

**Momentum Transfer in Process Engineering**  
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**Module 11**  
**Lecture No 51**  
**Solving Problems on Ergun's Equation (Contd)**

In the previous class we have solved one problem on packed bed, using Ergun's equation. What did we do? We assumed value of Delta p and from the Ergun's equation we because from that Delta p we found out the exact pressure and from that average here density between the inlet and outlet, so that average density we utilised and we got the assumed and actually calculated values by closed. And we said that if possible future we will also try to do similar problems or if we come across another problem then we can practice and utilise the Ergun's equation more meaningfully,. So let us do another problem, this is like this.

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Air at 390 K flows through a packed bed of cylinders having a diameter of 0.0127 m and length the same as diameter (this means, the cylinder can be treated as sphere). The bed void fraction is 0.45 and the length of the packed bed is 4 m. The air enters the bed at 2.5 atm abs at the rate of 3 Kg/m<sup>2</sup>-s based on the empty cross section of the bed. Calculate the pressure drop of air in the bed. Given, M=29 and  $\mu = 1.5 \times 10^{-5}$  Pa.s.

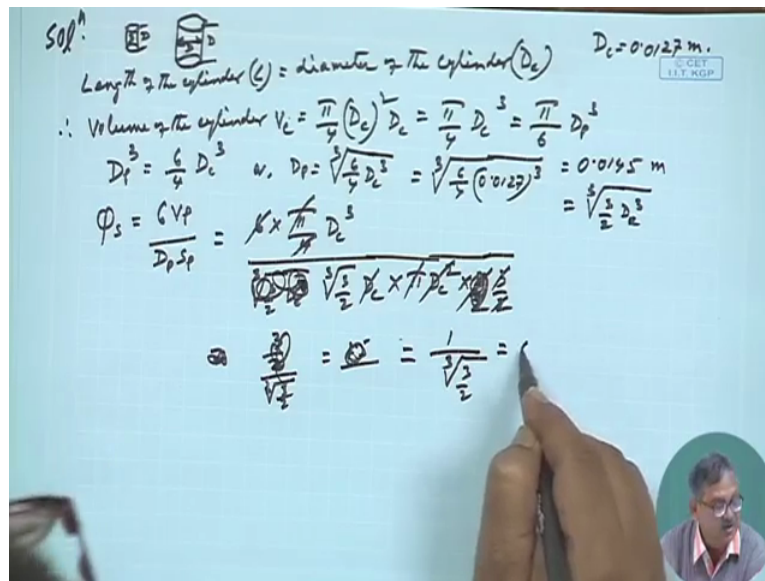
Sol. Since, length of the cylinder (L) = diameter of the cylinder ( $D_c$ ),  
 Volume of the cylinder,

$$V_c = \frac{\pi}{4} (D_c)^2 D_c = \frac{\pi}{4} D_c^3 = \frac{\pi}{6} D_p^3$$

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Air at 390 Kelvin flows through a packed bed of cylinders having a diameter of 0.0127 meter and length the same as diameter; this means the cylinder can be treated as a sphere. The bed void fraction is 0.45 and the length of the packed bed is 4 meters. The air enters the bed at 2.5 atmospheres absolute at the rate of 3 kg per meter square second, so mass flux is set based on the empty cross-section of the bed. Calculate the pressure drop of air in the bed given molecular weight is 29 and Mu 1.5 10 to the – 5 past seconds,. So I repeat, this is like that air at 390 Kelvin flows through a packed bed of cylinders having a diameter of 0.0127 meters and length the same as diameter that this means that the cylinder can be treated as sphere.

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The bed void fraction is 0.45 and the length of the packed bed is 4 meters. Air enters the bed at 2.5 atmosphere absolute at the rate of 3kg per meter square second based on the empty cross-section of the bed. Calculate the pressure drop of air in the bed given molecular weight of air is 29 and viscosity of air is  $1.5 \times 10^{-5}$  pascal seconds. So if you solve this, what is said that we have a packed bed of cylinders, now these cylinders do have this is the cylinder height, so height is equal to the diameter that is why this can be taken as d and this is d, so these 2 d are same, so this cylinder can be assumed to be sphere. But if you assume a cylinder to be a sphere than all cylinders can be equal to sphere, so then it appears that you may have to find out the sphericity, how close it is to a sphere, so that we will find out.

