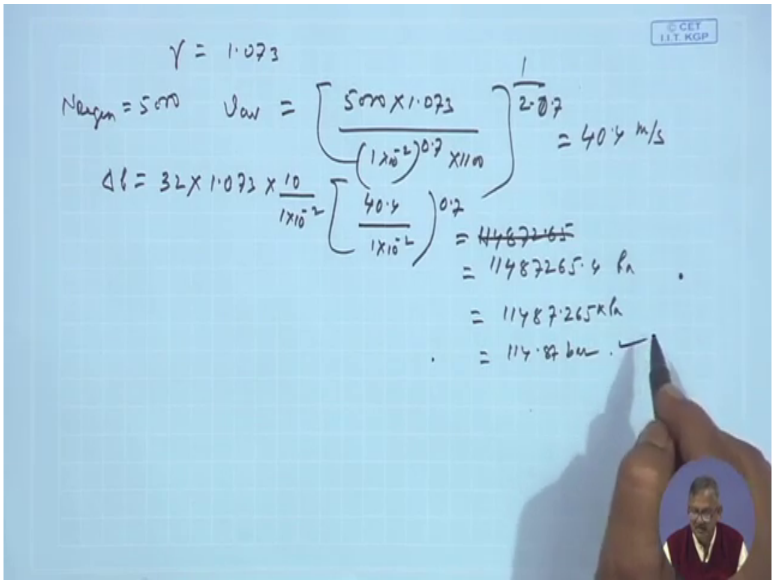
Course on Momentum Transfer in Process Engineering
By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur
Lecture 45
Module 9
Flow of Non Newtonian fluid through slit

So in the previous class we had done a problem and solution hopefully we have done for 1 Nre for the applesauce the second one two fluids where there, one was milk and the other was applesauce and hopefully milk we have done for both the cases 500 and 5000 Nre general, we found out both these velocity and pressure drop and for the other one that is applesauce for milk we (())(0:58) both the Nre general that is 500 and 5000 we have done both velocities and pressure drops. And for applesauce we have done for 500 Nre general that what was the velocity and what was the delta P because of the time constraint we could not do for 5000, hopefully you have done it.

So if you had any problem, so please bring to our notice and we will solve it or we will give you the values, okay because I have also not done but if you want we can do it here also because afterwards we will not get the time I hope let us do that, okay.

(Refer Slide Time: 2:00)

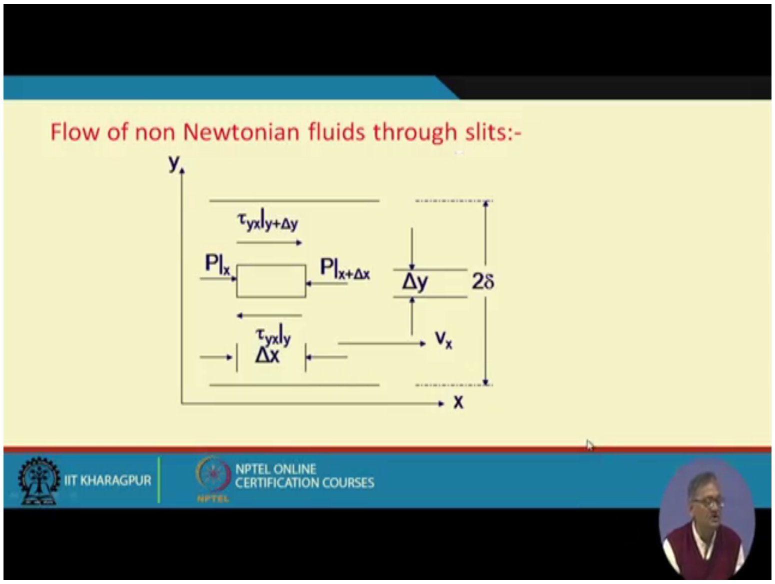


For applesauce we had gamma is equal to 1.073, right? Nre general given was 5000 so v average we can write to be 5000 into 1.073 divided by 1 into 10 to the power minus 2 and n was 0.7 and density was 1100 this to the power 1 by 2 minus 0.7 so this becomes equal to let us see that 5000 into 1.073 was this divided by 10 to the power minus 2 to the power 0.7 is this equal to this divided by 1100 is this to the power 1 by 2 minus 0.7, right?

