Course on Momentum Transfer in Process Engineering
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Lecture 4
Module 1
Equation of continuity in cylindrical coordinate system

Hello, we you remember that in the last class we had said that we are doing with Cartesian coordinate that is x, y, z. Now this time we will be doing with the cylindrical coordinate that is r, theta, z, right?

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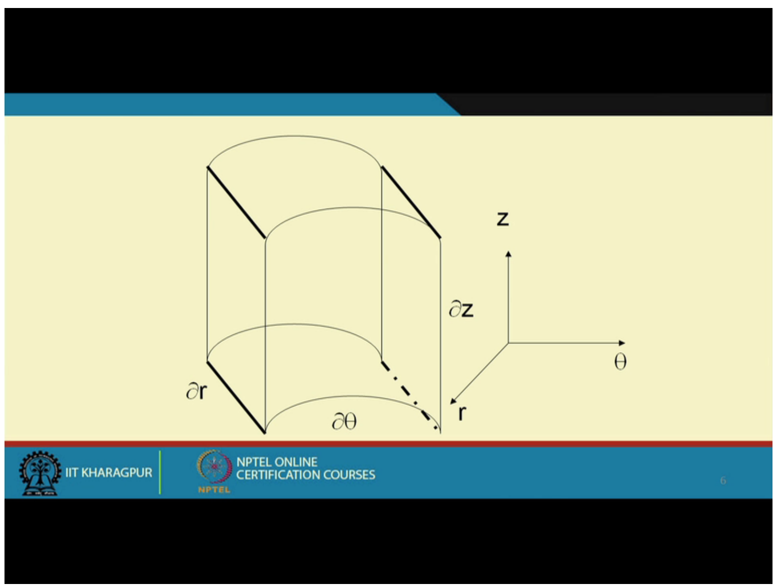

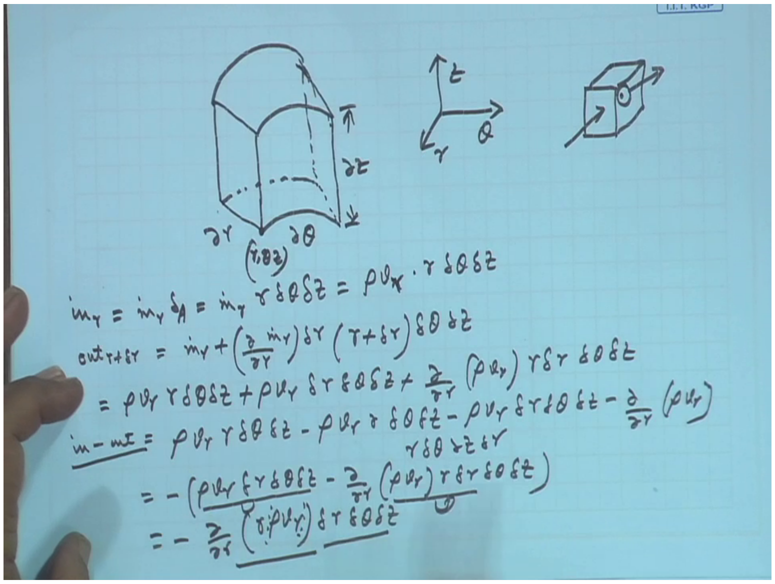

With that we will do, so let us look into so un unlike the previous one where we had said that x, y, z was the coordinate here we take some cylindrical coordinate like this and, right? And okay so this can be del theta, this can be del r and this is del z, right? So in the coordinate system we have r theta z so this is z, this is theta and this is r.

So if we take r theta z coordinate, then here we are taking this point as the r theta z coordinate, right? And in the previous one if you remember where we had said let me just recapitulate where we had said that we were taking a volume element at the center, right? We said if this is going like this and that is going like that so how much mass is coming here we took it as the balancing point. So here in this case if we take this balancing point to be at the r theta z point, right?

