

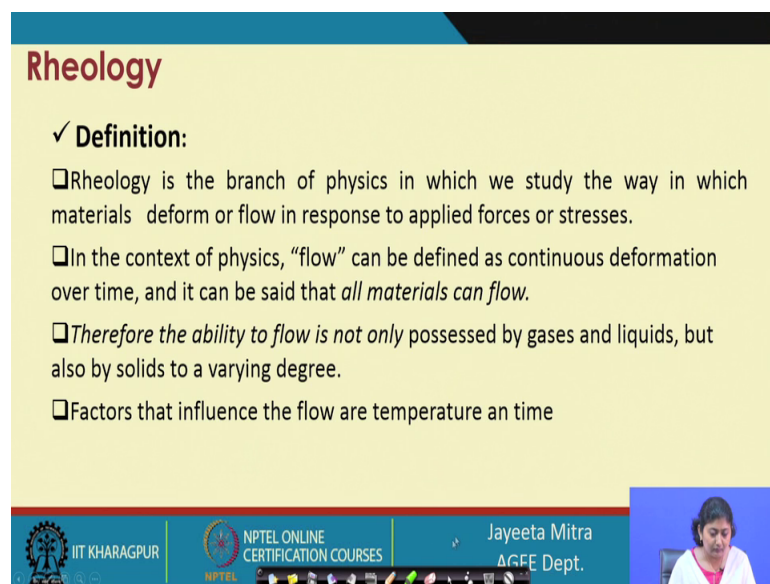
**Fundamentals of Food Process Engineering**  
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**Department of Agricultural and Food Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 02**  
**Food Rheology**

Hello everyone, welcome to NPTEL online certification course on Fundamentals of Food Process Engineering. So, our first lecture started with Food Rheology. Rheology is a very important parameter for you know to identify any material property. So, in today's class will see that what are the various kind of relation we can observe in the food rheology.

So, first let us see what is rheology actually.

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**Rheology**

✓ **Definition:**

- Rheology is the branch of physics in which we study the way in which materials deform or flow in response to applied forces or stresses.
- In the context of physics, "flow" can be defined as continuous deformation over time, and it can be said that *all materials can flow*.
- *Therefore the ability to flow is not only possessed by gases and liquids, but also by solids to a varying degree.*
- Factors that influence the flow are temperature and time

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The definition is rheology is the branch of physics, in which we study the way the materials deform or flow in response to the applied forces or stresses right. So, whenever we do any food processing operation; that means, the food which is either liquid, solid or semi solid that is exposed to certain kind of forces. So, forces again maybe of different types and that force will cause sudden stresses to it, and because of that stress the response of different food is in different way. So, that is the concern of our study.

Now, in the context of physics, flow can be defined as the continuous deformation over time ok. So, flow is the continuous deformation over time and it can we say that all

materials can flow. All materials can flow, but the difference is that the degree of flow will differ in case of different material ok. So, the ability to flow is not only possessed by the gases and liquid because we know obviously these materials flow, but also by the solids, but not with the same degree as it happens in case of the liquid and the gases.

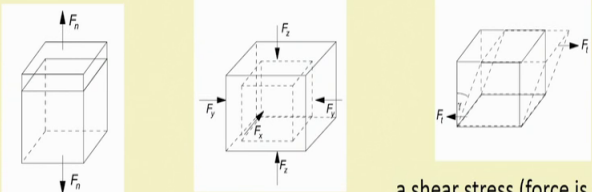
Now, the main factors that influence the flow are temperature and time ok. Because you know temperature is when we increase the temperature most of the cases in case of the liquid, their flow ability increases. That means there is a property that is providing the resistance of the flow that is going to decrease if we increase the temperature. Similarly if the same temperature increase can cause a reverse property change in case of the gases and since the food is mostly mixture of different composition, and different kind of material not only pure liquid there is suspended matter in it dissolved matter in it.

So, generally the behavior of food in terms of rheology if you want to assess, that is become a typically complex phenomena.

