

Cooling Technology: Why and How utilized in Food Processing and allied Industries

Prof. Tridib Kumar Goswami
Department of Agriculture Engineering
Indian Institute of Technology, Kharagpur

Module No 10

Lecture 49 Condenser (Contd.)

So, good afternoon my dear students and friends. We are doing condenser and we have done, though, it was, it is not due for condensers, because, the flow type, that comes under heat exchangers, as I told you that, I will try my best to give you as much information as possible. So, that is why, the flow type, depending on that, the heat exchangers are also different, that we have already shown. Now, we come back to condensers, under continued condition classes, right. So, we come to water cooled condensers. We come to water cooled condensers, and there we can get double tube type, shell and coil type, shell and tube type, whether, it is horizontal or vertical and evaporative condensers, right.

So, heat transfer related to water cooled condensers are like this, similar to air cooled condenser, log mean temperature difference is still valid, since, it is assumed that condensation occurs. Since, it is assumed that condensation occurs throughout the length of the condenser, and refrigerant temperature remains constant, that is, hot fluid, if water, and the refrigerant are in counter flow. Then we can write that Q is equal to rather Q is equal to $U_o A_o \Delta T_m$ which is $U_o A_o \Delta T_m$, means ΔT_2 minus ΔT_1 over \ln of ΔT_2 by ΔT_1 , or log min temperature difference, that can be told as ΔT_m , that is ΔT_m , log mean is ΔT_2 minus ΔT_1 over \ln of ΔT_2 by ΔT_1 right. Now, if we see, obviously, ΔT_m , this, I am not going to repeat because we have already said.

$$Q = U_o A_o \Delta T_m = U_o A_o \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}; \text{ or, } LMTD = \Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

where, $\Delta T_2 = t_c - t_{wo}$ and $\Delta T_1 = t_c - t_{wi}$; t_c = condenser temperature, t_{wi} = water inlet temperature, t_{wo} = water outlet temperature. Over all heat transfer coefficient U_o is

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{A_o}{A_i} \frac{r_i \ln\left(\frac{d_o}{d_i}\right)}{k_w} + \frac{1}{h_o}}$$

where, A_o = Outside area of water tube, A_i = inside area of water tube, h_o and h_i are outside water tube and inside water tube heat transfer coefficients respectively, r_i , d_i are inside water tube radius and diameter respectively, d_o is outside water tube diameter, k_w is thermal conductivity of water tube material. h_o can be obtained from heat transfer coefficient for condensation outside horizontal tube as:

$$h_o = 0.725 \left[\frac{k_{rf}^3 \rho_{rf} g h_{fg}}{N D_o \mu_{rf} \Delta t} \right]$$

where, rf = saturated liquid, properties are evaluated at $(t_{wo} + t_f) / 2$; N = average number of tubes per column. Inside heat transfer coefficient

Now, in this, there is a overall heat transfer coefficient, that is U_o , that outside overall heat transfer coefficient, we can say, that is equal to $1 / (1 / (h_i A_i) + A_o / (A_i d_i) + A_o / (A_i \ln(d_o / d_i)) + 1 / (h_o))$. Similarly, A_o is the outside area of water tube, A_i is the inside area of water tube, h_o and h_i are the outside water tube and inside water tube heat transfer coefficients, respectively, r_i , d_i are inside water tube radius and diameter respectively, d_o is outside water tube diameter, k_w is the thermal conductivity of water tube material. h_o can be obtained from heat transfer coefficient for condensation outside horizontal tube, and this can be obtained as h_o , outside heat transfer coefficient, that is 0.725 into this is a relation, k_{rf} to the power 3, ρ_{rf} into g into h_{fg} , rather, over N into D_o into μ_{rf} into ΔT . Obviously, rf is the saturated liquid properties, which are calculated at the average temperature of t_{wo} and t_f , right, and N is the average number of tubes per column inside heat transfer coefficient, and h_i can be calculated from this Sieder-Tate equation for laminar flow, like Nu is equal to $0.036 Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$. When, the flow is turbulent, this new average, which I had given you earlier also, no point of saying it again here, because, we are running out of time. So, what we can say that, cost of water, the total running cost of refrigeration system is the sum of the cost of compressor power and the cost of water, which includes the cost of municipal water, or the cost of running a cooling tower. We have seen that, the compressor power increases, as the condenser temperature or the pressure increases, for a fixed evaporator temperature, water from a cooling tower is usually available at a fixed temperature, which, is equal to the wet bulb temperature of air plus the approach temperature of the cooling tower.

