

Course on Momentum Transfer in Process Engineering
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Lecture 58
Module 12
Problem and solution (Continued)

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Okay, so you have done rather we have done here under different conditions if you remember that Brookfield viscometer the theory of that so it was like this the two concentric pipe or yeah concentric pipe and one was inner wall was fixed say and the outer one is rotating with an angular velocity of ω and we have found out what is the velocity profile, pressure drop, etcetera, right? So this is used for Brookfield viscometer or if it is closed if it is rather fixed if this is rotating then also we have found, right? We have found how the pressure drop or how the velocity profile is changing, right?

Now if you are asked, okay that was one type of viscometer, normally it is a viscometer by which viscosity is measured. So if you are measuring the viscosity, then no problem you have already seen for Brookfield viscometer how the theory is occurring and experimentally you can find out in knowing the all other parameters you can know you can find out the viscosity of some solutions of or some fluids, right?

Now if you are asked that okay you have another type of viscosity measuring instrument like say capillary we also have done if you remember the theory for a ball dropping from a height, right? Till it is coming to its terminal velocity what is the how the viscosity can be developed, right? So if you remember that expression we have also done, right? And if now the third case if you are asked that okay you are given a capillary, right? You are and you have heard the name of capillary viscometer there are many capillary viscometer manufacturers so if you are asked that if you are given a capillary and you have to find out the viscosity, right?



Now viscosity what does it mean? Viscosity means if the fluid is less viscos if this is the capillary say if this is the capillary and if through this if I have to pass the liquid the less viscos for the same height that will pass faster the more viscos for the same height that will pass at a lower speed or at a later time, right? So what will happen? The more viscosity means the both the wall as well as between the layers of the fluid they will have a drag force acting very high, right?

Now this if it is happening, that is why we need the value of viscosity to be known, right? For the importance of the viscosity is there that whether a fluid will move fast or a fluid will move slowly that will depend on the viscosity of the fluid, the higher the viscosity value, right? That is more the viscos the fluid, it will take more time for the same diameter same length of the pipe then compare to that of the less viscos fluid, right? Now you also have done we have done rather Stoke's law on that regard, right? That dropping of the ball, okay we have done the Brookfield also that one wall is fixed other wall is moving, right?

And then how the fluid characteristics flow characteristics are there. Now if you are said that okay you are given a capillary viscometer and find out the viscosity, right? So we cannot do experiment in this class because that is beyond the scope, but we can definitely do some problem on that, right? So let us look into how the values are that is why repeatedly we saying that values are very very important, okay.

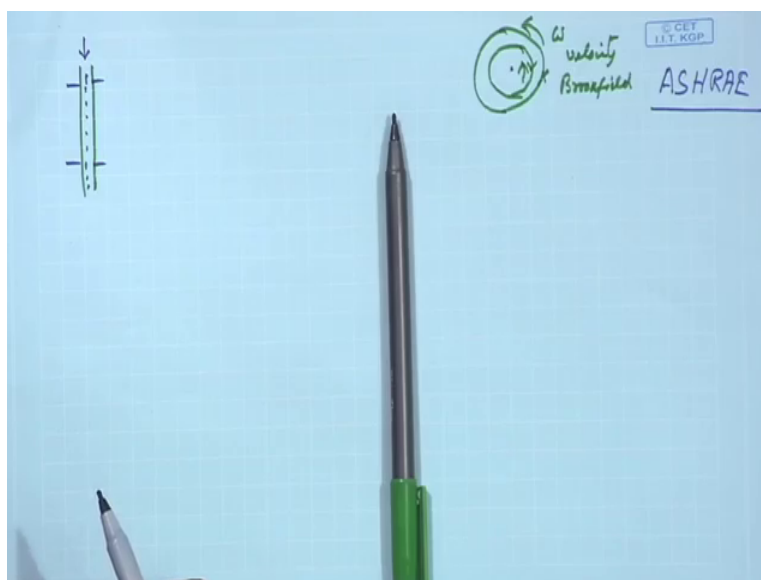
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Problem: A glass capillary viscometer has a height available for free fall is 9 cm and a capillary length of 7.5 cm. If the fluid used is distilled water at 24 °C, the efflux time measured is 35 s. Volume of water drains between two marks on the viscometer is 4 cm³. What is the radius of the capillary? Assume, density and viscosity of water at 24 °C to be 950 kg / m³ and 1.0 X 10⁻³ Pa s respectively.

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So let us look into this problem, this problem is saying for the capillary viscometer of glass a glass capillary viscometer has a height available for free fall is 9 centimeter and a capillary length of 7.5 centimeter. If the fluid used is distilled water at 24 degree centigrade, the efflux time measured is 35 second. Volume of water drains between two marks on the viscometer is 4 centimeter cube. What is the radius of the capillary? Assume density and viscosity of water at 24 degree centigrade to be 950 kg per meter cube and 1 into 10 to the power minus 3 Pascal second respectively, right?

