Course on Momentum Transfer in Process Engineering
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Lecture 49
Module 10
Ergun's equation-derivation (Part-2)

Now in the previous class we had a little summing up of the previous class we started with laminar flow and we started with the Hagen Poiseulle equation, right? And we ended up with equation number 1 and then we also used the that was for laminar for the turbulent flow that is using the friction factor and we had come up to another equation that was equation number 2, right? And in one equation we called it to be Blake Kozeny equation which was for the laminar flow and another was for Burke Plummer and that was for the turbulent flow, right?

And in both the (ca) of course from those equations ultimately we came to an derivation or relation where we utilized the mass velocity G prime factor and then in terms of G prime we have come to one point with equation 1 and equation 2, right?

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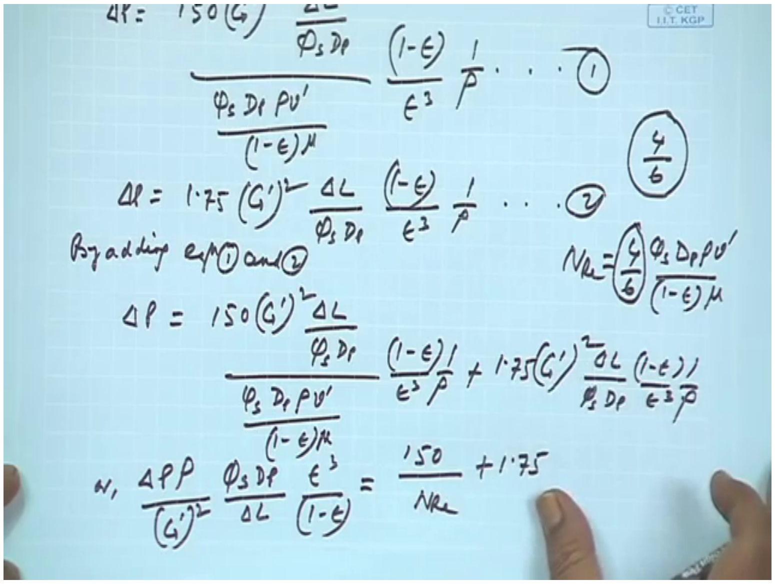

Now we let us start from there what we can do, right? So if we rewrite a little that delta P was equal to for equation 1 delta P was equal to 150 G prime square into delta L by phi s Dp over phi s Dp rho v prime over 1 minus epsilon into mu into 1 minus epsilon by epsilon cube into 1 by rho this was equation 1 and another was delta P equals to 1.75 G prime square into delta L by phi s Dp into 1 minus epsilon over epsilon cube into 1 by rho this we said to be equation 2.

Now if we add these two equations by adding equation 1 and 2 what we get? We get delta P is equals to 150 G prime square into delta L by phi s Dp over phi s Dp rho v prime over 1 minus epsilon into mu, right? Into 1 minus epsilon by epsilon cube into 1 by rho plus 1.75 G prime square into delta L by phi s Dp into 1 minus epsilon by epsilon cube, right? Into 1 by rho. So these two by adding we get these two equations, I mean the full form, right?

Now if we rewrite or rearrange this equation then we can say that delta P we can write this as delta P into this v goes up, right? So delta P into rho by G prime square, right? So into G prime square into phi s Dp over delta L into epsilon cube by 1 minus epsilon, right? This is equals to 150 divided by Nre, right? Plus 1.75, how? This here only one thing if you remember that in a Nre definition we had said that that was 4 by 6, now in 4 by 6 if we neglect this term, then phi s Dp rho v prime by 1 minus epsilon into mu is the Dv rho by mu for the packed bed Nre, right?

For Nre in the packed bed we had seen it was 4 by 6, right? phi s Dp rho v prime over 1 minus epsilon into mu this was for Nre. Now if we assume or neglect this 4 by 6, then this term we can write to be equal to Nre, right? And this we have brought down to this side delta P, right? G prime square in the denominator phi s Dp in the numerator delta L in the denominator this epsilon cube by 1 minus epsilon here, right? So this is equals to and this was common because G prime square which we could have taken in rather we should show you otherwise it maybe you have jumped the step.

