Course on Momentum Transfer in Process Engineering
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Mod 02 Lecture 07
Application of Navier Stokes Equation

Good morning, if you remember in the previous class we had dealt with equation of continuity and equation of motion. So we developed in both R component, theta component and z component that is in the cylindrical co-ordinates that is R, theta, z we had developed, also we developed in x, y, z co-ordinate and we also get the equations for the spherical co-ordinates r, theta, phi, right.

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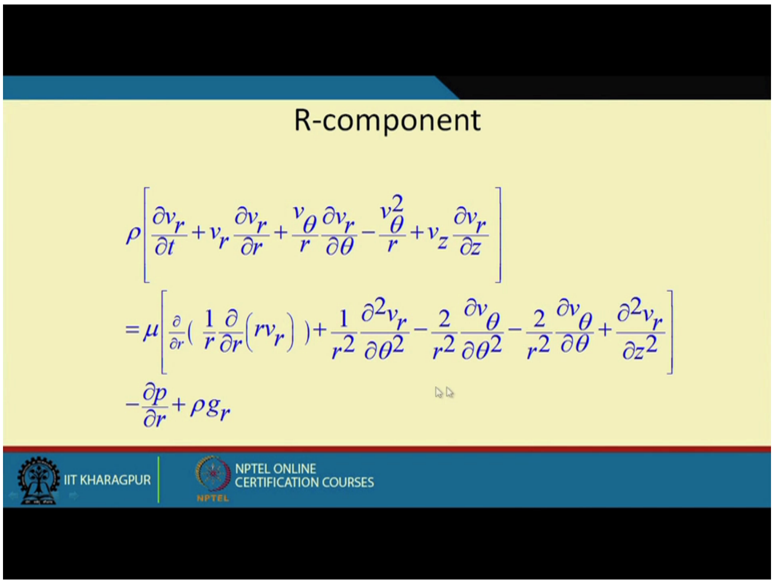

Now the question comes in the last class if you remember in the last slides where we had given these equations of motions that is Navier-stokes equations. These equations were not so much visible the reason being, the color in mismatch with the background was not was rather there for which it was not properly visible, right and besides if we recapitulate that what was the equation then we say that it was like this in the R-component that is Rho del Vr del t plus Vr del Vr del r plus Vtheta by r del Vr del theta minus V square theta by r plus Vz del Vr del z that is equals to Mu times del del r of 1 by r del del r of r Vr plus 1 by r square del 2 Vr del theta square minus 2 by r square, this also should be del 2 Vtheta del theta square minus 2 by r square del Vtheta del theta plus del 2 Vr del z square minus del p del r plus Rho gr. This is for the R-component, right and then we proceed to this is for R, theta, z in which R-component of the equation of motion is like that.

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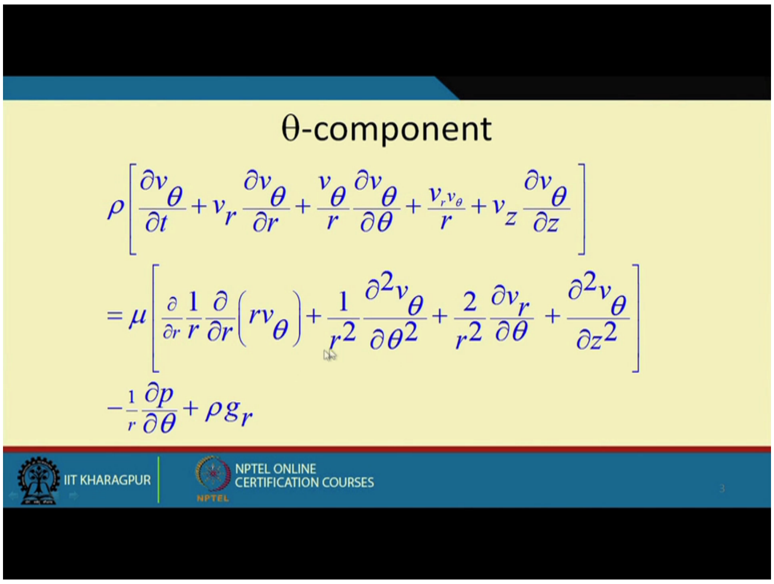

Now if you look into the other 2 components like theta component, then theta component looks like this that Rho into del Vtheta del t plus Vr del Vtheta del r plus Vtheta by r del Vtheta del theta plus Vr Vtheta over r plus Vz del Vtheta del z, this is equals to Mu times del del r of 1 by r del del r of r Vtheta plus 1 by r square del 2 Vtheta del theta square plus 2 by r square del Vr del theta plus del 2 Vtheta del z square minus 1 by r del del p del theta plus Rho gr, this is for the theta component, right.

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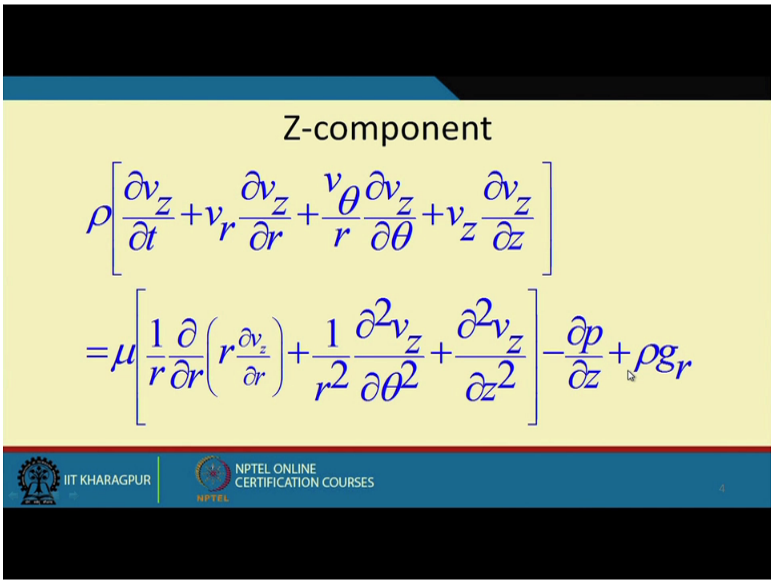

So if R and theta is done then we left with the third component that is the z-component, r, theta, z is our co-ordinate. So if you look at that z-components looks like this, Rho times del Vz del t plus Vr del Vz del r plus Vtheta by r del Vz del theta plus Vz del Vz del z, this is equal to Mu times 1 by r del del r of r del Vz del r plus 1 by r square del 2 Vz del theta square plus del 2 Vz del z square minus del p del z plus rho gr, right.

So this is these are rather all three co-ordinates or rather a three components of the co-ordinate system R, theta, z and why we recapitulated, because it was the properly seen in the previous class last slide, again I say that this was due to the mismatch of the color with the background so that was not visible; however it is rectified and now, once more reason of course is that in this is the basis of the further development further for follow up of these equations.

Now it is one of the best way of knowing the any anything is that until and unless you solve some problems you identify the parts of this equations identify individual components and then you implement it in the problem and solve the problem if you can do that then up it will appear that you have understood this equations I meaning of thus equations meaning of the individual components, how one component is affecting the other? These terms must be know and must be understood. So this is the reason we recapitulated and now we will try to solve some problem so that our understanding becomes more clear, right.

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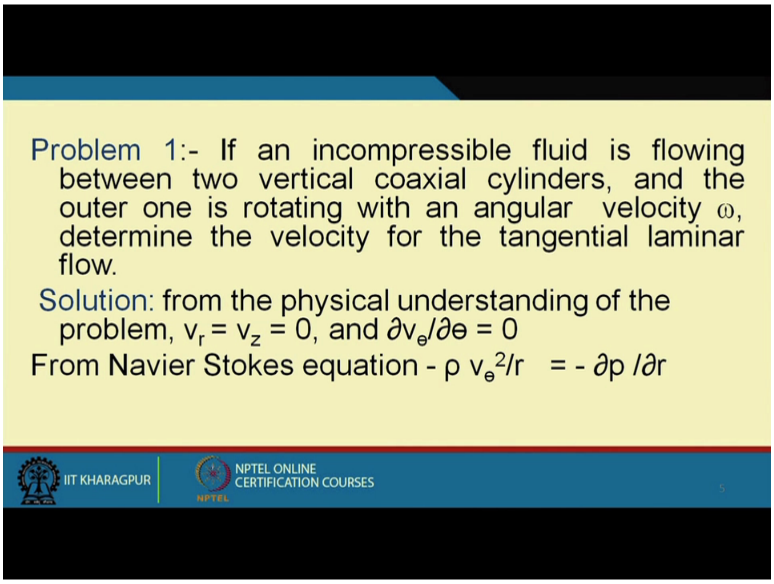

So let us go to the problem, say if we define the problem like this, if an incompressible fluid is flowing between two vertical coaxial cylinders, and the outer one is rotating with an angular velocity of omega then determine the velocity for the tangential laminar flow. I repeat if an incompressible fluid, so here we are taking the fluid to be incompressible is flowing between two vertical coaxial cylinders right, so we have to cylinders, so one cylinder like this and the other cylinder like this, these two cylinders are there and the outer one is rotating with an angular velocity of omega, so if this is the outer one, so this is the inner one, so inner value is fix, outer value is rotating with and angular velocity of omega then determine the velocity for the tangential laminar flow, right.