Then, this is of course r plus delta r, this is r plus delta r, this is theta plus del theta and this is z plus del z only one thing which is we are taking that area under this and area under this they are though there is a difference because in all the cases this will be all difference and in that case we will take this is z plus delta z, right? And this also will be different, this also will be r plus del r so all the changes whatever since we have said we are taking the volume element very small as small as possible.

So those changes will be ignoring, otherwise this will become too complicated so just to avoid the complication we are assuming that this area under this and the area under this they are more or less same. So the changes with that del quantity r plus del r or theta plus del theta or z plus del z this we are neglecting for the upper one, right? So if the on the similar fashion as we have done in the Cartesian coordinate so if we do for the cylindrical coordinate in the same way, then we can write in in the phase r, right? in in the phase r that is m dot r into del r (ra) or del a rather area del a so which is m dot r into the area is which one in the if this is r, right?

So r del theta del z r del theta del z is the area r del theta del z, so del z is this, right? And this one will be r plus del r but this is del theta plus del theta, right? So the phase which is having the area r del theta del z r is this arc, right? This arc r del theta is that and we can say that here it is m dot r r del theta del z, right? So now m dot r was what? Rho vr into r del theta del z, right? So this was in so out r or r plus del r so out r plus del r is at this point or at this point in this plain that will be equals to m dot r plus del del r out m dot r, right? Into the entire del r, right?

del del r of m dot r into entire del r, right? Into the area, area is r plus del r into del theta into del z r plus del r del theta del z, right? So this on simplification we can write that this is nothing but rho vr r del theta del z plus rho vr del r del theta del z plus del del r of rho vr r del r del theta del z, right? So we can say that in was like that earlier and in minus out if then we can write in minus out to be equals to rho vr r del theta del z minus rho vr r del theta del z, right? Minus rho vr del r del theta del z minus del del r of rho vr, right? Into r del theta del z del r.

So this on simplification gives us minus rho vr del r del theta del z minus del del r of rho vr r del r del theta del z, right? So this we can say in the previous class we had said that we expanded in the (())(9:28) method we expanded rho v, right? Keeping v constant rho we varied keeping rho constant we varied v if you remember that del del r of rho vr that how can we expand it. So here also we can expand it or we have already expanded form which if we do the other way then it we can make them as the compact one like this that this is equals to minus del del r of r rho vr into del r del theta del z, how?

If we expand this del del r of r rho vr if we expand this and take this rho vr and r as the uv product, right? As the product both r into rho vr if we take this as product then in one case r is constant, right? In one case r is constant, then it is (ex) on expansion it is (cam) coming r, right? Obviously in both the cases a negative sign is there, so in one case it will be r del r r, right? rho vr rho vr del del r of rho vr r del r del theta del z, right?

And in other case this del del r of rho vr is differentiated or rather this is one uv expansion and the other one if rho vr is constant, then del del r of r is one so in that case rho vr del r del theta del z that becomes so on. So this is u and this is say v if we make uv that del del of something of uv that expansion method if we apply here then that expanded form is like this which on which on clubbing we get r rho vr as the variable and del del r of r rho vr del r del theta del z is the in minus out of course with a negative, right?

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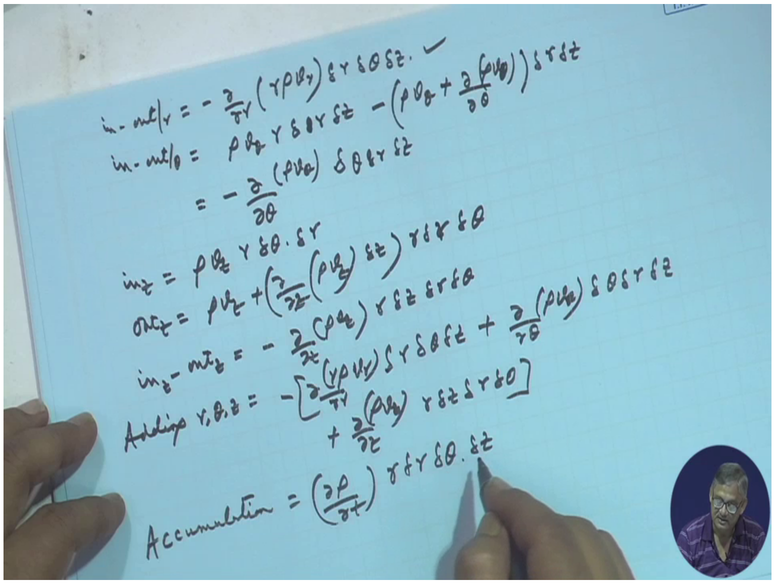

