goes like a I do not know whether you have seen that during your childhood you used to burn during Diwali some fireworks like where you put some fire and then it moves like this, right? With a sound and similar kind of thing when you have the balloon full with the volume of air and suddenly if slips it goes from your end to throughout the space where it is there and you do not know you have no control it goes there, right?

This is an example of this and this that you had the balloon full of air and now you allow it to or rather suddenly you can do it purposefully in many cases people do or at least kids do play with that that they blow the balloon and then allow it to move, right? So that purposefully you can do or by mistake or by slip it may go out of your hand this effect is same and this kind of flow is called the variable flow means you had the volume of the balloon now suddenly it is dropping down and this flow is known as the variable flow, right?

Now today we will discuss on this variable flow, okay. So variable is a flow which occurs when upstream pressure varies, important thing to be remember is that the p0 by p that is the tip to the inside or outlet to the inlet so p0 by p will always be at critical level until pa by p, now what is that pa? So we said that you had the balloon so in that balloon whatever pressure is inside that is the p at the tip of the balloon the pressure which will come out is the p0, now where it will go? It will go to the atmosphere, so this is pa that is atmospheric pressure, right?

So until pa by p increases p0 by p will have critical value, right? At its critical level until pa by p increases and this will happen only when p has dropped to a value of p equal to pa by 0.528 that is 1.894 atmosphere. So until it attains that 1.894 atmosphere this p value it drops this will be under critical condition that is p0 over p will be under critical condition that we have to keep in mind, right? So we said that p0 by p will be under critical condition until the value of p by pa, right? p is the inlet pressure and pa is the pressure of atmosphere where this p0 is delivering, right?

So this ratio p over p0 until it reaches it decreases of course and till the value p is equals to pa by 0.528 that is 1.894 until if the value of p comes down to p equal to 1.894 atmosphere till that time it will be under critical condition that is critical pressure ratio p0 by p will be under critical pressure ratio condition that is 0.528. So above 1.984 atmosphere p0 by p will always maintain at critical value, right?

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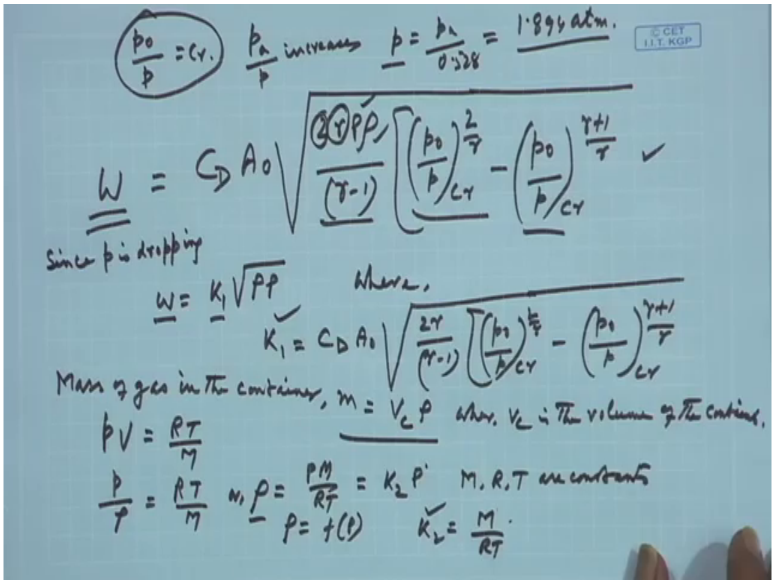

If that be true, then we can say that the discharge can be written W equals to Cd A0 under root 2 gamma p rho by gamma minus 1 into p0 by p to the power 2 by gamma under critical condition minus p0 by p to the power gamma plus 1 by gamma again under critical pressure ratio condition. This is valid this equation because we said that p0 by p will be under critical condition at critical condition till the pressure drops down to pa by p that is this increasing, right? Until this is increasing that means this will happen till when p is equals to pa over 0.528 so that means it is if we take p at 1 atmosphere, so then it is 1.894 atmosphere, right?

Until p comes down to 1.894 p0 by p will be under critical condition. So under that situation the discharge can be written as Cd A0 2 gamma p rho gamma minus 1 gamma p rho by gamma minus 1 into p0 by p to the power 2 by gamma under critical condition minus p0 by p to the power gamma plus 1 by gamma again under critical condition, right?