Now given this problem here we assume one thing that the flow is steady there is no unsteady component then, because we as we said that this is the outer one and this is the inner one, inner one is fixed outer one is rotating with an angular velocity of omega if that is there then the fluid inside between this two that is in the (())(8:29) space how the velocity profile what is the momentum distribution that if we find out then we will be able to understand an use the Navier-stokes equations properly, right. So for this, what we need to know first, what si the problem theoretically given and how it looks like on paper.

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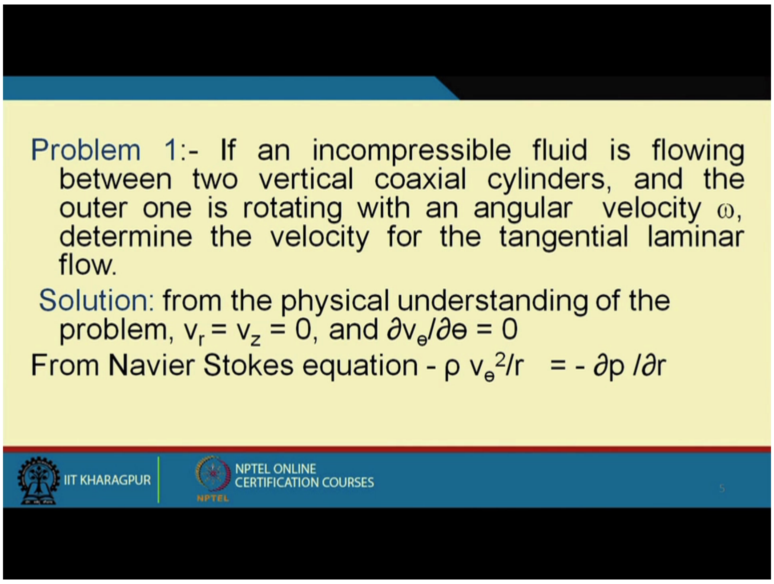

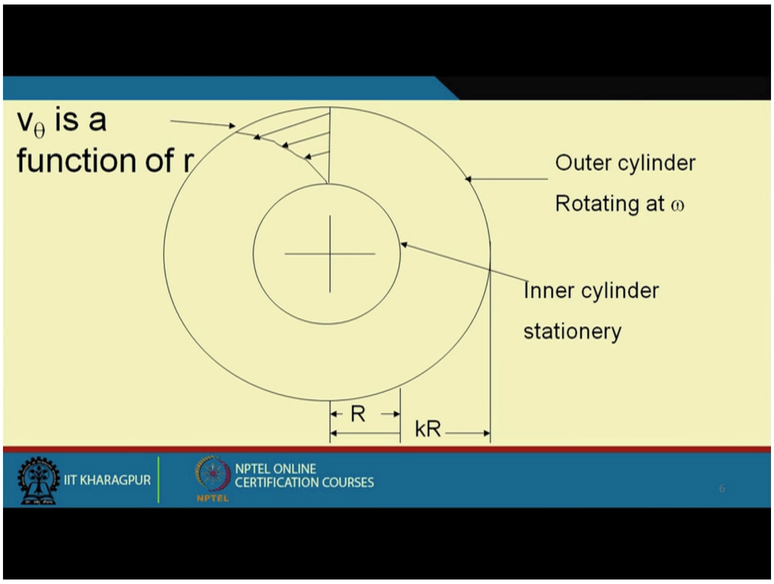

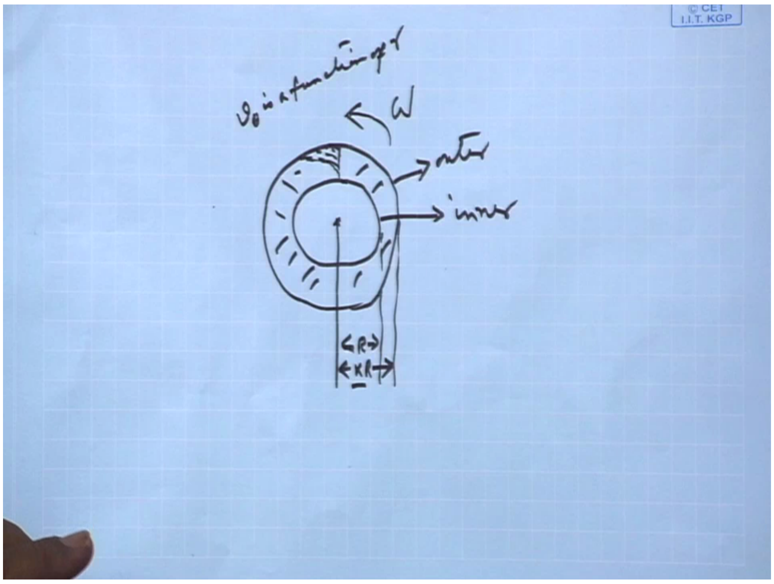

Now if you look at this, what we are given that we have two cylinders, this is the outer cylinder and this is the inner cylinder, right. So outer cylinder and inner cylinder, so if this is the inner cylinder and if this is the outer cylinder, right and if this is the center then we have the radii as this one is R and this one is some multiple of R say kR., right. So if this is true then of course this is the outer cylinder and this is the inner cylinder, right and we said that it will have an angular velocity of omega, right.

So if we look at the velocity component we will look into this looks like this, right where this Vtheta is function of r, right. So this and inner cylinder has an has as radius of R and the outer cylinder has the radius of a multiple of say kR. What that k is unknown; k is a constant which we do not know. So it can be anything twice of R, half of R, 0.3 of R, 1.8 of R whatever value you can assign too, right. So it does not matter, so we have to find out this. So this problem if again I am repeating that this is the outer cylinder, this is the inner cylinder and the fluid inside is air, right and this what we said, this inner cylinder is fixed and out outer cylinder suddenly started rotating with an angular momentum of rho omega. So at steady state, what is the velocity distribution and what is the momentum profile that if we can find out or say shear stress component that if we can find out then we will be able to understand how the Navier-stokes equation can be utilized to solve this kind of problem, right.

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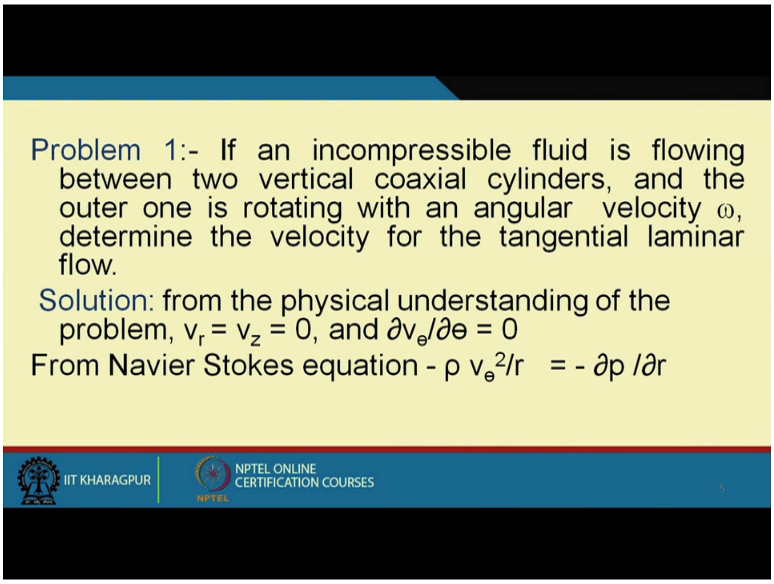

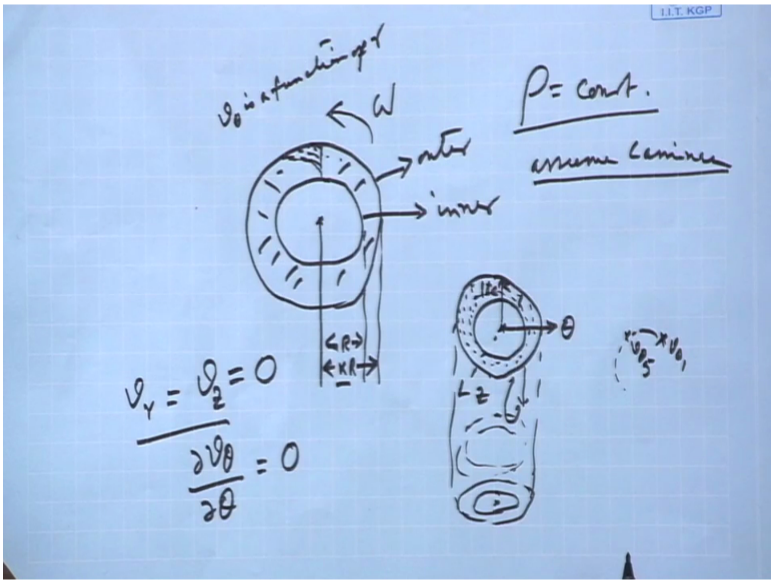

So if we look at whatever is given, now from this problem what we understand that number 1, it is incompressible fluid, so Rho is constant, right. It is incompressible fluid Rho is constant if Rho is constant then many factors will go out, number 1. Number 2, we said that there is this flow is of course we assume to be laminar, right. Why you are assuming it to be laminar? If we again look into this, this is there and this is the outer. So the layers which are like this, right layers which are like this, these are the liquid layers when they are moving like this, this interaction between the layers is not there, so that is why we assume it to be laminar that if we assume the flow to be laminar we assume that the density of the fluid is constant then if that be true then there is no interaction between the layers. So when there is no interaction between the layers that means one layer with the other layer, they are not having any mutual interaction, right. They are just moving to (())(13:32) freely and this we have depicted like this, right okay.

