Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 13
One Dimensional Heat Transfer Through Cylinders (Contd.)

Good afternoon. So, we have done on cylindrical coordinate both solid cylinder as well as hollow cylinder; how to do analytical solution that we have done, 'right'. And today is the 13th class on that One Dimensional Heat Transfer Through Cylinders, this will do today also, 'right', ok.

So, one dimensional heat transfer through cylinders, this will do today and we said that we will come to the solution of problems, 'right'. So, with let us go to that we have done this problem earlier and this problem also. Now, let us do this one which we have not done.

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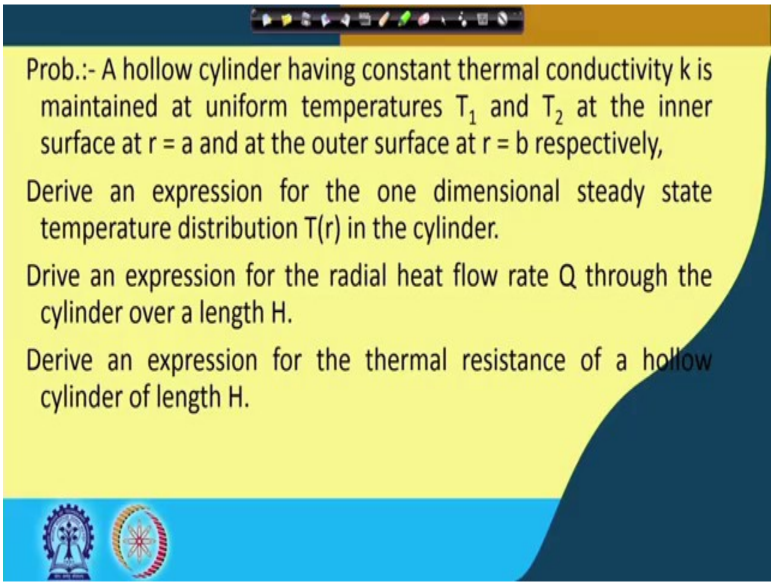
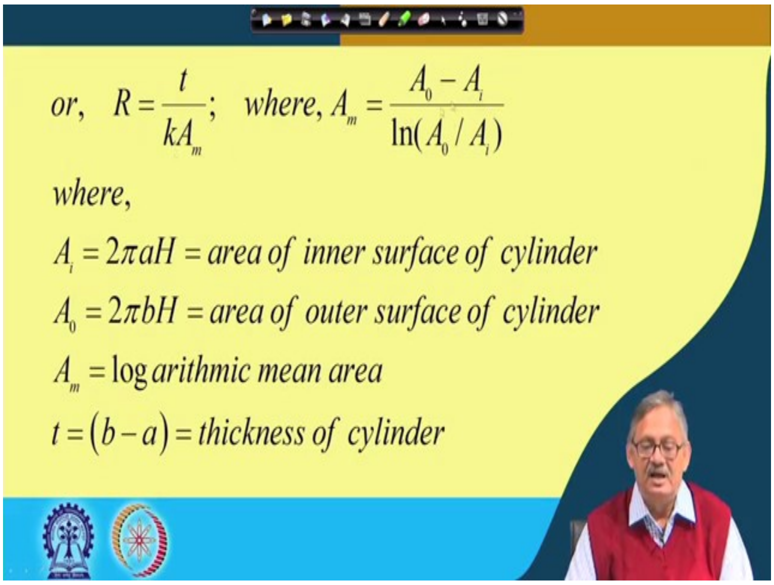
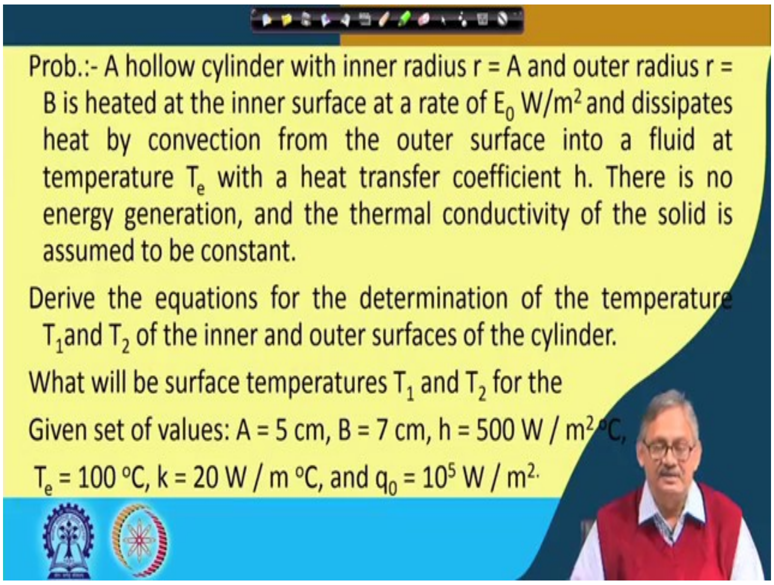