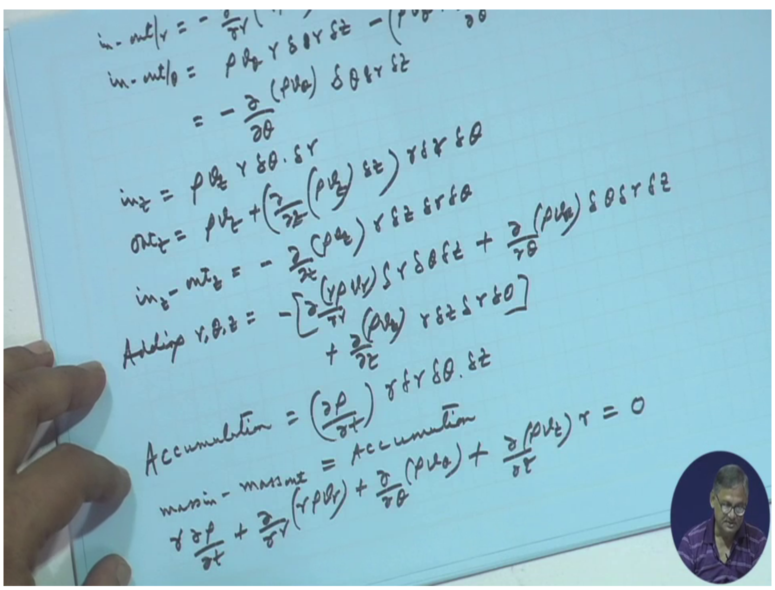

So similarly we can say for the other two cases that in minus out for this in minus out for r coordinate we can write that this is nothing but minus del del r of r rho vr into del r del theta del z. So in similar way exactly in a similar way we can also write in minus out that is mass flow in minus mass flow out at the coordinate theta, then we get similar to that okay let us do this way that in minus out this way in will be rho v theta r del r del z rho v theta r del r del z and out will be rho v theta plus del or minus this is del del theta rho v theta del del theta of rho v theta into del r del z, right?

So that means this in minus out we can write to be equals to say this rho be theta into del r del z rho v theta into del r del z this goes out then we can write this way that minus del del theta of rho v theta, right? rho v theta into del theta del r del z this is that in minus out minus del del theta of rho v theta del theta del r del z, right? So in was this was for r coordinate in the theta coordinate in was rho v theta r del r del z r del r is the r del r is the this is (ma) two del has come that is why.

So r del r into del z that was for in and for the out it was rho v theta plus del del theta of rho v theta into del r del z, right? So if we remove that, then we can say that this we can write del del theta of rho v theta into del theta del r del z is the in minus out at the theta coordinate, okay. Similarly, for z coordinate in at the z we can write rho vz into the area r del theta into del r and out at the z we can write rho vz plus del del z of rho vz, right? del del z of rho vz into del z times the area this area is r del r into del theta, right?

So this we can then write in the z coordinate in minus out at z this is equals to minus del del z of rho vz, right? Into r del z del r del theta, right? so by adding in minus out in all the three coordinates we get in all three coordinates r theta and z in all three coordinates if we add, then in minus out we get minus del del r of rho vr times r r rho vr, right? times the volume del r del theta del z plus del del theta of rho v theta times the volume that is del theta del r del z plus del del z of rho vz, right? Times r del z del r del theta, right?