Now if that be true from this condition we can say that Vr is equals to Vz that means there is no velocity in the R-component, right. In the R direction have there been any velocity then it would have mix like this, right which it is not because we assume that there is a stream line like this as it will can right. So these stream lines are (())(14:19) there, there is no R component.

Similarly and if this is the R the other one is theta and the third one is z, right. Similarly this is one and you in just extend it like this, okay if this is the z component, right. So this is the z component, so there is no no vertical, there is no horizontal that flow since R is the or R component of the velocity is not there. Similarly z component of the velocity is also not there that is there is no mixing within the fluid, right. So that is what we know it to be laminar. So like this then another like this. So these kinds of layers are there, so that is one layer is not mixing with the other. So that is why we can assume we can say the Vr is equals to Vz and that is equals to zero and also here, one is must say that this seems this is moving like this. Now since this is moving like this, this one is not going and mixing with that one, right. So this there is no velocity component in the theta direction also. This is the theta direction.

So there is no velocity component in the theta direction also, so in that case we can say del Vtheta del theta is equals to zero, because this theta component or Vtheta say 1 and say this Vtheta of say 5 is not they are one is over the other, there is not mixing of this Vtheta also, right Vtheta 1 and Vtheta 5 we can whatever be the value remaining (())(016:49) the same and there is no change in the theta component in this direction, right. So that is why we can say del Vtheta del theta is zero if this is true then we if we remember that the Navier-stokes equation which we had originally given, right.

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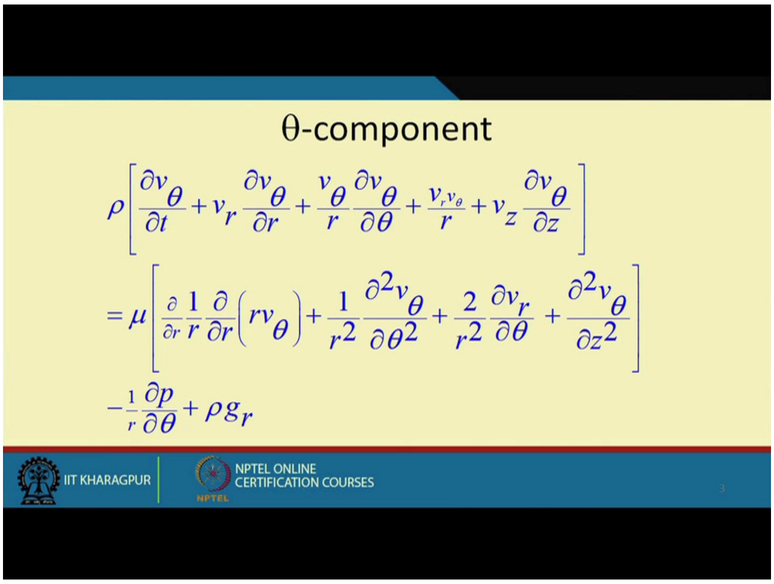