So is equal to that so this is equals to 40.39 so (())(4:20) 40.4 meter per second is average velocity and delta P we can write to be 32 into 1.073 into 10 length L by D into 1 into 10 to the power minus 2 into v average that is 40.4 divided by 1 into 10 to the power minus 2 whole to the power 0.7, so this comes to equal to let us see 40.4 divided by 10 to the power minus 2 this whole to the power 0.7 is equal to this into 10 to the power 3 so 1000 into 1.073 into 32 is equal to 114872.65 sorry not point 11487265.4, right?

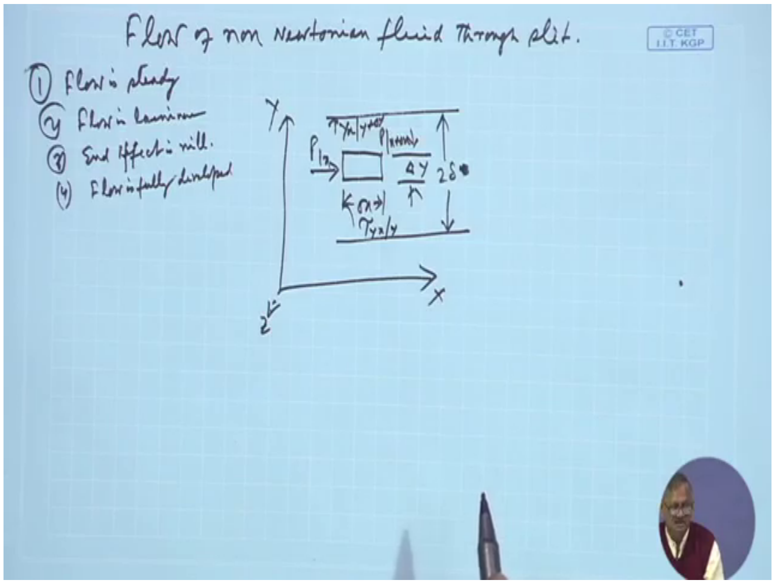
That means 1, 2, 3 so much Pascal so is equal to 11487 no 11487.265 kilo Pascal is equals to 114.87 bar if we take Pascal 1 bar, right? So this we have done so you please check hopefully you will get the same, right? Now let us move to another very important Non-Newtonian fluid flow, right?

(Refer Slide Time: 7:15)



And that is let us look into where it is, yeah here it is flow of Non-Newtonian fluid through slit, flow of Non-Newtonian fluid through slit, right? Now if you remember that in earlier case we had done so this also we are doing in the same way similar way, so here what we are doing? We are taking in say x, y two dimensional third dimension is also perpendicular to this so we have taken a volume element, right? And the thickness of this is del y and this is del x, right?

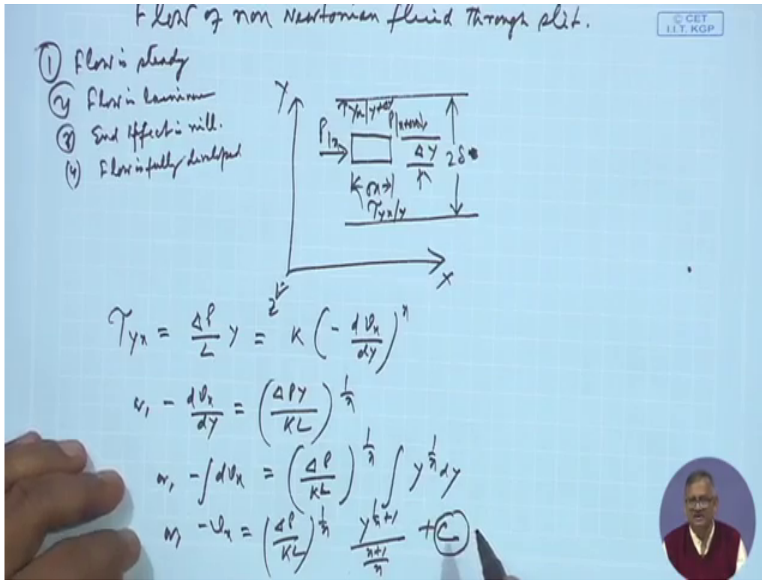
(Refer Slide Time: 8:34)



