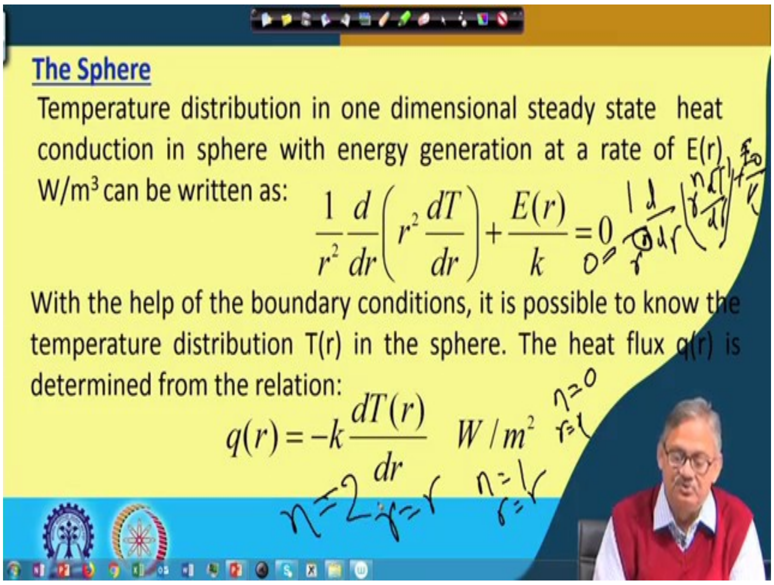
Thermal Operation in Food Process Engineering: Theory and Applications
Prof. Tridib Kumar Goswami
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
One Dimensional Heat Transfer

Good morning. So, we have done on cylindrical coordinate, also earlier on Cartesian coordinate One Dimensional Heat Transfer, 'right'. Now, we will do that one dimensional heat transfer in spherical coordinate, 'right'. So, this is the lecture number 14 and we are now moving towards the sphere, 'right'.

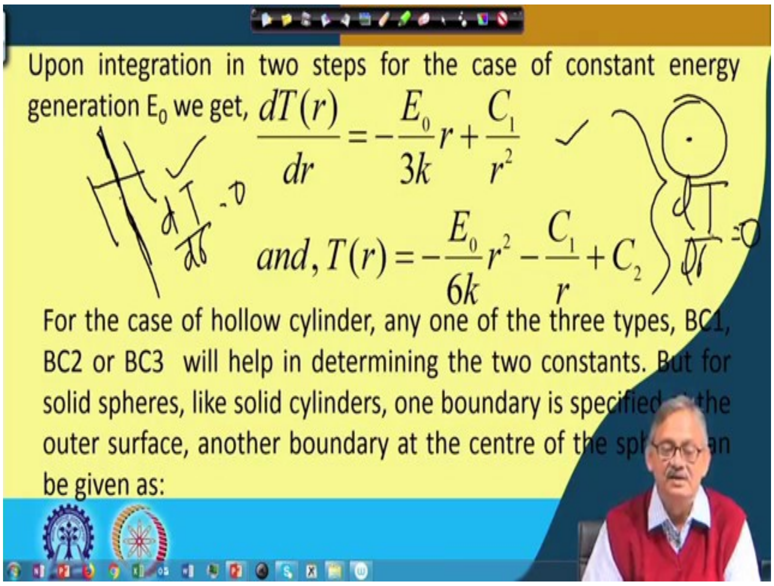
(Refer Slide Time: 00:49)



So, in this case the temperature distribution in one dimensional, steady state, heat conduction in sphere with energy generation at the rate of E(r) Watt per meter cube this can be written as $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{E(r)}{k}=0$ So, this again I am saying so, it is a repeat, but still our general equation was $\frac{d}{dr}\left(r^n\frac{dT}{dr}\right)=0$ if conductivity is constant, energy generation is 0,. And, it is steady state condition.

And, here when we were with Cartesian coordinate that time n was equal to 0, n was equal to 0 and we also took that r to be equal to x, 'right', but when into a cylinder we had taken n is equal to 1 and r of course, equal to 1. But, when it is now in the spherical coordinate then; obviously, r is equal to r, but n is equal to now 2. And, that is why these terms had come $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{E(r)}{k},=0$ and this is the basic governing equation, 'right'.