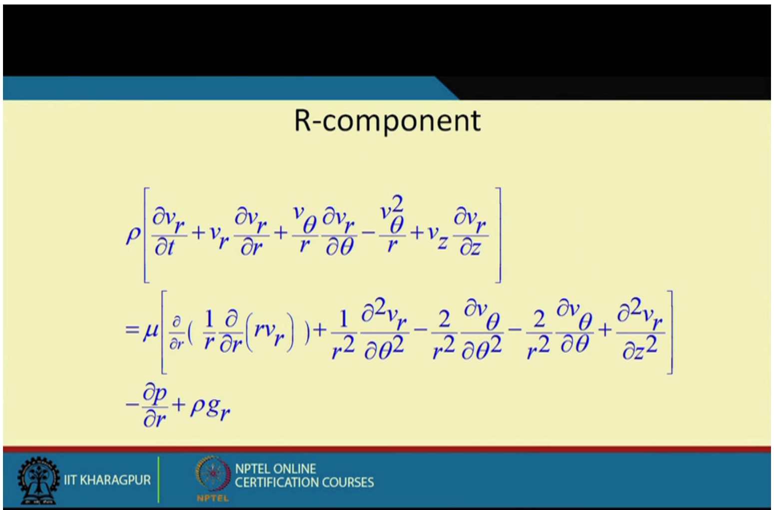

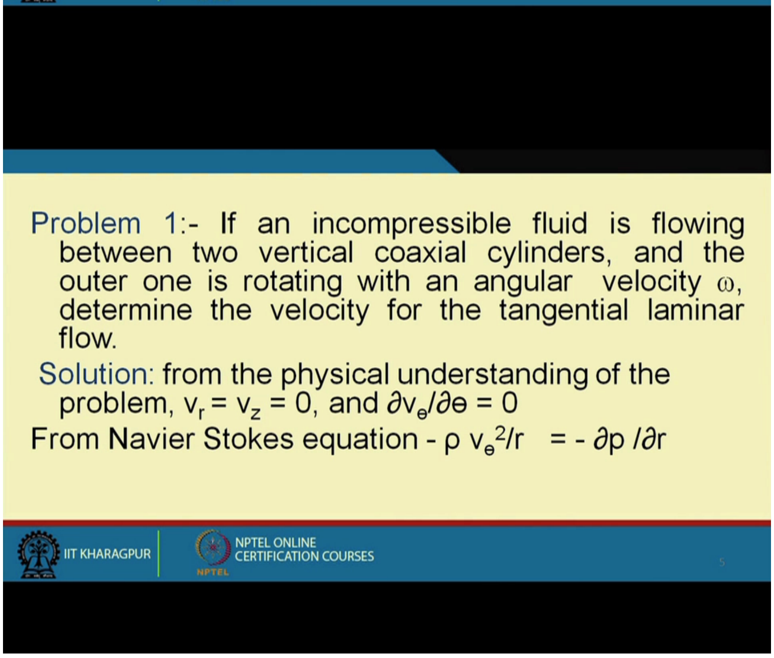

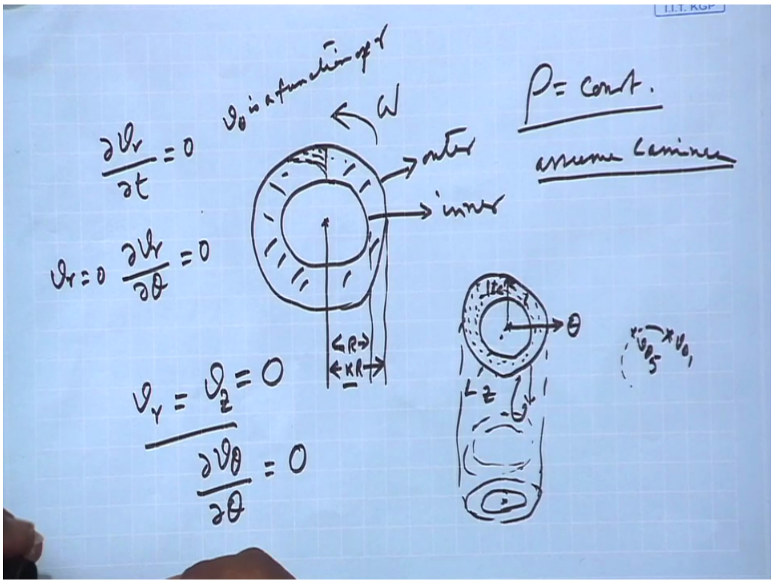

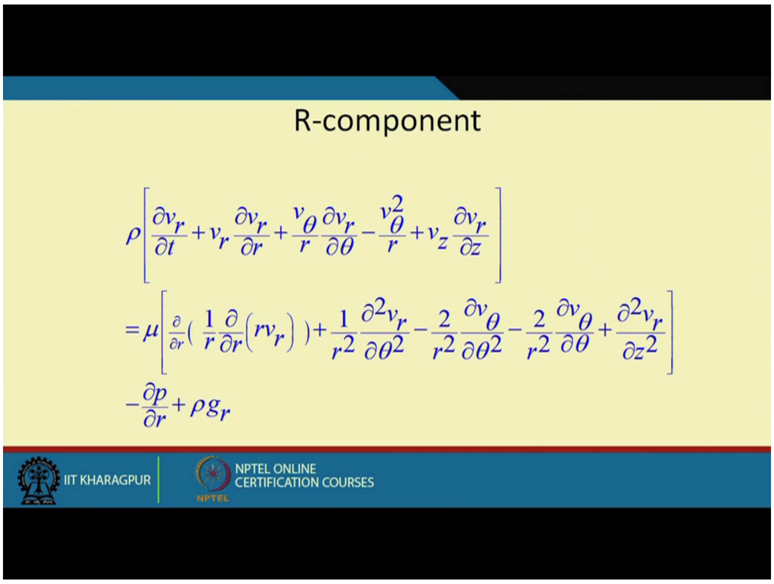

So this is the R component and in R component Vr is zero, first Rho constant and also it is under steady state, so del Vr del t this is also zero, del Vr del t is zero Rho is constant, right. So we can say that this component del Vr del t is zero, Vr is zero. So this component goes out, right del Vr del theta, right del Vr del theta that is change in Vr with theta. So that can be there, so that is why we can say Rho the V zero square by r del Vr del theta del Vr del theta if there been any Vr component also with the theta, right. With (())(18:21) the theta direction change in theta with Vr that is not also there. So we can say since Vr is zero, so del Vr del theta is also zero Vr is zero, right. So we can say this component also is not there minus Vtheta square by r. So it remains, so it is Rho Vtheta square by r with negative. Now Vz is also zero, so this component remains zero then, this is equals to Mu del del r of 1 by r, since Vr is zero. So this again is zero del 2 Vr del theta square, since Vr is zero then del 2 Vtheta del del 2 or Vr del theta square that remains zero. Then since del Vtheta del theta that also is zero. So this term also is not there then this term del Vtheta del theta is also not there and since Vr is zero, so del 2 Vr del z square is zero, but minus del p del r plus rho gr is there.

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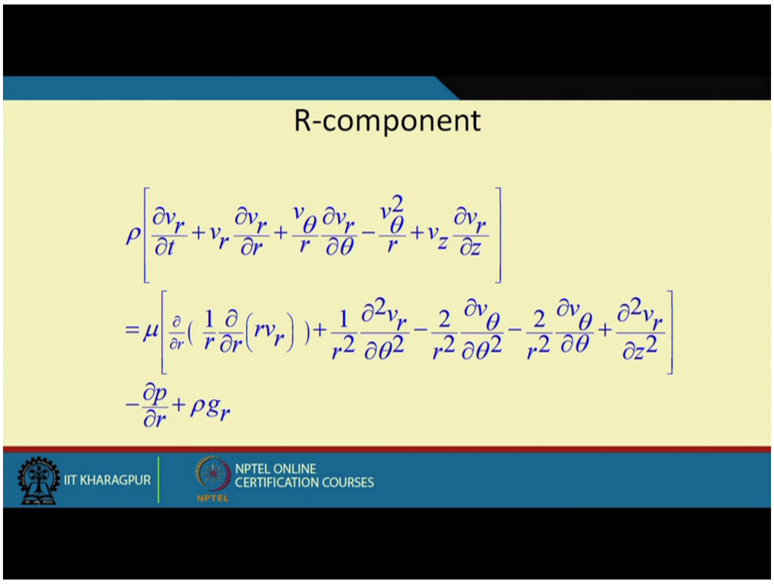

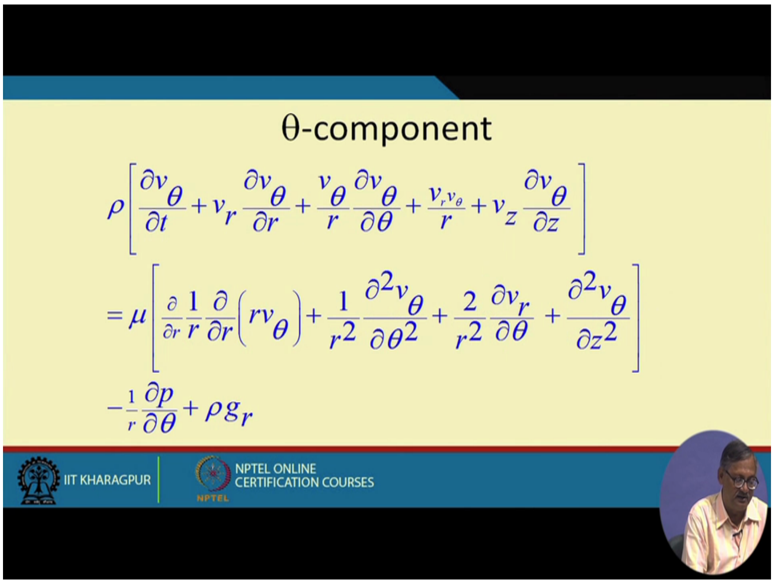

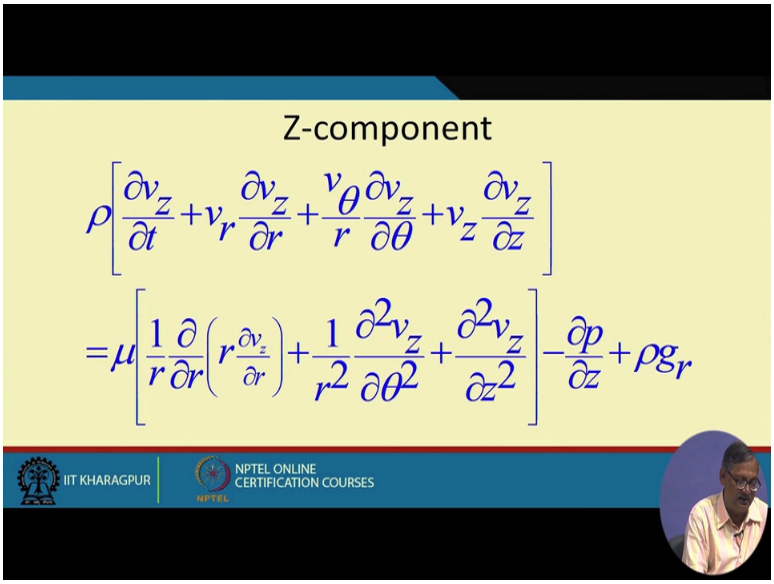

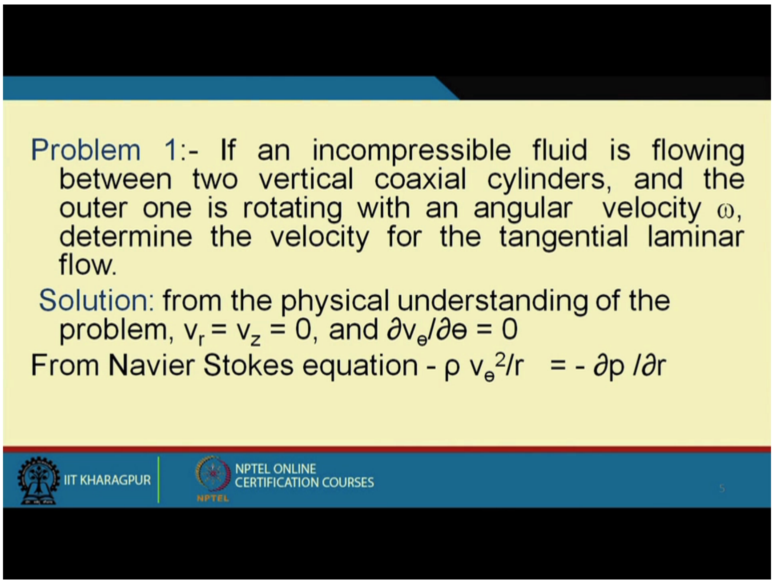

Similarly this is if that is true then we can write that for the first equation minus Rho Vtheta square by r that is equals to minus del p del r. So here one thing is the Rho gr, right rho gr that is the gravitational value gr since r is there we assume that gr is negligible and gr component is not there, right. So we than get that minus rho right into Vtheta square by r. This is equals to minus del p del r, right Rho gr gr is assume to be negligible and that can be neglected.