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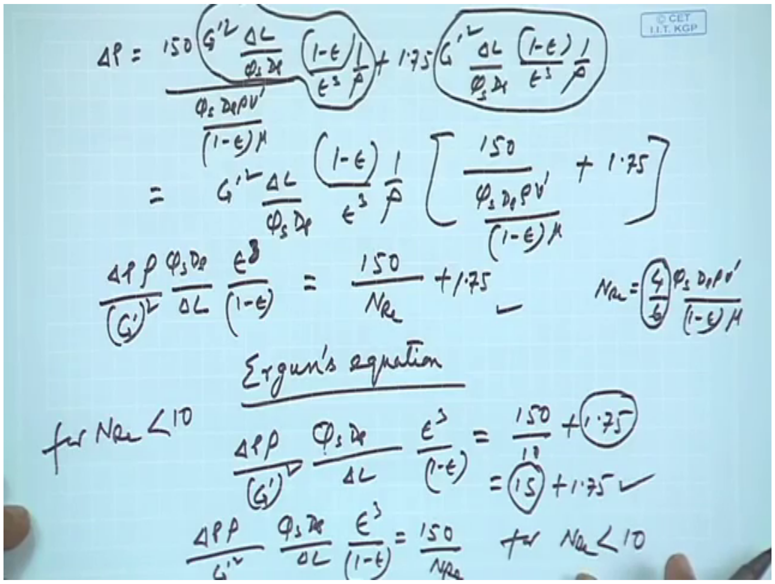

So our original was 150 G prime square, right? G prime square divided by into delta L by phi s Dp by phi s Dp rho v prime 1 minus epsilon into mu, right? 1 minus epsilon by epsilon cube into 1 by rho plus 1.75 G prime square delta L by phi s Dp 1 minus epsilon by epsilon cube into 1 by rho, right? So from there if we take that of common of G prime square which is common in both the cases delta L by phi s Dp, right? Into 1 minus epsilon by epsilon cube, right? Into 1 by rho up to this it is common in both the cases, then it remains 150 over phi s Dp rho v prime divided by 1 minus epsilon into mu plus 1.75, right? This is there we have taken this portion as common, right? We have taken this portion as common like this, right? From this we have taken common, right? And that has come G prime square delta L phi s Dp 1 minus epsilon by epsilon cube into 1 by rho so remained 150 by phi s Dp rho v prime by 1 minus epsilon into mu plus here remained 1.75.

Now if we divide and rearrange, then only we can write that delta P into or G prime square let us write this way G prime square, so here it is phi s Dp by delta L here it is epsilon cube by 1 minus epsilon, right? So here it was delta P and this rho went up, right? So delta P rho by G prime square phi s Dp by delta L epsilon cube by 1 minus epsilon, so this can be made equals to 150 divided by now we said Nre by definition we had said 4 by 6 phi s Dp rho v prime divided by 1 minus epsilon into mu, right? If we just ignore that 4 by 6 is roughly equal to 1, then we can say that this is nothing but Nre so 150 over Nre plus 1.75 this is the relation, right?

So in this relation what you see that one this side expression this is epsilon cube one this side expression delta P rho by G prime square phi s Dp by delta L epsilon cube by 1 minus epsilon this is equal to 150 by Nre plus 1.75, right? Now you just think this equation combination when we have done this was done by one scientist called Ergun and the equation is known as Ergun's equation this is known as Ergun's equation this was developed by adding that equation 1 and equation 2 this was done by Ergun and according to his name this is named as Ergun's equation where both the equations 1 and 2 where added up.

Now we will see one good thing here that we said that Blake Kozeny equation was valid for Nre less than 10 and we also said that Burke Plummer equation is valid for Nre greater than 1000, so what about if Nre is within less 10 and 1000. Now you see this Ergun's equation when you are using both, right? The whole expression you were using, right? Then between 10 and 1000 anywhere it is applicable or even less than 10 greater than 1000 also it is applicable.