1<sup>st</sup> you have to find out that air as we have said that if this is a cylinder if this height and this diameter are same, then it theoretically diameter and height of cylinder if they are same then that appears to be equal to a sphere. But whether it is a sphere or not or what is the sphericity of that sphere, sphericity means how close it is to the sphere that 1<sup>st</sup> you have to find out and then you can use the other things. So let us find out that the length of the cylinder that is L is equal to diameter of the cylinder say D cylinder. Therefore, volume of the cylinder V cylinder that should be equal to  $\frac{\pi}{4} D^2 L$  of cylinder square into D of cylinder which is the height that means  $\frac{\pi}{4} D^3$  cube. So this should be equals to  $\frac{\pi}{6} D_p^3$  cube that is the actual volume of the sphere having particular diameter D p.

So our given thing was the cylinder has a diameter of D cylinder = 0.0127 meter, so if that be true then D p cube = this  $\frac{\pi}{4} D^3$  goes out so  $\frac{\pi}{6} D_p^3 = \frac{\pi}{4} D^3$  or  $D_p = \sqrt[3]{\frac{6}{4} D^3}$  is 0.0127 so much cube. So let us look into how what the

value is, so  $0.0127$  cube  $x$  to the power  $y$  or  $x$  is cube, so this is that into 6 divided by 4, so it is cube root, cube root is equal to  $0.0145$  so that means particular diameter is  $0.0145$  meter. Then we can say that  $\Phi_s$  is  $6 V_p$  by  $D_p S_p$ , this you have done in the packed bed class beginning,  $\Phi_s$  is  $6 V_p$  by  $D_p S_p = 6$  into  $\text{Pie}$  by  $4 V_p$  is  $\text{Pie}$  by  $4 D_c$  cube divided by  $D_p$  we have already found out to be say this is the value but this we can also write to be cube root of  $3$  by  $2$  into  $D_c$  cube.

So we can also write this to be cube root of  $3$  by  $2 D_c$  cube no  $D_p$  is  $D_c$  cube root so it is  $D_c$  not cube root cube root of this, here  $D_p$  cube is  $6$  by  $4 D_c$  cube so  $D_p$  is cube root of okay  $D_c$  cube is that  $6$  by  $4 D_c$  so cube root if it goes out so it is  $3$  root  $3$  by  $2 D_c$  we write this way cube root  $3$  by  $2 D_c$  because this  $D_c$  cube, cube root it goes out okay. So  $D_p$  is so much and  $S_p$  this into  $S_p$ ,  $S_p$  is how much? It is  $\text{Pie} D_c$  square  $3$  by  $2 \text{Pie} D_c$  square  $S_p$  is  $3$  by  $2 \text{Pie} D_c$  square, so this becomes equals to this is  $D_c$  cube so  $D_c$  square, so this goes out, this  $\text{Pie}$  this  $\text{pie}$  goes out and this is  $6$  by  $84$  this is  $6$  by  $4$  that is  $3$  by  $2$  so this  $3$  by  $2$  this goes out so then it remains  $3$  by  $2$  under root  $3$   $3$  by  $2$  under root  $3$ .

$3$  divided by  $2$  is equal to so much, cube root of that so this becomes cube root of  $3$  by  $2$  okay  $D_c$  into  $3$  by  $2 \text{Pie}$  by  $S_p$  is  $\text{Pie} D_c$  square clear, so it is  $\text{Pie}$  this is square so this  $3$  by  $2$  may not be going out so  $6$  by  $4$  so it becomes  $3$  by  $2$  okay because  $S_p$  is how much  $\text{Pie} D_c$  square so well from this  $3$  by  $2$  comes, so then this  $D_c$  cube goes out then  $3$  by  $2$  and this divided by under root  $3$  by  $2$  it is cube root. So it is  $1.5$  divided by that came to be  $1.14$ ,  $6 B_p B_p$  is  $\text{Pie} = \text{Pie}$  into  $D_c$  cube find and  $D_p$  is from here cube root of  $3$  by  $2 D_c$  fine and this is  $\text{Pie} D_c$  square that is the  $S_p$  that is the surface area of this particle that is  $\text{Pie} D_c$  square.