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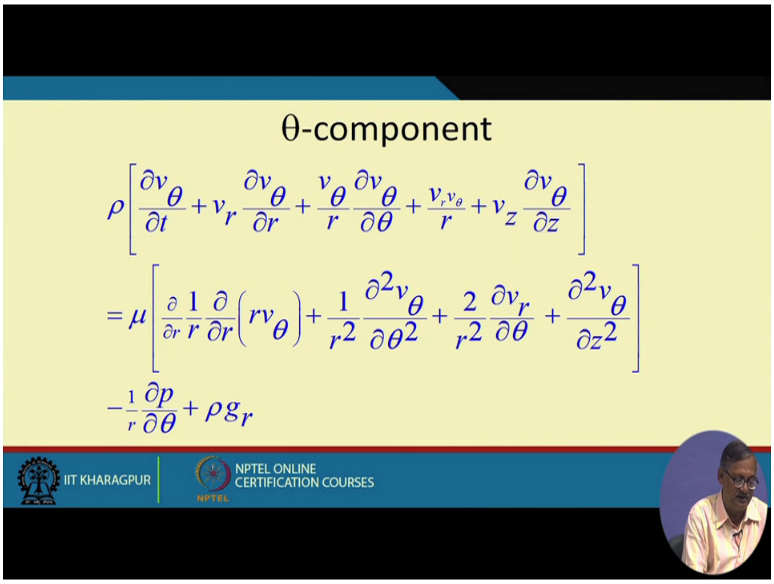

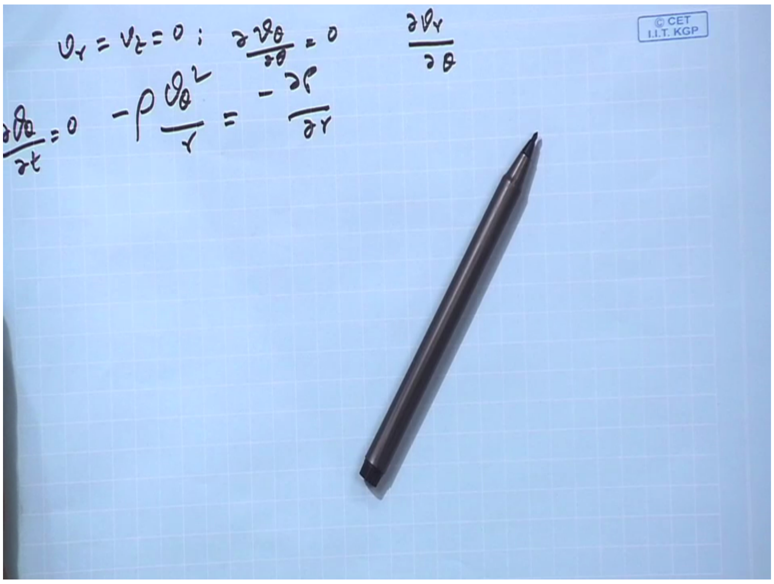

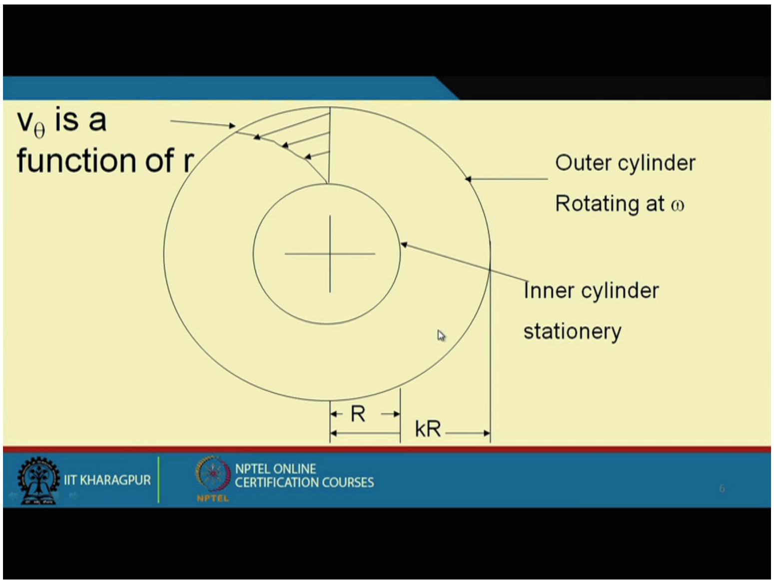

So if this is r component, similarly for theta component we can say stay on the same way since Rho is constant and since it is steady state. So we can say that del Vtheta del t is equals to zero, right we also said Vr is equals to Vz is equal to zero, this was our initial del Vtheta del theta was also equals to zero, right. So if that be true then we can write del Vtheta del r, right Vr into del Vtheta del r that is there. So Vtheta by r del Vtheta del theta that goes out then Vr Vtheta v over r since Vr is zero, so that it goes out. So this goes out Vz is zero this goes out right Vr is zero. So this also goes out but Vtheta by r del Vtheta by del theta. So this goes zero, so this is zero this is zero this is zero this is zero and this is also zero.

So left side is zero, right side is Mu del del r of 1 by r del del r of r Vtheta, right which (()) (22:21) the function of r. So this results, so one by r square del 2 Vtheta del theta square. So since del Vtheta del theta is zero. So del 2 Vtheta del theta square is also zero. So this term goes out. The third one del Vr del theta right, So 2 by r square del Vr del theta. So this for this is Vr that is R component of the velocity Vr Vtheta del Vtheta del Vr del theta, right. So this also goes out, because since Vr is only function of r so 2 by r square del Vr del theta okay, now del Vr del theta this can be there, right. So this can be there.

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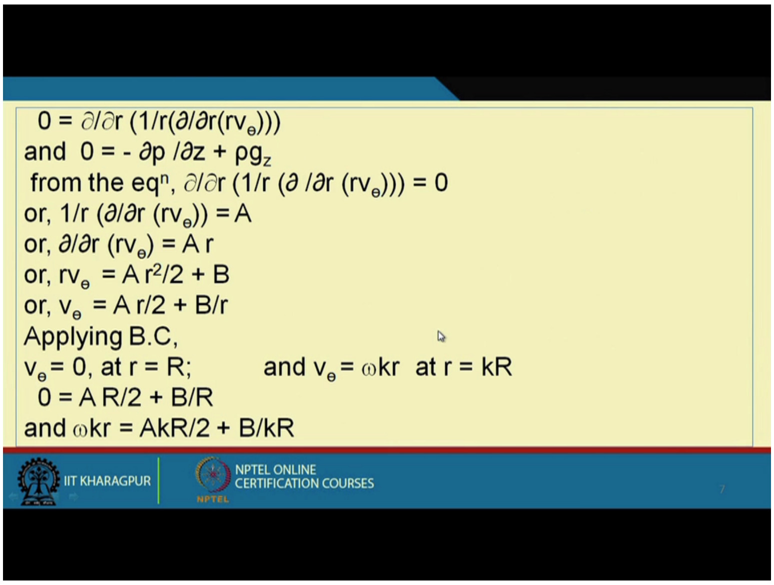

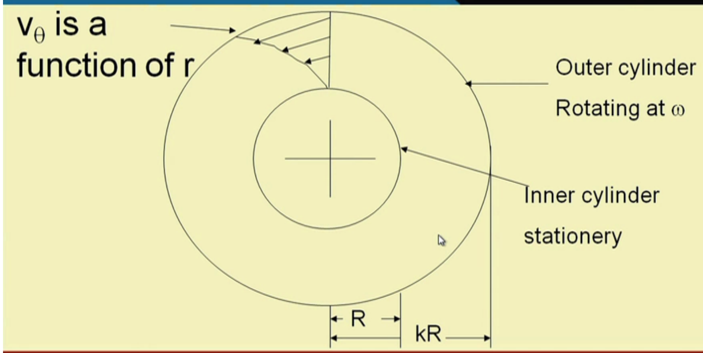

Problem 1:- If an incompressible fluid is flowing between two vertical coaxial cylinders, and the outer one is rotating with an angular velocity ω, determine the velocity for the tangential laminar flow.

Solution: from the physical understanding of the problem, $v_r = v_z = 0$, and $\partial v_\theta / \partial \theta = 0$

From Navier Stokes equation $- \rho\, v_\theta^2 / r = - \partial p / \partial r$

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Z-component

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right]$$

$$= \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_r$$

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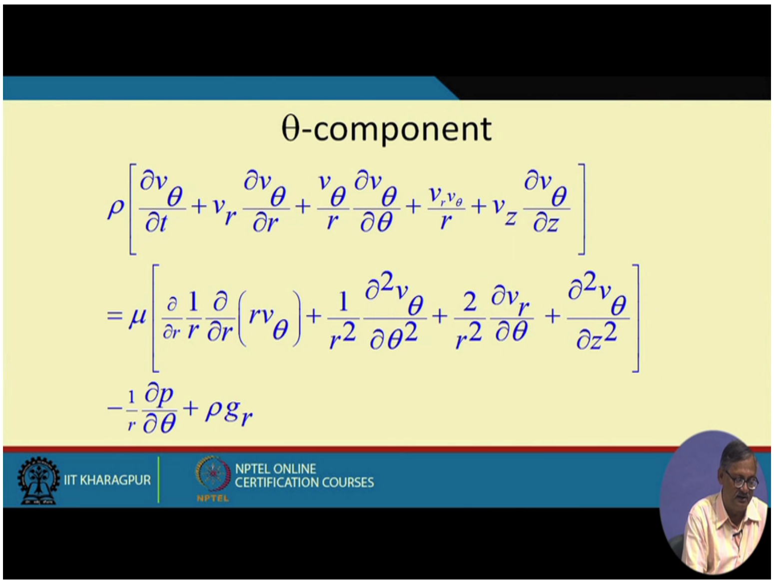