So this is on adding r theta z all three coordinates, right? Then, we have to also find out the accumulation, is it? We have to find out the accumulation, is it? We have to find out the accumulation and in that case what is the accumulation that we have to find out, right? So accumulation is del rho del t in the entire volume element volume element r del r there is one side del theta another side and del z the third side, right? So this r del r is one arc and del theta del z the other two, so this is the total volume, okay.

So if we now put that mass in minus mass out is equal to accumulation this balance mass balance if we do, then we can write r del rho del t that plus del del r of del del r of r rho vr plus del del theta of rho v theta plus del del z of rho vz into r this is equals to 0, right? Because this r del r del theta del z r del r del theta del z that goes out and we get this, okay.

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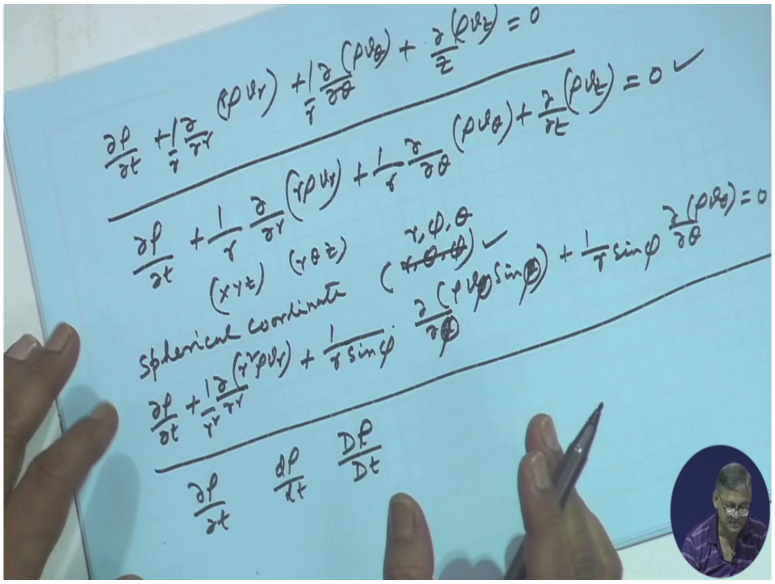

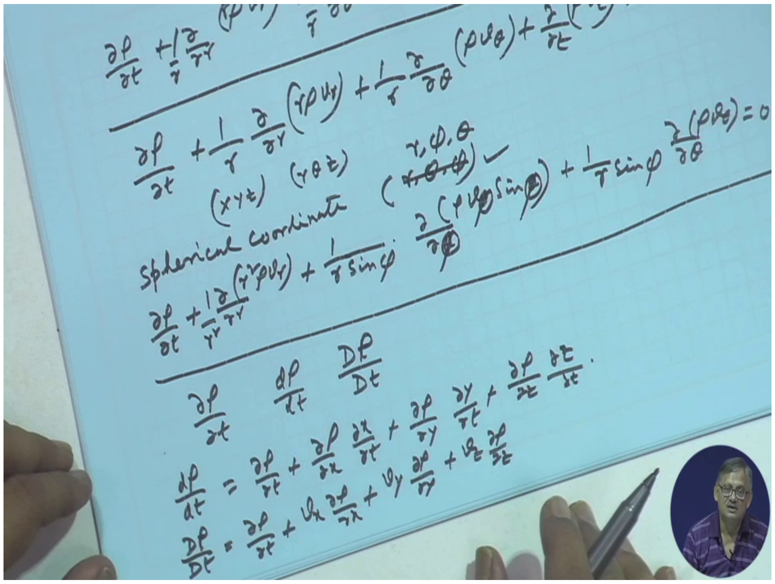

Then we can write this to be equals to we can write then this to be equals to del rho del t plus del del r of rho v r rho vr, right? del del r of r rho vr plus del del theta of rho v theta, right? Plus del del z of rho vz, right?