So the we assume that flow is steady and flow is laminar and there is no end effect and there is flow is fully developed, right? These all assumptions remain same. So now as we had taken earlier here also we have taken volume element, so this volume element is having a thickness of delta y, right? And it has a length of say delta x, right? And the plate or slit is having a this slit is having a thickness of 2 delta, right? 2 del is the thickness, so it is acting the pressure force at the phase x and the pressure force at the phase x plus delta x and tau at the phase tau yx at the phase y and tau yx at the phase y plus delta y, right?

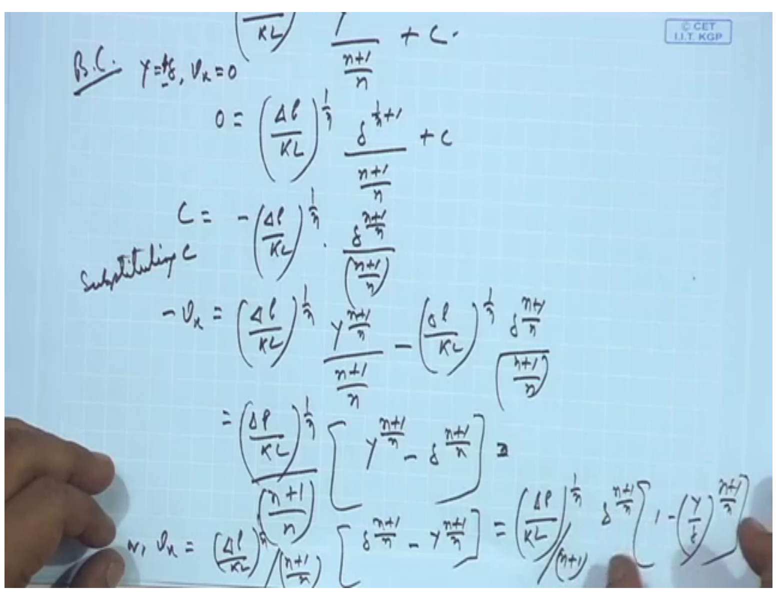
So if this be true, then if this is the xy coordinate obviously the third coordinate is z, okay. If we take these, right? And sorry, like in the previous which we had done the slit so let us here also take that from the previous as we have said that there is no this is all steady state so no accumulation all some of the forces equal to 0. Then we also assumed we also said that since it is a steady state so bulk flow is getting nullified all these taking together we had said that in if you remember in earlier class very very beginning there we had said that we will start in all practical purposes from this relation of tau, right?

(Refer Slide Time: 11:41)



So now tau yx this is equals to delta P by L into y, right? This is equal to k minus dvx dy to the power n, right? Or minus dvx dy is equals to delta Py over kL to the power 1 by n, right? Or integral of dvx, right? Is equals to delta Py or delta P by kL to the power 1 by n integral of y to the power 1 by n dy, right? So this on integration gives vx with negative is equals to delta P by kL to the power 1 by n and y to the power 1 by n plus 1 divided by n plus 1 by n, right? Plus C. Now we have to find out the constant C value of the constant C what it is coming that we have to find out, right?

(Refer Slide Time: 13:23)



