Cooling Technology: Why and How utilized in Food Processing and allied Industries

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Module No 07

Lecture 33
Problem Solving with Carnot System

Good morning my dear students and friends. Do you remember, in the previous class, we have finished the Carnot Refrigeration System, its difficulties and one of its remedies, right. So, today let us do, a problem and its solution, right. Let us do a problem, and its solution, right and the problem is like this. Let me take it to the other side here, I could rightly, here, we are saying that, the compression ratio is 5, right, so that means, the difference between the two pressures, is 5.

So, one is 5 times more than the other. So, which, one it can be? We have already studied a lot, who is, what is compressor doing? Compressor is receiving, some earlier, it was liquid now, it is no longer liquid, it is vapour. So, from vapour, a pressure of evaporator pressure, of P e to a condenser pressure of P c. It is compressing, right. So, that is what we can say, the compression ratio is 5 right.

So, what we are seeing, here is that, let me take it to this side a little, that becomes easy for you yeah. So, if we have taken to this side a little, then the problem is like this, in a reversed refrigeration Carnot refrigeration system, of one ton of refrigeration cooling capacity, running on perfect gas. Heat is absorbed at minus 10 degree centigrade and rejected at plus 50 degree centigrade. Find the states at all the points of the cycle, heat transfer and work done in all the processes, mass flow rate, volume flow rates and the COP.

So, so many things, we have been asked to do. The maximum pressure, at inlet to the isentropic compression or compressor is standard atmospheric pressure. We have been given, some values like C p is 1.005 kilo joules per kg kelvin, universal gas constant R as 0.287 kilo joules per kg kelvin and the heat capacity ratios that is gamma to be 1.4. Basically, you know that, for diatomic gases, the heat capacity ratio, is 1.4. However, to understand the problem, let us repeat the problem. It is saying that, the compression ratio is 5, which we have just discussed. In a reverse Carnot refrigeration system of 1 ton of refrigeration cooling capacity running on perfect gas heat is absorbed at minus 10 degree centigrade and rejected at 50 degree centigrade.

Find the states at all the points of the cycle, and also heat transfer and work done in all the processes, mass flow rate, volume flow rates and the COP. The maximum pressure at inlet to the isentropic compression is standard atmospheric pressure. Standard atmospheric pressure means, one atmospheric pressure, that we can also convert it into bar, you know there are many ways of expressing the pressure. So, one other unit, could be bar one is atmosphere, one could be millimeter of mercury, things like that. So, we are also given with some values like Cp is 1.005 kilo joules per kg kelvin and universal gas constant R as 0.287 kilo joules per kg kelvin. The heat capacity ratios that is gamma is 1.4. So, hopefully, you will solve it your before looking into the solution, which, we are providing, or which, I am providing.

I hope, you will do this first, and then, look into the solution, right. So, first thing, which we do is that, what are the things given? Our things given is p1 is 1.01325 bar, 1.01325 bar, that is the standard atmospheric pressure. You are also given, T1 as minus 10 degree centigrade, that is 263 kelvin, right 263 kelvin.

Kelvin has come out here so, 263 kelvin and T2, we are also given as 50 degree centigrade, which, we can convert into kelvin as this is by mistake, cut and paste, this is 263. So, it was again 263 right. So 273 plus minus 10 is 263, but 273 plus 50 is equal to 323. So 323, perhaps, afterwards, I have corrected it exactly right. So, it is not 263 it is 323 kelvin right.

We are given that, the pressure ratio is p2 is to p1 is equal to 5. Now, I tell you that, what we have done in T-s diagram. If you remember that, we have this dome, like this, right. We started with here, as point 1, went to this place at point 2, or we have been asked, we have been said, rather, if you remember, if you look at, if you look at this, that in a reversed Carnot refrigeration system, right. Already, we have been given, a reversed Carnot refrigeration system, no modifications have been said, right. So, that means, instead of, instead of that, we can say, we can say that, we have, sorry, we have this T-s diagram, and we have the dome like this,, and we started with here, there, there, there, this was point 1, this is point 2, this is point 3 and this is point 4, right.