A hollow cylinder having constant thermal conductivity k is maintained at uniform temperature T_1 and T_2. This we have done. This solution of it we have done and we have seen that the area is coming Log mean area, 'right' and the solution it came to be like this, 'right'.



This was the solution for that. So, where R was t/kA_m and A_m was

$$A_m = \frac{A_0 - A_i}{\ln(A_0/A_i)}$$

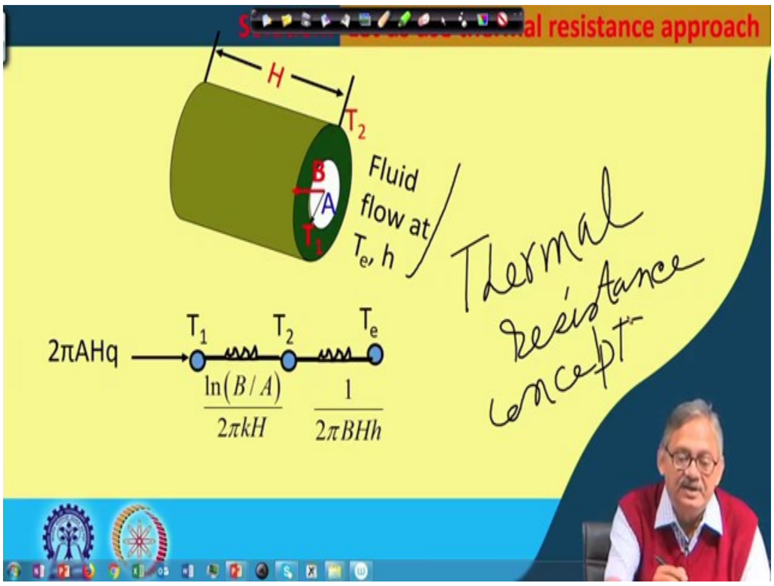


So, now we will do another problem. A hollow cylinder with inner radius R is equal to A and outer radius R is equal to B is heated at the inner surface at a rate of E_0 W/m^2 and dissipates heat by convection from the outer surface into a fluid at temperature T_e with the heat transfer coefficient h. There is no energy generation, and the thermal conductivity of the solid is assumed to be constant.

So, we have to derive the equations for the determination of temperature T_1 and T_2 of the inner and outer surfaces of the cylinder. And then, we have to also quantify what will be the surface temperature T_1 and T_2 for the given set of values; A is equal to 5 centimeter, B is equal to 7 centimeter, h is 500 W/m^{2o}C, T_e is 100ºC, k conductivity to be 20 W/mºC and the heat flux q_0 is 10^5 W/m^2. Then, we have to solve it.