So, with the help of boundary conditions, it is possible to know the temperature distribution T(r) in the sphere and the heat flux q_r is to be determined from the relation, q r is equal to minus k dT /dr of at any r; so, much of W/m^2. So, if we want to solve it then we need to do the integration and then put the boundary conditions, 'right'.



So, if we want to solve it then on integration, we get, first integration, we get

$$\frac{dT(r)}{dr}=-\frac{E_0}{3k}r+\frac{C_1}{r^2}$$

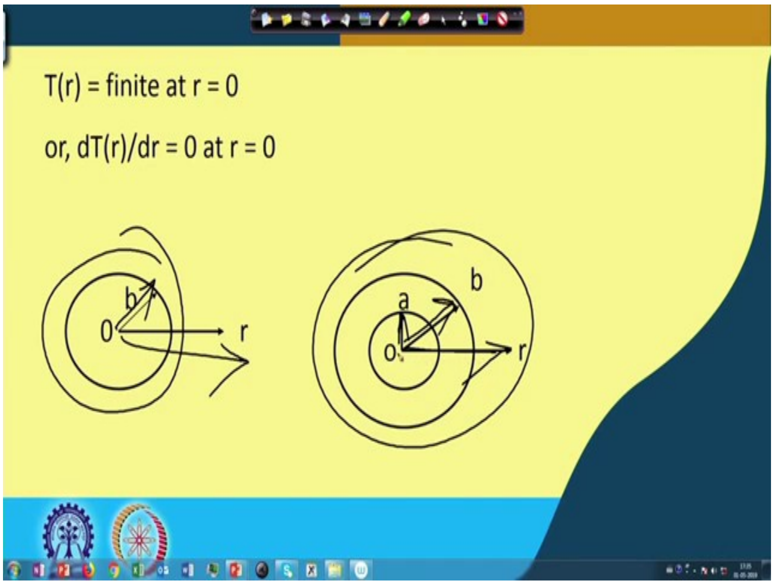
$$and\,,T(r)=-\frac{E_0}{6k}r^2-\frac{C_1}{r}+C_2$$

So, these are the two solutions these are the two integration constants and after two integrations we get it, 'right'.

Now, it is very earlier as you have seen that when it was a solid cylinder, then we had taken at the center to be we had taken the center to be under constant boundary condition. That was the dT/dr is equal to 0, 'right', dT/dr is equal to 0; that means, this was assumed to be insulated. Similarly, for sphere also if it is a solid sphere then at the center there also, we can assume the same boundary that is if it is a solid dT/dr at r is equal to 0.

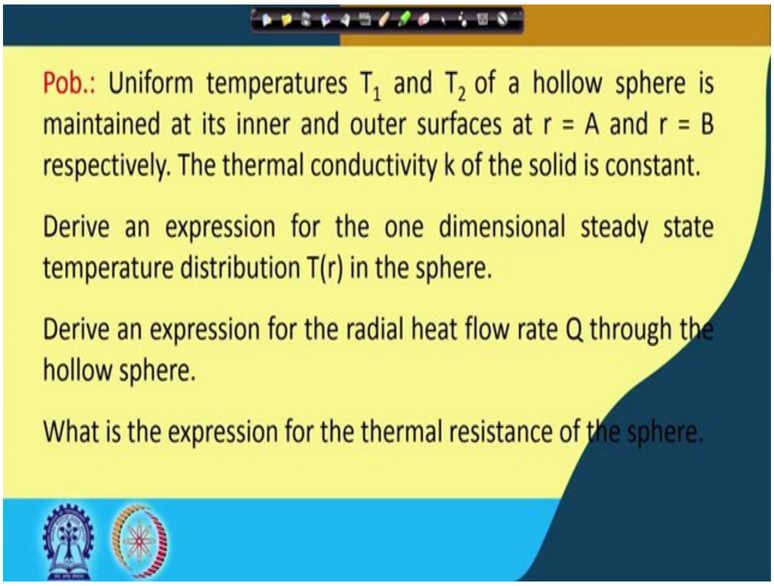
But, if it is a hollow sphere, 'right', then two dimensional picture will appear like this only if it is a hollow sphere, 'right'. So, the else it will have 1 a and it will have 1 a area in one radius another radius, 'right'. So, we have to get the boundaries also accordingly 'right'. So, for the cases of hollow cylinder one of the three types of boundary conditions, that is boundary condition of first kind, or boundary condition of second kind, or boundary condition of third kind, that is boundary condition under constant temperature condition, or boundary condition under constant heat flux boundary condition, or the boundary condition under convective boundary condition.