And we have been asked that, p2 is to p1 is 5 right, p2 is to p1 is 5. This is the p, this is the p diagram, and this is the, this is the p diagram, by p1, and this is the p2 diagram, right. So, p2 to p1 is 5, that is given, and this is by mistake, I have made to 63, but it is 3 2 3 Kelvin, that we have just shown, we have been given p2 is to p1 is 5. So, we can write p2 is to p1 is equal to T2 is to T1, to the power gamma by gamma minus 1, that pV gamma is constant, from there right. So, p1 V1 to the power gamma by T1 is equal to p2 V2 to the power gamma by T2. So, from there, we can write that, p2 is to p1 is equal to T2 is to T1 to the power gamma by gamma minus 1, right.

So, this we have to, every time whenever, we are clearing 3 2 3 Kelvin. So, it is T2 is also known T1 is known. So, 323 over 263, here we have corrected, that is, to the power gamma that is 1.4 by 1.4 minus 1. So, it is 2.05288 is the p2 is to p1 ratio, right and it is said that p2 is to p1 is 5. So, p2 can be written as 2.05288 into 1.01325. So, it is 2.08 bar right. Now, we also know that p3 is to p2 is nothing, but p3 is to p1, over p2 is to p1, right. So, this is p0 p3 p2 is to p1 over, no p3 is to p1 over p3 is to p1, over p2 is to p1 right. And you know p3 and p2 are same, this was, this was 0.1. This was for 0.2, and after isothermal, this is also 0.3. So, the pressure remains same, however, that is different, we are not looking into that, now. So, we can write it is, 5 is to, sorry, 5 is to 2 5 is to 2.03 2.05 over, my goodness, 5 is to 2.05288. So, this is 2.4356, and this is also same as p4 is to p1 right.

Therefore, p3 is 5 times 1.01325, that is 5.06625 bar, and p4, can be written as 2.4356 into 1.01325, that is 2.4678717 bar right. So, we can say that q1 to 2, that is, this point q1 to 2 is 0 right. So, we can now say, that, minus W1 to 2 is H2 minus H1, which, we have shown earlier, utilizing it now, is Cp into T2 minus T1, and this is equal to 1.005 into 60 that is 60.3 kilo Joules per kg. Then q2 to 3 is W 2to 3. It is minus RT2 ln p3 over p2, and this is equal to minus 0.287 into 323 into ln of 2.4356, this comes equal to 82.52179 kilo Joules per kg right. So, we can write now, that W3 to 4 is H3 minus H4. So, that is equal to Cp times T3 minus T4 and that is, 60.3 kilo Joules per kg and therefore, again q3 to 4 is also 0.

So, 1 to 2 was this and 3 to 4 was this, right. So, this was 1, this was 2, this was 3, and this was 4. So, q3 to 4 is also 0, then, we can write COP is, as it is defined, COP is T1 over T2 minus T1. So, that is 263 by 323 minus 263 and that comes to be 4.383. It is a very high COP 4.383. For a system of 1 ton of refrigeration capacity, we can write, this is m dot q4 to 1 is 1 ton of refrigeration, because, this is giving this one, 4 to 1 right. So, that 4 to 1 is that refrigeration effect, and that is 1 ton right. So, and this is 3.5167 kilo Watt right. So, if that be true, then we can say that the mass flow rate m dot is 3.5167 over 67.19266. So, this is 0.05233 kilo Joules per second right. Therefore, the volume flow rate, at state point 1, we can write is m dot RT1 by P1 and this on substitution of the values, we can say, it is 0.03898 meter cube per second. Then, we can also say that, volume flow rate at state point 4. So, at state point 1 that is, here at state point 1, the volume flow rate was 0.03898 and at state point 4, that is, this, the volume flow rate is m dot RT4 over P4. So, this on substitution of the values is 2 4 6.78717 is P4.

So, is 0.01689 meter cube per second. So, we can write that, specific volumes at various points, and they are like this V1 is RT1 by P1 and that is equal to 0.287 into 263 by 101.325.

So, it is 0.7449 meter cube per kg right. Similarly, V2 is RT2 by P2 and is 0.287 into 323 over 208 that is 0.445 meter cube per kg right. Now, the third one is volume V3.

