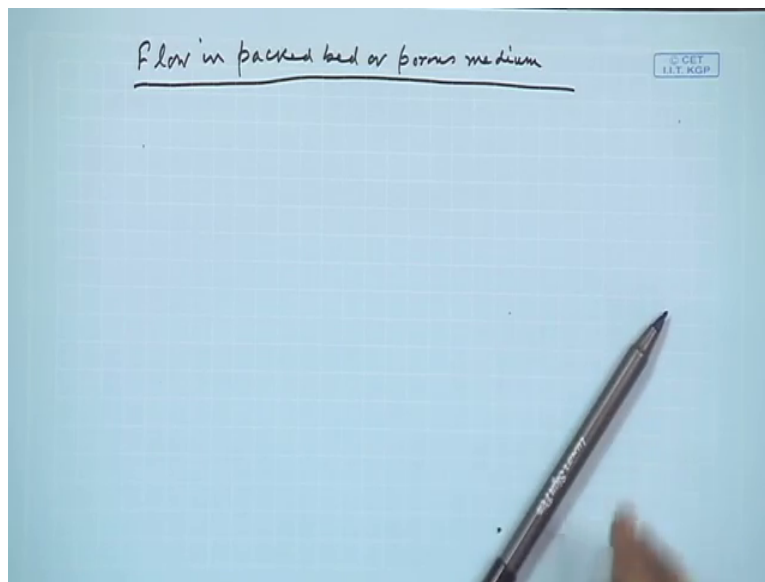


**Course on Momentum Transfer in Process Engineering**  
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**Lecture 47**  
**Module 10**  
**Flow through packed beds**

Okay, we have done flow through pipe, flow through slits for both Newtonian and Non-Newtonian fluids, right? So in Non-Newtonian fluids we also have to develop many things like generalized coefficient of viscosity or generalized Reynolds number things like that, right? Now let us move to another very important aspect that is flow in Packed Beds or Porous medium, right?

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Flow in packed bed so flow in packed bed or porous medium this is very important because in many cases when you are handling I am not saying only food material because this is not restricted to food material only this is a general thing where in many cases maybe for minerals, maybe for metals, maybe for chemicals, maybe for many others metallurgical applications these are possible because you have the packed bed.

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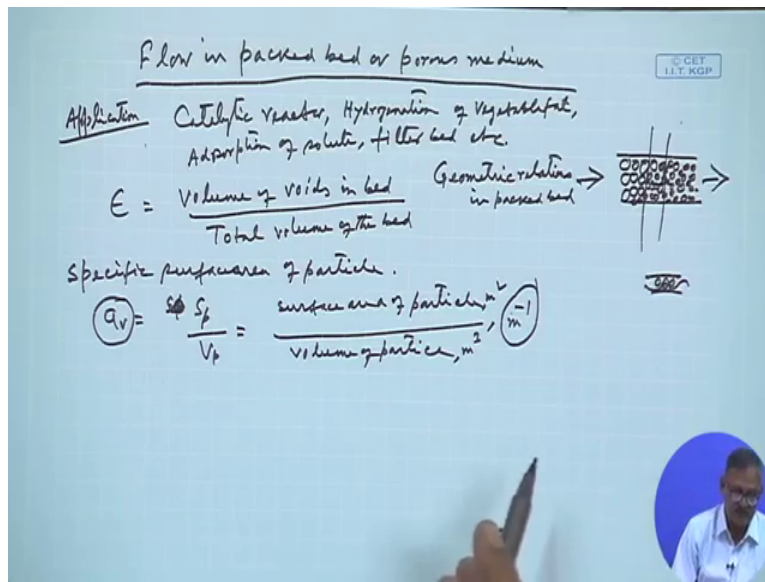
Flow in packed bed or porous medium

Application Catalytic reactor, Hydrogenation of vegetable fat, Adsorption of solute, filter bed etc.

$\epsilon = \frac{\text{Volume of voids in bed}}{\text{Total volume of the bed}}$  Geometric relation in packed bed  $\rightarrow$  Geometric relation in packed bed

Specific surface area of particle.

$a_v = \frac{S_p}{V_p} = \frac{\text{surface area of particles, } m^2}{\text{volume of particles, } m^3}$



Now what we understand by packed bed? That is a bed say this is a pipe which is filled with certain material like this, right? And depending on the material we call it to be a size of the material we call this condition to be under packed condition, right? And depending on how these materials are sized and in what way in what density they are packed, so this we call to be a packed bed if your fluid is flowing through this and going out of this then we call it to be flow through packed bed or in many cases this is also termed as porous medium, why porous medium?

