Fundamentals of Food Process Engineering
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Lecture - 07
Measurements of Rheological Properties (Contd.)

Hello everyone, welcome to the NPTEL online certification course on Fundamentals of Food Process Engineering. We will continue today with measurement of rheological properties. Today we will move on to the measurement using the rotational viscometers.

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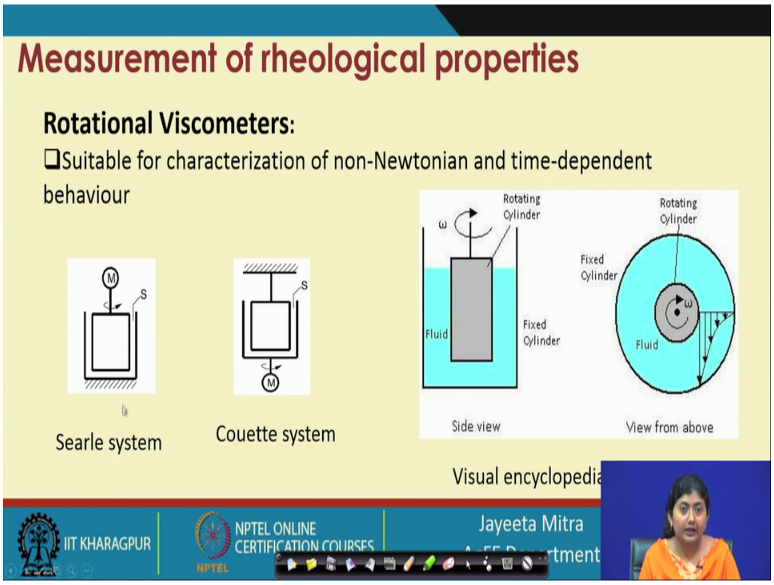

So, rotational viscometer this is very important measurement method of viscosity, because it can handle very well the Newtonian as well as the non-Newtonian fluids. The specialty is that the measurement method is like that in these rotational viscometers, the fluid is generally getting shared between 2 component of the measuring system. And they if the agitation is being done in the rotational system so, the shear rate will be actually depend or will be proportional to the rotational speed of the system ok. So, using this we can measure the shear stress for different changing shear rates and also what we can do is we can apply the shear rate for varying length of time.

So, the effect of time will also be observed on the fluid; that is why we can handle the different kind of non-Newtonian fluid which can behave differently over a range of shear rate or maybe constant shear rate for the prolong time. So, that is why this is very useful;

and this kind of system the rotational viscometers are of different kind of geometries, but the two most common that we will discuss. So, one is the kind of rotational system where is two concentric cylinders are there, the inner cylinder is rotates is generally rotated and the outer is fixed.

So, this is called Searle type systems and the other where the inner cylinder is fixed and the outer is being rotated that is called the Couette type system ok. So, now, we will see that any of this kind of rotational system we used for measurement. The basic principle will remain same. That is the torque required to move the inner cylinder at a constant speed and the shear stress that the fluid will exert on the rotating element will be balanced and that will be similar for both kind of system. So, we will see that how we can measure those thing.

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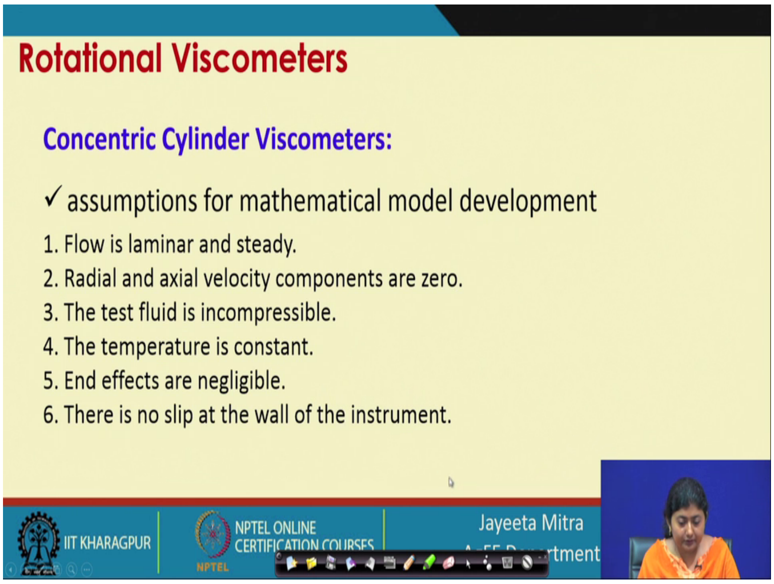

So, rotational viscometer or the cylindrical concentric cylinder viscometer; again based on certain assumptions which are more or less we have discussed in the previous measurement methods. So, flow will be laminar and steady so that we can get the constant velocity and in the laminar flow. There will be no radial and axial velocity components here only the rotational viscosity will be considered. The test fluid should be incompressible so, that we can take the density row is constant.