Only the thing why it is applicable because you see when Nre is very small for laminar Nre is very small, so if Nre is very small less than 10, then this value comes to be 15 and this is 1.75. So compare to this value 150 1.75 can be neglected, right? So we write for Nre less than 10, right? Delta P rho by G prime square into phi s Dp over delta L, right? Epsilon cube over 1 minus epsilon this can be written as 150 over 10 if it is less than 10 maximum 10 so let us take it to be maximum so less than 10 plus 1.75 so this is 15 plus 1.75.

So compared to 15 1.75 is negligible, right? So for laminar flow we can neglect the second term 1.75 so this expression then comes to be delta P rho by G prime square into phi s Dp by delta L into epsilon cube by 1 minus epsilon this is equals to 150 over Nre and we neglect the term 1.75, right? Then this is valid for Nre less than 10, right? Now what about the other case?

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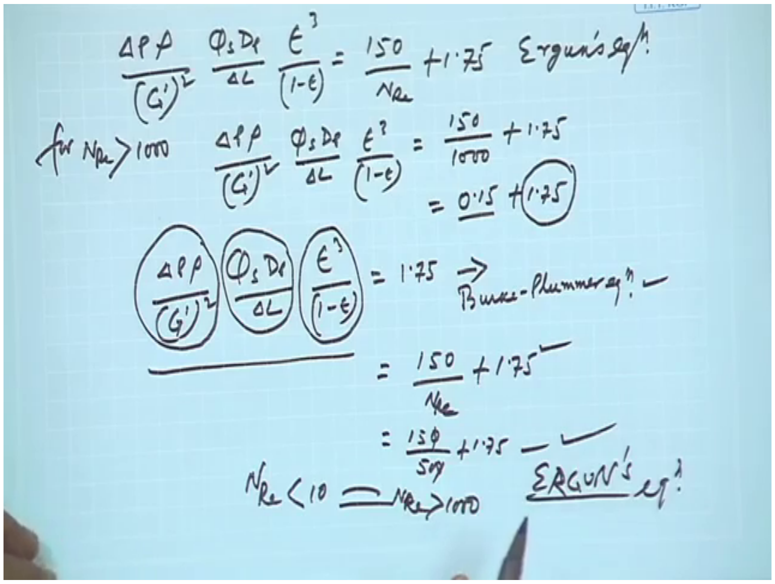

The other case is that when Nre is greater than 10 our expression Ergun’s equation was delta P rho by G prime square phi s Dp by delta L epsilon cube by 1 minus epsilon this is equals to 150 over Nre plus 1.75. So we have already shown when Nre is less than 10 what is the effect, so we can come back to the Blake Kozeny equation from the Ergun’s equation, Ergun’s equation was that by neglecting the second term 1.75 because compare to 150 by Nre this value when Nre is very low this 1.75 can be neglected because this term is high.

So our equation comes to delta P rho by G prime square phi s Dp by delta L epsilon cube by 1 minus epsilon 150 by Nre, right? That was the valid for Nre less than 10 and this we know knew as this equation to be Blake Kozeny, right? So this is Blake Kozeny equation which we have seen. Now for the other one when Nre is very high greater than 1000, right? If Nre for Nre greater than 1000, what we can say? We can say that when Nre is greater than 1000, then delta P rho by G prime square phi s Dp by delta L, right? Into epsilon cube by 1 minus epsilon, right? This is equals to 150 by 1000 plus 1.75.

So that means this is 150 by 1000 that is 0.15 plus 1.75 so compare to 1.75 1.5 is very low, is it? So it is 1 10th of that it is compared to 1.75 0.5 is very low 0.15 or less than that as the Nre value will be 1000 or greater than that. So when it is 1000 or greater than that then it will be less than 0.15 or lower value and we can say that 1.75 is much much higher than that point 175 when Nre value is greater than 1000. So we can say that compared to 1.75 if we neglect this 0.15, then the equation comes down to delta P rho by G prime square into phi s Dp by delta L into epsilon cube by 1 minus epsilon, right? This is equals to 1.75, right? And this is we have seen to be known as Burke Plummer equation, right? So from the our equation which we have said to be Ergun's equation we can come down to Burke Plummer equation or we can go back to Blake Kozeny equation, right?