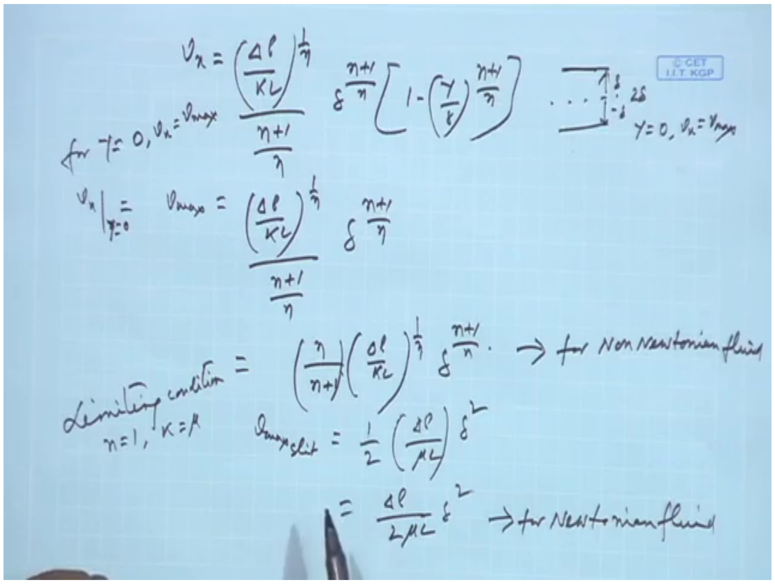
To find out that constant C let us, okay our expression is vx minus is equals to delta P by kL whole to the power 1 by n y to the power 1 by n plus 1 over n plus 1 by n plus C, right? Now the boundary condition for getting the value of C is like that at y is equals to del vx is equals to 0 this is plus minus del, so we could take plus or we could take minus del if you remember this we have taken 2 del, right? Total thickness, so plus del minus del so if it is at this center axis so it is plus del and this is minus del, right?

So that is why it is 2 del, so boundary is vx is rather vx is equals to 0 at y is equals to del that is at the wall, right? This is at the wall so if it is plus del or minus del in either case it is 0 vx is 0 here also vx is equals to 0, right? So if that be true, then we write y is equals to plus del for all practical purposes so it is 0 delta P by kL whole to the power 1 by n and this y is now del to the power 1 by n plus 1 divided by n plus 1 by n so this is plus is equal to C, right? So C is equal to minus delta P by kL whole to the power 1 by n into del to the power n plus 1 by n divided by n plus 1 by n, right?

This is C, so by substituting C we can write minus vx substituting C value of C of course vx is delta P by kL whole to the power 1 by n, right? y to the power n plus 1 by n divided by n plus 1 by n minus delta P by kL whole to the power 1 by n del to the power n plus 1 by n by n plus 1 by n. So if we take common is equal to delta P by kL whole to the power 1 by n if we take common and if we also take common n plus 1 by n, right? So we can write y to the power n plus 1 divided by n minus del to the power n plus 1 divided by n or this is equal to this, right? Or we can write vx is equals to delta P by kL whole to the power 1 by n divided by n plus 1 by n, right? Into del to the power n plus 1 by n minus y to the power n plus 1 by n, right?

So this we can rewrite is equals to delta P by kL whole to the power 1 by n over n plus 1 times if we take del to the power n plus 1 by n common so 1 minus y by del whole to the power n plus 1 by n, so this is the vx, right? So vx comes like this so that is the instantaneous velocity at any point at any time of course time is independent we have taken steady state, so instantaneous velocity at any place is vx and that is equal to delta P by kL over n plus 1 to the power 1 by n times del to the power n plus 1 by n times 1 minus y by del whole to the power n plus 1 by n, so that is the vx, right?

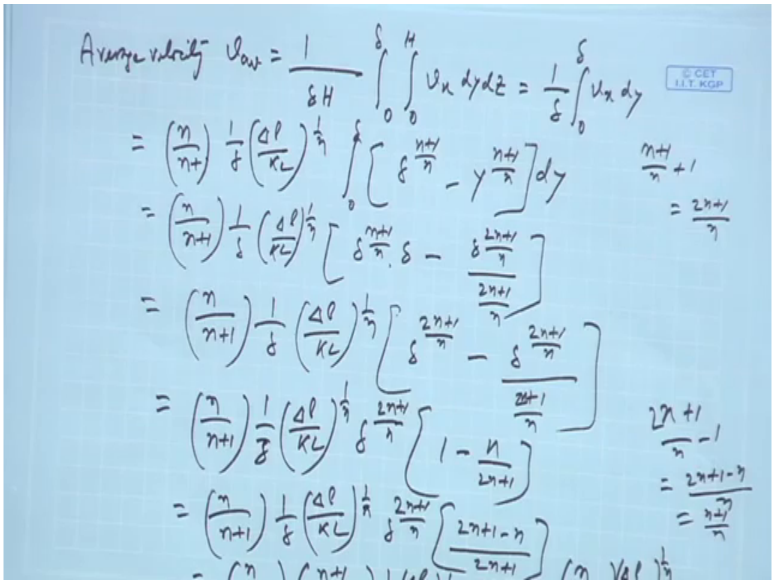
(Refer Slide Time: 19:00)