Now, since p is dropping so we can say W is equals to K1 1 constant under root rho into p, right? Under root rho into p where this K1 this is equals to Cd A0 under root 2 gamma by gamma minus 1 into p0 by p to the power 2 by gamma under critical minus p0 by p to the power gamma plus 1 by gamma again under critical condition. So this is K1 because these are constant 2 constant, gamma constant, so this is constant, this is also having a definite value under critical condition, this is also having a definite value so variable is only p and rho, rho and p are only variable so that is why we can write that W discharge is a coefficient K1 under root p rho where

K1 is this Cd A0 under root 2 by gamma by gamma minus 1 into p0 by p under critical condition to the power 2 by gamma minus p0 by p to the power gamma plus 1 by gamma at critical condition, right?

So if this be true, then if we say that mass of gas in the container if that is assumed to be m, then m can be written as Vc into rho, right? Where, of course Vc is the volume of the container, volume of the container is the Vc, right? Then, if m is Vc then we can from this gas relation we can write pV is equals to RT by M or p by rho is equals to RT by M, right? So p by rho is RT by M or we can write rho is equals to PM by RT, right? Where, rho is a (function) variable of p so we can write this is nothing K2 into p because M, R and T are constants, right?

If M, R and T are constants, then we can write rho is a function of p, rho is a function of p only where M, R, T being constant is K2 and this K2 is equals to M by RT, right? So we know K1 we know K2, right?

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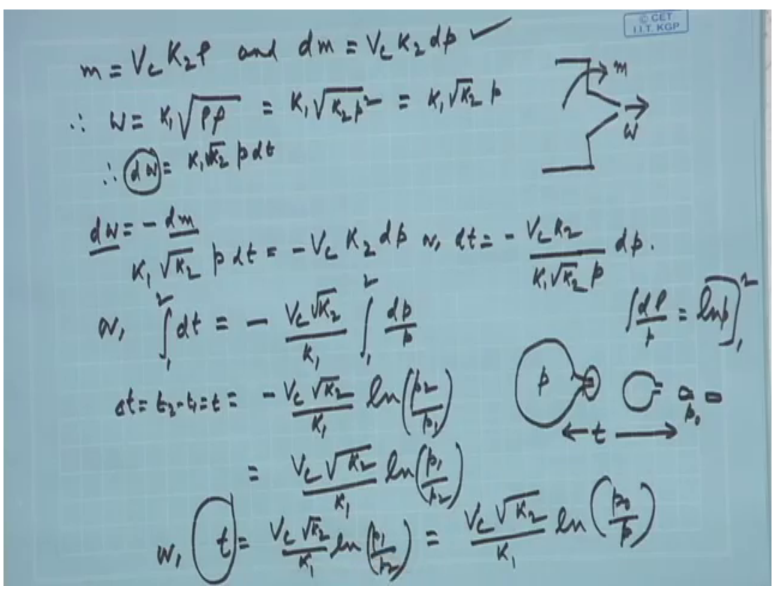

Then, we can write that M is equals to Vc into K2 p, right? And from there if we differentiate we can write dm is equals to Vc K2 dp because this c and K2 are constant so we on a differential of M is equals to this constant times differential of p, right? Therefore, we can write from W is equals to K1 under root p rho which we have seen is earlier is equals to K1 under root K2 p square, right? Because we have seen K2 is that rho is K2 p, right? rho is K2 p so K2 p square K1 under root K2 p square, right? This is what we can say and that can be simplified as K1 under root K2 into p because p square from the root it comes out and it is p K1 under root p, right?

Therefore, from this we can write a small change in W or small discharge W is dw that is nothing but K1, K2 under root and this is into p into dt, right? So a small discharge dw can be written as K1 under root p K1 under root K2 into p times dividing that period of delta t dt whatever change has been made, right? Now if there is a discharge we our thing was like this if there is a discharge from there, right? So if W be the discharge, then corresponding m quantity will be depleted from the container.