Finally, the temperature is also constant here because the temperature fluctuation can cause viscosity fluctuation as well and end effects are negligible. There is no slip at the wall of the instrument is being considered.

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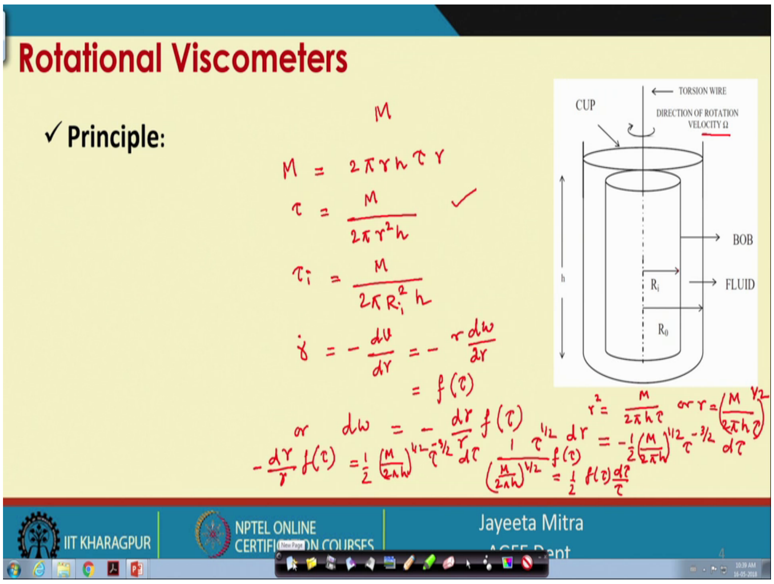

So, the principle we will see this is a cup which is steel here that is being fixed with the with some arrangements, but the inner cylinder or the bob, that is rotating with the constant angular velocity and inner radius of bob is R i outer radius of the outside cup is the inner radius of the outside cup that is ro. That means, fluid will be in between this R i to R o the fluid which will be getting sheared that will be in between these 2 concentric cylinder; so therefore, as the bob will rotate.

So, continuous shearing or the change of the shear rate can be provided on the fluid kept in between and also for different time it can be applied. Now we will see the torque required to rotate this inner cylinder at a constant angular velocity will be M let us say this torque is capital M and the force that will be opposed that torque will be the shear force, which is the liquid held here will provide. So, let us first see that if you want to make a force balance we can write that this torque M will be equivalent to 2 pi r h. This is the area where the shear stress tau is being applied and it is acting let us say at a distance r from the centre. So, this will be the torque.

So, we can write that torque will be equal to M by 2 pi r square h. Now since the inner cylinder is inner cylinder is rotating and the outer is fixed, just if we go back to if we go back to the previous diagram, here we can see that the shear stress or the shear rate we not be similar in all the annular spaces. So, it will be higher at the inner radius because that is the inner cylinder is rotating with a constant angular speed; however, the stress at the outer wall will be or the outside of that liquid level, which is which is in contact with the sixth fix cylinder that will be less. So, here if you want to measure the stress that is coming on the inner cylinder; so we can write this as tau i this is equal to M by 2 pi R i square into h, where h is the height of the liquid film ok.

Now with that we also try to get the expression of shear rate, because to have the viscosity, we want to get the relation between the shear stress and shear rate. So, the expression of shear rate gamma dot it is this one dV by dr, because as we move on to radial direction the velocity will be decreasing, because the outer cylinder is still and the inner is rotating that is why gamma dot will be like this.

Now we can replace this velocity v to tangential velocity with the angular speed. So, we can write this equal to minus d omega by dr into r ok. Now we also know that gamma dot will be a function of tau that is shear stress. So, we can write the differential angular speed dw d omega that is equal to dr by r into f of tau right; now what we do is, we need to find the expression of dr from this relation. So, what we can do is, since r square this is equal to M by 2 pi h into tau.