$$Nu = 0.036 Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

when the flow is turbulent

$$\bar{Nu}_d = 3.66 + \frac{0.0668 \left(\frac{D_i}{L} \right) Pe}{1 + 0.04 \left[\left(\frac{D_i}{L} \right) Pe \right]^{2/3}}$$

Assignment:- Determine the length of tubes in a two – way pass 10 TR shell and tube water cooled condenser with 48 tubes arranged in 12 columns and R22 as refrigerant. The heat rejection ratio is 1.3. The condensing temperature is 40 °C. The water inlet and outlet temperatures are 23 and 30 °C respectively. The tube inner and outer diameters are 12 and 14 mm respectively. The average properties of the refrigerant and water are as follows:

Water

$$\mu_w = 7.5 \times 10^{-4} \text{ kg / m s}$$

$$k_w = 0.7 \text{ W / m K}$$

$$\rho_w = 1000 \text{ kg / m}^3$$

$$c_{pw} = 4.2 \text{ kJ / kg K}$$

$$1/h_s = 0.000176 \text{ m}^2 \text{ K / W} \quad k_{\text{copper}} = 390 \text{ W / m K}$$

$$h_o = 0.725 \left[k_f^3 \rho_f^2 g h_{fg} / (N d_o \mu_f \Delta t) \right]^{0.25}$$

R22

$$\mu_w = 1.8 \times 10^{-4} \text{ kg / m s}$$

$$k_{rf} = 0.08 \text{ W / m K}$$

$$\rho_{rf} = 1100 \text{ kg / m}^3$$

$$h_{fg} = 165 \text{ kJ / kg}$$

$$Nu = 0.023 Re^{0.8} Pr^{0.4},$$

Ans.:- Heat rejection in the condenser for a 10 TR plant, $Q_c = 1.3 \times 10 \times (211 / 60) = 45.7 \text{ kW}$. This heat is rejected to water. The temperature of water goes up by 7 oC. The specific heat of water is given and hence the mass flow rate of water can be found out. Water passes through 24 tubes at a time with a mass flow rate, say, . Then,

$$Q_c = \dot{m}_w c_{pw} \Delta t_w = \dot{m}_w \times 4.2 \times (30 - 23) = 45.7 \text{ kW}$$

$$\text{or, } \dot{m}_w = 45.7 / (4.2 \times 7) = 1.55 \text{ kg / s}$$

$$\therefore \text{Water flow per tube, } \dot{m}_{wt} = 1.55 / 24 = 0.065 \text{ kg / s}$$

$$\text{Reynolds number, } Re = \frac{4 \dot{m}_w}{\pi d_i \mu_w} = 4 \times \frac{0.065}{3.14 \times 0.012 \times 7.5 \times 10^{-4}} = 9200.3$$

Since, Reynolds number is greater than 2300, hence, the flow is turbulent and the inside heat transfer coefficient hi may be found by the Dittus – Boelter equation

$$\text{Pr} = \frac{C_{pw} \mu_w}{k_w} = \frac{4.2 \times 1000 \times 7.5 \times 10^{-4}}{0.7} = 4.5$$

$$\text{Nu} = \frac{h_i d_i}{k_w} = 0.023 (9200.3)^{0.8} (4.5)^{0.4} = 62.24$$

$$\therefore h_i = 62.24 \times \frac{0.7}{0.012} = 3630.67 \text{ W / m}^2 \text{ K}$$

