Cooling Technology: Why and How utilized in Food Processing and allied Industries

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Module No 04

Lecture 17
Basics of Thermodynamics Contd.

Good morning my students. In the previous class, if you remember, at the end of the class, we were giving you lot of relations, which are basically non-dimensional, and used both heat transfer and mass transfer. And as I told you also there that in this basics of thermodynamics continuation class, some relations, I will be providing you, which are very very helpful. And as and when you come across you will be utilizing it more, say it becomes a ready reference right. So, some more relations are like this that Lewis number, this is L e that is alpha over d that is, ratio of mass diffusivity and thermal diffusivity, right ratio of mass diffusivity and thermal diffusivity, Lewis number alpha is the thermal diffusivity and d is the mass diffusivity right. L, a number that is R a this is shown as g beta delta L L cube over V a right.

This of course, as a very elongated definition that this is associated with buoyancy driven flow also known as free convection in natural convection, if be below the critical value not that fluid heat transfer is primarily in the form of conduction and it exceeds the critical value, heat transfer is primarily in the form of convection. So, depending on whether it is convective heat transfer, or conductive heat transfer this Rayleigh number gives a very very good idea right. Similarly, some other numbers, like Stanton number, S t this is of course, ratio of two non dimensional numbers, that is N u over R e in into P r, N u over R e over P r right. So, this can be that is N u is Nusselt number, R e is the Reynolds number, as of now we have seen and P r is the Prandtl number, and this can be said that this is the ratio of heat transfer into a fluid to the thermal capacity of fluid right.

Rayleigh number, another very important P e and of course, this is a product of two non dimensional parameters, and that is R e into P r. Obviously, as we have used earlier, R e is the Reynolds number and P r is the Prandtl number. So, this can be said, as ratio of the rate of advection of a physical quantity to physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient. Now, another very misleading very misleading, non dimensional number is Biot number B i, B i is known as Biot number or Biot number whatever, you call it, is B i o t. It is defined, as h d by k, if you remember, in the previous class we said Nusselt number, N u that was also h d by k and this is also h d by k then what is the difference? Obviously, there is a difference, otherwise, they would not have been the same number right in the Nusselt number where we said it was h d by k there, d in both the cases are diameter.

There h was the heat transfer coefficient of the convective medium, and k was also the conductivity, of the convective medium. All the properties were, with the heat transfer, from where, medium it is coming to the product or base whatever, but here it is coming into the body. So, h is the outside heat transfer coefficient, d is the diameter of the matter and k is the conductivity of the material. So, h d by k for this and h d by k for Nusselt number are absolutely different and this difference is in the thermal conductivity k one is of the medium or fluid another is of the body through which the heat is getting transferred. And this is defined as the ratio of internal conduction resistance and surface convective resistance to heat transfer.

You will see, if you have done heat transfer that, this Biot number is very very helpful in Hessler chart. Of course, we cannot bring it here, because that becomes really more, beyond the purview of the course, as such these, are also, but since all of you are our student. So, more information you have it is better off. Then Fourier number, again this is very much useful in heat transfer typically useful in Hessler chart, using the chart beard number, Fourier number these are very much and non-dimensional temperature, they are used in the chart of Hessler to find out the temperatures from non-dimensional temperature. So, it is defined as ratio of the heat conduction rate to the rate of thermal energy storage ratio of the heat conduction rate to the rate of thermal energy storage alpha T by L square right.

Another thing since it has come, let me also tell that this, L is the half of the length right if the thickness is say 2 L, then half is the L, and that is used right. So, you have to be careful, using that L, which one the entire L, or L by 2 or normally it is, if it is a symmetric body, is written plus minus L, right. So, that part you have to be careful. Now there are certain relations for heat transfer to find out the heat transfer coefficient inside the tube and many other cases. Obviously, for different conditions, it will be different, like if the flow is laminar then Nusselt-Grace correlation says that with parabolic velocity profile and constant wall temperature.