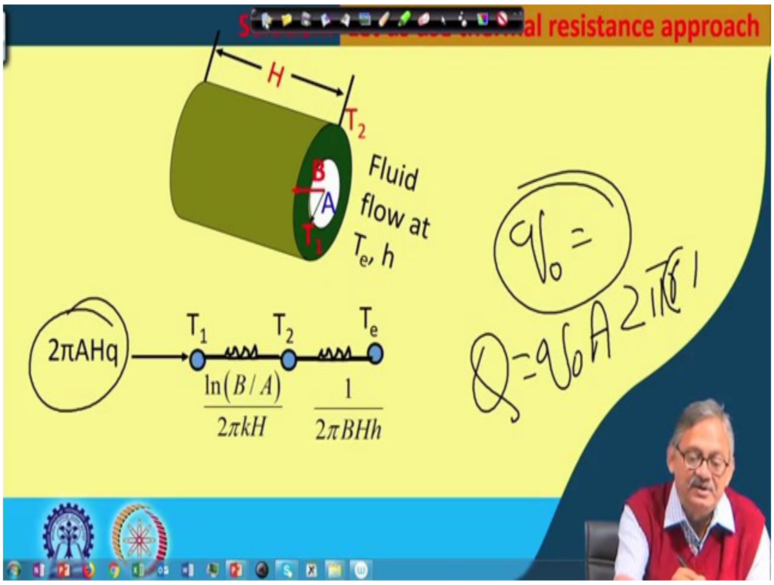
Now, here again there is no internal energy generation, 'right'. So that means, we can also use the thermal resistance concept. So, we read once again quickly a hollow cylinder with inner radius R is equal to A and outer radius R is equal to B is heated at the inner surface at a rate of E_0 Watt per meter square and dissipates heat by convection from the outer surface in to a fleet at temperature T_e with a transfer coefficient H. There is no internal energy generation and the thermal conductivity of the solid is also assumed to be constant.

Derive the equation for the determination of the temperature T_1 and T_2 of the inner and outer surfaces of the cylinder. Also what will be the surface temperature T_1 and T_2 for the given set of values A equal to 5 centimeter, B equal to 7 centimeter, h is 500 W/m^{2o}C, T_e is 100ºC, k is W/mºC and q_0 heat flux to be 10^5 Watt per meter square.

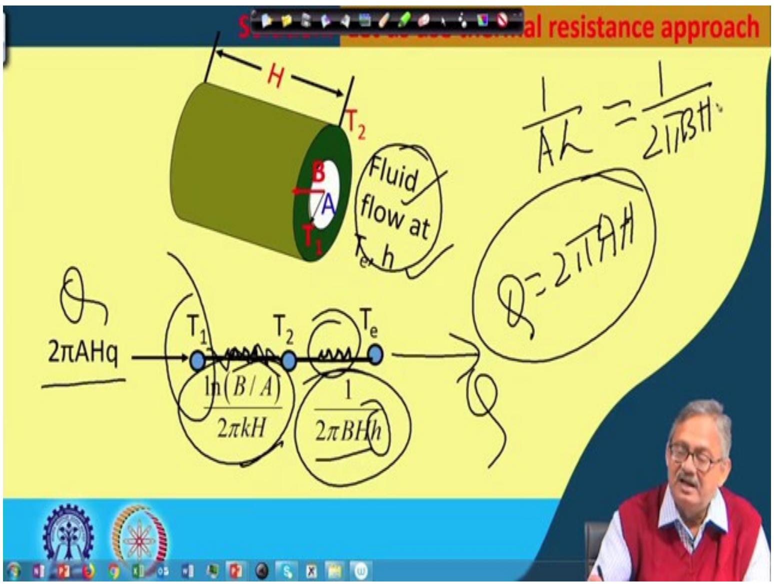


So, if we see this we can make this like that. As you see that we have taken a cylinder, this is a hollow cylinder, 'right'. This white portion is saying to be hollow, 'right' and it has a radius of B and also a radius of A and this surface is T_1 at this inner radius and outer radius is T_2, 'right' and the length of the cylinder is H. The environment is outside is fluid flow at temperature T_e and the heat transfer coefficient of h, 'right'.

Then, we can do this with the thermal resistance concept, 'right'. With the thermal resistance concept, we can do this solution of this problem, 'right'. So, what we do? We do the analogous of the thermal resistance, 'right'.