So, or r will be equal to M by 2 pi h into tau to the power half. So, dr we can take this as a constant, and then can take tau to the power minus half and differentiate this with respect to d tau. So, the expression we will get that get is some constant M by 2 pi by h to the power half into tau to the power minus 3 by 2 and then we put that expression here. So, this will be ok. So, d dr by r, f of tau minus that will half of M by. So, this will be M by 2 pi h to the power half tau to the power minus 3 by 2 into d tau and r we can get from this expression. So, 1 by this will be M by 2 pi h to the power half and this tau we can take it here. So, this will be tau to the power half.

So, finally, this will be getting cancelled and we can get and here also was f of tau. So, we are getting finally, one half of f of tau into d tau by tau ok. So, this expression finally, we got.

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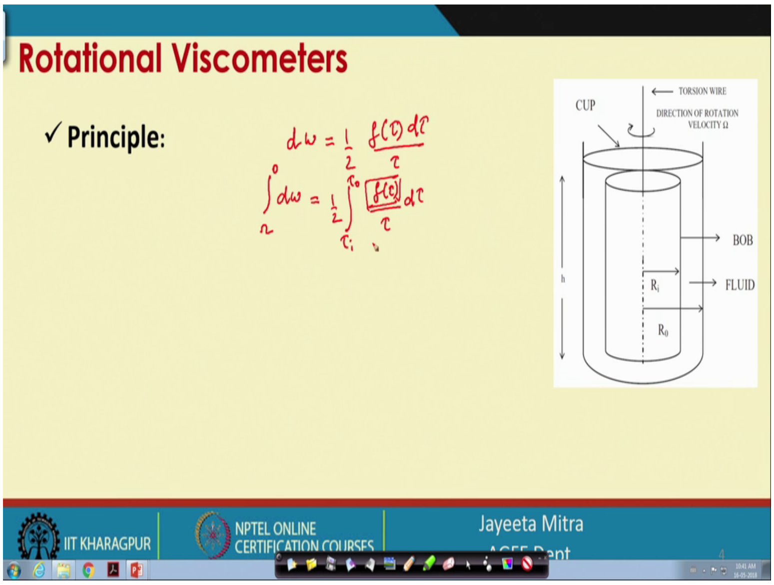

So, let us now write it clearly that D omega this is equal to one half of f tau d tau by tau that.

We are getting now since omega which is angular velocity of the inner cylinder, that is that may vary from ohm that constant at R i 2 0 at the R o. So, similar things will happen with the stress as well. So, stress will be max at R i and it will be minimum at R o. So, we can integrate this as. So, half of tau i to tau o, then f of tau by tau into d tau ok. Now solution of this kind of a problem will depend on function of tau function of tau now function of tau will again vary with different kind of solution. This if it is Newtonian then there will be shear rate will be shear stress by dynamic viscosity; if it is power law fluid then the relation will be different from shear stress divided by k to the power 1 by n like that. So, as f t will vary for different kind of a solution different kind of fluid food sample, then solve of this equation the final result of this equation will be varying.

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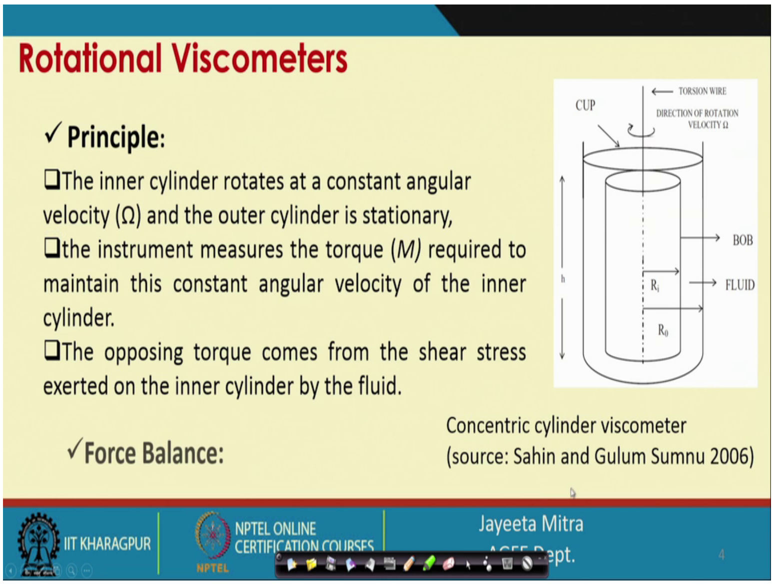

So, now, we will see the cases ok. So, this we have we have already solved this doing the force balance so we have come to this stage now.