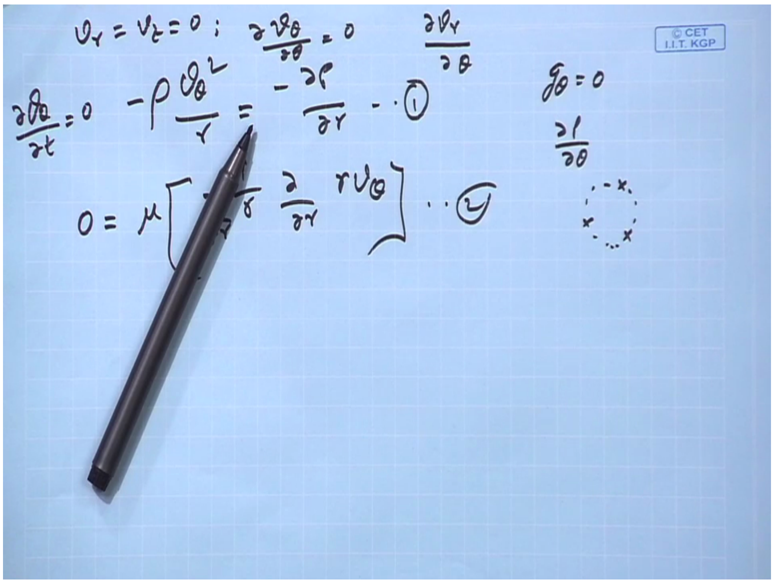

So second one this del Vtheta del theta is there. So theta component Vr 2 by r square r is there del Vr, Vr is zero, right. So del Vr is zero, so 2 by r square to del Vr del theta is also out and the z del 2 Vtheta del z square, since it is the double derivative and there is no Vtheta Vz component. So that is also zero, so we can write and remaining 1 by r del p theta del theta 1 by r del p del theta plus Rho gr.

Now here you Rho sorry, this will be Rho g theta as I said that there be some (())(24:28) mistakes, so here it should have been Rho g theta, right. Now here also theta g theta we can assume to be zero and del p del theta what does it mean? A physically del p del theta, so pressure with the theta. So this is the theta component, so whatever pressure is here same pressure is there the assume pressure is there. So del p del theta from the physical understanding is also zero. Now so we cannot say that del p del theta is also there. Then we get the from their left hand side is equals to zero, because all these components have been zero. So zero is equals to Mu del del r of r 1 by r del del r of 1 by r del del r of r Vtheta, this is the second equation.

The first equation we got there as to remember (())(25:50) that it was Rho minus Mu minus Mu ya this was Rho Vtheta square, this that is Rho Vtheta square by r is del p del r right, but here that it is zero Mu this is the second and the third component if we look at is Rho is constant Vz is zero. So V and not only Vz zero del t is also not there that is it is not (())(26:28) with type, so del Vz del t is zero Vr is zero.

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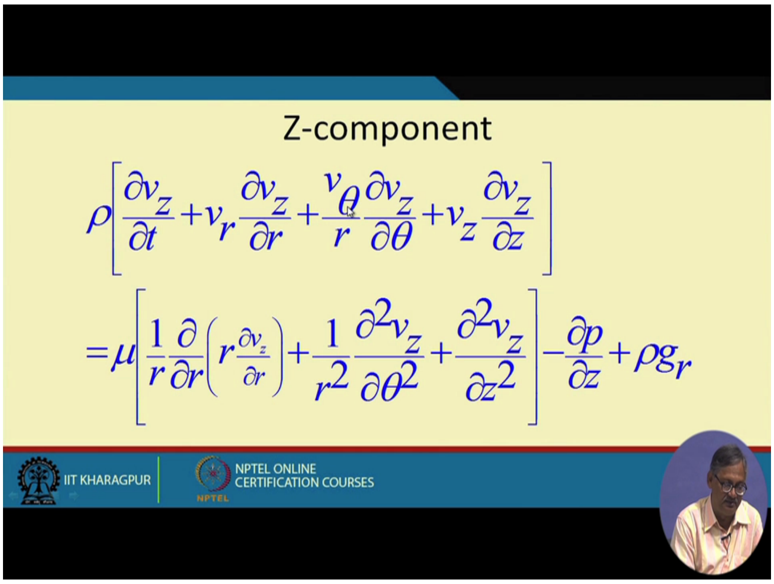

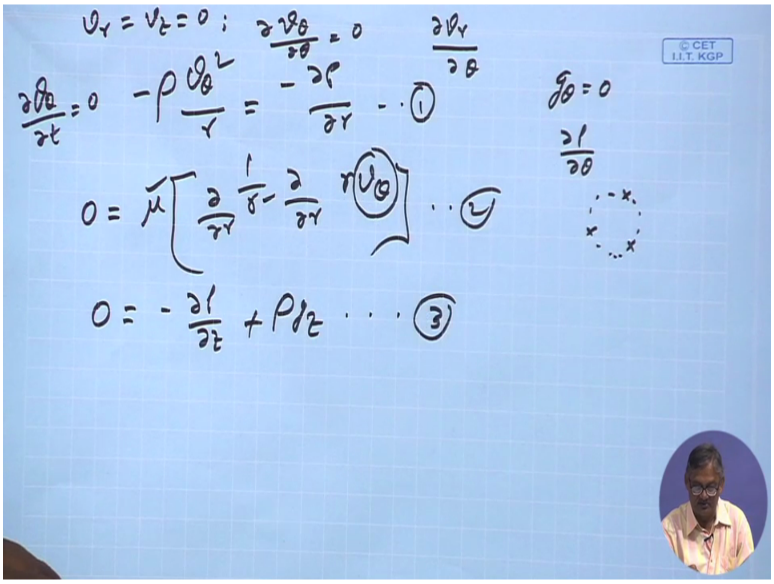

So del Vtheta del Vtheta del Vz del theta this is also zero, because Vz is zero Vz this term is also zero Mu one by r del del r of r Vz del Vz del r, so Vz V zero this term will goes out del 2

Vz del theta square. This was also zero del 2 Vz del z square is zero, but minus del p del z plus Rho gr, this term is again where it should be Rho gz, right. So this term is there, so we can write zero is equals to minus del p del z plus Rho gz, this is equation 3, right. So we need to find out Vtheta, so only one equation is good (())(27:32), because one unknown other Mu is known for a given (())(27:38), right r is known for the given diameter of the cylinder. So if they are known then with unknown is Vtheta.

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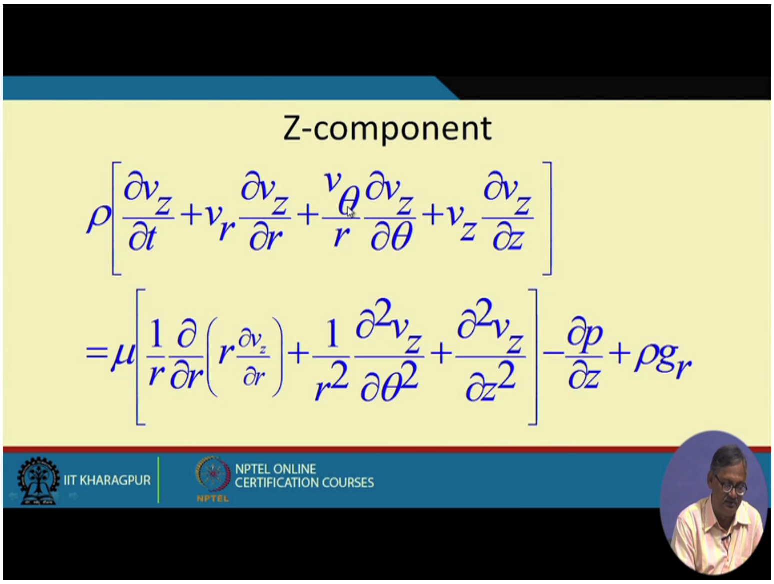