So we can write dw is equals to minus dm because when W will be discharge corresponding m quantity from the container will deplete. So this depletion of the mass is equivalent to the discharge, but that is why it is negative because that will be gradually depleting, so to incorporate that fact this negative sign is introduced so dw is minus dm, right? So we can write that dw already we have seen how what it is, it is K1 under root K2 into p into dt this is equals to minus Vc into K2 into dp that is from the dm, right? This is from dm that Vc K2 dp with a negative and here W was K1 under root K2 into p and we said that small quantity dw is discharge at the pressure p for a time small quantity of time of dt.

So total quantity is discharge is this K1 K2 were constants so this was the total quantity which is discharged, right? And since this discharge has to be equivalent to the depletion of the mass from the container so it is dw is equals to minus dm, right? Now if this we rearrange and write in terms of integral and integrate for a definite integration of say point 1 to 2 or time t1 to time t2 integral of dt this can be written as minus Vc is constant, then root K2, right? Over K1 because this comes out, right? K2 okay let us rewrite this.

Or dt is equals to minus Vc K2 over K1 root K2, right? Over p, right? This into dp, right? So Vc this root K2 and K2 goes off so 1 root K2 remains so minus Vc root K2 over K1 so it is between point 1 to 2 dp over p, right? Now dp over p if we integrate between the points 1 to 2, then dp over p is integral of this is lnp with the domain domain is 1 to 2, right? So we can write that lnp 2 by p1 so this is equals to minus Vc root K2 over K1, right? Over K1 times this is ln p2 over p1, right? ln p2 over p1 and this we can this minus we can take in like this Vc is equals to so this integration is t2 minus t1 or delta t or whatever we call t delta t t2 minus t1 is equals to t whatever we call it, right? Is equals to delta t anything we write that is equals to this minus we are taking off Vc under root K2 over K1, right? ln of since we have taken there, so it is p1 over p2, right? So ln of p1 over p2.

Therefore, the time require time t is equals to Vc under root K2 over K1 ln of p1 over p2 this tells us that what is the time that is required for the pressure to drop from a pressure p1 to a pressure p2, right? Under critical condition because we have taken all this K2 K1 everywhere it is this pressure under critical condition that is pressure ratio p1 by p2 or p0 by p whatever we call it to be that is the pressure ratio is under critical.

So this we can also write in that earlier in the same fashion that K under root K2 K1 ln of p0 over p, right? So this is also there, okay. So we can tell what is the time that is required for the pressure to deplete from a pressure p to a pressure p0, right? From a pressure p or this was of course no this is in that case p1 to p to p1 1 is lower and 2 is higher limit, so when we have given here this is one lower limit to a higher limit, okay. So lower limit is 0 p0 and higher limit is p2 okay p, right?

So much time it will require to deplete from a pressure p to p0 or from a pressure p2 to p1 whatever we call or from a pressure p outlet to p inlet that time require we can find out, right? This is how we can determine the time required, right? You remember we had given the volume of the balloon like this, right? And here you normally tie with something, right? Here we tie with something but if you do not tie or if it removes, then after some time it reduced by volume, right? And then after some time further reduces by volume and then it becomes a small balloon like this, right?

So this pressure p if it is reducing to the pressure po, how much time it will take for this t to happen the total time required t how much it is that we can also determine only we have to keep in mind this is called variable flow and whenever variable flow is there, that means we assumed that p0 by p is under critical pressure ratio condition, right? And we also said this will happen till the pressure increases from p I mean this ratio increases to p by pa, right? Where pa is the atmospheric pressure, right?

And we also said this means that the pressure p is still it is coming to the number that is p atmosphere 1 atmosphere if it is 1 by 0.528 that is 1.894. So till the pressure comes to the 1.894

atmosphere this critical condition will remain, right? So upto 1.894 atmosphere, right? This pressure ratio will remain under critical condition and in that situation we can find out what is the time required for the pressure to drop from a pressure p1 to a pressure from a pressure p rather okay if we tell one to be the inlet from a pressure p1 or in this case since we have integrated so this is the lower value this is the higher value so in that case we say that p1 to be the outlet and p2 to be the inlet.

So this is the higher this is the lower, right? And in any case we can say that p1 is the say if it is inlet then if p2 is the outlet better to say that po p outlet and p there is the inside pressure, so until this pressure ratio is 1.894 the p by p0 that is p by p0 it is coming p by p0 by atmosphere p0 by p is 1.894 till that p is 1.894 it is under critical pressure and we can find out that the time required for the pressure drop from a initial pressure to a final pressure, okay. Thank you.