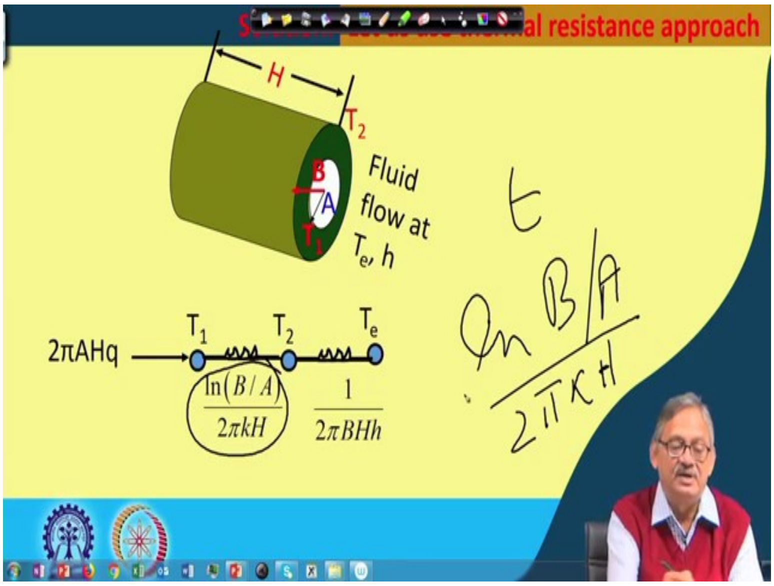


So, heat flux is coming we are given q_0 equal to some value afterwards ok. Now, q_0 is coming; so, Q is equal to q_0 times area, 'right'. So, area is here 2πrH 'right' or 2π yes, in this case it is A; instead of r, we are writing A, because our given is Q is equal to 2 π A into H; H is the length, 'right'. So, our radius given is A.

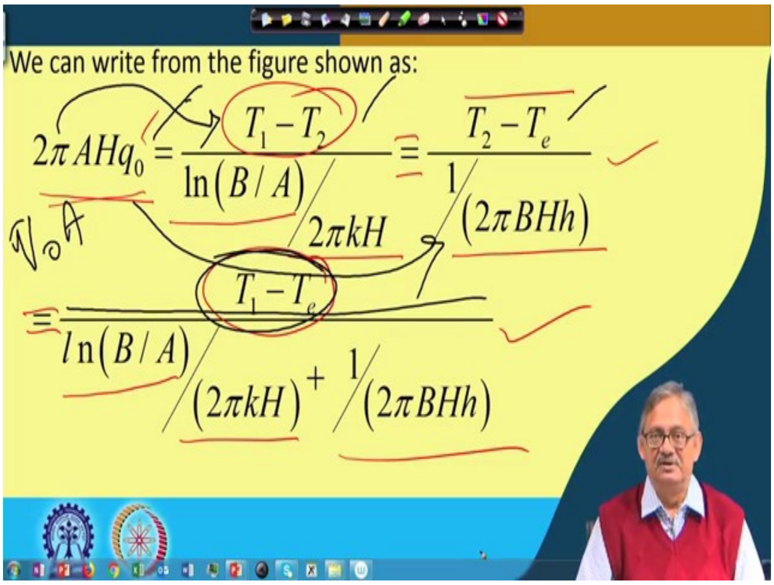


So, inner radius is that; so, that is what we are supplying Q, 'right'. The same Q is going out through the other side and the inner temperature is T_1, 'right'. So, there is a resistance and this resistance is equivalent to ln(B/A)/2πkH which we have seen earlier and the outer surface is at the boundary. It is a fluid at temperature T_e with a heat transfer coefficient of H, 'right'.

So, this is the resistance which is equivalent to 1/Ah, where A is 2πBHh, 'right'. So, 2πBH is area here and h is a transfer coefficient. So, we have seen that this is equivalent to 1/Ah, 'right' and this 1/Ah is equal to 1/2πBH is the area and h is the heat transfer coefficient, 'right'. So, this is what we have.



And this through the body, we have seen that earlier it is t, 'right' that was t by B H, this was ln(B/A) that is the 2 areas over 2πkH, 'right'.



So, if these be true, then we can write that solution of T_e to be equal to the solution of it we can write from the figure which we have given $2\pi AHq_0$, 'right';

$$2\pi AHq_0 = \frac{T_1 - T_2}{\ln(B/A)/2\pi kH} = \frac{T_2 - T_e}{1/(2\pi BHh)}$$

$$= \frac{T_1 - T_e}{\ln(B/A)/(2\pi kH) + 1/(2\pi BHh)}$$

So, this we got and now the solution of this we get equal to by equating the first and the last term, we get that first and last term if we equate; then, we get T_1 is

$$T_1 = \left(\frac{A}{k}\ln\frac{B}{A} + \frac{A}{Bh}\right)q_0 + T_e$$

This is by equating first and third 'right'; first and third means, let us go back to that, first and third means here, 'right'. So,

$$T_2 = \frac{A}{Bh}q_0 + T_e$$

Now, if we put the values by putting the numerical values, we get T_1 which is given 'right' the value of A, value of B and A those are given like A is given 5 centimeter let us take that this black color. These goes up A is 5 centimeter; B is 7 centimeter; h is 500 $W/m^2{}^oC$. T_e is 100^oC; k is 20 W/m^oC and q_0 is 10^5, 'right'. If that be true, then we can rewrite and say that T_1 is equal to 0.05 by 20 because our expression was like that. If we look at our expression was like that, it was $T_1 = \left(\frac{A}{k}\ln\frac{B}{A} + \frac{A}{Bh}\right)q_0 + T_e$