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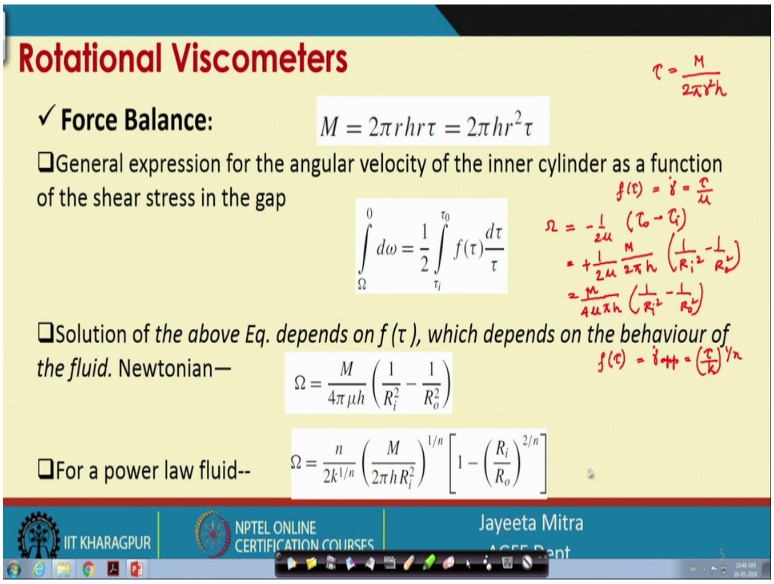

And if it is Newtonian, then what we can do is we know that for Newtonian fluid the function of tau. So, that is such that gamma dot will be tau by mu. So, will put it here integrate this over ohm to 0 from for tau i at R i and tau o at Ro. So, here it will be minus 1 by 2 mu into tau 0 minus tau i. Again this tau is related with the torque. So, the expression was tau will be equal to M by 2 pi r square h. So, we can write it like this 1 by

2 mu into M by 2 pi h 1 by r i square minus R o square if we take this as plus right. So, finally, we are getting this equation that angular speed that is equal to 1 by 4 mu pi h into 1 by R i square minus 1 by R o square ok. So, from this we can get the idea of the dynamic viscosity mu and next we can see the case.

So, here was M finally, we are getting this equation ohm equal to M by 4 pi mu h 1 by R i square minus 1 by R o square. Now let us see if it is Newtonian if it is non-Newtonian then considering power law fluid, we will get this kind of expression. So, for this what we need to do is, we will take this f tau the function that is gamma dot ok. So, this is actually shear rate so, that that we can take in this way tau by constant k to the power 1 by n. So, then putting this into this equation function ft and in a similar way integrating putting the values from this equation tau, and the torque required we are getting this equation that is ohm equal to n which is the flow behavior index of the power law by 2 k to the power 1 by n the consistency index. Then M divided by 2 pi h r i square to the power 1 by n into 1 minus r i by r o to the power 2 by n.

So, in this way we can we can now plot the shear stress and shear rate and we can we can find that what will be the relation between them and then calculate the dynamic viscosity.

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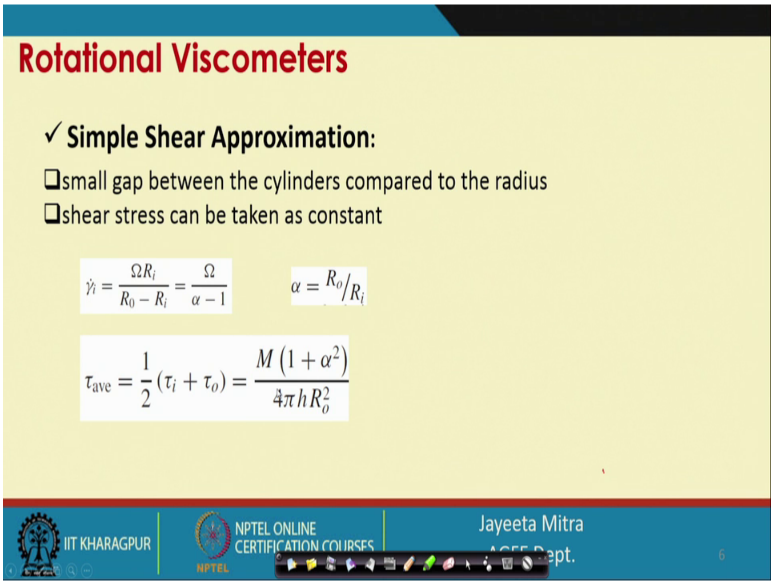