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**Elastic Properties:**

✓ Force and deformation behaviour:



The slide contains three diagrams illustrating different types of force and deformation behavior on a solid body (represented as a cube):

- Uniaxial loading:** A cube is shown with two vertical arrows, one pointing up and one pointing down, labeled  $F_n$ . This represents a normal stress.
- Bulk compression:** A cube is shown with four arrows pointing inward from the top, bottom, left, and right faces, labeled  $F_c$ . This represents a volume change in the solid body.
- Shear stress:** A cube is shown with two horizontal arrows on opposite faces, one pointing right and one pointing left, labeled  $F_s$ . This represents a bending or twisting of the body.

“axial loading” in an uniaxial direction (causing a normal stress).

bulk compression loading, results in a volume change in the solid body.

a shear stress (force is acting parallel to the surface on which it is applied). deformation is in the form of a bending or twisting of the body.

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So, first we will see the elastic properties; so elastic property which is basically seen by the solid material ok. So, what is that this is a force and deformation behavior and if we want to you know assess that what are the different kind of forces. The first thing is that the first kind of force that we can analyze is that, when the food or any material is exposed to uniaxial loading ok. That means, in one direction in axial direction either we

are pulling the body from both the side or we are compressing it from you know axial direction.

So, in that case what happened that, there is some elongation we can observe on the body in the axial direction or in the direction of the applied force ok. So, this kind of force causes this is called the normal the force is acting in the normal direction. So, the stress generated is called the normal stress. Then if we look into the second picture where from all the side forces acting on a body ok. From all the 3 direction that is x y and z the force is acting on the body and therefore, causing the decrease in the volume.

So, this kind of phenomena this is called the bulk compression loading, which results in volume change in the solid body ok. So, the first thing first force that is axial force or uniaxial loading that can cause change in one dimension one direction. However, the bulk compression changes the body from all 3 sides all 3 directions.

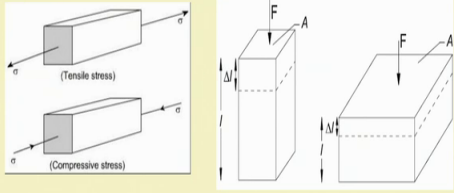
Now, the next one kind of force may act on a body that is called the Shear force ok. So, what is that? Let us say first we can consider a block resting on a on a floor or on a ground; and if we apply a force  $F_t$  parallelly on the top surface ok. So, top surface will try to pull it on the x direction because of this force  $F_t$ , whereas the bottom surface which is fix on the ground will have a reverse directional stress, because this will not try to you know flow with the with the upper layer. So, because of because of inertia this kind of behavior we can observe.

So, in this case what will happen? The body will deform body will deform, but not the similar way it has happened in case of axial loading or bulk compression, but it will you know some kind of bending or twisting will be observed and the body will be deform from its longitudinal position by an angle  $\gamma$ . And that angle  $\gamma$  will be the you know from using that  $\gamma$  we can measure the deformation by the distance, the layer has moved from its equilibrium position divided by the distance between these 2 layer ok. So, that is basically  $\tan \gamma$  right. So, so then these are the different kind of forces that can act on a body and because of these 3 different kinds of deformation we can see.

(Refer Slide Time: 07:40)

### UNIAXIAL STRESS

✓ Tensile and compressive stress:



• Opposite directions of uniaxial compression or tension would simply be assigned as positive (extension) or negative (compression) directions along the same uniaxial path.

• Force and deformation behaviour of a specimen

✓ Material rheological property?

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Now the thing is as we have mentioned that when we apply the uniaxial loading that time it may be you know when we are pulling the body apart it is tensile force. And when we you know compress the body from both the side it is compressive force and we know that force divided by the area on which it is acting, give us the concept of the stress. So, we can get the tensile stress or compressive stress ok.

Now, why this concept of stress is more important than the force and deformation behavior? Because if we look into these 2 diagram let us say that these are of the same material ok. So, this is also material A and this is also material A ok; however, their shape is different ok.

So, in the first one we can observe that length is high compared to the width, height is more you can considered in that way here the height is less and width is high. On both the material we are applying the same axial force  $F$ . So, since the area is difference and the geometry of the figure are different. So, both the cases of deformation we can observe in terms of  $\Delta l$  that may vary. I mean they that will definitely vary because these 2 shapes are different ok.