The refrigerant condenses outside the tubes. 48 tubes are in 12 columns. Hence, there are 4 tubes (48 / 12) in each column. So, N = 4.

$$\begin{aligned} \therefore h_o &= 0.725 \left[\frac{k_{rf}^3 \rho_{rf}^2 g h_{fg}}{N D_o \mu_{rf} \Delta t} \right]^{1/4} \\ &= 0.725 \left[\frac{0.08 \times 1100 \times 9.81 \times 165 \times 1000}{4 \times 0.014 \times 1.8 \times 10^{-4} \Delta t} \right]^{0.25} \\ &= 0.725 \left[\frac{1.4 \times 10^{13}}{\Delta t} \right]^{0.25} \\ \text{or, } h_o &= \frac{0.725 \times 1938.85}{\Delta t^{0.25}} = \frac{1405.66}{\Delta t^{0.25}} \end{aligned}$$

As the compressor temperature increases, the overall log mean temperature increases. As a result of a lower mass flow rate of cooling tower, or cooling, rather, cooling water is required as a result, low mass flow rate is required. This reduces the cost of water at highest condenser temperatures, or higher condenser temperatures. From the figure of variation of total running cost of refrigeration system with condensing pressure and hence, temperatures, it can be observed that, there is a condenser pressure, at which the running cost is minimum, and it is recommended that the system should be run at this pressure. This process, this poses there are some problem during winter operations. The complete analysis of the cost should include, the initial cost of the whole system, the interest on capital, the depreciation, the maintenance cost, the operator cost, and so on. The final selection of the system and the operating conditions should be such that the cost is less over the running life of the system.

Now, this could have been an assignment, but, you have seen, right, hopefully, you have taken a picture, and this is the properties, values, so you can find out the problem solution. I am not going to it. Yes, here, I have also given the answer, obviously, and you

have seen, how we have done this, and please, you do it at your home, right. This is, I am not repeating. Prandtl number, Nusselt number, then, internal heat transfer coefficient, all these, we have formulated, we have calculated outside heat transfer coefficient, that also, we have found out, and ultimately, we have come to some solution. Now, very quickly, we shall go to the extension of the surface, right, this is part of, this is not there, in the condenser, but this may be both, in the condenser, as well as the evaporator, that is why, it is, its function is that, if it is a condenser tube, and if the area, by chance, you are not able to improve, then, what you do, you add some surface, like this, and the area of heat transfer is increased, that is why, this extension of the area, to know, it is very important, and that is why, I have not left it, I have included it in this. So that, you know, this is true for both condenser as well as evaporator, this is more likely to do in evaporator, than that of the condenser, but still, I am continuing it, because, it is useful for both, ok. So, here, we have taken one pictorial view, not pictorial, this is a schematic drawing, where, as you see that, the surface, as you see that, this is the main thing, right. This is the main thing, and here, we are extending the area by this, right we are extending this area, this is a rectangular piece, but, we have also done for the cylindrical piece, right.

So, for both the things, what is the base, there are some preconditions, like, temperature is a function of x , or rather, yes, it is a function of x , here, x is the distance of the fin, right and we have taken a small volume of that, where, the thickness of the fin is ' t ', and the length is Δx , in the direction of x , right, which, has a perimeter of p , and a cross sectional area of A , and the base is always at T_0 temperature base, means, where, it is connected, right, this is the fin, and this is the wall, or whatever. So, it is connected there, if it is a tube, then, this, the base is this one, right, like, if this is the extended surface, this, one, then, this is the base, ok. Then, what we do, we solve it, with an assumption, that it is one dimensional steady state heat transfer for fins of uniform cross section, and the governing energy equation is like this, that, the net rate of heat gain by conduction in x direction, into volume element Δx , plus net rate of heat gain by convection through internal surfaces, into volume element Δx , is equal to 0 right. Then, we can say that, the net gain by convection, is one, which, we have denoted, that is, minus $d dx$ of $q A$ into Δx , right, that is equal to $k A d^2 T x dx$ square, by defining q as minus $k A d T dx$, right. So, it becomes $k A d^2 T x dx$ square into Δx , and net heat gain by convection, that can be written as h into T_e minus T_x into p into Δx , right, where, the cross sectional area A , the perimeter, p the heat transfer coefficient h , and the thermal

conductivity of the fin material k are constant.