So, V3 is RT3 by P3. So, it was 0.287 into 323 over 506.625 that is, 0.18297meter cube per kg. And V4 is RT4 by P4 that is 0.287 into 263 over 246.78717. This is equal to 0.30585meter cube per kg. So, specific volumes are also found out. We have also found out COP. If you remember, what are the things, you are asked to find out, if you remember, what are the things you are asked to find out, is like thi,s that we are asked to find out, what are the that, find the state points, at all the points, of all the cycle, and heat transfer rate and work done in all the processes. Now, work done we have said, heat transfer will be doing, and state points finding out, will be doing, but mass flow rate, we have done, volume flow rates, we have done, COP we have done, maximum pressure at inlet to the isentropic compression, it is already given.

So, many of them we have already done right. Now, we have to find out the state points and we have to find out the state points, and this is the reverse Carnot cycle, which we are referring to right, which we are referring to is this, that, Q 4 to 1, we have said it to be 0, W 4 to 1, Q 1 to 2 is equal to also 0, we have said, we have found out, Q 3 to 4 is 0 right. So, W 1 to 2, W 2 to 3 and W 3 to 4, these things we have found out, and W 4 to 1 right. So, this is the reverse Carnot cycle which we have done with two compressors. Now the thing which we need is to find out the state points right. If we plot in a T-S diagram then you see that, dome is here this is the triple point, we have from point 1, it is P 1, point 2 it is P 2, point 3 it is P 3 and point 4 it is P 4, right and we have been also given T 1 is 263 and T 2 is 323 right.

So, all the pressures and temperatures, we know. So, two properties, we know P 1 and T 1 and the other one P 2, T 2, P 3, T 2 and P 4, T 1, we know. So, we can draw the T-S diagram right. So, we have done the T-S diagram, like this, and if we want to do it to be P-V diagram, P-V diagram, then unfortunately, perhaps, I could not do it properly, because this was manually done by me right. So, obviously, V 1, V 2, V 3 and V 4 are done, you please check whether, V 3 and V 4 are in the right place, or all are in the right place, right, then, all Ps are, they are known. So, if you plot a P-V diagram, you take a P-V diagram, right, you know the values of P 1, V 1, you know the values of P 2, V 2, you know the values of P 3, V 3, and you know the values of P 4, V 4, right.

So, all the values you know. So, take a graph paper and solve it and draw it properly, maybe here, I could not have drawn because, this was drawn by me, and obviously, maybe not that much conversant with this kind of drawing in this kind of board. So, I have a doubt, whether this diagram is correct or not, please, you draw P-V diagram, and see how the processes are getting changed, right, and show all the state points 1, 2, 3, 4 and you know P-Vs. So, you can identify, all 4 points. So, this way you can obviously, find out the P-V diagram ok. So, perhaps, with this we complete the ideal situations that is Carnot's.

Now, afterwards, we will go to the actual, which is in the market, or in your commercial level, things are being done, ok. So, thank you all.

Sol:- Given, p_1 = 1.01325 bar, T_1 = -10 °C = 263 K, T_2 = 50 °C = 263 K

$$\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{\gamma}{\gamma-1}} = \left[\frac{323}{263}\right]^{\frac{1.4}{1.4-1}} = 2.05288$$

Since p_2 / p_1 = 5, hence,

So, p_2 = 2.05288 X 1.01325 = 2.08 bar

$$NOW, \frac{p_3}{p_2} = \frac{p_3/p_1}{p_2/p_1} = \frac{5}{2.05288} = 2.4356 = \frac{p_4}{p_1}$$

p_3 = 5 X 1.01325 = 5.06625 bar, and p_4 = 2.4356 X 1.01325 = 2.4678717 bar. ∴ Q_{1-2} = 0

$- W_{1-2} = h_2 - h_1 = C_p (T_2 - T_1)$ = 1.005 (60) = 60.3 kJ / kg

$Q_{2-3} = W_{2-3} = - RT_2 \ln (p_3 / p_2)$ = - 0.287 (323) ln (2.4356) = 82.52179 kJ / kg

$W_{3-4} = h_3 - h_4 = C_p (T_3 - T_4)$ = 60.3 kJ / kg, ∴ Q_{3-4} = 0

COP = $T_1 / (T_2 - T_1)$ = 263 / (323 – 263) = 4.383

For a system of 1 TR cooling capacity, $\dot{m} q_{4-1} = 1\,TR = 3.5167 kW$

$$\dot{m} = \frac{3.5167}{67.19266} = 0.05233\ kg/s$$

Volume flow rate at state 1 $\frac{(\dot{m}) RT_1}{p_1} = \frac{0.055233 X 0.287 X 263}{101.325} = 0.03898\ m^3/s$