The relation is valid for thermal entrance length having parabolic velocity profile and constant wall temperature. So, in that case Nusselt number, x, Nu x, means at any position x, is given as 1.007 into peclet number into Di that is internal diameter over x to the power 1 by 3, right and this is valid for peclet into internal diameter over x equal to or greater than 10 to the power 2. And, is equal to 3.

66. If peclet number into internal diameter over x is less than 10 to the power 2, right.

So, another relation for this is Nu x, that is Nusselt number at any position x, is 1.61 into peclet number into internal diameter Di over L, L, obviously, is the length to the power 1 by 3, that is valid for peclet number into internal diameter over L and if it is greater than 10 to the power 2, and is equal to 3.66, if peclet number into internal diameter over L is less than 10 to the power 2. Obviously, the peclet number, Pe, that is equal to Reynold's number into Prandtl number right.

So, this is meaning, rho into D into V by mu into Cp into mu by k right. So, it becomes rho Cp V D by k because mu cancels out. Now, other relations, like Herson's correlation, this is valid for developing hydrodynamic and thermal boundary layer and for constant wall temperature. This is saying that Nusselt number average is equal to 3.

66 plus 0.0668 into internal diameter Di over L length into peclet number, Pe over 1 plus 0.04 into Di over L into peclet number to the power 2 by 3. And another relation could be Nusselt number average, equal to 3.66 plus 0.19 into peclet number into internal diameter over L to the power 0.

5 sorry 0.8 over 1 plus 0.0117 into the initial diameter Di over length L into peclet number to the power 0.467. Some relations are very useful and mostly it is used. One of them is Sieder-Tate equation or Sieder-Tate correlation.

This is very useful for pipe flow in terms of heat transfer and this is valid for constant wall temperature where, mu W is the viscosity at the wall temperature. If obviously, the wall temperature is changing the viscosity is also changing that is why it is valid for constant wall temperature for which the viscosity mu W that becomes also constant. Now, Nud based on diameter average, is equal to Nud equal to 1.86 into peclet number to the power 1.3 into Di over L to the power 1 by 3 peclet number to the power 1 by 3 not peclet number, is also to the power 1.

3 and the this is called non dimensional length Di over L, that it is also to the power 1 by 3 into the viscosity correction that is mu by mu W to the power 0.14. This tells that if there be any viscosity correction, because wall will have either higher or lower temperature than the inside. So, if you take the viscosity inside one then for the wall there will be some other temperature than the core temperature. So, there, the viscosity correction factor is introduced and that is mu by mu W to the power 0.

14. Another one is also very much used, that is called slender correlation. This is also valid for constant wall temperature and the relation is Nusselt number averaging equal to 3.66 to the power 3 plus 1.61 to the power 3 times peclet into individual or the internal diameter over length to the power 1 by 3. So, that is another relation, but again valid for

constant wall temperature.

For constant heat flux, I hope flux you know. In the earlier class, I had said flux. Flux means, anything or any parameter per unit time per unit area is the flux of that. If it is for flow of fluid it is called momentum flux. If it is for heat transfer it is called heat flux.

If it is for mass transfer, it is also there used as mass transfer flux right. So, that flux you have to use as the parameter per unit time per unit area. So, Nusselt number is 1.302 into peclet number into d i that is diameter of the internal pipe over x, at any point to the power 1 by 3 where, peclet number into initial diameter d over x is greater than 10 to the power 4 whereas, Nusselt number again based on the diameter d is 4.36, if peclet number into internal diameter d i over x is less than 10 to the power 3, whereas, Nusselt number is 1.

953 into peclet number into d i, that is initial diameter of the system over x to the power 1 by 3 and is valid for peclet into d initial over x and that should be greater than 10 to the power 2. For 10 to the power less than 10 to the power 2 the other relation is 4.36 where peclet number into internal diameter d over x is less than 10 to the power 2. Now, some more relations are like, if it is a annular passage right. So, you see this can be said annular passage, that this arc, you can see and this arc, you can see.

