Course on Momentum Transfer in Process Engineering
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Lecture 43
Module 9
Problems and solutions of Non Newtonian fluid – Part-1

So in the previous class we have developed generalized viscosity, right? Coefficient of viscosity that we have done, right? Now let us go and find out that, yeah so let us derive now relationship for generalized coefficient of viscosity, right?

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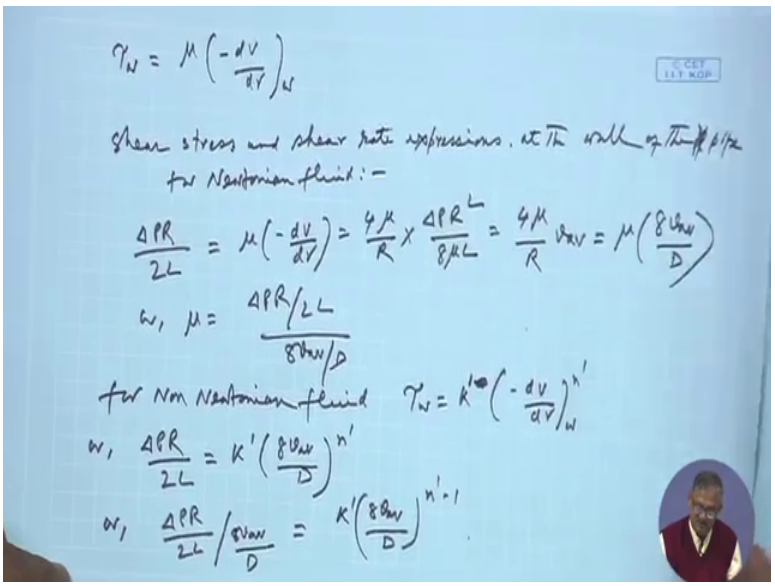

So if we see we that tau w was mu dv dr at wall, right? So let us use the shear stress and shear rate expressions at the wall of the pipe, right? For Newtonian fluids and if we do Newtonian and from that analogy we can also find out for the Non Newtonian, right? So if we do it for the Newtonian fluid, then it was delta PR by 2L or delta PD by 4L this was mu minus dv dr, right? This was 4 mu by R into delta PR square by 8 mu L is equals to 4 mu by R into v average is equals to mu into 8 v average by D, right?

This relation we have already done or mu can be written as delta PR by 2L over 8 v average over capital D, right? This we can write for Newtonian fluid, right? But for Non Newtonian fluid we know that tau w or shear stress at the wall was minus k dot dv dr k rather not minus 1 k dot into minus dv dr at the wall n to the power prime or k prime minus dv dr at the wall to the power n prime and also we have seen that delta PR by 2L this was equal to k prime 8 v average over D to the power n prime, right? So we can write that delta PR divided by 2L over 8 v average by D this is nothing but k prime 8 v average divided by D to the power n prime minus 1, right?

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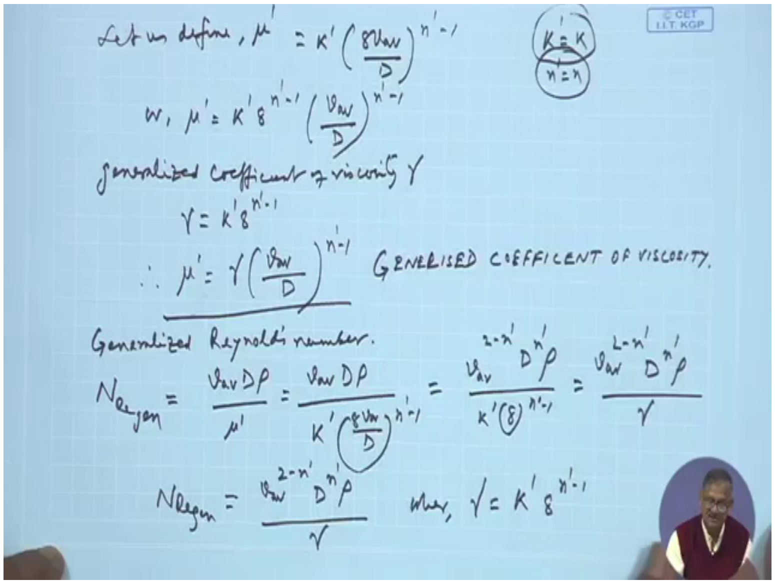

Now let us define mu prime that is equals to k prime into 8 v average over D to the power n prime minus 1 or we can also write mu prime is equals to k prime 8 to the power n prime minus 1 into v average over D to the power n prime minus 1, right? So if we define now generalized coefficient of viscosity if this we say to be gamma, right? Then gamma is equals to k prime 8 to the power n prime minus 1 and we also had earlier found the relation between k prime and k and we defined that n prime was n, right?