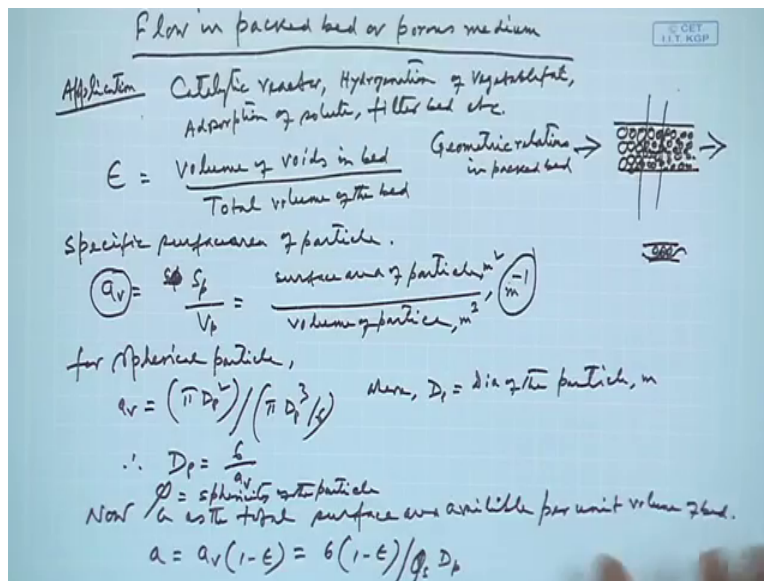
Because if you look this section, then when it is a porous means this material, this material, this material so within this the fluid is flowing like that, right? So that means there is a gap between the materials, so the fluid can move this way that way like that and when it is moving like that it is called a porous medium, right? That means in the medium there are pores through which the fluid can move, right? Now to go for this let us now define certain things so its applications are applications of this systems is that catalytic reactor, hydrogenation of vegetable fat, absorption of solute, filter bed, etcetera, right?

Now let us define certain geometric definitions like the geometric relations like epsilon this is called volume of voids in bed over total volume of the bed, right? Certain this is epsilon, right? And this is called geometric relation in packed bed, right? Specific surface area of particle this is

if  $a_v$  is  $S_p$  over  $V_p$  that is surface area of a particle to volume of particle. So this specific surface, surface area of particle to volume of particle that means it has unit a unit of meter inverse, right?

So this is in meter square, this is in meter cube so the  $a_v$  comes to be meter inverse, right? So ((  
 (6:59) specific surface area but since it is specific surface area that is why this unit is meter inverse, right?

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Now for spherical particle we can say that  $a_v$  is equals to surface area of the particle that is  $\pi D$  particle square divided by  $\pi D$  particle cube by 6, right? So this  $D$  particle is  $D_p$  is diameter of the particle in meter, right? Therefore we can say diameter of particle is nothing but 6 over  $a_v$  that is specific surface area.

Now if we define another term called  $a$  as the total surface area available per unit volume of bed if we say that, then  $a$  becomes equals to  $a_v$  into 1 minus epsilon, right? Which is 6 times 1 minus epsilon, right? And if this is 1 if  $a$  is that and if  $\phi_s$  is the sphericity of the particle, then we can write 6 time into 1 minus epsilon divided by  $\phi_s$  into  $D_p$ , right?

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$$\phi_s = \text{sphericity of the particle}$$
$$= \frac{S_s}{V_s} \frac{V_p}{S_p}$$

$S_s$  = surface area of a sphere having same diam as the particle  $D_p$ ,  $m^2$ .

$V_s$  = volume of the same sphere,  $m^3$ .

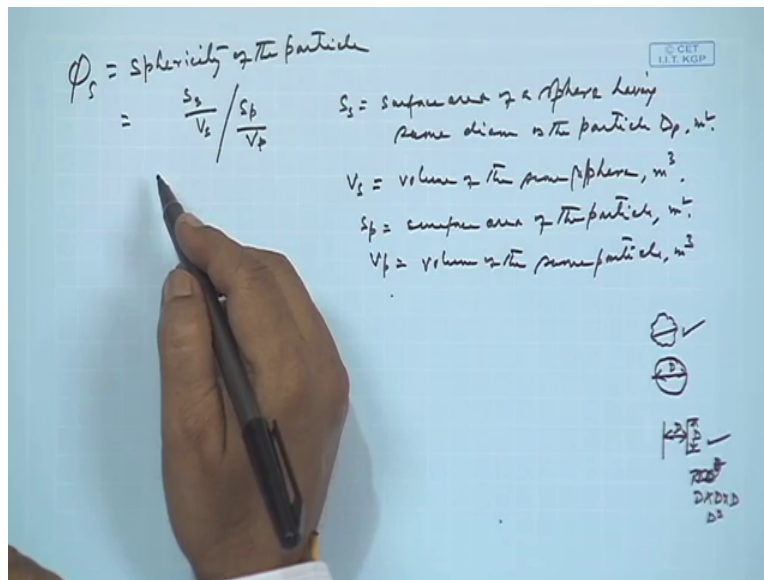
$S_p$  = surface area of the particle,  $m^2$ .