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So this is also giving us a chance to see if it is a capillary, right? And a liquid water is flowing through this, right? From one definite height to another definite height the time required to pass is known, then how can we find out the viscosity of water, this is the basic fundamental question we are given that we have given you a capillary through which water is flowing and we have given you the property values of water at a definite temperature of course property values are a function of temperature so that is why it is mentioned, right?

And for your when it has come, so let us also say there is a very good book called ASHRAE, right? So this I am (8:42) for heating, refrigeration and air conditioning engineering that general or that is not general that is a bound where I should say that a bound volume and data book that data book you can find out any fluid properties at many different temperatures, right? Like steam table you get property values, similarly that book hand book that is hand book, so this handbook many other books are also available, so you can get the property values from there.

So what where we have been asked that we have been given a capillary and water at definite temperature and that through the capillary water is dropping, right? And we have to find out what is the viscosity when we know that what is the time required to travel that water from one height or one distance to the other distance, right? We have been given the values, we have to find out.

So let us reread that problem and then solve, it is a glass capillary viscometer has a height available for free fall is 9 centimeter and a capillary length of 7.5 centimeter. If the fluid used is distilled water at 24 degree centigrade, the efflux means what is coming out time measured is 35 seconds. Volume of water drains between two marks on the viscometer is 4 centimeter cube. What is the radius of the capillary? That is a fundamental question that what is the radius of the capillary that we have to find out, not the viscosity, so if it is the radius is given then we could find out the viscosity, we can try if one is done we can if we have time we can do the other way that one value is given what is the other value, right?



So that means one is given the other has to be found out, given that at 24 degree centigrade the value of viscosity and density or density and viscosity of water are 950 kg per meter cube and 1 into 10 to the power minus 3 Pascal second, right? This we have to find out.

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From Hagen Poiseuille equation,

$$v_{av} = \frac{(P_{in} - P_{out}) R^2}{8 \mu L} = \frac{\Delta P R^2}{8 \mu L}; \text{ or, } \mu = \frac{\Delta P R^2}{8 v_{av} L}; \text{ and, } \Delta P = \rho g H$$

$$\text{Now, } v_{av} = \frac{\text{volume}}{\pi R^2 t_e}; \therefore \mu = \frac{\rho g H R^2 \pi R^2 t_e}{4 \times 10^{-6} \times 8 \times L}; \therefore R^4 = \frac{(4 \times 10^{-6}) \times 8 \times L \times \mu}{\rho \times g \times H \times \pi \times t_e}$$

$$\text{or, } R = \left(\frac{1.0 \times 10^{-2} \times 4 \times 10^{-6} \times 8 \times 7.5 \times 10^{-2}}{950 \times 9.8 \times 9 \times 10^{-2} \times \pi \times 35} \right)^{\frac{1}{4}} = 0.001237 \text{ m} = 1.237 \times 10^{-3} \text{ m}$$



Hagen-Poiseuille eqn.

$$\Delta P = \frac{32 \mu v_{av} L}{D^2}; \text{ or, } v_{av} = \frac{\Delta P D^2}{32 \mu L} = \frac{\Delta P R^2}{8 \mu L} = \frac{\Delta P R^2}{8 \mu L}$$

$$\therefore \mu = \frac{\Delta P R^2}{8 v_{av} L}; \Delta P = \rho g H$$




$$= \frac{\rho g H R^2}{8 v_{av} L}$$

$$\text{Now, } v_{av} = \frac{\text{Volume}}{\pi R^2 \times t_e}; \mu = \frac{\rho g H R^2 \pi R^2 t_e}{8 \times \text{Volume} \times L}$$

$$R^4 = \frac{8 \times \text{Volume} \times L \times \mu}{\rho g H \pi t_e} = \frac{8 \times 4 \times (10^{-2})^3 \times 7.5 \times 10^{-2} \times 1.0 \times 10^{-2}}{950 \times 9.8 \times 9 \times 10^{-2} \times \pi \times 35}$$

$$R = \left[\frac{8 \times 4 \times (10^{-2})^3 \times 7.5 \times 10^{-2} \times 1.0 \times 10^{-2}}{950 \times 9.8 \times 9 \times 10^{-2} \times \pi \times 35} \right]^{\frac{1}{4}} = 7.14 \times 10^{-4} \text{ m} = 0.714 \text{ mm}$$

ASHRAE

Now here since it is flow through a capillary so we can assume that we can use Hagen Poiseuille's equation. So from Hagen Poiseuille equation what we have seen that delta P is equals to 32 mu v L by D square, right? So from there if we have to find out the v, then we can say that v average so this is average v average is equals to delta P D square by 32 mu L, right? Or this we can write if it is not D, then delta P so D is 2 R so 4 R square D is 2 R so 4 R square by 32 mu L or this can be written as delta P R square by 8 mu L, right? So v average is like that, therefore we can say that mu is delta P R square by 8 v average into L, where delta P is nothing but rho g and H, right? Height of the liquid, right?