So, the  $\Delta l$  here is much higher compared to this  $\Delta l$ ; that means, if you analyze the force and deformation that cannot give you the property which is a material property, because this  $\Delta l$  will change from different shape of the material. So, we want something that will be of the material property that does not change when the geometry

changes right. So, because of that to understand the material rheological property force and deformation will be changed to some other parameter that is stress and strain ok.

(Refer Slide Time: 09:57)

**Elastic Properties:**

- ✓ **Stress –strain relation:**
- ✓ The quotient of force over area is called stress ( $\text{N.m}^{-2}$ ):  $\sigma = \frac{F}{A}$
- ✓ The relative change in length is called strain (dimensionless):  $\epsilon = \frac{\Delta l}{l}$
- ✓ **stress–strain diagram :**

When a material exhibits a linear relationship between increasing stress and strain, it is said to be a classic elastic material that follows Hooke's law.

$\sigma = E \cdot \epsilon$  *E is a material property named the modulus of elasticity,*

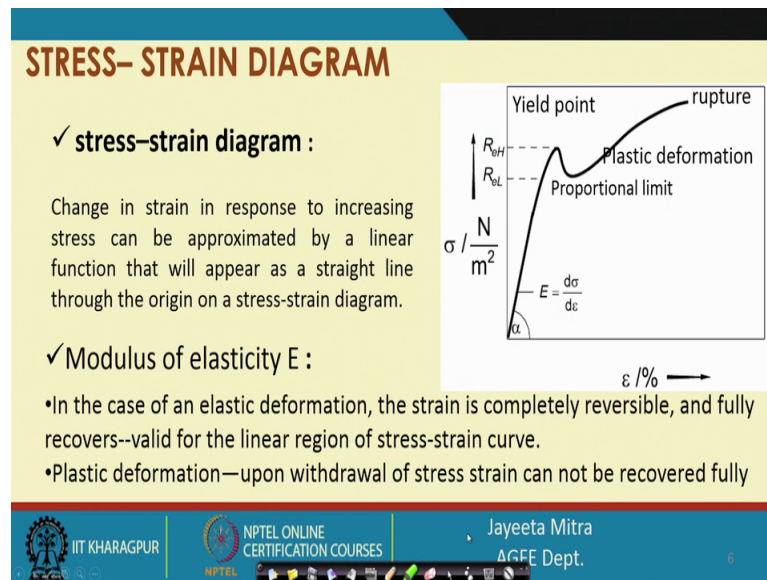
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So, the elastic property of a material can be best understood by the relation between the stress and strain, and the coefficient of the force over area is call the stress expressed in Newton per meter square. So, sigma that is equal to F by A whereas, the relative change in length is called the strain, which is the dimensionless parameters of epsilon that is equal to delta l by l; so relative change of the length.

So, when a material exhibit a linear relationship between increasing stress and strain, it is said to be a classic elastic material that follows Hooke's law ok. So, Hooke's law if we can plot the stress strain diagram that will do now. So, that is expressed by the Hooke's law that is stress; stress is proportional to strain and the proportionality constant is E which is modulus of elasticity.

Now, this modulus of elasticity is the material parameter ok. So, whatever may be the shape and size of the same material, we can get the modulus of elasticity fix ok. So, that is the advantage of using the stress strain analysis of any material.

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So, the next stress we will now look into the general stress strain diagram. So, let us see that here we have express the strain in percentage in the x direction, and stress in Newton per meter square in the y direction, and we try to increase strain and we measure the record stress continuously. So, along the graph if you can follow initially we are getting a straight line portion ok. So, in this straight line zone, the stress is proportional to strain ok. So, the material obeys the Hooke's law material obeys Hooke's law here.

Now, after this portion there the curve starts deviating from linearity right. And till this again we can we can consider the proportional limit, because till then we can considered more or less straight line behavior beyond this point, it is totally starts changing the slope is starts changing ok. So, in between these zone in between these zone if we want to withdraw the stress at any moment. So, the material will regain it initial shape ok.

Now, beyond that if further we increase the stress what will happen that, a point will come where without increasing the shear, shear without increasing the stress strain will start increasing ok. That means, the material will yield at this point, and beyond which even with less or no change of the stress the material starts flowing.