So this is equals to 0 so on accumulation we get this del rho del t plus del del r of r rho vr plus del del theta of rho v theta plus del del z of rho vz this is equals to 0, right? So one thing we have to keep in mind of course this r which was there this r which was there if we divide all the three terms, then here in the del del theta of rho v theta here we should have 1 by r, is it? That should come out there and that is equals to because in all the three out of this we had here at 1 r and we had here at 1 r at 1 r here, right?

So in that case this r and this r if we divide them, then they should be 1 by r del del r of r rho vr and this should be 1 by r of del del theta of rho v theta and this is like that, right? Now is the real expansion, so if we see that this is quite little not little quite different than that of the x, y, z or Cartesian coordinate. So we can write that in the r theta z or cylindrical coordinate the actual term is del rho del t plus 1 by r del del r of r rho vr plus 1 by r del del theta of rho v theta plus del del z of rho vz this is the equal to 0 is the equation of continuity for the cylindrical coordinate that is r theta z, right?

So this is quite different then that what we had done for the x, y, z that is the Cartesian coordinate if you remember that was absolutely different then this so many additional terms have come. Now this will be also useful when you are going for the equation of motion that derivation these relations will be helpful substituting for expanded or contracted form. So in that case there will be using these relations either in the Cartesian or in the cylindrical coordinate, right?

So we have done both then x, y, z that is the Cartesian coordinate and theta z that is the cylindrical coordinate these two system we have shown how to derive the equation of continuity and what is the form of equation of continuity, right? Now, this we will not derive but we can say for the spherical coordinate we can say that del rho del t this is del rho del t this plus del del r of r square rho vr r square rho vr, right? del del r of this into 1 by r square plus 1 by r sin phi if our coordinate is coordinate system is r theta phi, right? Then we can write like this, 1 by r sin phi into del del theta of rho v theta sin theta, right? Plus 1 by r of sin phi del del theta of rho v theta, right?

So this is equals to 0 for the r theta phi coordinate that is spherical coordinate. So spherical coordinate we are not deriving because r theta phi that will be too much complicated and will take really a good time and to avoid that we are giving the final form of the r theta phi that is spherical coordinate the final equation is del rho del t plus 1 by r square del del r of r square rho vr plus 1 by r sin phi del del theta of rho v theta sin theta, right? rho v theta sin theta and del del 1 by r sin phi del del theta of rho v this is, okay this is theta is there so if you mix up this is phi rho v theta, right? rho v phi sin phi del del so r phi theta then we should write in this form r phi theta, right?

Second term we are writing in terms of phi, so 1 by r sin phi, right? To del del phi of rho v phi sin phi plus 1 by r sin phi del del theta of rho v theta is the spherical coordinate, right? So we have already seen that partial coordinate that partial derivative was del rho del t, then (sa) total time derivative that was d rho dt and we also had shown that the substantial time derivative was capital D rho Dt, right? So these three and their respective expressions also del rho del t is okay, but d rho dt we had also shown this will be required for when we were doing the equation of motion that time again it will be required.

So d rho dt we had shown this was del rho del t, right? plus del del x of rho or del rho del x into del x del t plus del rho del y into del y del t plus del rho del z into del z del t this was for the del z del t this was for the this partial derivative but this was for the total derivative and for the substantial time derivative we had shown that this was d rho dt this is equals to del rho del t, right? Plus this will be again required vx del rho del x plus vy del rho del y plus vz del rho del z, right?

So this will be requiring when we are going for the equation of motion that set of equations we said earlier that this we call very very useful equations Navier-Stokes equation so when we will do that, all these relations will be very much useful and they are required, right? So in the next class we will do the equation of motion. Keeping in mind that continuity equation and the in the all three coordinates that is Cartesian coordinate or cylindrical coordinate or spherical coordinate in all these three coordinates we have out of those two, two we have derived and one we have shown you the final form.

So these forms will be utilized for equation of motion, but again equation of motion you will see that it is as we have progressing it is bigger and bigger, so equation of motion will be even bigger, right? So in that case it may not be possible that all the three coordinates we will be showing but at least for the Cartesian coordinate we will show, okay thank you.