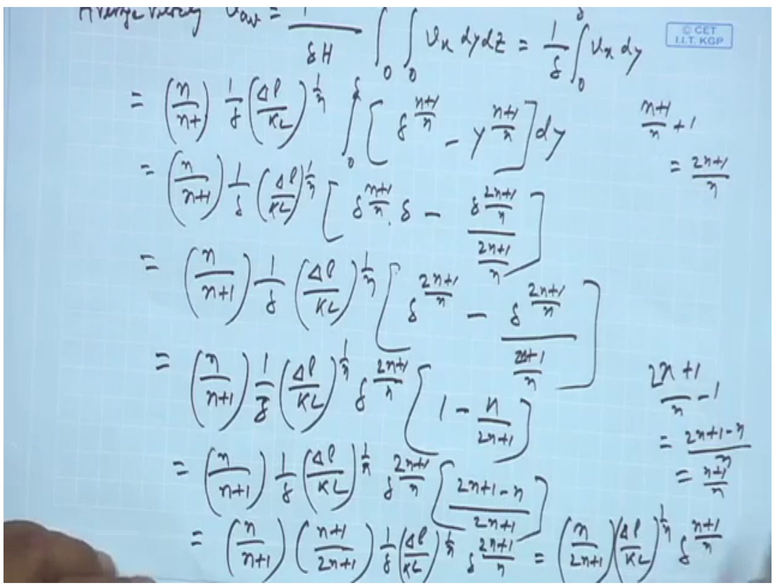


So if this vx if we tell now we also can write that, okay vx is equals to delta P by kL whole to the power 1 by n over n plus 1 by n, right? This into del to the power n plus 1 by n times 1 minus y by del whole to the power n plus 1 by n, right? So for y is equals to 0 vx is equals to v max, true because this was our del, 2 del this is our center so this was our 1 del plus del this was our minus del so it is 2 del, right? So at y is equals to 0, vx is equals to v max. So that value is v max then is equals to or vx at y is equals 0 is equals to v max is equal to delta P by kL whole to the power 1 by n divided by n plus 1 by n, right? Into del to the power n plus 1 by n so y0 so it is 1, so this becomes equals to this which on simplification we can write n by n plus 1 times delta P by kL to the power 1 by n times del to the power n plus 1 by n is the v max, right?

So for slit again you see the difference now for a limiting condition if n is equals to 1 and k is equals to mu so v max or slit flow is that becomes equals to n becomes 1 so it is 1 by 2 delta P by mu L, right? And this becomes 1 and this is del to the power 1 plus 1 so del square, right? So that is delta P by 2 mu L del square if you go back to the previous classes you will see that flow through slit for Newtonian fluid this is for Newtonian fluid and this is for Non-Newtonian fluid, right? So if you go back and see the previous class there we had done for Newtonian fluid this derivation and there it was delta P by del square by 2 mu L as the v max, right?

(Refer Slide Time: 22:55)