This relation k prime with k earlier we have found out defined and we assumed that n to be n prime, right? So gamma then is equals to k prime 8 to the power n prime minus 1, therefore we can write mu prime is equals to gamma v average divided by D to the power n prime minus 1, right? So this we know and we have also done this for Non Newtonian fluid and this is then the generalized coefficient of viscosity, right? So generalized coefficient of viscosity, now if we define generalized Reynolds number, right? So that Nre general so this we can write is nothing but v average D rho by mu prime.

So this is equals to v average D rho this is mu prime is k prime into 8 v average over D to the power n prime minus 1. So this we can write v average to the power 2 minus n prime into D to the power n prime into rho over k prime 8 to the power n prime minus 1, right? So this we can write to be equals to v average to the power 2 minus n prime, right? Into D to the power n prime rho divided by gamma, right? So the Nre general we can write this is equals to v average to the power 2 minus n prime D to the power n prime rho over gamma, right? Where of course gamma is equals to k prime 8 to the power n prime minus 1, right?

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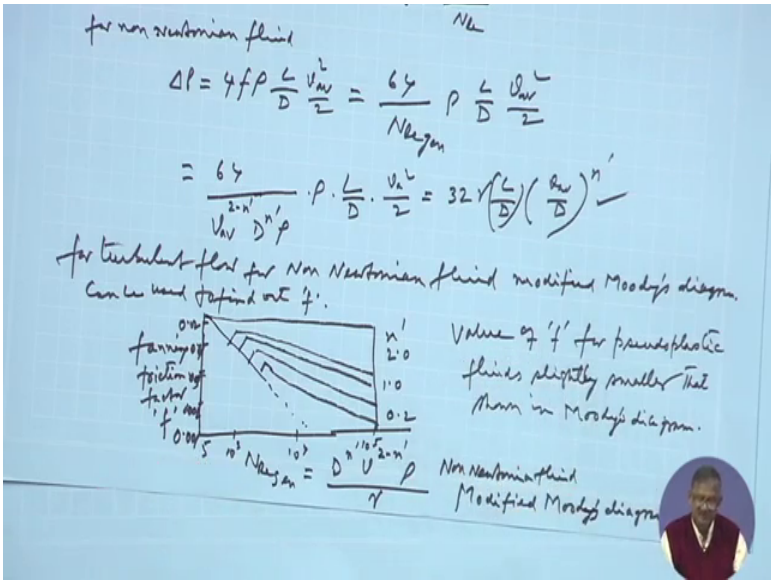

So if we define like this, then we have seen for Newtonian liquid for Newtonian liquid for Newtonian fluids we have seen that friction factor and Nre I mean Reynolds number the relation between relationship between friction factor and Reynolds number was like this f was equals to 16 by Nre this was true, right? Now for the same pressure drop of Non Newtonian fluid and flowing through the pipe etcetera same identical situation we can write delta P is equals to 4 f rho L by D v square average by 2 this was for Newtonian fluid which we can write 64 by Nre general into rho into L by D into v average square by 2, right?

This we can rewrite as 64 over v average to the power 2 minus m prime D to the power n prime to rho into rho into L by D into v average square by 2. So this becomes 32 gamma L by D into v average by D to the power n prime, right? So delta P with f is for Non Newtonian fluid can be related like this, right? For Non Newtonian fluid delta P is 32 gamma L by D v average by D to the power n prime, right? So now if we see for turbulent flow modified Moody's diagram for turbulent flow for Non Newtonian fluid modified Moody's diagram because we have seen

Moody's diagram for Newtonian fluid this can be used to find out the fanning friction factor f, right?

And if we look at how it looks like the Nre general this is equals to D to the power n prime v to the power 2 minus n prime rho divided by gamma this is Nre and this side is fanning friction factor f, right? So if we see this is a log log plot 10 to the power 3, 10 to the power 4, 10 to the power 5, etcetera and this side if we see say 0.001, 0.002, 0.003 and 0.002 0.005 say this is 2, this is 5 this is 0.01 this is 0.02 like that, right?

So the plot is like this, right? And for this side is like that so for different n prime so n prime can be 0.22 say 2.0, right? In between say 1.0 things like that. So from this generalized Reynolds number versus friction factor for Non Newtonian fluid this is the modified Moody's diagram, right? So this is modified Moody's diagram, okay. So fully this is not visible so it is modified Moody's diagram, right?