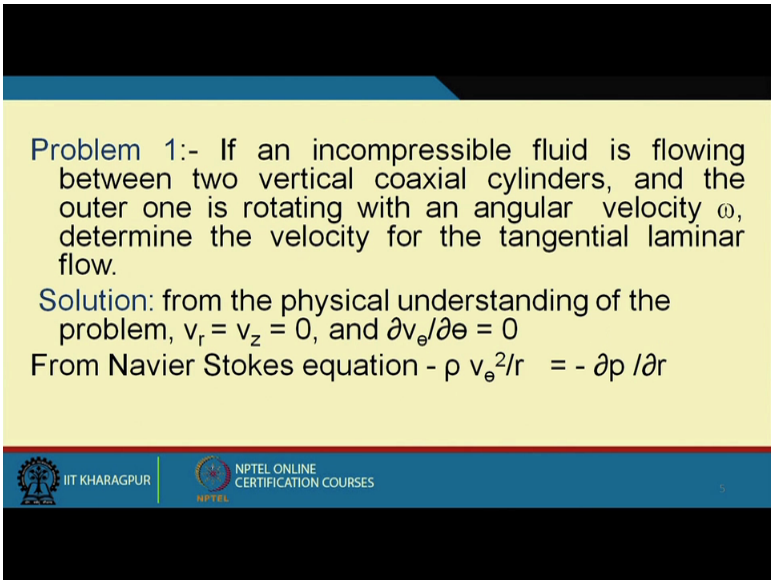

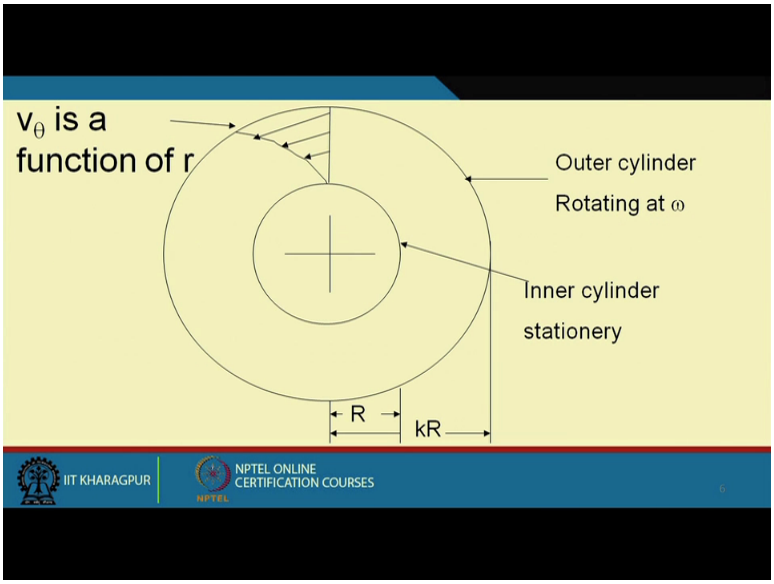

$0 = \partial/\partial r\,(1/r(\partial/\partial r(rv_\theta)))$

and $0 = -\partial p/\partial z + \rho g_z$

from the eq^n, $\partial/\partial r\,(1/r\,(\partial/\partial r\,(rv_\theta))) = 0$

or, $1/r\,(\partial/\partial r\,(rv_\theta)) = A$

or, $\partial/\partial r\,(rv_\theta) = A\,r$

or, $rv_\theta = A\,r^2/2 + B$

or, $v_\theta = A\,r/2 + B/r$

Applying B.C,

$v_\theta = 0$, at $r = R$; and $v_\theta = \omega kr$ at $r = kR$

$0 = A\,R/2 + B/R$

and $\omega kr = AkR/2 + B/kR$

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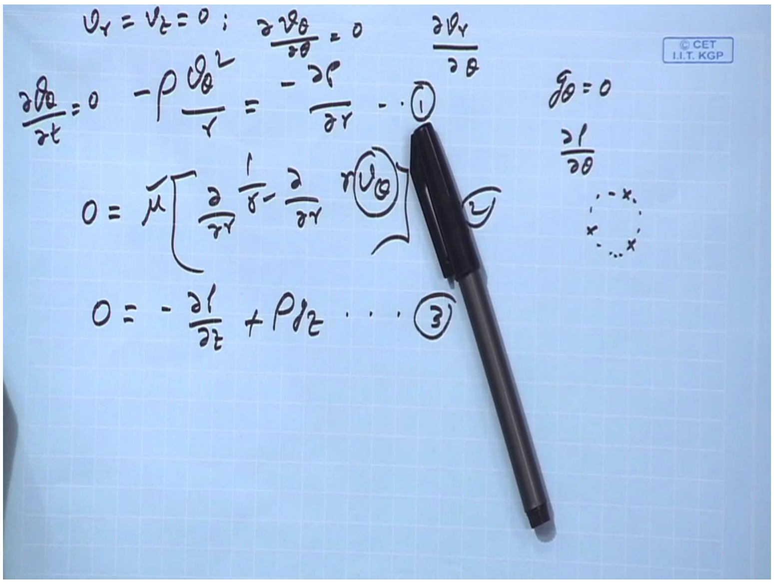

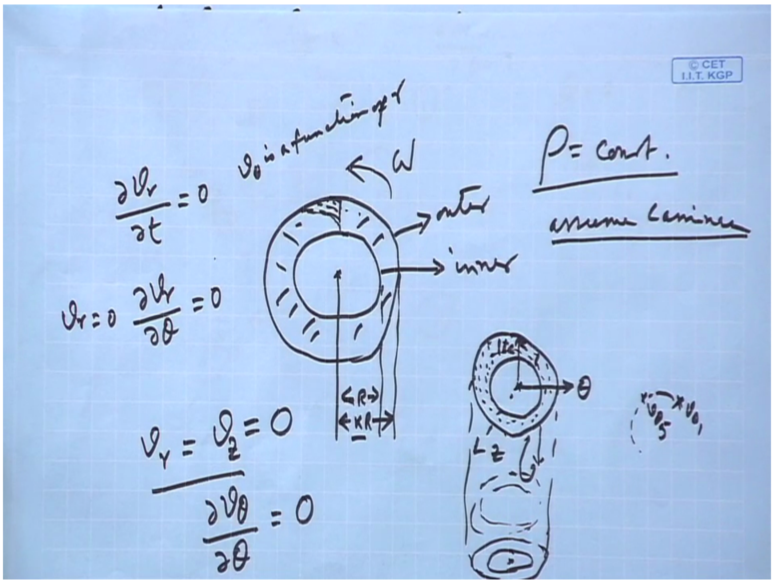

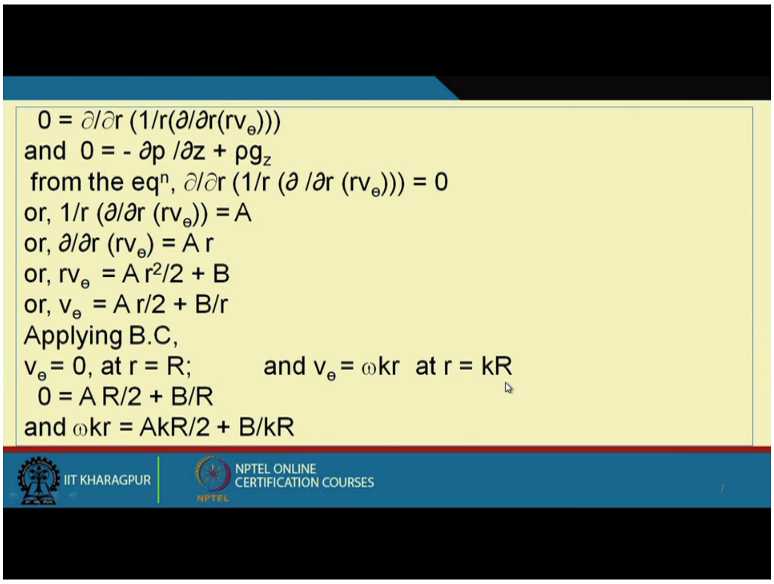

So to find out this Vtheta unknown if we solve this problem then we can say that which start (())(28:01) that second equation that is zero is equal to del del r of 1 by r del del r of r Vtheta, right an second equation which we got that was minus del p del z plus Rho gz this one, right. So this one was the and the third one which we had already shown that minus Rho Vtheta square by r is equals to minus del p del r, right. This out of these three equations let us start with the second equation that del del r of 1 by r del del r of r Vtheta is equals to zero on first integration it gives one by r del del r of r Vtheta that is equals to r integral constant, so A.

So on simplification this can be writurn that del del r of r Vtheta is equals to Ar, right. So on again second integration we can say r Vtheta is equals to A r square by 2 plus B which on simplification can be writurn that Vtheta is equals to A r A by 2 plus Br, right. Now we have two 2 equations and two boundary constants that is integral constant B and C, right. So we have to find out what is the A and B what is the boundary. Boundary is Vtheta is equals to zero at r is equals to r that is what we have been said Vtheta is equals to zero r is equals to r, this at r is equals to r if this is constant. So Vtheta is equals to zero at r is equals to r. so in that case we can write then the second is the outer cylinder is rotating with an angular velocity of omega.

So we can say Vtheta is equals to omega kr at r is equals to KR. So using these two boundaries and solving we can write zero is equals to AR by 2 plus B by R and omega kr is equals to AkR by 2 plus B by kR.