So, by substituting the values, $T_1 = \left(\frac{0.05}{20} \ln \frac{0.07}{0.05} + \frac{0.05}{0.07 X 500} \right) X 10^5 + 100$

$= 327.98\ ^oC$

I repeat again and again in all the classes, I am repeating that whenever you are doing this calculations, please also check whether the values are correctly calculated or not; if it is not, so please bring to our notice; so, that we also can rectify accordingly. Hopefully it is ok, but yes can be said.

Now, I cannot do this calculation here because that will take unnecessarily some more time and which may not allow us to do some new and new things, 'right'. So, that is why it is already solved and you are requested to calculate it again and check and if there is anything wrong, then bring to our notice. If it is a let of course, nothing to be said. So, T_1 came to be 327.98 ºC and T_2 and from the expression of T_2, if we see that whatever is T_2?

So, $T_2 = \frac{A}{Bh} q_0 + T_e$

$T_2 = \frac{0.05}{0.07 X 500} X 10^5 + 100 = 242.86\ ^oC$

So that means, that the if you remember we had taken this, 'right', this was our A and according to this it is 5 centimeter and this was B which is according to this is 7 centimeter, 'right'. And we have found out that the temperature at the inner one; so, here the temperature at the inner one T_1 is equal to 327.98; whereas, the temperature at the outer one is 242.86.

Whereas, our outside ambient temperature was hundred degree; that means, this heat if you if you plot, the thickness versus temperature; if it is thickness, 'right' thickness versus temperature. So, this is the 0; so, 0th your temperature was around 327 and from the temperature has come down to 242 not 48, 242, 'right'. 242.86. So, it was 242.86 and from there the temperature went down to this, where it is it was 327. So, T_1 was 327 sorry.

So, T_1 was 327. So, if we plot this that 327.98and this came down to 242.86. So, 242.86; that means, the inner one had higher temperature than the outer one and it was dissipating heat to the ambient at 100ºC and heat transfer coefficient of 500W/m^{2o}C. So, this way we can solved the problems.

Now, if we look at some other problem, if we look at; so, this is how we are doing. So, if we recapitulate that what we have done on T_1, T_2 of the thermal resistance concept as well we have done on the internal energy generation, where internal energy was generated, 'right'. So, there we have done and also the thermal resistance concept, 'right'.

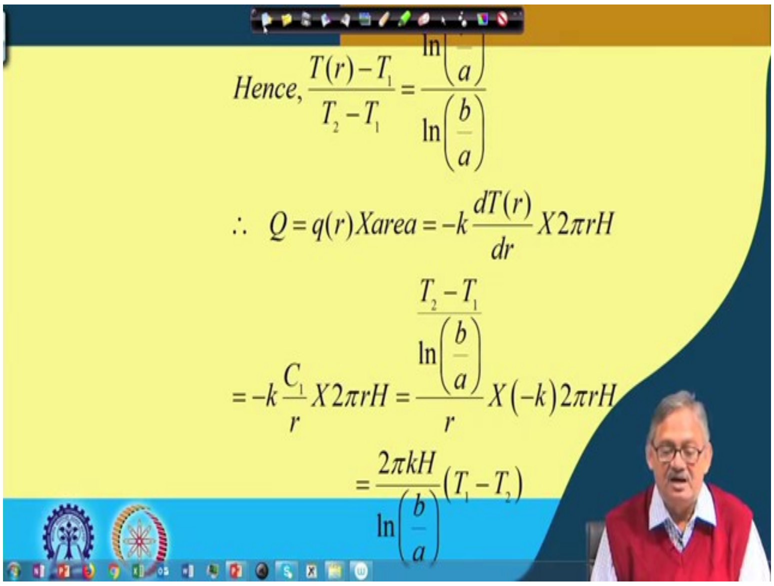
So, in that thermal resistance, here it was much more easier where it was general that T_1 was $C_1 \ln a + C_2$ and this was T_2 was $C_1 \ln b + C_2$ and we found out C_1 to be