So, if they are 2, then this internal is called annular passage,right and there Stefan's correlation based upon the results of housen is like this, d h is equal to d o minus d r into Reynolds number is equal to u d h over v and Nusselt number is h d h over T a. And for that, we can write Nusselt number, equal to Nusselt number as a function of d i over d o times, or it can be said that not function it can be also said integral or cyclic integral is 0.19 peclet number into d i that is initial diameter over L to the power 0.8 by 1 plus 0.117 into peclet number into d i initial diameter over L to the power 0.

667 0.467 rather. Hence, if 0.1 is less than Prandtl number is less than 10 to the power 3, 0, d initial over d o outside is less than 1 and Re is also less than 2300, then Nusselt number equal to 3.66 plus 1.2 into d i over d o to the power 0.5. This is case 1 for outer wall of annulus is considered if you are and Nusselt number is 3 into 1.

2 into d into d o to the power 0.5. Then, it becomes case 2 and inner wall of annulus is insulated. The third one is Nusselt number is 3.66 into or rather plus 4 into 0.102 over d i over d a or d o rather it should be plus 0.2 and this is case 3, where none of the walls is insulated.

More relations are like this the function f that is d i over d o is as follows for the 3 cases case 1 is a function of d i over d o equal to 1 plus 0.14 into d i over d o to the power 0.5 case 2 is a function of d i over d o is equal to 1 plus 0.14 into d i over d o to the power 1 by 3 and case 3 is a function of d i over d o and is equal to 1 plus 0.

14 d i over d o to the power 0.1. Now, for turbulent flow the most common use is deters voltage equation in heat transfer. So, it is valid for fully developed turbulent flow and the properties are evaluated at the bulk temperature. Now, the basic equation is N u is equal to 0.023 into Reynolds number to the power 0.8 into Prandtl's number to the power N and this is valid for Prandtl number 0.

6 to 100. For heating N is 0.4 and for cooling N is 0.3 and the Nusselt number is h d i over k into Reynolds and Reynolds number is equals to rho d v i by into v by mu and Prandtl number is C p mu by k. So, if all are known then deters voltage equation is very very in full for turbulent flow. Sider-Tate correlation could be written as variation of viscosity with temperature is included.

Nusselt number is 0.036 into Reynolds number to the power 0.8 into Prandtl number to the power 1 by 3 into mu by mu w to the power 0.14, where mu w is the viscosity of the wall temperature. This means that viscosity is also a function viscosity is also calculated at the wall right. So, this is the last perhaps that Nusselt coordination is like N u equal to 0.

036 Reynolds number to the power 0.8 Prandtl's number to the power 1 by 3 mu by mu w which I have said many times that this is the wall correction in terms of viscosity d i over L that is you see all are non-dimensional this is also non-dimensional in a in a diameter to length and this is valid between 10 to 400, where mu w viscosity at wall temperature and all other parameters are evaluated at the bulk temperature right. So, with this let us stop today and hopefully we have completed the basics of thermodynamics including all these relations. Some more, many relations are there, but obviously, I do not want to make you or give so much pressure on it, because there are many other things, but these are very helpful relations, mind it, ok. So, thank you very much.

Lewis Number	$Le = \frac{\alpha}{D}$	Ratio of mass diffusivity and thermal diffusivity
Rayleigh Number	$Ra = \frac{g\beta\Delta t L^3}{va}$	associated with buoyancy driven flow (also known as free convection or natural convection), if be below the critical value for that fluid, heat transfer is primarily in the form of conduction and it exceeds the critical value, heat transfer is primarily in the form of convection
Stanton Number	$St = \frac{Nu}{Re\,Pr}$	Ratio of heat transferred into a fluid to the thermal capacity of fluid
Peclet Number	Pe = Re . Pr	Ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient
Biot Number	$Bi = \frac{hD}{k}$	Ratio of internal conduction resistance and surface convective resistance to heat transfer.
Fourier Number	$Fo = \frac{\alpha t}{l^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage

Heat transfer coefficient inside tubes:-

Laminar flow:- *Nusselt – Graetz correlation*:– valid for thermal entrance length with parabolic velocity profile and constant wall temperature.