So, if we look into the simple shear approximation cases. So, the small gap between the 2 concentric cylinder is taken very small compared to the radius and then what we do is, we take the shear stress as constant and we can measure the shear rate gamma i dot as equal to as equal to ohm into R i that is if we are measuring at a distance R i and the angular speed is constant that is ohm.

So, the distance it can move is ohm into R i divided by R o minus R i that is the that is the radial distance were the liquid is. So, that is equal to ohm by alpha minus 1, where this alpha is equal to R o by R i. And here we can take the shear stress as average that is half of tau i which is at the inner cylinder plus tau o which is at the outer cylinder, and that will be equal to M into one plus alpha square by 4 pi h into R o square. So, tau i and tau o we have taken from that relation where we are relating torque with the shear stress.

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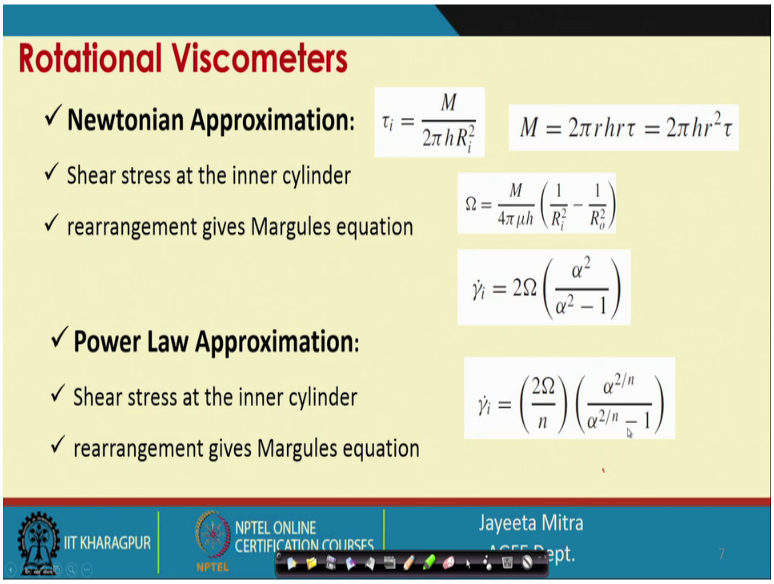

And if for the Newtonian approximation we can take tau i as M by 2 pi h R i square by that relation ok. So, shear stress at the inner cylinder will be governed by this law M by 4 pi mu h 1 by R i square minus 1 by R o square. And rearranging this we can get the expression of gamma dot i that is called the Margules equation gamma i dot that is equal to 2 into ohm in to alpha square by alpha square minus 1 and if we take the power law approximation. So, shear stress at the inner cylinder will be gamma i dot 2 into ohm by n into alpha to the power 2 by n divided by alpha to the power 2 by n minus 1 and again ok.

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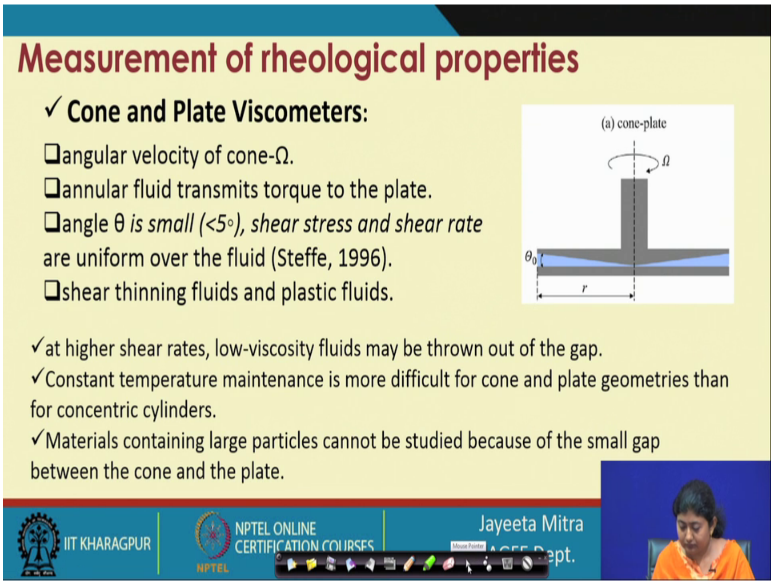

So, like this way we can get the shear rate versus shear stress data in case of the rotational viscometer and can measure the viscosity, we will solve a problem at the end of the class and next will move on to another measurement method, that is cone and plate viscometer.