$\text{Pie}$   $\text{Pie}$  goes out,  $6$  by  $2$  remains,  $3$  by  $2$  by root  $3$  cube root of  $3$  by  $2$ , so this is  $3$  by  $2$   $1.4$  so  $3$  by  $2$  cube root is  $1.144$  so  $1$  by  $1.5$  by  $1.144$ , so  $1.144$  divided by  $1.5$  no no no no, I think I have made any mistake that is  $3$  by  $2$   $1.5$ ,  $1.5$  cube root of this inverse of this yes so it was no  $3$  by  $2$  is there. Then it becomes equals to this  $3$  by  $2$  goes out,  $1$  by cube root of  $3$  by  $2$  so that becomes equals to  $0.873$  so  $\Phi_s$  is  $0.873$ .

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$D_c = 0.0127 \text{ m}$   
 Length of the cylinder ( $L$ ) = diameter of the cylinder ( $D_c$ )  
 $\therefore$  Volume of the cylinder  $V_c = \frac{\pi}{4} (D_c)^2 L = \frac{\pi}{4} D_c^3 = \frac{\pi}{6} D_p^3$   
 $D_p^3 = \frac{6}{\pi} D_c^3$  or  $D_p = \sqrt[3]{\frac{6}{\pi} D_c^3} = \sqrt[3]{\frac{6}{\pi} (0.0127)^3} = 0.0145 \text{ m}$   
 $\phi_s = \frac{GVP}{D_p S_p} = \frac{GVP}{\frac{\pi}{4} D_c^2 L} = \frac{GVP}{\frac{\pi}{4} D_c^3}$   
 $\phi_s = \frac{3}{\sqrt[3]{\frac{6}{\pi} D_c^3}} = \frac{3}{\sqrt[3]{\frac{6}{\pi} (0.0127)^3}} = 0.873$   
 $Re = \frac{\phi_s D_p G'}{\mu (1-\epsilon)} = \frac{0.873 \times 0.0145 \times 3}{1.5 \times 10^{-5} (1-0.45)} = 4031 \checkmark$   
 $\rho_{in} = \frac{pM}{RT} = \frac{2.5 \times 101325 \times 29}{8314 \times 390} = 2.266 \text{ kg/m}^3$

Then  $\phi_s$  is 0.873 then we can write  $Re = \phi_s \times D_p \times G' / \mu (1-\epsilon)$ , so this becomes equals to  $\phi_s$  is 0.873,  $D_p$  is  $D_p$  we have found out to be the cube root of this no okay so that we put  $D_p$  not given,  $D_p$  we have to find out that is  $6 / \pi \times 0.0127^3$  so that goes out  $D_c$  that is cube root of  $3 / 2 \times D_c$ , so cube root of  $3 / 2 \times D_c$  we have been given that so it can be 0.0145 and uh  $G'$ ,  $G'$  is it given?  $G'$  is given as 3 KG per meter square, into 3 divided by  $\mu$  given is  $1.5 \times 10^{-5}$  and  $\epsilon$  given is  $1 - 0.45$ , so this becomes equal to 0.0127 is given oh it is just having a diameter of 0.0127 yes that  $D_p$  is given 0.0127 so not required, it is 0.0127, so this is 3.

So we can write that 0.873 into 0.0145 into 3 divided by this divided by  $1.5 \times 10^{-5} (1 - 0.45)$  divided by 10 to the  $-5$ , 4031. So that means that our energy is very very high, so in the outlet pressure depends on  $\Delta p$  so we assuming as 1<sup>st</sup> approximation density is equal to (22:55) inlet pressure. So inlet pressure given is 2.5 atmosphere, so  $\rho_{in} = pM / RT$  or  $p_{in} / RT = 2.5$  atmosphere into  $1.01325 \times 10^5$  to the power 5,  $M$  is 29,  $R$  is 8314 and  $T$  is given 390  $T$  given is 390 so it comes to be 2.266 kg per meter cube.