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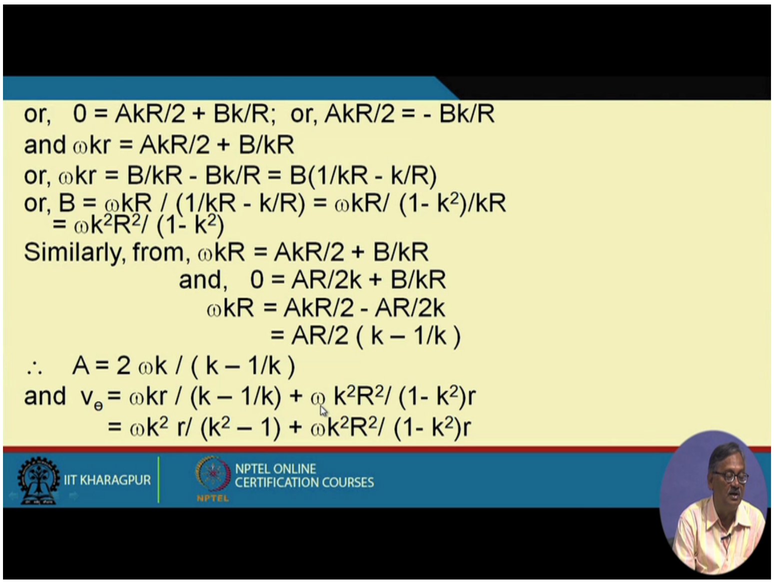

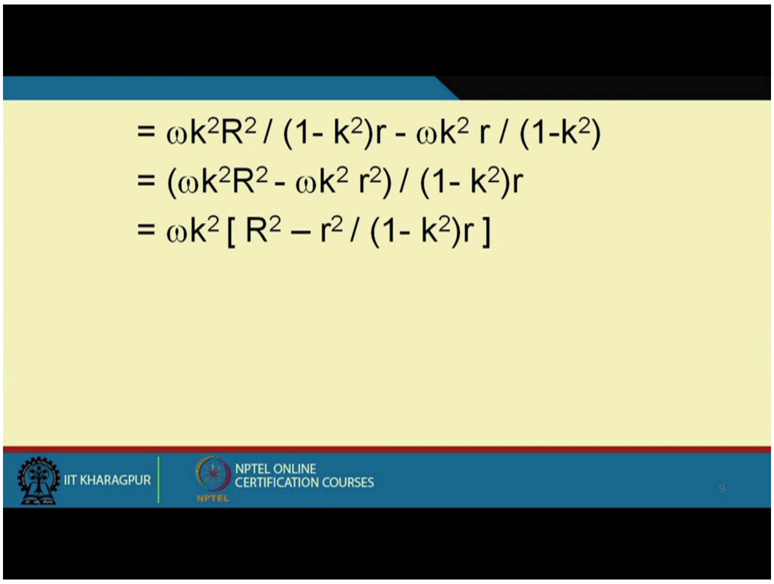

So in that case we have these two equations zero is equals to AkR by 2 plus Bk by R and AkR by 2 is equals to minus Bk by R and omega kR is equals to AkR by 2 plus b by kR. So on simplification and solving we can write omega kR is equals to B by kR minus Bk by R that is equals to B into 1 by kR minus k by R that is equals to in that case we can say B is equals to omega kR by 1 by kR minus k by R, this is equals to omega kR by 1 minus K square by kR that must be is equal to omega k square R square by 1 minus k square. So similarly from that equation omega kR is equals to be kR by two plus B by kR.

So from there we can write zero is equals to AR by 2 plus AR by 2 k plus B by kR and omega kr is equals to AkR by 2 minus AR by 2k that is equals to AR by 2 into k into 1 minus 1 by k. So in that case A becomes 2 omega k over k minus 1 by k. So Vtheta can be writurn as omega kR. by k minus 1 by k plus omega k square R square into 1 minus k square into R that is omega k square R over k square minus 1 plus omega k square R square over 1 minus k square. So this on simplification gives omega k square r square by 1 minus k square into R minus omega k square R by 1 minus k square. So omega k square R square minus omega k square R square over that was capital R, this small r 1 minus k square r so this means on simplification we can write omega k square into R square minus r square over 1 minus k square into r. So Vtheta is equals to the omega k square R square minus r square 1 minus k square r. So we have come down from the problem and understanding of the problem what is the velocity component which is acting in the theta direction, right.

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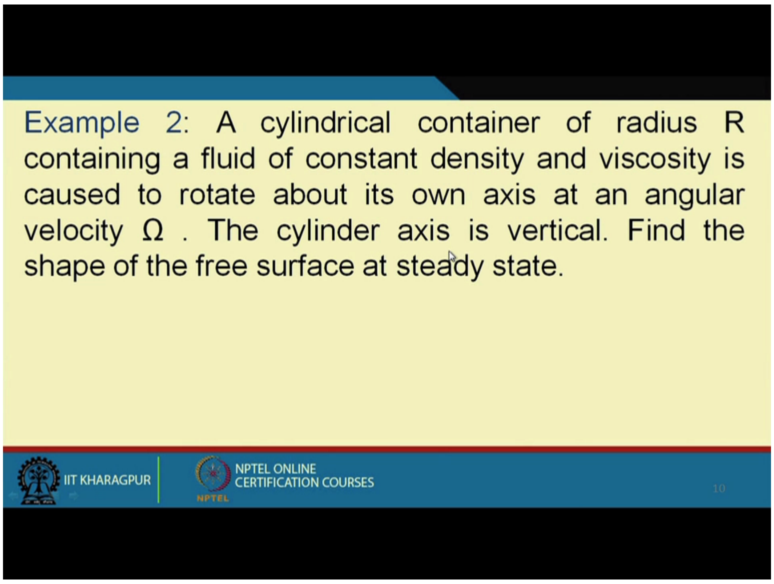

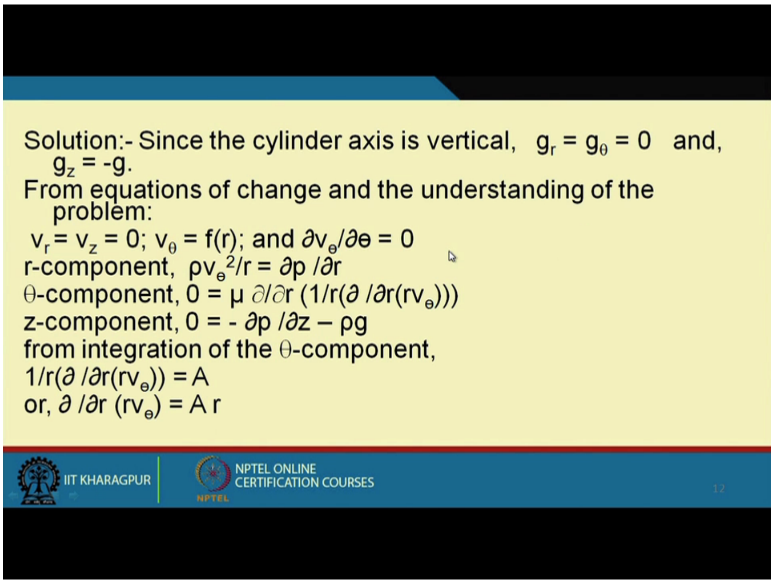

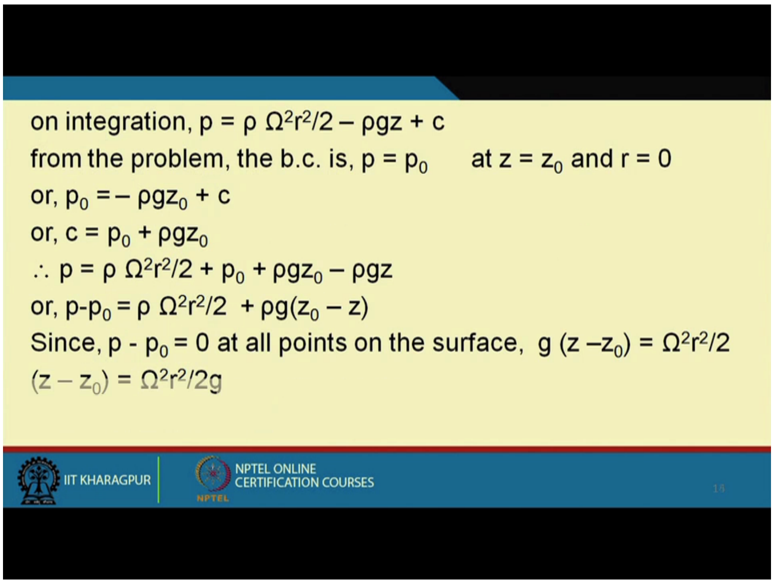

So this way if you solve many problems you try the other one. In this case what we have given that inner one is fixed and the outer one is rotating. Now you try to do that the inner one is rotating and the outer one is fixed. So inner one rotating in that case what will be the velocity component in the theta directions. So when it there is no Vr there is no Vz, because we assumed it to be say laminar (())(33:45), so that is why there is no Vr there is no Vz as an it is steady state.

So del t part is also not there. So implementing all these you can find out how the equation of motions that is Navier stoke can be utilized, obviously we will do some more problem otherwise it really difficult it will be to implement and understand, okay. So that is why we do here for this class, thank you.