$V_p$  = volume of the same particle,  $m^3$ .

Where this  $\phi_s$  is the sphericity and this is defined as  $\phi_s$  is the sphericity of the particle, right? Sphericity of the particle which is defined as  $S_s$  over  $V_s$  over  $S_p$  over  $V_p$ , right? Where  $S_s$  is the surface area of a sphere having same diameter as the particle  $D_p$  in meter square  $V_s$  is equal to volume of the same sphere, right? In meter cube  $S_p$  is equal to surface area of the particle in meter square and  $V_p$  is the volume of the same particle in meter cube.

Now what we understand by the term sphericity? Mathematical we have said something, but physically what we understand that is most fundamental thing which we have to understand, right?

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Say this is a material, right? Does this material have a diameter same as that of this D? No it is not everywhere D maybe at some places it could be D but many others it is not D. So in that case this will not be a sphere but how close it is to the sphere having the same diameter as that of the particle, so that we would like to know and that is the sphericity of the particle.

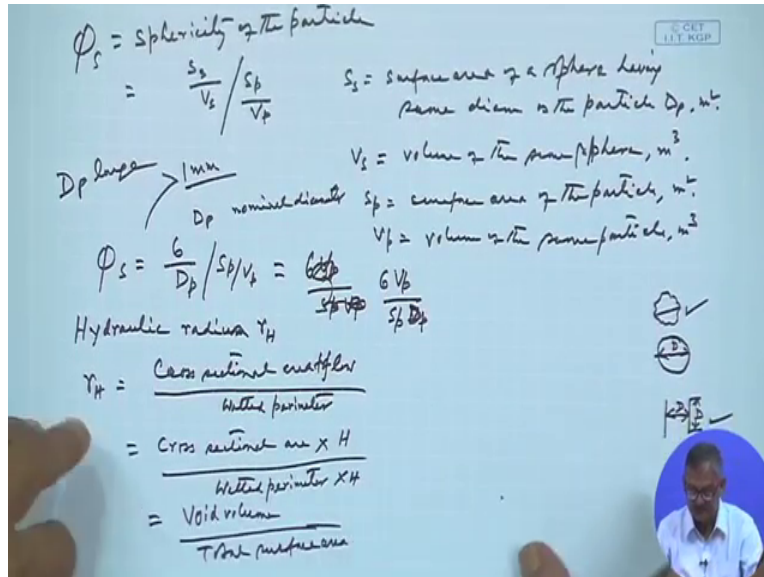
That means how close it is to the particle having the diameter  $D_p$  close to that of the sphere with the same diameter  $D_p$ , right? This is what exactly sphericity means that whenever you are doing this you must know the sphericity of the particle. Now you know that if this is a cylinder having a diameter say D, right? What is the area of this cylinder?  $\pi D$  square or there is a surface area, right? Now if this is D, right? And diameter is D and if the height of that is also D, right? Then all the three dimensions of this will be D so it is D by D by D so it is D cube.

So it should be a sphere because all the dimensions of the 3 dimensions are and equal to D so it should be same as D or sphere, but it is not, right? Does it look like that it will be a sphere? It does not, right? Though the dimension wise we can say that it could be same as that of the sphere, but it is not so, right? So that understanding that how close it is to the sphere, how we can define this that particle closeness to the sphere that is the sphericity, right?

So that is why sphericity is defined as  $S_s$  by  $V_s$  over  $S_p$  over  $V_p$ ,  $S_s$  is the surface area of the sphere having the same diameter of the particle  $D_p$  in meter square,  $V_s$  is the volume of the same sphere in meter cube  $S_p$  is a surface area of the particle again in meter square and  $V_p$  is the

volume of the same particle in meter cube, right? Now for fine particles  $D_p$  is the (())(16:19) analysis diameter or nominal diameter, right?

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For okay for is  $D_p$  is large, then it can be said that diameter of the particle is the diameter of the sphere having equal volume as that of the particle.