$$C_1 = \frac{T_2 - T_1}{\ln\left(\frac{b}{a}\right)}$$

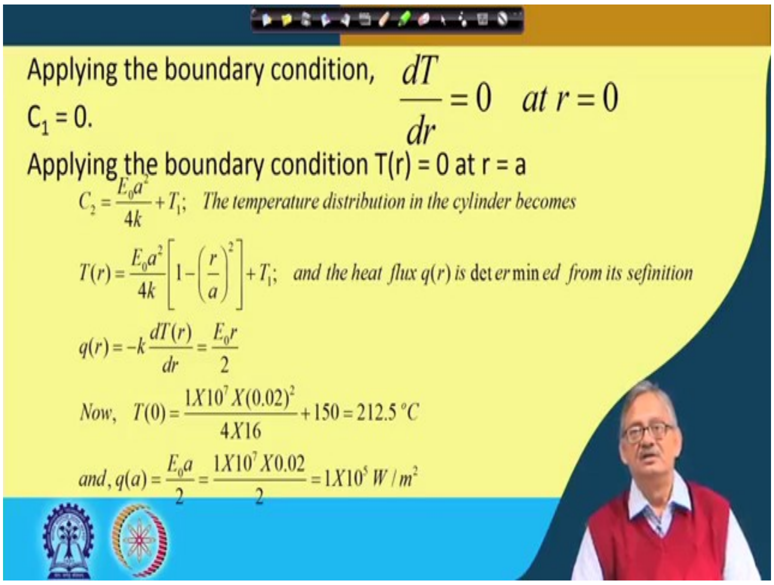
and we also found out C_2 to be

$$C_2 = T_1 - \left(T_2 - T_1\right)\frac{\ln a}{\ln\left(\frac{b}{a}\right)}$$

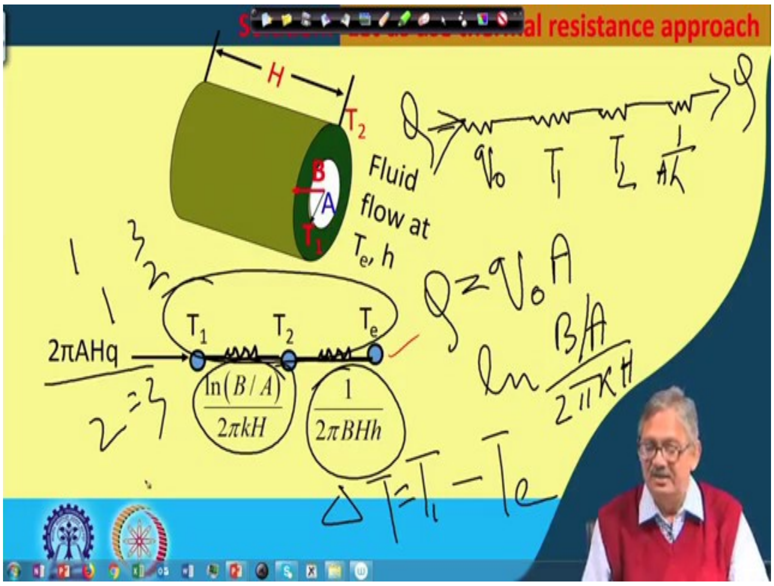
So, once we did this, then we also did that this that T(r) is equal to T_1 minus and so, what I would like to highlight here.



In this is that here we had done based on this, 'right', this was based on this firmly did here by integrating this that, but when we came to the yes t was k by A_m, fine A_0 minus A_0 by l; so, hollow cylinder when we came to this another hello we had done, 'right'. This one no, I hope with a resistance concept we had done one only and with the internal energy generation we have done another one only.



However, so in the internal energy generation, if there is no. So, in the energy concept, then you have to do like this that from here let us let us go to the other one, 'right'.



So, when we are doing internal energy concept, here it is only three there can be some port. So, in that case, so what do you need; so, these resistances whatever you have, you go on adding, 'right'. So, and these resistances you have to know, 'right'; so, these resistances you have to know as if that Q quantity of heat is coming and that Q quantity of heat is going out, 'right'.