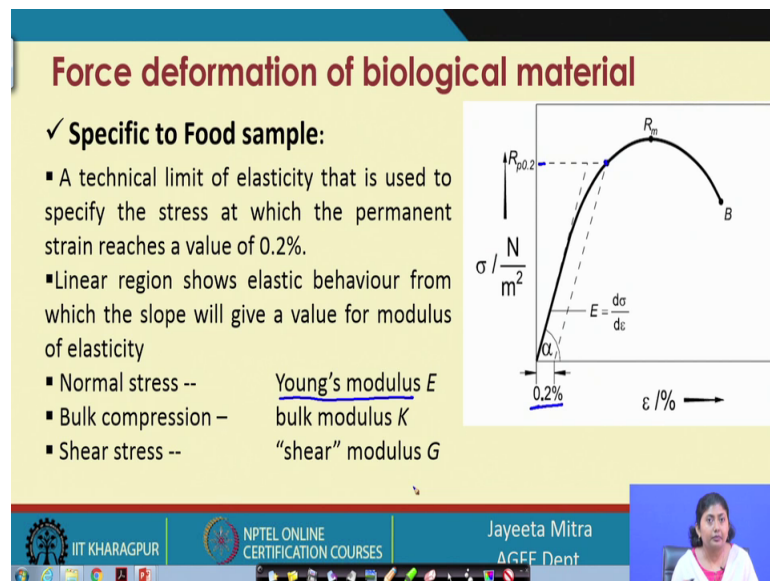
So, beyond this if we want to withdraw the stress, we can it get back to the initial condition. So, beyond that zone the plastic deformation will be there, again the stress will increase up to a maximum point that we normally defined as sigma m and after that the

rupture will happen; that means, the material have when fully exhaust that cannot withstand the stress to a higher extent. So, it will break ok.

So, this is a general nature of a material if we if you draw the stress strain diagram. So, here as I mentioned that change in strain in response to increasing stress can be approximated by the linear function that will appear as a straight line through the origin on a stress strain diagram. So, modulus of elasticity is the slope of the straight portion that is equal to  $D \sigma$  by  $D \epsilon$  that is the differential stress divided by differential strain ok

Now, the condition is that we have understood that elastic and plastic deformation. So, these general phenomena we may not observe in case of the biological material.

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What happened in that case? For example, any food material the sharp change from the you know elastic to plastic state is not visible in case of the biological material. So, for that what we do is that if we see that how the plot is look like here the strain in percent we have plotted and stress is there. So, we can see that a gradual dome has been generated there is no transition point, there is no fixed yield point ok.

So, initially there was a linear region and then it starts deviating and then we are reaching a maximum stress and then rupture happened at point B. So, what we do is we fix at

technical limit of elasticity that is used to specify the stress at which the permanent strain reaches a value of 0.2 percent this reaches a value of 0.2 percent of the strain ok.

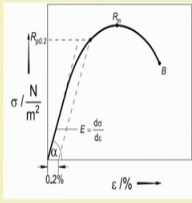
So, that point we consider as yield point ok. So, this linear region shows the elastic behavior from which the slope will give the value of the modulus of elasticity. And here at this point we will consider the yield point based on the 0.2 percent strain if it generate in the in the body. So, the similar fashion we are calculating the modulus of elasticity or this is also called the Young's modulus. We can also do the same thing for the bulk compression and shear stress because we have know by now there are forces where we can apply the axial loading or we can apply the bulk compression or shear force. So, we will eventually come on those studies.

(Refer Slide Time: 17:20)


## Young's Modulus


✓ **Young's Modulus:**  
Young's modulus gives an indication of easily a material can be stretched or contracted, and is sometimes referred to as "stiffness."

$$E = \frac{d\sigma}{d\epsilon} = \tan \alpha$$




$$\frac{F}{A} = E \cdot \frac{\Delta l}{l} \quad \text{or} \quad F = \frac{E \cdot A \cdot \Delta l}{l} \quad \text{or} \quad F = D \cdot \Delta l \quad \text{D is Hooke's constant in } \text{N}\cdot\text{m}^{-1}$$


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So, Young's modulus as we have seen for any biological material if you plot the stress and strain curve like this. So, we will get a straight lined portion. So, from the slope of the straight line we can get the idea of Young's modulus. So, Young's modulus gives an indication of how easily a material can be stretched or contracted right and is sometime referred to as stiffness of the material. So, it is a property that is defined as stiffness of the material right.