And from this diagram we can say that the value of f for pseudo plastic fluids, right? Is slightly smaller than that shown in Moody's diagram, right? So this is the difference that modified Moody which we had use Moody we had used for this not value of f this is that for pseudo plastic the in rough use, yeah this is that value of f for pseudo plastic fluid is slightly less than or smaller than that in the Moody's chart we had shown this in the Moody's chart for Newtonian fluid if you remember when it was flowing through a pipe, right?

If we just recapitulate a little that it was flowing through a pipe and the pipe was the genesis of Moody's diagram was that for pipe flow when we developed the relation called from the Navier Stokes or for the pipe flow with the shell momentum balance developed and the equation was Hagen Poiseuille equation and in that Hagen Poiseuille equation whatever we found out we said that the length of the pipe was L subsequently when the friction factor was defined and it was said by Moody from Moody's diagram which saw that the friction factor when it is included and length which we assumed to be L that was not originally L because of the roughness of the pipe through which the fluid was flowing, right?

And to give an example of that barrel of the ice cream freezer, right? It is absolutely smooth, right? That is for of course different reason but it is absolutely smooth so the travelling of the liquid is exactly the same as that in the pipe more or less but if the pipe is not smooth if it is rough that depends on the construction of the pipe, then Moody showed that length which we actually taken as L is no longer L it is something beyond L, right? Which we had shown also and that extra L could be taken into from the roughness of the pipe and that relative roughness equivalent roughness those things came up and we showed that the friction factor versus Moody's Reynolds number or rather Reynolds number friction factor versus Reynolds number from Moody's chart we had shown how friction factor can be found out, right?

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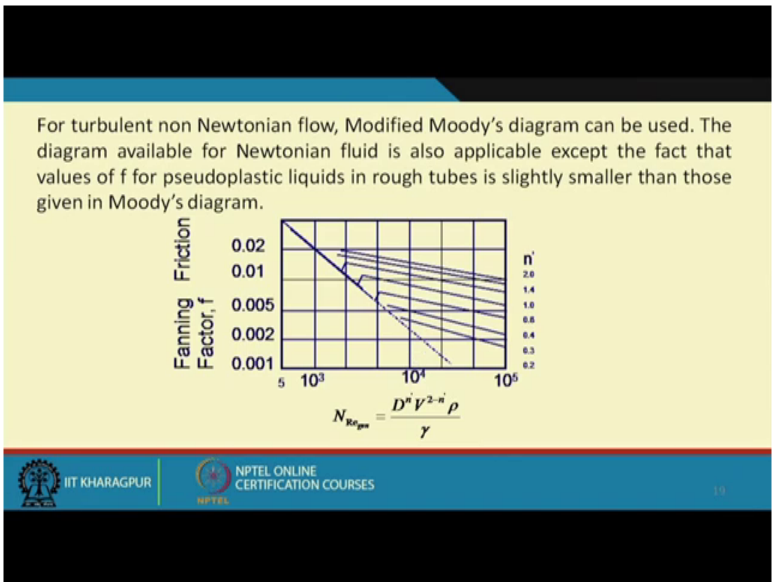

Now here, for Non Newtonian fluid slightly modified that Moody's chart was slightly modified and as we have shown if we look at this here this looks like more or less like that, okay this is of course taken from any published literature, right? Because drawing it on the paper is little difficult so we had taken it from there so it is in the log log scale so 10 here it is Nre general that is D to the power n dot n prime v to the power 2 minus n prime rho by gamma so this is plotted versus fanning friction factor again it is, say here it is given 10 to the power 3, 4, 5 here it is 0.01, 0.001, 0.002, 0.005 like that 0.01, 0.02 this range and this side is n prime, right?

So n prime also varied from 0.1, 0.2, 0.3 like that 1, 1.4 to like that as many n prime we can, right? And for a corresponding n prime we find out which one it is and then for a given Nre general or generally general you obtain from there you can find out the value of friction factor, right? What is the problem here is for turbulent Non Newtonian fluid flow modified Moody's diagram this can be used, right? Where the diagram available for Newtonian fluid is also applicable except the fact that values of f for pseudo plastic liquids in rough tube is slightly smaller than those given by the Moody's diagram, okay.

That slightly smaller is definitely it has impact, but not for all engineering purposes not so high such that your prediction will be upside down, right? So all practical purposes this chart or this diagram is used and for Non Newtonian fluids the okay for dilatant or pseudo plastic these fluids are these rather values are utilized and you get the value of f or the friction coefficient or friction factor, right?