Any one of them or in combination can be the two boundaries on the hollow sphere 'right'. So, that will help in determining the two constants. But for solid spheres, like the solid cylinder just now as I said, one boundary is specified at the outer surface and another boundary at the center of the sphere.



And this can be given like this can be given like this that at r is equal to T(r) is equal to finite at r is equal to 0, that is one boundary, or it can be said in other way that dT/dr is equal to 0 at r is equal to 0, 'right'. Where your boundary though it is given, it is like that you have this and this is the direction of the heat flow and this is the radius solid, 'right'.

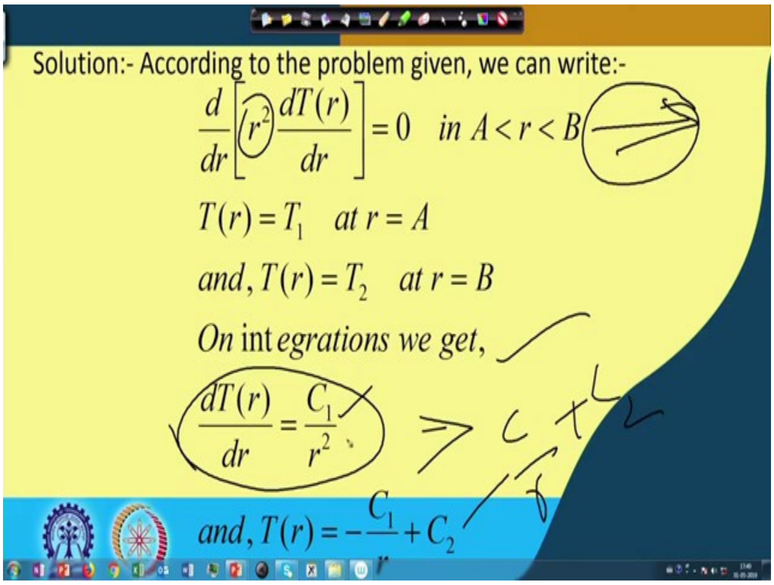
So, that is one and if it is for hollow then; obviously, you have a radius here a you have a radius here b, 'right' sorry a radius b and the direction of the flow is this, then we can depending on what are the boundaries you are given accordingly you can solve them, 'right'. So, it is easy as earlier also we have done for Cartesian, we have done for cylinder and now we are doing for the sphere, 'right'.



So, let us now do for the sphere that with a problem. Problem is like this uniform temperatures T_1 and T_2 of a hollow sphere is maintained at it is inner and outer surfaces at r=A and at r= B respectively. The thermal conductivity k of the solid is constant.

Derive an expression for the one dimensional steady state temperature distribution T(r) in the sphere; also derive an expression for the radial heat flow rate Q through the hollow sphere. And thought, what is the expression for the thermal resistance of the sphere?

So, I repeat uniform temperatures, T_1 and T_2 of a hollow cylinder hollow sphere is maintained at it is inner and outer surfaces, at r is equal to A and r is equal to B respectively. The thermal conductivity k of the solid is constant. Derive an expression for the one dimensional steady state temperature distribution T(r) in the sphere. Derive an expression for the radial heat flow rate Q through the hollow sphere. And, third what is the resistor, what is the expression for the thermal resistance of the sphere?



So, these three we can do easily from the experience what you have again, in case of rectangular coordinate or cylindrical coordinate, 'right'. Now for this, but the thing is similar that according to the given problem we can write d/dr, because here we are said that you have a hollow sphere, but steady state no internal energy generation. So, these conditions if it is applied and conductivity is constant, if these are applied then from the governing equation straightway we can write

$$\frac{d}{dr}\left[r^2\frac{dT(r)}{dr}\right]=0 \quad in\ A<r<B$$

$$T(r)=T_1 \quad at\ r=A$$

$$and,\ T(r)=T_2 \quad at\ r=B$$

On integrations we get,

$$\frac{dT(r)}{dr}=\frac{C_1}{r^2}$$

$$and,\ T(r)=-\frac{C_1}{r}+C_2$$

$$T_1=-\frac{C_1}{A}+C_2 \quad and, \quad T_2=-\frac{C_1}{B}+C_2$$

Solving these two simultaneous equations for C_1 *and* C_2

$$C_1=-\frac{AB}{B-A}(T_1-T_2) \quad and, \quad C_2=\frac{BT_2-AT_1}{B-A}$$

$$\therefore \quad T(r) = \frac{A}{r}\frac{B-r}{B-A}T_1 + \frac{B}{r}\frac{r-A}{B-A}T_2$$