So, here if we measure the angle that straight line has been drawn with respect to the horizontal so, that will be tan alpha. Now stress is F by A that is force per unit area that is equal to E which is Young's modulus into delta l by l that is the relative change in the



length or we can write  $F$  equal to  $E$  into  $A$  into  $\Delta l$  by  $l$  and  $F$  is also written as  $D$  into  $\Delta l$  where  $D$  means Young's modulus into area  $a$  divided by length  $l$  initial length  $l$ . So,  $D$  is also called the Hooke's constant  $ok$ .

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Terms for interpretation of $\sigma - \epsilon$ diagrams 			
Description	Characteristic feature		
		$\sigma$	$\epsilon$
Elastic range	$\frac{d\sigma}{d\epsilon} = \text{const}$		strain is completely reversible
Non-elastic range	$\frac{d\sigma}{d\epsilon} \neq \text{const}$		strain is not completely reversible
	$\frac{d\sigma}{d\epsilon} = 0$	yield stress (flow starts)	strain continues without increasing stress
Maximum of curve	$\sigma = \text{max}$	Maximum strength of material	strain at maximum stress
Technical limit plasticity	$\epsilon = 0.2\%$	stress for 0.2% strain permanent strain	strain goes to 0.2% permanent
Rupture (breakage)	$\sigma$ shows sudden drop	stress at rupture	strain at the point of rupture

So, next in a very concise manner we will just see that the different terms for interpretation of stress strain diagram  $ok$ . So, what will happen is that if we consider if we first plot the plot the diagram like you know the way we have done that, if you if you remember. So, first a straight line first a straight line will be there then it start decreasing  $ok$ .

So, the elastic range; that means, the range where we are getting  $D$  sigma by  $D$  epsilon that is the differential change in the stress was with the ratio of the strain that is constant, that signify that strain is completely reversible  $ok$ . So, this is the status of the strain in case of the elastic range and non elastic range; that means,  $D$  sigma by  $D$  epsilon will not be constant  $ok$ .

So, in this zone strain is not completely reversible then  $D$  sigma by  $D$  epsilon that is equal to 0 indicate the yield stress. So, the point at which it happened the slope will be 0 so, that that stress will be yield stress. So, from there the flow starts and the strain will be strain continues without increasing stress. So, from this point onwards the strain continues without having change in the stress. Now sigma max when we when we reach the peak value where we can observe the highest stress. So, that is the maximum strength

of the material and the corresponding strain we can measure that will be the strain at maximum stress.

This is not the maximum strain which is strain at maximum stress and technical limit of plasticity because plasticity is actually that the term is used when the material needs to flow, some materials are there where as instantly the force applied on them and they start flowing those are called the viscous material, but certain material are there where they need an initial stress to flow ok. So, beyond that initial stress only they will flow below that stress they does not flow. So, the technical limit of plasticity in case of the biological material we considered as epsilon that is equal to 0.2 percent; if the strain reaches this value. So, this deformation we can consider that yield point has been occurred and the flow starts.

So, sigma will be the stresses for 0.2 percent strain, which is considered as permanent strain and strain goes 0.2 percent of the permanent strain that we can measure now rupture point where sigma shows sudden drop ok. At this point when sigma shows sudden drop that is the rupture point and stress we can measure that what is the stress we have applied here, and strain at the point of rupture we can measure what was the strain at this condition when the rapture has happened ok.

So, this is all about now next is next is bulk modulus ok.

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**BULK MODULUS**

**Bulk compression:**  
A body is compressed from all directions (*x*-, *y*- and *z*-directions in a Cartesian coordinates system).

- elastic range of compression and a nonelastic range.
- positive stress → pressure, → reduction in volume of the sample
- negative stress → expansion → increase in volume or volume dilatation
- This can happen when a sample is placed under vacuum, or is suddenly released from high pressure to a lower pressure (expansion).