If that be true, then we can use this ΔP here as ΔP is given as $\rho g H$, right? And v_{average} is $\rho g H$, okay into R^2 by $8 v_{\text{average}} L$, right? Now from other side v_{average} can also be written as volume over surface sectional area that is πR^2 times the required to flow that is t_{efflux} , right? So this volume by this means we can substitute that volume from there from the given value, right? So we can write that this expression μ is equals to $\rho g H R^2$ divided by 8 now v_{average} is this 8 into π , this is 8 volume by this v_{average} , right? v_{average} is this 8 into volume, right? Into π into R^2 into t_e , right?

So if we say that volume to be q or something, right? Then, we can write that this is $\rho g H R^2 \pi R^2 t_e$ by 8 times L is also there, right? $\pi R^2 L v_{\text{average}}$ this is okay, so $8 v_{\text{average}}$ into L , right? So we can write from there that R to the power 4 is equals to 8 times volume times L times μ over $\rho g H \pi t_e$, right? So this is correct, so now if we substitute the values 8 remains volume whatever was given if we remember that was 4 centimeter cube, right?

That means 8 into 4 centimeter cube means 10 to the power minus 2 cube, right? Into L , L was given 9 centimeter, right? L given, L is that 7.5 centimeter, okay. 7.5 into 10 to the power minus 3 into μ , μ given is 1 into 10 to the power minus 3 over ρ , ρ given is 1000 , no 950 into 9.8 into H height it was given 9 centimeter so 9 into 10 to the power minus 2 into π into t_e was 35 seconds, right?

So now if we do this, then let us find out what is the value R to the power 4 so we can write R is equals to 8 into 4 into 10 to the power minus 2 whole cube into 7.5 into 10 to the power minus 3 into 1.0 into 10 to the power minus 3 by 950 times 9.8 times 9 times 10 to the power minus 2 times π times 35 to the power 1 by 4 , right?

So let us look into this that 8 into 4 into 10 to the power minus 2 square, right? 10 to the power minus 2 square is equal to this, no something wrong 8 into 4 into 10^x to the power y 2 plus minus, right? That was the mistake why it could not be, okay 8 into 4 into 10^x to the power 2 plus minus so now you square it this is that into 7.5 into 10 to the power minus 3 into 10 to the power minus 3 another divided by 950 divided by 9.8 divided by 9 divided by 10 to the power minus 2 divided by π divided by 35 is equal to this x to the power y 1 by 4 7.14 10 to the power minus 4 7.14 10 to the power minus 4 so much meter, right?

So that means 7.14×10^{-4} meter, so into 1000 that is 0.714 millimeter, right? So 0.714 millimeter is a capillary we can say, okay let us see how much it came up it was no 1.237×10^{-3} not 7.14 I think somewhere we have made a mistake so it is 8 into 4 okay 8 into 4 1×10^{-2} , that was another mistake 1×10^{-2} this is 1×10^{-3} we have written but taken here as 10^{-2} in both the cases 10^{-2} has been so this is something wrong here this is wrong I think we are right, right? So however the value you see it is so low, right? So that means, okay now if we look at now if we look at that this R we said this is for the capillary what is the radius of the capillary, right?

Now if the radius of capillary is given, right? Then we can also find out the viscosity from this expression we can also find out the viscosity, so in that case it would have been that the radius of this capillary is given all other parameters are given and μ has to be found out, right?

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$$\mu = \frac{\rho g H \pi R^4}{8 \text{ volume} \times L} = \frac{950 \times 9.8 \times 9 \times \pi \times 35 \times (7.14 \times 10^{-4})^4}{8 \times 4 \times (10^{-2})^3 \times 7.5 \times 10^{-2}}$$

For example so here if we take that for 7.14×10^{-4} meter radius, what is the value of μ ? So μ should be equal to from this $\rho g H \pi R^4$ by 8 into volume into L , right? So if we take ρ was given 950 g is 9.8 H 9 π t 35 seconds R we have found out to be 7.14×10^{-4} to the power 4 , right? Over 8 volume if we remember volume was given 4 centimeter cube so 4 into 10^{-2} whole cube into L , L was 7.5 .

So $7.5 \cdot 10^{-3}$, right? So if we solve it, let us see of course it should come the same unless we have done something somewhere some mistake. So $950 \cdot 9.8 \cdot 9 \cdot \pi$ into $35 \cdot 7.14 \cdot 10^{-4} \cdot 10^4$ might be we have done something wrong divided by $8 \cdot 10^2$ plus minus, right? Cube into $7.5 \cdot 10^{-3}$, 39.90 I said no somewhere we have done something wrong maybe here we have done something wrong, let us quickly try I think our time is up.

So you try here hopefully this part we have not done correctly, so you try it should come the same value of μ , okay thank you.