So large particle means where particles whose diameter can be assured or is greater than or can be measured or can be assured in terms of measurement or can be measured or is greater than 1 millimeter, right? Now for small particles  $D_p$  is the sip size or is the sieve analysis size or it can be a nominal diameter, right? So we can say  $\phi_s$  is equals to 6 over  $D_p$  over  $S_p$  over  $V_p$  or this can be rewritten as 6  $V_p$  over  $S_p$  into  $D_p$  not sorry sorry, 6  $V_p$  over  $S_p$  into  $V$  by  $D_p$  (())(18:16)  $S_p$  into  $D_p$ , right? 6  $V_p$  over  $S_p$  into  $D$ , if that be true then we can define another thing which is called hydraulic diameter hydraulic radius, right? Hydraulic radius, so hydraulic radius  $r_h$  that is defined as  $r_h$  is equals to cross sectional area available for flow over weighted perimeter, right?

So this we can write cross sectional area times Capital H over weighted perimeter times H, right? Now this means this is nothing but void volume, right? And this means weighted perimeter into H total surface area, right?

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$$r_H = \frac{\text{void volume} / \text{total volume}}{\text{Total surface area} / \text{total volume}}$$

$$= \frac{\epsilon}{6(1-\epsilon)} = \frac{\epsilon \phi_s D_p}{6(1-\epsilon)}$$

Equivalent diameter  $D = 4 r_H = \frac{4 \epsilon \phi_s D_p}{6(1-\epsilon)}$

$V'$  is the velocity based on empty cross sectional area of the bed  
 $v$  is the actual velocity through void space

$$V' = \epsilon v$$

$$\therefore N_{Re_p} = \frac{D v \rho}{\mu} = \frac{v^2 (4 r_H) \rho}{\epsilon \mu} = \frac{4 v' \epsilon \phi_s D_p \rho}{6 \mu (1-\epsilon)}$$

$$= \frac{4 \phi_s D_p v' \rho}{6 \mu (1-\epsilon)} = \frac{\phi_s D_p v' \rho}{\mu (1-\epsilon)}$$

So if now we do a little jugglery  $r_H$  is to void volume divided by total volume over total surface area divided by total void volume by total volume total surface area by total volume, right? So void volume over total volume by total surface area by total volume.

Now void volume by total volume we have seen this is equals to epsilon by 6 into 1 minus epsilon divided by phi s  $D_p$ , right? So this is equals to epsilon phi s  $D_p$  divided by 6 into 1 minus epsilon, right? Now for that we can write equivalent diameter capital  $D$  is nothing but 4  $r_H$ , right? So this can be written as 4 epsilon, now if we introduce the term we have introduced that phi s into  $D_p$ , right? Divided by 6 into 1 minus epsilon, right? Now that  $V'$  prime is the velocity if  $V'$  prime is the velocity based on empty cross sectional area of the bed and  $v$  is the actual velocity through void space, right?  $V'$  prime is the velocity based on the empty cross sectional area of the bed and  $v$  is the actual velocity through void space, right?

So if that is true then we can write  $V'$  prime is equals to epsilon into  $v$ , right? Therefore, we can define  $N_{Re\ particle}$  is equals to  $D v \rho$  by  $\mu$  by common definition now if we substitute this  $D$  with the equivalent or hydraulic radii and equivalent diameter, so this becomes  $v$  double prime 4  $r_H$  into  $\rho$  divided by epsilon into  $\mu$ , right? So this is nothing but 4  $v$  say prime or double prime does not matter here we have given  $v$  prime, right? So  $v$  prime 4  $v$  prime epsilon phi s  $D_p$  rho divided by 6 into  $\mu$  into epsilon into 1 minus epsilon, right?

So this we can rewrite as  $4 \phi s D_p V_p \rho$  divided by  $6 \mu^{1-\epsilon}$ , so this is  $\phi s D_p v \rho$  by  $\mu^{1-\epsilon}$ , right? So this is  $N_{re}$  particle not  $Dv \rho$  by  $\mu$ , right? So till now then we have defined many things for packed bed, number 1 we have said that the what is the area, then what is the specific surface area, then what is the hydraulic radius, what is the hydraulic diameter, then we also have said the specific rather, okay specific surface area we have said and also we have said the sphericity defined sphericity, then after that definition when we came to  $N_{re}$  that is Reynolds number, right?

So  $N_{re}$  of the particle normally is it is  $Dv \rho$  by  $\mu$ , but since it is for particle and  $D$  is not for the  $\phi$ , so that equivalent or hydraulic radius or hydraulic diameter that has to be found out and for which it has become  $v \rho$ , okay  $v$  is also not  $v$ ,  $v$  is also not  $v$  it is also  $v$  prime that is the velocity based on empty cross section, right? So that is why the Reynolds number is not  $Dv \rho$  by  $\mu$  so simple it is some a little complicated as  $\phi s D_p v \rho$  by  $\mu^{1-\epsilon}$ , right?

So till now we have started this packed bed so we have also to go a long and today of course we have come to the end because time is not there, so we finish it today thank you.