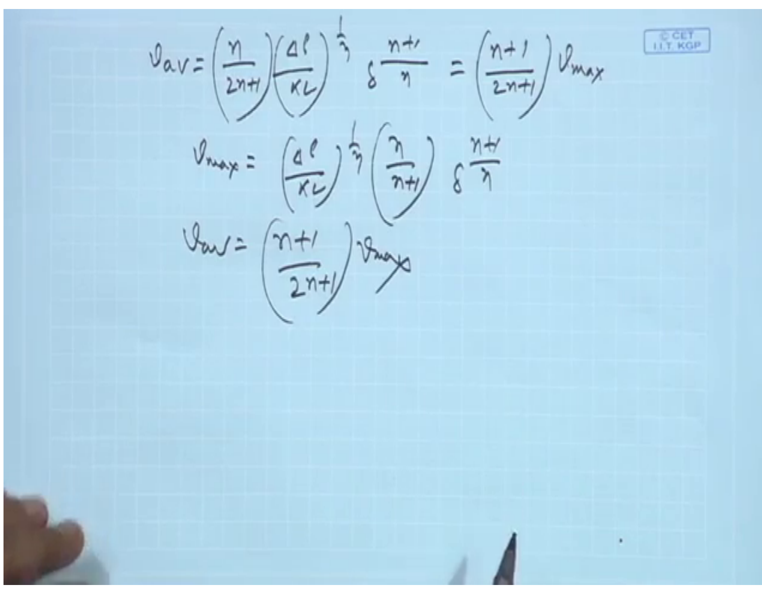


Then it comes the average velocity, so average velocity v average we can write 1 by del if height is the H, then 0 to del, 0 to H, right? vx dy dz is area, so this is equals to 1 by del 0 to del vx dy, right? So this we can write vx we have already found out is n by n plus 1, right? Into 1 by del into delta P by kL whole to the power 1 by n 0 to del, then this was del to the power n plus 1 by n minus y to the power n plus 1 by n into dy, right? So this on simplification can be written n by n plus 1, right? Into 1 by del into delta P by kL to the power 1 by n and this is 0 and del so this is y, right? So del to the power n plus 1 into y on putting del so we can write this to be del to the power n plus 1 by n into del that is y, right? Minus this is on integration gives y and plus 1 by n plus 1 that is 2 n plus 1 by n, right? So this is that is del to the power 2 n plus 1 by n divided by 2 n plus 1 by n, right?

So this is dy so like that, so on simplification this we can write n by n plus 1, right? 1 by del delta P by kL to the power 1 by n so this becomes del to the power n plus 1 plus n that is 2 n plus 1 by n, right? Minus this is del to the power 2 n plus 1 by n over 2 n plus 1 by n, right? So if we, sorry take if we take del n by n plus 1, 1 by del delta P by kL whole to the power 1 by n into del to the power 2 n plus 1 by n if we take common, so 1 minus 1 by or n by 2 n plus 1, right? So this can be written as n by n plus 1 to 1 by del delta P by kL to the power 1 by n del to the power 2 n plus 1 by n this becomes 2 n plus 1 minus n by 2 n plus 1.

So that means 2 n minus n, so n plus 1 by 2 n plus 1. So it is n by n plus 1, right? Into this becomes n plus 1 by 2 n plus 1, 1 by del delta P by kL whole to the power 1 by n and this is 2 del 2 n del to the power 2 n plus 1 by n, right? So n plus 1 and n plus 1 goes off, so this is equals to n by 2 n plus 1, right? Into 1 by del so this is del to the power minus 1, so we can write delta P by kL whole to the power 1 by n and this del goes there, so del to the power minus 1. So that is 2 n plus 1 by n minus 1 is equals to 2 n plus 1 minus n by n is equals to 2 n minus n, so n plus 1 by n so del to the power n plus 1 by n, right?

(Refer Slide Time: 28:32)



So the average velocity this has comes to the average velocity has come v average is equals to n by 2 n plus 1, right? Into delta P by kL to the power 1 by n, right? And del to the power n plus 1 by n by n plus 1 by n, right? So this is also nothing but equal to v max what we had shown earlier v max was this, right? Delta P by kL into n by n plus 1 so this was v max is delta P by kL to the power 1 by n into n by n plus 1 into del to the power n plus 1 by n. So this was v max and now we have got this new v average is equals to so v max was this, what did we write just now, yeah so vx v max was this.

So delta P by kL, so delta P kL by this one n by n plus 1 this is n plus 1 or 2 n plus 1 n by 2 n plus 1 this will be del n by n plus 1 vx is delta P by kL 1 by n n plus 1 by n, so 1 minus this so this we can write, this is nothing but n plus 1 by 2 n plus 1 v max, right? So that was n by n plus 1 so this is also n by 2 n plus 1 so v max was originally delta P by kL to the power 1 by n, n by n plus 1, right? del to the power n plus 1 by n. So v average has become equals to n plus 1 by 2 n plus 1 v max, right? So today we are going out of time so let us stop it today, thank you.