The flow rate Q is determined as:

$$Q = \left(4\pi r^2\right)\left[-k\frac{dT(r)}{dr}\right] = \left(4\pi r^2\right)\left(-k\frac{C_1}{r^2}\right) = -4\pi k C_1$$

$$or, \quad Q = 4\pi k\frac{AB}{B-A}\left(T_1 - T_2\right) = \frac{\left(T_1 - T_2\right)}{R}$$

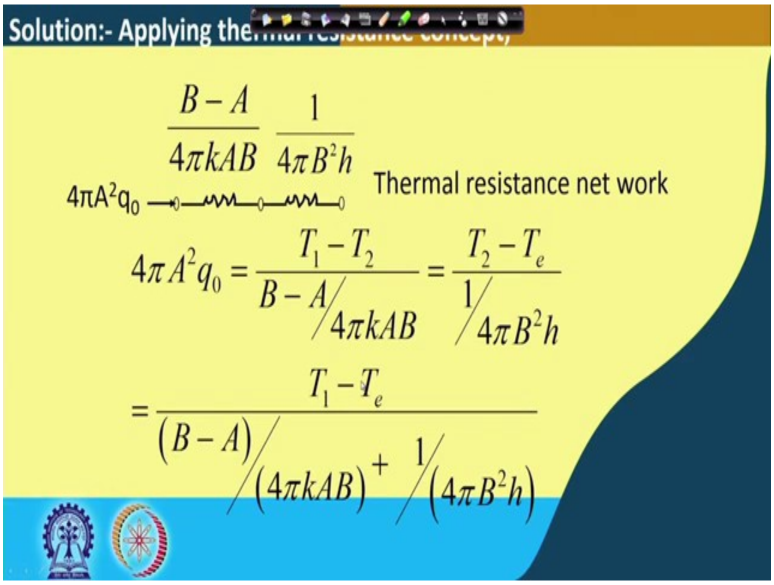
$$where, \quad R = \frac{B-A}{4\pi k AB}$$

Now, a hollow sphere of radius r is equal to A and outside radius r is equal to B is electrically heated at the inner surface at a constant rate of q_0 W/m^2. At the outer surface it dissipates heat by convection into a fluid at temperature T_e with a heat transfer coefficient h. The thermal conductivity k of the solid is constant.

Derive expressions for the determination of the inner and outer surface temperatures T_1 and T_2 of the sphere. Calculate the inner and outer surface temperatures for A is equal to 4 centimeter and B is equal to 6 centimeter, h is 500W/m^{2}ºC, T outside or ambient our environment T_e is equal to 90 ºC, conductivity of the material is 20 W/mºC and the heat flux q is equal to q_0 is equal to 10^5 W/m^2, 'right'.

So, we repeat quickly hollow sphere of inside radius r is equal to A and outside radius r is equal to B is electrically heated at the inner surface at a constant rate of q_0 W/m^2. At the outer surface it dissipates heat by convection in to a fluid at temperature T_e with high heat transfer coefficient h. The thermal conductivity k of the solid is constant. Derive expressions for the determination of the inner and outer surface temperatures T_1 and T_2 of the sphere.

Calculate the inner and outer surface temperatures for A is equal to 4 centimeter B is 6 centimeter h is 500 W/m^{2}ºC, T_e is 90 ºC, k 20 W/mºC and q_0, 10^5 W/m^2.