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So, bulk modulus as I have mentioned already, that the body is compressed from all the directions x, y and z if you consider in a Cartesian system. So, the elastic range of compression and the non-elastic range of compression may be there. So, if the positive stress we apply that is that causes the pressure. So, that will result in reduction in the volume of the sample or if the negative stress generated that is expansion so, that increases the volume or the volume dilatation may happen.

So, when the volume dilatation can happen; that means, if the product is under pressure, you are performing sudden operation and suddenly the pressure has been released. So, that is called a negative pressure and then the volume will be increased. For example, if some of you have known the concept of extrusion what happens in extrusion? This is some kind of technique where we combine many unit operations like hitting, cooking and we also apply shear force on the body of the material that is being extruded.

So, in the die, when the food is coming out of the die; so, it suddenly the pressure is released on the body. Because initially in the inside of the barrel in the inside of the screw of the extruder there was excessive pressure and shear force will be there. So, when it comes out from the die to the atmospheric pressure. So, pressure release that is why they expand and we are getting a very high you know porous structure you might have seen in the kurkure and then many kinds of this extruded snack items. So, that happened because of this negative pressure that is a case of a bulk expansion.

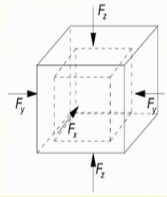
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**BULK MODULUS**

➤ Applying Hooke's law :  $-\frac{dV}{V} = k \cdot dp$  or  $k = -\frac{1}{V} \frac{dV}{dp}$

$\frac{1}{k} = K = -V \frac{dp}{dV} = -\frac{dp}{dV/V}$

➤ Where,  
 ➤  $V$  volume in  $m^3$        $p$  pressure in Pa;  
 $K$  bulk modulus in Pa       $k$  compressibility in  $Pa^{-1}$



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So, next is now if we consider that we consider that a element a block of the food sample, and then we are applying bulk compression from all the side on this. So, what will happen? Volume will decrease because this is a compressive force in nature. So, positive force so, volume will be decrease that is why we have written in this manner that is minus delta V that is the decrease in the volume of the initial volume V that is equal to small k into dp. So, dp is the pressure change that is needed to cause dV amount of volume reduction in the sample.

So, we can write small k that is equal to minus 1 by v into dv by dp right now again we can write it as if we write 1 by small k if we write 1 by small k that is equal to capital K and that is equal to minus v into dp by dV. So, here what we are what we can think of is that. So, k is equal to minus 1 by V into dV by dp k is termed as compressibility whereas, when we write 1 by small k that is equal to capital K that is bulk modulus bulk modulus. So, that unit of the bulk modules is in Pascal and k is a compressibility in per Pascal right.

So, this we can think of s dp by dc by v. So, the pressure required to cause the relative change in the volume. So, it is the ratio of these two ok. So, that is why we can we can termed it as the bulk modulus and it is decreasing because increase in the positive pressure will cause the reduction in the volume. So, minus sign has been given.

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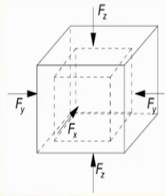
## BULK MODULUS

➤ Applying Hooke's law :  $-\frac{dV}{V} = k \cdot dp$  or  $k = -\frac{1}{V} \frac{dV}{dp}$

$K = -V \frac{dp}{dV}$

➤ Where,

➤  $V$  volume in  $m^3$        $p$  pressure in Pa;  
 $K$  bulk modulus in Pa       $k$  compressibility in  $Pa^{-1}$



➤ The bulk modulus  $K$  characterizes how a material can withstand an elastic compression, and is sometimes referred to as "firmness" of a material.

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So, the bulk modulus  $k$  characterizes how a material  $k$  can withstand and elastic compression and is sometime referred to as firmness ok. So, first one that is Young's modulus that is the concept of stiffness we are getting there. Here what we are getting that is concept of firmness, which is how a material can withstand and elastic compression elastic compression.

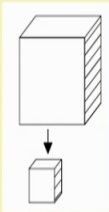
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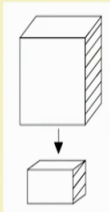
## BULK MODULUS

### Isotropic and anisotropic material:

For isotropic solids and normal fluids the modulus is simply  $K = K_{ij}$  and  $k = 1/K$

$$K = \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix}$$





- An isotropic material shows the same reduction in length in all directions (left).
- An anisotropic material (right) does not, and shows a change in shape.