So, from the diagram we can see that it is another kind of rotational viscometer. So here, it is another kind of rotational viscometer and here a cone is there on a circular flat plate; in a way that access will be perpendicular on the plate, and the epics is will be on the on the plate on the surface of the plate. This cone is moving with the angular velocity and the gap between the cone and plate will be filled by the liquid.

So, this kind of arrangement is very much suitable for the shear thinning behavior and the plastic fluid behavior. The gap is usually very less the angle which it makes the cone make the angle with the parallel plate and that angle is generally very small theta generally kept lower than 5 degree ok. So, if it lower than 5 degree then it is considered that the shear stress and shear rate in the in the gap or the angular space or the space for the fluid is, that will be exposed to the constant shear rate and constant shear stress.

So, we can handle the shear thinning fluid and plastic fluid here, but then also carefully we have to do this because there is a chance that the shear thinning fluid will come out when we operate at a higher shear stress or shear rate. So, constant temperature maintenance here is another problem, because of the plate geometry and material containing the large particle cannot be used here, because the place available to put the liquid is very small.

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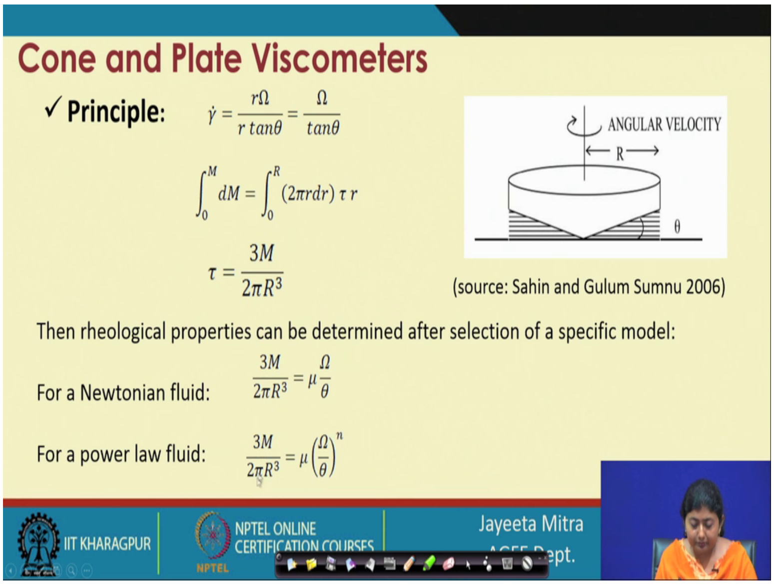

So here also we will we will see that; what is the principle of determining the viscosity here. So, here the cone is rotating with the angular velocity constant angular velocity and we measure the shear rate gamma dot by the amount it moves in the angular direction that is if we measure at a distance at a radial distance r. So, r into ohm divided by r into tan theta that the tan theta where theta is the angle between the cone and the plate and if the theta is very small instead of tan theta, we can take only theta here ok. So, we are getting ohm by tan theta as the shear rate here. So since, here also the torque will not be same in all the places, it will vary from centre where R equal to 0 to the R equal to R where the where the outside radius or the whole radius of the cone; so 0 to M dm if we indicate the torque.

So, here it will be 0 to R again 2 pi r dr. So, dr a small fluid element here we are considering and the tau which is the shear stress that is being applied by the fluid to oppose the rotational movement, and it is acting at a distance r. So, this will be there. So, then from here we can find this expression tau will be equal to 3 M by 2 pi r cube. Here we can get pi r square dr. So, it will be r cube by 3 that 3 will come here. So, we are getting 3 M by 2 pi r cube. So, this will be the stress ok. So, then the rheological properties can be determined after selection of the specific model. So, from the specific model, if we plot the shear stress versus shear rate gamma dot we can get a curve from which the viscosity can be calculated. So, for the Newtonian fluid for the Newtonian fluid we can get the relation 3 M by 2 pi r cube that is equal to.