Volume flow rate at state 4 $\frac{(\dot{m}) RT_4}{p_4} = \frac{0.055233 X 0.287 X 263}{246.78717} = 0.01689\ m^3/s$

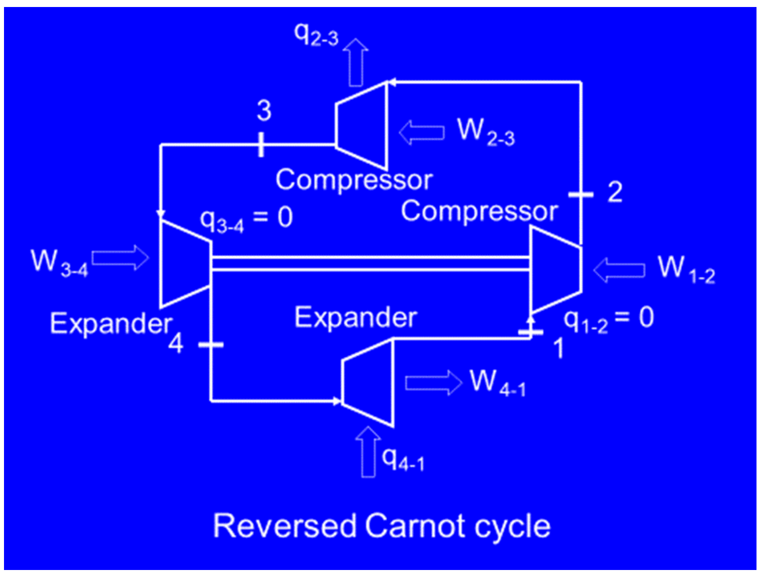
Specific volumes at various points are:

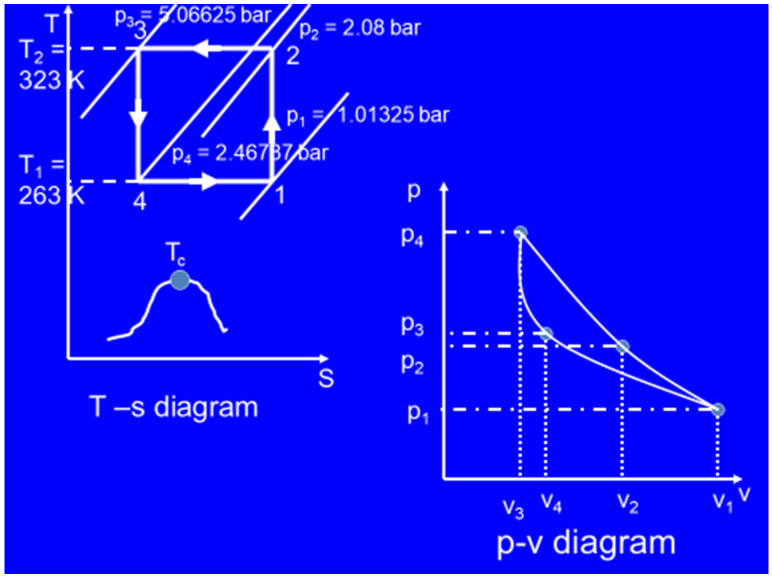
$$v_1 = \frac{RT_1}{p_1} = \frac{0.287X263}{101.325} = 0.7449\ m^3/kg$$

$$v_2 = \frac{RT_2}{p_2} = \frac{0.287X323}{208} = 0.44567\ m^3/kg$$

$$v_3 = \frac{RT_3}{p_3} = \frac{0.287X323}{506.625} = 0.18297\ m^3/kg$$

$$v_4 = \frac{RT_4}{p_4} = \frac{0.287X263}{246.78717} = 0.30585\ m^3/kg$$





T
p_3 = 5.06625 bar
p_2 = 2.08 bar
T_2 = 323 K
3
2
p_1 = 1.01325 bar
p_4 = 2.46737 bar
T_1 = 263 K
4
1
T_c
S
T –s diagram
p
p_4
p_3
p_2
p_1
v_3
v_4
v_2
v_1
v
p-v diagram