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from Ergun's eqn.

$$\frac{\Delta p \rho}{G'^2} \frac{\phi_s D_p}{1 - \epsilon} \frac{\epsilon^3}{\epsilon^3} = \frac{150}{N Re + 1.75}$$

$$\Delta p = \frac{G'^2}{\rho} \frac{\Delta L}{\phi_s D_p} \frac{1 - \epsilon}{\epsilon^3} \left[ \frac{150}{N Re + 1.75} \right]$$

$$= \frac{3^2}{2.266} \frac{4}{0.873} \frac{(1 - 0.45)}{0.45^3} \left[ \frac{150}{4036 + 1.75} \right]$$

$$= 15.439 \text{ kPa}$$

$$p_{in} = 2.5 \times 1.01325 \times 10^5 \text{ Pa} \quad \Delta p = 15.439 \text{ Pa}$$

$$p_{out} = 2.5 \times 1.01325 \times 10^5 \text{ Pa} - 15.439 \text{ Pa} = 268.7515 \text{ kPa}$$

$$p_{av} = \frac{p_{in} + p_{out}}{2} = 261.032 \text{ kPa}$$

$$\rho_{av} = \frac{p_{av} M}{R T} = 2.334 \text{ kg/m}^3 \quad \Delta p = 14.58 \text{ kPa}$$

very close to assumed.

$$\Delta p = 15.069 \text{ kPa}$$

Now from Ergun's equation we have seen that  $\Delta p \rho$  by  $G$  prime square  $p$  s  $D$  p by  $\Delta L$  Epsilon cube  $1 - \text{Epsilon}$   $150$  by  $N Re + 1.75$  so from this we can write that  $\Delta p$  to be equals to  $G$  prime square by  $\rho$  into  $\Delta L$  by  $\Phi$  s  $D$  p,  $1 - \text{Epsilon}$  by Epsilon cube into  $150$  by  $N Re + 1.75$ . So this if we write that means this is  $3$  square by  $\rho$ ,  $\rho$  we found out  $2.266$ ,  $\Delta L$  will given is  $4$  meter here it is  $4$  meter, so  $4$  by  $\Phi$ ,  $\Phi$  we have found out  $0.873$  we have found out or say  $4$ ,  $3$  something was there so rounded of to  $0.874$ . So  $\Delta p$  so  $1 -$  this has been given as  $0.45$  divided by  $0.45$  cube into Epsilon  $150$  by  $N Re$ ,  $N Re$  is  $4036 + 1.75$ , this is equal to  $15.439$  kilo pascal, so  $\Delta p$  is  $15.439$  kilo pascal.

Therefore, since  $p_{inlet} = 2.5$  into  $1.01325 \times 10^5$  Pascal, there and  $\Delta p$  is  $15.439$  pascal, so therefore  $p_{exit}$  or outlet =  $2.5$  into  $1.01325 \times 10^5 - 15.439$  pascal, this is also pascal, so this means that  $\Delta p$  comes to be equals to or  $p_{outlet}$  comes to  $268.7515$  kilopascal. So with this outlet as again one, so then and this is  $p_{inlet}$  so  $p_{average}$  can be found out as  $p_{inlet} + p_{outlet}$  over  $2 =$  this comes to be  $261.032$  kilopascal. Therefore,  $\rho_{average}$  that can be written as  $p_{average} M$  by  $R T$  and that can be said to be equals to  $2.34$  or  $2.334$  KG per meter cube. And if we do the same way  $\Delta p$  calculation, then  $\Delta p$  comes to be  $14.589$  kilopascal, which to very close to the assumed value or what we have found out  $p_{average}$  from there and  $1^{st}$  approximation.

So very close to the  $\Delta p$  assumed, so we can say  $\Delta p$  to be equals to  $15.069$  KPa finally. So this way by doing trial and error you can find out that what is exactly  $\Delta p$  or if one is given  $p_1$ , we can also find out the outlet whatever given depending on the situation.

So here you have seen that we have taken both, the laminar part, length part, that is the entered Ergun's equation and from there if  $\mu$  is given like inlet pressure is given, outlet is not given and in this case we assume that both inlet, outlet they are different true what we assumed that density to be at the inlet pressure and we found out  $\Delta p$ . With that  $\Delta p$  we found out pressure outlet and from there again average and from that average again we found out what is the  $\Delta p$ , so this way by trial and error we can find out the actual value okay thank you.