This is shear stress that is equal to mu which is the viscosity into ohm by theta, here we can we have put directly the shear rate gamma dot, tan theta has been reduced to theta as the angle is very small and if it is for the for the non Newtonian fluid that is a power law fluid. So, we will take shear stress 3 M by 2 pi R cube that is equals to mu into ohm by theta to the power n.

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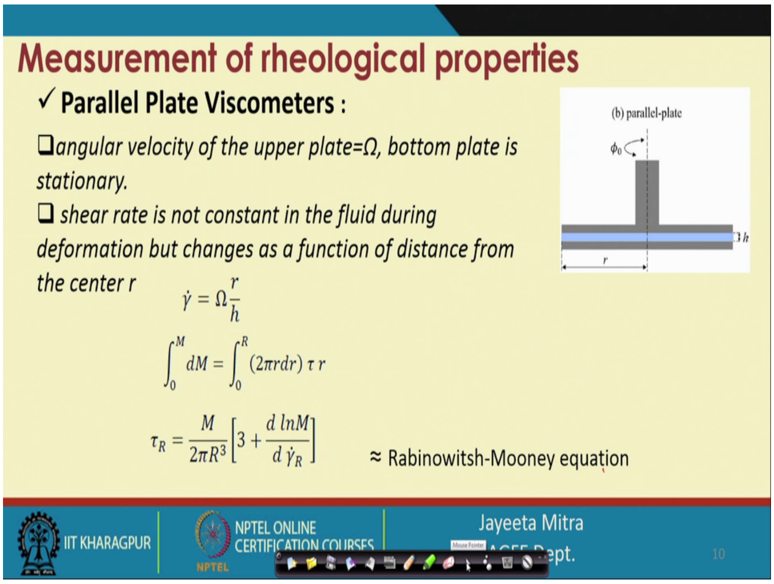

So, another similar kind of measurement method is the parallel plate viscometer, here the 2 parallel plates are there; one is a rotating plate which is having a constant rotational movement and the lower plate which is fixed, in between there is liquid of height h is kept and the radius is r. So, as the upper plate is circular plate that is rotating, the stress is offered by the liquid which is kept in between. So, here the shear rate is not constant in the fluid during the deformation, but changes as a function of distance from the centre. So, again gamma dot will be r divided by h.

So, again we will integrate in a similar fashion as we did for the cone and plate viscometer, and here we can get the expression of this kind where tau R that is the shear stress that is equal to M by 2 pi r cube into 3 plus d of lnM by d of gamma dot R. So, tau R that is the shear stress at the extreme outer radius and this gamma dot R is the shear rate there. M is the torque that is equal to move the upper circular plate at to rotate it at a constant angular speed ohm.

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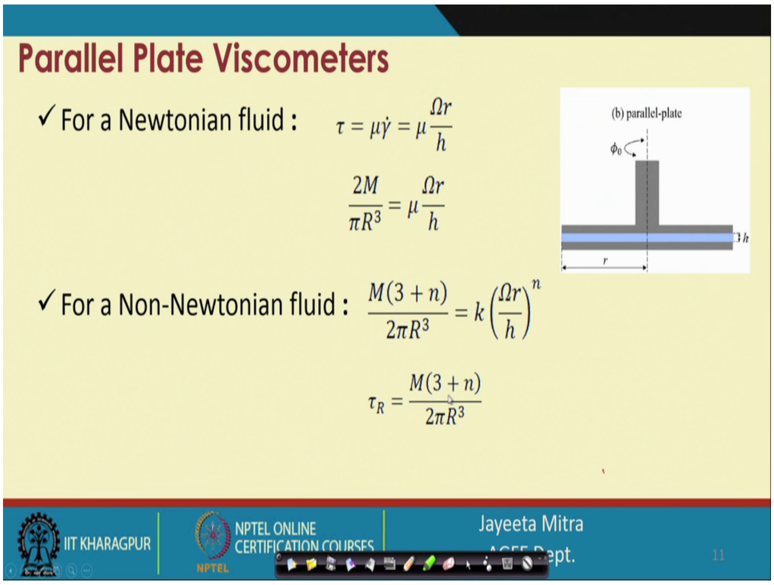

So, this is called the Rabinowitsh Mooney equation and for a Newtonian fluid we will just apply the linear relation between shear stress and shear rate put it in that equation. So, we are getting this expression finally. And if we are dealing with a non Newtonian field, we are getting this kind of expression where we take con consistency index into ohm into r by h to the power n. So, tau R we getting as M into 3 plus n by 2 pi R cube.