$Nu_x = 1.007\,(Pe.\,D_i / x)^{1/3}$: $Pe.D_i / x > 10^2$

$= 3.66$: $Pe.Di / x < 10^2$

$\overline{Nu_x} = 1.61\,(Pe.\,Di / L)^{1/3}$: $Pe.Di / L > 10^2$

$= 3.66$: $Pe.Di / L < 10^2$

Where,

Pe is Peclet number: Pe = Re.Pr =

$$\left(\frac{\rho V D_i}{\mu}\right) X \left(\frac{C_p \mu}{k}\right)$$

Hausen's correlation:- valid for developing hydrodynamic and thermal boundary layer and for constant wall temperature

$$\bar{Nu}_d = 3.66 + \frac{0.0668(D_i/L)Pe}{1+0.04\left[(D_i/L)Pe\right]^{2/3}}$$

$$and,\ \bar{Nu} = 3.66 + \frac{0.19(Pe.D_i/L)^{0.8}}{1+0.117\left[(D_i/L)Pe\right]^{0.467}}$$

Sider Tate correlation:- valid for constant wall temperature, μ_w is the viscosity at the wall temperature.

$$Nu_D = 1.86\left(Pe^{1/3}(D_i/L)^{1/3}(\mu/\mu_w)^{0.14}\right)$$

Schlunder's correlaiton:- valid for constant wall temperature

$$\bar{Nu} = \left[(3.66)^3 + (1.61)^3(Pe.D_i/L)\right]^{1/3}$$

for constant heat flux

$$Nu_x = 1.302(Pe.D_i/x)^{1/3} \qquad : Pe.D_i/x > 10^4$$

$$Nu_D = 4.36 \qquad : Pe.D_i/x < 10^3$$

$$\bar{Nu} = 1.953(Pe.D_i/x)^{1/3} \qquad : Pe.D_i/x > 10^2$$

$$\bar{Nu} = 4.36 \qquad : Pe.D_i/x < 10^3$$

Annular Passage:- Stefen's correlation based upon the results of Hausen is $D_h = D_o - D_i$, $Re = UD_h/v$, and $Nu = hD_h/k$, and

$$Nu = Nu_\infty f\left(D_i/D_o\right)\frac{0.19\left(Pe.D_h/L\right)^{0.8}}{1.0+0.117\left(Pe.D_h/L\right)^{0.467}}$$

$$0.1 < \Pr < 10^3, 0, D_i/D_o < 1, Re < 2300$$

$Nu_\alpha = 3.66 + 1.2\,(D_i/D_o)^{0.8}$: Cae I:- Outer wall of annulus is insulated

$Nu_\alpha = 3.66 + 1.2\,(Di/Do)^{0.5}$: Case II:- Inner wall of annulus is insulated

$Nu_\infty = 3.66 + \left[4 - \dfrac{0.102}{\left(D_i/D_o\right)+0.2}\right]$: Case III:- None of the walls is insulated

The function $f(D_i/D_o)$ is as follows for the three cases:

Case I : $f(D_i/D_o) = 1+0.14\,(D_i/D_o)^{-0.5}$

Case II : $f(D_i/D_o) = 1+0.14\,(D_i/D_o)^{1/3}$

Case III : $f(D_i/D_o) = 1+0.14\,(D_i/D_o)^{0.1}$

for ***Turbulent Flow***: ***Dittus-Boelter equation***:-

Valid for fully developed turbulent flow. Properties are evaluated at the bulk temperature: $\mathbf{Nu = 0.023\,Re^{0.8}\,Pr^n}$ valid for Pr = 0.6 to 100. For heating : n =0.4, and for cooling : n = 0.3, and $Nu = hD_i/k$, $Re = \rho D_i v/\mu$, $\Pr = \frac{C_p\mu}{k}$

Sider and Tate correlation :- Variation of viscosity with temperature is included: $Nu = 0.036\,Re^{0.8}\,Pr^{1/3}\,(\mu/\mu_w)^{0.14}$ where, μ_w is viscosity at wall temperature.