The net heat gain by convection is

$$I = -\frac{d(qA)}{dx} \Delta x = kA \frac{d^2T(x)}{dx^2} \Delta x$$

The net heat gain by convection is

$$II = h[T_e - T(x)]P\Delta x$$

Where, the cross sectional area A , the Perimeter P , the heat transfer coefficient h , and the thermal conductivity of the fin material k are constant.

∴ The governing equation can be written as,

$$\frac{d^2T(x)}{dx^2} - \frac{hP}{Ak}[T(x) - T_e] = 0$$

This can be rearranged in a more compact form

as: $\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \dots(A)$

$$\text{where, } m^2 = \frac{hP}{Ak}$$

$$\text{and, } \theta(x) = T(x) - T_e$$

Equation (A) is known as one dimensional fin equation for fins of uniform cross section and is a linear, homogeneous, second-order ordinary differential equation with constant coefficients.

General solution of equation (A) is

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx} \quad \text{for long fins}$$

Alternatively,

$$\theta(x) = C_1 \cosh mx + C_2 \sinh mx$$

$$\text{or, } \theta(x) = C_1 \cosh m(L-x) + C_2 \sinh m(L-x)$$

for fins with finite length

For determining the constants, C_1 and C_2 , two boundary conditions, one at the fin base and the other at the fin tip are required to be known. Customarily, the temperature at the fin base $x = 0$ is considered known

The fin base condition is: $\theta(0) = T_0 - T_e = \theta_0$
Where, T_0 is the fin base temperature.

Several different conditions may arise, such as (a) long fin, (b) negligible heat loss from the fin tip, and convection at the fin tip.

Long fin

For a sufficiently long fin, it can be assumed that the temperature at the tip of the fin approaches the temperature T_e of the surrounding fluid. Then, the mathematical formulation for one dimensional steady state heat transfer in a long fin can be written as:

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } x > 0$$

$$\theta(x) = T_0 - T_e = \theta_0 \quad \text{at } x = 0$$

$$\text{and, } \theta(x) \rightarrow 0 \quad \text{as } x \rightarrow \infty$$

$$\text{where, } m^2 = \frac{Ph}{Ak}$$

We take the solution in the form

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx}$$

Applying the second boundary condition

$$\text{we get, } C_2 = 0$$

Applying the first boundary condition

$$\text{we get, } C_1 = \theta_0$$

Hence, the solution of the differential equation

can be written as:- $\theta(x) = \frac{T(x) - T_e}{T_0 - T_e} = e^{-mx}$

The heat flow through the fin can be determined as:

$$\theta_0 = \frac{T_0 - T_e}{T_0 - T_e} = e^{-mx} \quad \text{or, } \theta(x) = \theta_0 e^{-mx}$$

(1) Either by integrating the convective heat transfer over the entire fin surface as:

$$Q = \int_{x=0}^L h p \theta(x) dx \quad \text{or, by evaluating the heat flow at the fin base as:}$$

$$Q = -Ak \frac{d\theta(x)}{dx} \Big|_{x=0}$$

Results obtained from these two equations are identical since heat flow through the lateral surfaces by convection is equal to the heat flow at the fin base by conduction. Assuming the conduction equation and with the help of temperature distribution obtained as $\theta(x)$, we can write the heat flow through the fin as:

$$Q = Ak\theta_0 m = \theta_0 \sqrt{PhkA} \quad W$$

$$\text{since, } m = \sqrt{\frac{Ph}{Ak}}$$

Fins with negligible heat loss at the tip:-

Heat transfer area at the fin tip is generally small compared with the lateral area of the fin for heat transfer. Under these situations, the heat loss from the fin tip is negligible compared with that from the lateral surfaces, and the boundary condition at the fin tip characterising this situation can be taken as $d\theta/dx = 0$ at $x = L$. The mathematical formulation of this fin problem can be written as,