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Now isotropic and anisotropic material; because sometimes we handle the food material those are not homogeneous in nature. So, because of food having you know the

composition of carbohydrate, fat, protein and many other nutrient nutrients vitamins and all and their distribution in the food matrix may not be same all the time right. So, for those kind of isotropic material where the homogeneous property we can see and there are certain ok. So, for the isotropic solid and normal fluid the modulus  $k$  can be expressed in this manner ok.

There are 9 components  $K_{xx}$   $K_{xy}$   $K_{xz}$   $K_{yx}$   $K_{yy}$   $K_{yz}$   $K_{zx}$   $K_{zy}$  and  $zz$  ok. So, bulk modulus in the normal direction and also in the shear shear modulus I mean in the in the direction of the other 2 phases, if you are considering the  $x$  directional directions. So,  $x$  directional change that will also have effect on the other 2 direction; so this is simply written as  $K$  equal to  $K_{ij}$  which is the bulk modulus, and small  $k$  that is equal to  $1/3$  by capital  $K$  which is bulk compression.

Now, what happen that if it is if the material is isotropic? So, when we are applying bulk compression or compressive force from all the side. So, it will deform uniformly from all the side ok. So, the structure the size and shape that we are we will get the size will be lower, but the shape will be same.

However, in case of an anisotropic material it will not be same, because the compression the bulk compression that we are giving that will not felt by the material in all the side equally, because of their different property they will behave in a different fashion.

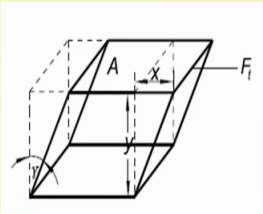
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## Shear Modulus


✓ **Shear deformation :**

- This is not an axial change in length, but a rotational change (torsion or twisting).
- The angle of deformation  $\gamma$  is called shear deformation.


Shear strain: The tangent of the angle of deformation is shear strain. In the elastic range of the material, the elongation  $x$  with respect to the sample height  $y$  is the ratio  $x/y$ , which is  $\tan \gamma$




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And another thing which is the shear modulus; so, what happened that as we have mentioned that if a dotted line dotted block is a stationary block kept on a surface and then we start pulling the upper plate with a with a force  $F_t$ , then what will happen its try to bend towards the you know horizontal side it will it will be pulled down and an angle will be made from the equilibrium position. So, that is gamma.

So, this is a rotational change or torsional change and the angle of deformation is called here the shear deformation. So, if it has you know moved X direction in the direction of the applied force and the height of height between the 2 layer is Y. So, X by Y that will be the expression of the deformation here that is equal to tan gamma.

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**Shear Modulus**

- ✓ Shear stress is  $\tau = \frac{F_t}{A}$  and elasticity is  $\tau = G \cdot \tan \gamma$
- ✓ small deformations it can be approximated as  $\tan \gamma = \gamma$
- ✓ Therefore,  $\tau = G \cdot \gamma$
- Solids with ideal elastic behaviour show a linear relationship between shear stress and shear strain.
- The material property G is called the **shear modulus G** ( $\text{N m}^{-2}$ )
- It gives an indication of how easily the material will bend or twist in response to an applied shear stress, which is sometimes called "rigidity".

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So, shear stress is expressed as tau equal to  $F_t$  by A ok. So, this is a you know tangential force is  $F_t$  by area. So, this force is acting parallel to the surface where as the normal forces that x on the perpendicular to the surface and the elasticity is tau equal to g into tan gamma. For the small deformation we can approximated tan gamma as equal to gamma therefore, we can write tau is equal to g into gamma ok.

So, solids with ideal elastic behavior show a linear relationship between the shear stress and shear strain. So, solids with ideal elastic behavior will can give this kind of a equation. So, the material property g is called shear modulus and unit is Newton per meter square. So, we have by now familiar with the Young's modulus, Bulk modulus and

Shear modulus. So, it gives an indication of how easily the material will be bend or twist in response to an applied stress which is some time call rigidity ok.

So, this is all about this different kind of modulus.