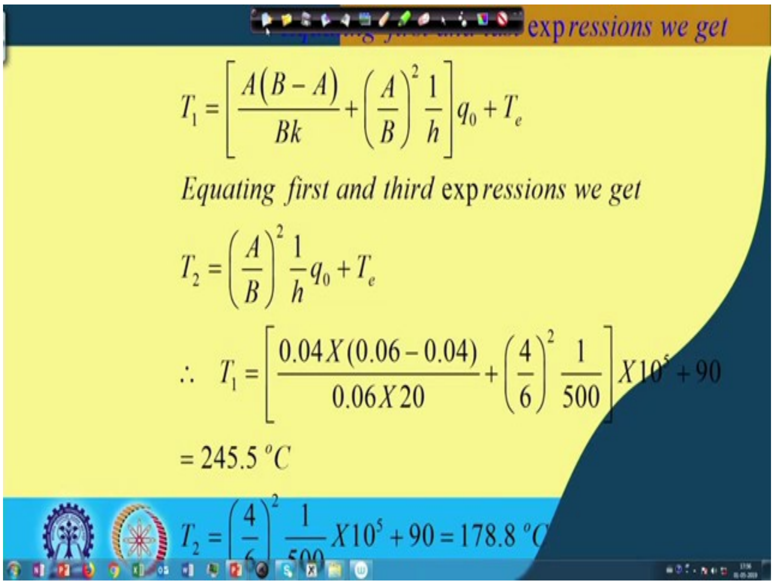
Then from the thermal analogy concept, or thermal resistance concept, or electrical analogy concept whatever we call, we can solve it like this, 'right'. (B-A) by that your first thing is your first thing is that this is the heat flux, which is getting in to the area. So, area is $4\pi A^2$ is the area or $4\pi A^2$ square is given here into q_0 the first. Then this Q is flowing through this resistance or metal resistance that is $\frac{B-A}{4\pi kAB}$, then it is also flowing through the other resistance that is $\frac{1}{4\pi B^2 h}$, 'right', that is the convective boundary, 'right'.

So, if this is true then by applying thermal resistance network, we can write

$$4\pi A^2 q_0 = \frac{T_1 - T_2}{B-A \big/ 4\pi kAB} = \frac{T_2 - T_e}{1 \big/ 4\pi B^2 h}$$

$$= \frac{T_1 - T_e}{(B-A) \big/ (4\pi kAB) + 1 \big/ (4\pi B^2 h)}$$

So, like the previous one we can here also say previous means I mean in the in the cylindrical coordinate where we did it similar.



$$T_1 = \left[\frac{A(B-A)}{Bk} + \left(\frac{A}{B}\right)^2 \frac{1}{h} \right] q_0 + T_e$$

Equating first and third expressions we get

$$T_2 = \left(\frac{A}{B}\right)^2 \frac{1}{h} q_0 + T_e$$

$$\therefore \quad T_1 = \left[\frac{0.04X(0.06-0.04)}{0.06X20} + \left(\frac{4}{6}\right)^2 \frac{1}{500} \right] X10^5 + 90$$

$$= 245.5\ ^oC$$

$$T_2 = \left(\frac{4}{6}\right)^2 \frac{1}{500} X10^5 + 90 = 178.8\ ^oC$$

178.8 degrees centigrade is the T_2. So, that whether it is coming correct or not 178.8 degrees centigrade or 245.5 degrees centigrade, please you check and then let us know whether there is any mistake or not, hopefully there should not be any mistake, 'right', but you can never tell because the calculator when you are using, 'right' there could be some.

So, you please check it and let us let me know with that the values are correct or not, but again on the thermal resistance concept that when we are using you have to keep in mind irrespective of the irrespective of the coordinate system, whether it is Cartesian coordinate, or cylindrical coordinate, or spherical coordinate means, whether it is a rectangle or slab or a cylinder or a sphere, wherever it is if there is no internal energy generation.

And, generally the conductivity is also constant, because if conductivity is not constant what it can be conductivity can be a function of temperature, ‘right’. Generally conductivity is a function of temperature, but for the given range of temperature many materials may not have any change in the conductivity or if they are really change that is so, in appreciable that it may not be considered. So, that is why conductivity constant taken is easier.

So, if it is taken then whenever you are doing it the there should not be any internal energy generation. So, once there is no internal energy generation, then you can do the thermal resistance concept and solve it, ‘right’. So, our today’s time is over. So, we will do in the next class subsequent problems and maybe this resistance because here we are doing all in series. So, there could be some resistances where it is not all series so, that we will try to do in the next class, ok.

Thank you.