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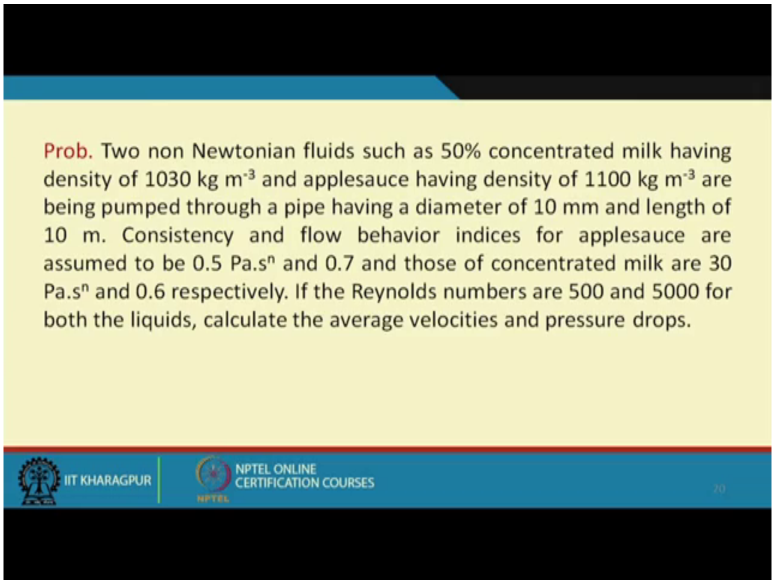

This you have to find out, right? Now to solve it to see that whether we have understood correctly or not let us go for a solution of a problem that is like this, two Non Newtonian fluids such as 50 percent concentrated milk having density of 1030 kg per meter cube and applesauce having density of 1100 kg per meter cube are being pumped through a pipe having diameter of 10 millimeter and length of 10 meter, right? Consistency coefficient and flow behavior indices for applesauce are assumed to be 0.5 Pascal second to the power n and 0.7 and those of concentrated milk are 30 Pascal second to the power n and 0.6 respectively, right? If the Reynolds number are 500 and 5000 for both the liquids, calculate the average velocities and pressure drops, right?

So I repeat, two Non Newtonian fluids such as 50 percent concentrated milk having density of 1030 kg per meter cube and applesauce having density of 1100 kg per meter cube are being pumped through a pipe having a diameter of 10 millimeter and length of 10 meter. Consistency and flow behavior indices for applesauce are assumed to be 0.5 Pascal second to the power n and 0.7 and those of concentrated milk are 30 Pascal second to the power n and 0.6 respectively, right? If the Reynolds number are 500 and 5000 for both the liquids, calculate the average velocities and pressure drops, right? Now of course we have come to the end session of the this class it is also very difficult to solve it at this very moment, right?

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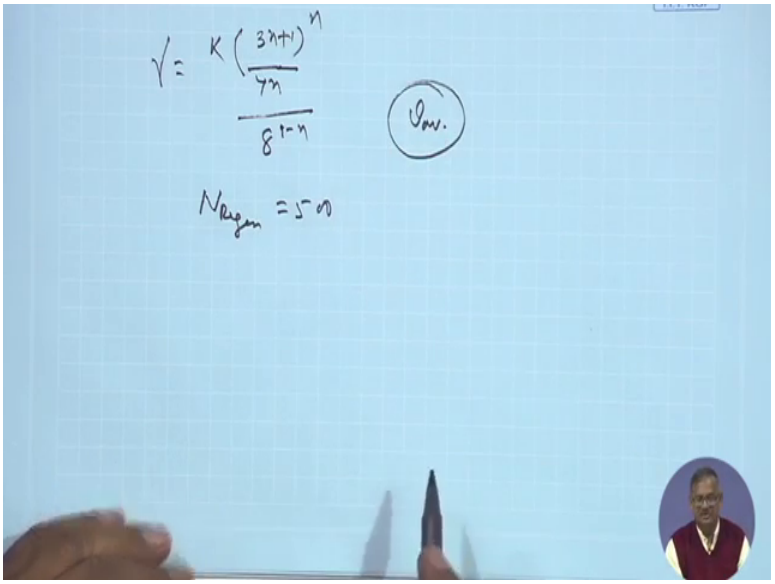

So what we do that we let you know or we ask you that you try we give some hints that the generalized coefficient of viscosity that gamma if we write this to be k 3 n plus 1 over 4 n to the power n divided by 8 to the power 1 minus n, right? So if we see that and if we use Nre general that is given 500 so find out average velocity so if we do this at home maybe next time when we are in the class we can solve it in the class and do it, but you try at home and see how much you are getting, right? But read carefully the problem given what has been given and what has been asked to find out, right? And all the things I have already been told and shown so you can do it, right? Okay thank you.