So, this has to be kept in mind that Q quantity of heat is coming in and Q quantity of heat is going out and these resistances depending on whether it is heat flux or with that it is a temperature or it is a another temperature or it is a convective boundary. So, whatever we the situation, you have to find out the resistances, 'right' and then, since you know that Q is flowing. So, Q is equal to like here it is $2\pi AHq_0$; since it is a heat flux, 'right'. So, $2\pi AHq_0$ that is q_0 into A that you know, 'right'.

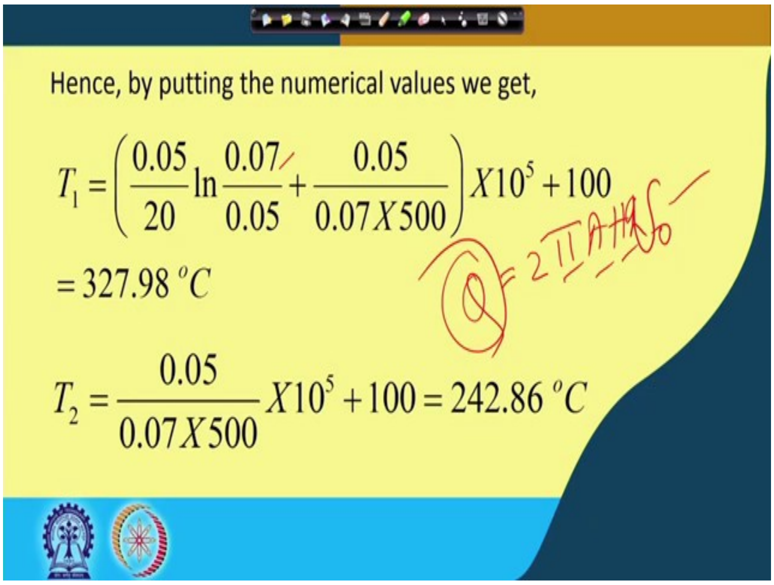
Similarly, here you know the resistance, 'right' ln of B over A/2πkH or 2πkH 'right'; so, this you know. So, similarly the convective boundary that is also known that is 1/2πBH, 'right'. So, had it been that one more resistance is there. So, that also you have to add, 'right'. So, then that he like here (T_1-T_e) is the driving force or ΔT 'right'; so, T_1-T_e is the driving force or delta T.

So, that by adding all or as we have said that adding 1 and 3 or are equating 1 and 3 or 1 and 2 or 2 and 3 depending on how you are solving it what you need to solve, 'right', which we have done here. In this case also we have done here that, we have taken that this one this one this one and this one, 'right'. So, we added them when we added them, we got T_1 minus T_e over ln(B/A), 'right'.

Over 2πkH +1/2πBHh. This is by adding everybody, 'right'. So, that is q_0 into A, this is equal to all these, 'right'. Then, from there may be equating this and that and maybe equating this and that you can get T_1, T_2 all equations and then, by solving them means you it cannot be that it will be all the time very simple, 'right'.

So, it may not be all the time it is very simple equations, but it is also known that number of unknown and number of equations if they are same, then it can it is solvable. Yes, it may take some time, it may take some iterations or it may take some simplification. So, that may be required, but you may have to do that equating 1 and 2 or 1 and 3 or depending on what it is coming that you have to make and then you have to solve them, 'right'. And as earlier you have seen that in this problem itself it came that you are given some data and based on that the numerical values you have to find out, 'right'. So, this is how you need to solve and you need to do the solutions and numerals also, 'right'.

So, this we have done T_1 from there equating this and we got the T_2 also and then, by substituting the values 0.05, 5 centimeter and 7 centimeter were the 2 radii given, 'right' and we got it. Then, we can also find out if it is required what is the value of capital Q, 'right'.



What is the value of capital Q that also we can find out because we know Q is equal to 2πAHh, 'right'.

So, that can be easily found out or this was 2πAH and not h q_0, 'right'. 2πAHq_0. So, that can be easily found out. q_0 if it is given; H is known, A is known, π is known. So, what is the value of Q that can be found out, 'right'.

With this, we come to the end of this one dimensional heat transfer, 'right'; one dimensional heat transfer on cylindrical coordinate, 'right'. So, we will do next time the spherical coordinate, 'right' on the sphere, ok.

Thank you.