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**Shear Stress and Stress tensors**

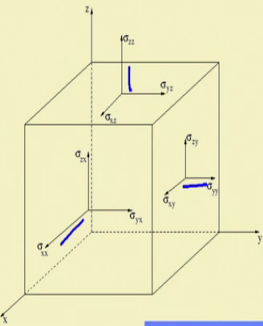
Stress defined as force per unit area and usually expressed in Pa(N.m<sup>-2</sup>) may be tensile, compressive or shear.

Nine separate quantities are required to completely describe the state of shear in a material.

Stresses indicated as  $\sigma_{ij}$

$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$  is the normal stress

$\sigma_{xy}, \sigma_{xz}, \sigma_{yx}, \sigma_{zx}, \sigma_{zy}, \sigma_{yz}$  are the shear stress



The diagram shows a 3D rectangular element in a Cartesian coordinate system with x, y, and z axes. On the top face (perpendicular to the z-axis), normal stress  $\sigma_{zz}$  is shown as a downward arrow, and shear stresses  $\sigma_{zx}$  and  $\sigma_{zy}$  are shown as arrows pointing in the x and y directions respectively. On the right face (perpendicular to the y-axis), normal stress  $\sigma_{yy}$  is shown as a rightward arrow, and shear stresses  $\sigma_{xy}$  and  $\sigma_{zy}$  are shown as arrows pointing in the x and z directions respectively. On the front face (perpendicular to the x-axis), normal stress  $\sigma_{xx}$  is shown as a rightward arrow, and shear stresses  $\sigma_{xy}$  and  $\sigma_{xz}$  are shown as arrows pointing in the y and z directions respectively.

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And next is just I will talk about a strain stress tensor for a moment, because you might be knowing this concept in your fluid mechanics class or transport phenomena, but we are not going to cover all these heat transfer, mass transfer momentum in our this fundamental course. However, just to give you an idea that how we can how we can express them because this term you may get some time in the course. So, that is why we are expressing here is that stress is defined as force per unit area and usually expressed in Pascal, maybe tensile compressive or shear we know that.

So, nine separate quantities are required to completely describe the state of shear in a material basically this is in case of a fluid in a in a flowing situation we need this. So, if this element we consider here. So, we are getting that there is sigma xx sigma yy and sigma zz. So, these are the normal stresses. So, there are we can see that there is 2 suffix ij that is needed to express the stresses in this case, and the other stresses which is called the shear stress are xy xy yx zx xz and yz zy.

So, the point is that if we write the stresses in terms of sigma ij, i and j will be same for the normal stresses and i and j will defer for the shear stresses. Now how we can explain



this sigma xx and sigma xy like that normal and shear stresses. Sigma xx will be the stress acting on a plane perpendicular to the x direction, because of x directional force whereas, if we consider sigma yx that is stress acting in y direction because of the x directional force ok. So, all this we can we can categorize all such forces and when whenever we want to discuss or we can analyze any such process. We can discuss that in which direction the flow is actually happened because most of the cases we are handling unidirectional flow. And, that time the other direction we may not need to take into account for the calculation so that we need to keep in mind right.

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**Stress tensors**

State of stress at a point inside a fluid in the deformed state, can be summarised as a stress tensor written in the form of a matrix

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Generally this tensor is of symmetrical in nature under equilibrium condition.

$\sigma_{ij} = \sigma_{ji}$   
 $\sigma_{12} = \sigma_{21}$   
 $\sigma_{23} = \sigma_{32}$   
 $\sigma_{31} = \sigma_{13}$

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So, basically stress tensor is state of stress at a point inside a fluid the deform fluid in the deform state can be summarize as stress tensor, written in the form of a matrix in this way sigma 1 ij equal to sigma 11, sigma 1 2, sigma 13, sigma 2 1, sigma 2 2, sigma 2 3 and sigma 3 1, sigma 3 2 and sigma 3 3. So, generally this tensor is of symmetrical in nature under equilibrium condition. So, sigma ij will be equal to sigma ji, in that way sigma 1 2 is equal to sigma 2 1 sigma 2 3 is equal to 3 2 and sigma 3 1 is equal to 1 3 and the diagonal elements are the normal stresses.

So, we will stop here. We will continue in the next class.

Thank you.