This fundamental thing if we keep in mind, then keeping or remembering the equation because this equation is very helpful particularly that Ergun's equation is very very helpful for finding out delta P in any packed beds situation, right? So for packed bed we have seen and we can establish that Ergun's equation can predict both for less than 10 Nre or Nre greater than 1000 or if the Nre value lies in between 10 and 1000 then also we can utilize that, right?

This we have established and we have seen, right? So Burke Plummer and Blake Kozeny these two equations on combination gives us that Blake Kozeny and rather Ergun's equation and in from Ergun's equation we can also find out the other one that is by neglecting the two terms we got, right? One expression equal to one term plus another so neglecting one we get to the less than 10 Nre or lower value for laminar flow that we can see and we can neglect the other (()) (21:50) term 1.75 compared to that and compare to the higher term 1.75 was low for laminar flow so we neglect it 1.75 and for turbulent flow compared to 1.75 this was very low that is that factor by Nre 150 by Nre was very low so neglected that Nre term in that and we came back to the Burke Plummer equation, right?

So if we remember this way that delta P rho by G square this is on term and the dimensional term is phi s Dp delta L, so phi s Dp by delta L another term and third one is the how much void fraction, right? So epsilon cube by 1 minus epsilon is the third term if we remember this way, right? So this was equals to 150 over Nre in general for Ergun's equation plus 1.75, right? So this we can easily say that this 150 by Nre when we take this, then it is Blake Kozeny and when we neglect this 150 by Nre then it is Burke Plummer.

And when we do not neglect both of them, then it becomes Ergun's equation, right? So if we have also shown that these three can be derived this expression we have derived, right? So we can say that from the very basic that is the equation for we have said that from the equation which was for pipe flow that is for the that Hagen Poiseulle, right? From the Hagen Poiseulle's equation we have derived that Blake Kozeny equation and we said that this is valid for less than 10 Nre, right? We also showed from the other that is when the flow is laminar but the flow is turbulent, then we started with the not from Hagen Poiseulle but from the frictional pressure factor relation that delta P 4 f, right?

4 f rho L v square L by D v square by 2 that this relation we had started with and from there we came back to Burke Plummer and that was valid for Nre greater than 1000. Then the question came when Nre is less than 10 we have an expression for predicting delta P. When Nre is greater than 1000 we have an expression for predicting delta P, right? But how to express the pressure drop relation when Nre is between 10 and 1000 that is neither it is laminar nor it is turbulent in between that is in transition zone what can be done.

So there we have shown that this also can be done when Nre is less than 10, greater than 1000 or in between with the help of another equation that was developed by adding these two equations and then rearranging of course that rearrangement was done by them and ultimately they landed up with these parameters or these expressions as delta P rho by G prime square this is one, phis s Dp by delta L this is another and epsilon cube by 1 minus epsilon, right?

So we have to keep in mind one is pressure drop, density and mass velocity these three factor in a one then the sphericity then particle size and the length of the bed and third one is the voids fraction. So in this way these three terms was connected with 150 by Nre plus 1.75 as the Ergun's equation. So when we neglected the second term we came to the first that is laminar and when we neglected the first we came back to the turbulent.

So when it was in between, then the Ergun's equation also can be used using both of them. Say Nre is greater than 10, but less than 1000 say 500, so in that 500 it becomes 150 by 500 plus 1.75, right? So it is what? This goes down and this 15 so it is roughly one third that is 0.33. So 0.33 or 0.34 say, say 0.4 roughly so 0.4 and 1.75 they are not so big, it is three four times only, right? So in that case it cannot be neglected, so that is why these two lies between when Nre is less than 10 or greater than 1000 both or in between that entire thing can be used by this Ergun's equations, right? So today let us stop it here, thank you.