Nusselt's correlation:- valid in the entrance region

$$Nu = 0.036\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\mu/\mu_w\right)^{0.14}\left(D_i/L\right)^{0.055}: \quad 10 < D_i/L < 400$$

where, μ_w is viscosity at wall temperature and all other properties are evaluated at bulk temperature.

Petukhov's correlation:- valid for fully developed turbulent flow:

$$Nu = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)}\left(\frac{\mu_b}{\mu_w}\right)^n$$

$0.5 < \mathrm{Pr} < 2000$, $10^4 < \mathrm{Re} < 5 \times 10^6$, $0.8 < \mu_b / \mu_w < 40$
$n = 0.11$ for $T_w > T_b$, $n = 0.25$ for $T_w < T_b$, and $n = 0$ for constant heat flux $f = (1.82 \log_{10} \mathrm{Re} - 1.64)^{-2}$. Property values are evaluated at $T_f = (T_w + T_b) / 2$.

Accuracy: $0.5 < \mathrm{Pr} < 200$: 6% and $200 << 2000$: 10%

Heat Transfer Coefficient from flow over a horizontal flat plate:- ***Laminar flow***:-

$Nu_x = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$: valid for isothermal plate

$Nu_x = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$: valid for constant heat flux

Turbulent flow:- $\overline{Nu}_L = \frac{\bar{h}L}{k} = \mathrm{Pr}^{1/3}\left(0.037\,\mathrm{Re}_L^{0.8} - 850\right)$: *Isothermal*

Critical Reynolds Number has been taken to be 500,000. For constant heat flux, $\overline{Nu}_L$ is 4% more than that of isothermal.

Free Convection heat transfer coefficient:- ***Isothermal case*** : Vertical flat plate and cylinder

$Nu = 0.55\,(\mathrm{Gr\,Pr})^{1/4}$: $10^4 < \mathrm{Re} < 10^9$
$\quad = 0.1\,(\mathrm{Gr\,Pr})^{1/3}$: $10^9 < \mathrm{Re} < 10^{13}$

For air, $Nu = 1.42\,(\Delta t / L)^{1/4}$: $10^4 < \mathrm{Re} < 10^9$
$\quad = 1.31\,(\Delta t)^{1/3}$: $10^9 < \mathrm{Re} <$

***Horizontal cylinder*:-**

Nu $= 0.53\ (Gr\ Pr)^{1/4}$: $10^4 < Re < 10^9$

$= 0.13\ (Gr\ Pr)^{1/3}$: $10^9 < Re < 10^{12}$

For air, Nu $= 1.32\ (\Delta t / d)^{1/4}$: $10^4 < Re < 10^9$

$= 1.24\ (\Delta t)^{1/3}$: $10^9 < Re < 10^{12}$

Upper surface of heated plate or lower surface of cooled plate, Nu $= 0.54\ (Gr\ Pr)^{1/4}$: $2 X 10^4 < Re < 8 X 10^6$

$= 0.15\ (Gr\ Pr)^{1/3}$: $8 X 10^6 < Re < 10^{11}$

For air, Nu $= 1.32\ (\Delta t / L)^{1/4}$: $2 X 10^4 < Re < 8 X 10^6$

$= 1.52\ (\Delta t)^{1/3}$: $8 X 10^6 < Re < 10^{11}$

Lower surface of heated plate or upper surface of cooled plate, Nu $= 0.27\ (Gr\ Pr)^{1/4}$: $10^5 < Re < 10^{11}$

For air, Nu $= 0.59\ (\Delta t / L)^{1/4}$: $10^5 < Re < 10^{11}$

***Constant heat flux*:-**

Vertical plate: $Nu_x = 0.6\ (Gr^*\ Pr)^{1/5}$: $10^5 < Gr^* < 10^{11}$

Horizontal cylinder: $Nu_d = 0.6\ (Gr^*\ Pr)^{1/4}$: $10^6 < Gr^* < 10^{13}$

where, $Gr^* = (g\beta q_w\ x^4) / (kv^2)$