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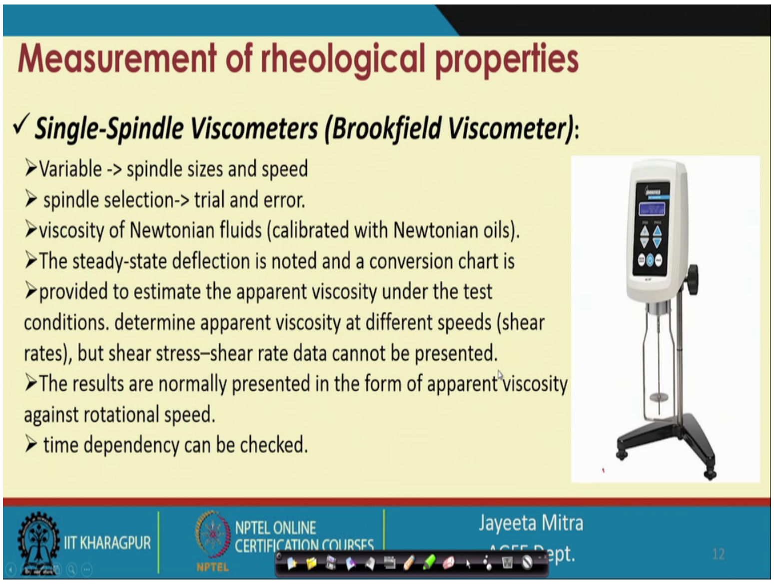

So, there are other methods like Single Spindle Viscometres which is also called the Brookfield viscometer this is the pictorial view of that.

This spindle is rotating continuously in the in the liquid; spindle size and the speed can be varied here to measure its affect and the spindle selection is done by trail and error by doing some initial measurement. And viscosity of the Newtonian fluids is used to calibrate this generally the Newtonian oil is used, and the viscosity of Newtonian fluid can be measured very well by this Brookfield viscometer. The steady state deflection is noted and a conversion chart is provided to estimate the apparent viscosity under the test condition. So, the determination of the apparent viscosity at different speed that is different shear rate can be done, but the shear stress shear rate data cannot be presented; so only apparent viscosity that can be calculated.

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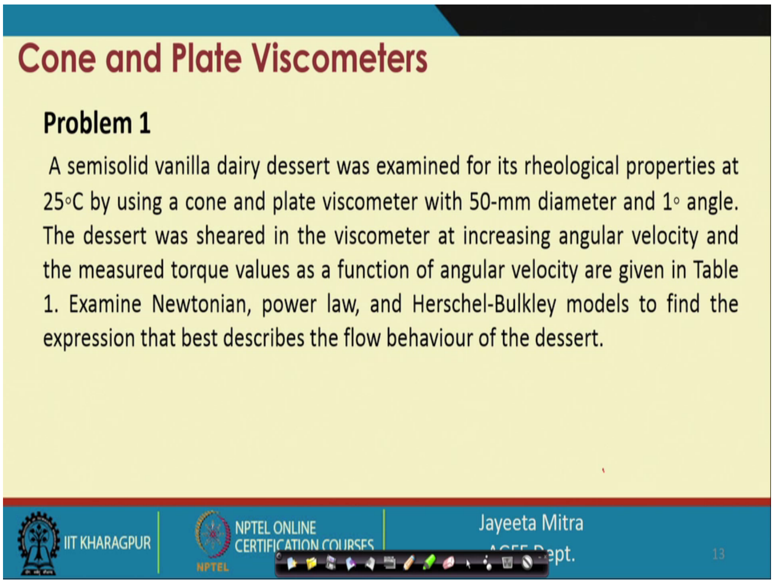

So, now we will do one problem quickly on that. Let us take a semisolid vanilla dairy dessert. This was examined for the rheological. Shall I stop here?

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Ok.

(Refer Time: 39:58) next lecture we will continue query mean.

Ok.

(Refer Time: 40:01) Ok.

(Refer Time: 40:02).

Ok.

(Refer Time: 40:05) we will continue in the next lecture.

Ok.

In next lecture (Refer Time: 40:12).

Ok.

Alright (Refer Time: 40:15).

So, we will stop here. And, in the next class we will do some problem solving on measuring the rheological parameter of the fluid food.

Thank you.