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } 0 \leq x \leq L$$

And, the boundary conditions are:

$$\theta(x) = T_0 - T_e = \theta_0 \quad \text{at } x = 0$$

$$\text{and, } \frac{d\theta(x)}{dx} = 0 \quad \text{at } x = L$$

Let us choose the solution in the form as

$$\theta(x) = C_1 \cosh m(L-x) + C_2 \sinh m(L-x)$$

Applying the second boundary condition

$$\text{we get, } C_2 = 0$$

and, putting the first boundary condition

$$\text{we get, } C_1 = \frac{\theta_0}{\cosh mL}$$

The solution can be written as

$$\frac{\theta(x)}{\theta_0} = \frac{T(x) - T_e}{T_0 - T_e} = \frac{\cosh m(L-x)}{\cosh mL}$$

The heat flow rate Q through the fin is

$$Q = Ak\theta_0 m \tanh mL = \theta_0 \sqrt{PhkA} \tanh mL$$

If mL is sufficiently large, $\tanh mL \rightarrow 1$ and the expression for Q reduces to that for long fin, e.g., $\tanh mL$ is equal to 0.76, 0.96, and 0.99 for mL = 1, 2, 3 respectively.

Fins with convection at the tip:- most realistic boundary condition – Mathematical formulations are:

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } 0 \leq x \leq L$$

$$\theta(x) = T_0 - T_e = \theta_0 \quad \text{at } x = 0$$

$$\text{and, } k \frac{d\theta(x)}{dx} + h_e\theta(x) = 0 \quad \text{at } x = L$$

The solution can be written as :

$$\theta(x) = C_1 \cosh m(L - x) + C_2 \sinh m(L - X)$$

Application of boundary conditions lead to

$$\theta_0 = C_1 \cosh mL + C_2 \sinh mL$$

$$\text{and, } -kC_2m + h_eC_1 = 0$$

since,

$$\begin{aligned} \frac{d\theta}{dx} \Big|_{x=L} &= [-C_1m \sinh m(L - x) - C_2m \cosh m(L - x)]_{x=L} \\ &= -C_2m \end{aligned}$$

Determining the constants C_1 and C_2 and replacing them into the solution, the temperature distribution in the fin becomes equal to

$$\begin{aligned} \frac{\theta(x)}{\theta_0} &= \frac{T(x) - T_e}{T_0 - T_e} \\ &= \frac{\cosh m(L - x) + (h_e/mk) \sinh m(L - x)}{\cosh mL + (h_e/mk) \sinh mL} \end{aligned}$$

and the heat flow rate through the fin is

$$Q = \theta_0 \sqrt{PhkA} \left[\frac{\sinh mL + (h_e/mk) \cosh mL}{\cosh mL + (h_e/mk) \sinh mL} \right]$$

If $h_e = 0$ which corresponds to no heat loss from the tip leads to the solution obtained earlier.

So, therefore, the governing equation can be written as $d^2 T_x / dx^2 - m^2 (T_x - T_e) = 0$, this is equal to 0 right. So, this can be rearranged in a more compact form as, $d^2 \theta_x / dx^2 - m^2 \theta_x = 0$, where, of course, m^2 is equal to $h p / A k$ and θ_x is equal to $T_x - T_e$, i.e., T at any position x minus T_e , that is environmental temperature, right. So, this equation that, $d^2 \theta_x / dx^2 - m^2 \theta_x = 0$, this is known as one dimensional fin equation for fins of uniform cross section and is a linear, homogeneous, second order, ordinary differential equation, with constant coefficients. Then, the general solution of this kind of equation is, $\theta_x = c_1 e^{-m x} + c_2 e^{m x}$. This is true for long fins, and it may have other solutions also, for example, we can write that $\theta_x = c_1 \cosh(m x) + c_2 \sinh(m x)$, or $\theta_x = c_1 \cosh(m l - x) + c_2 \sinh(m l - x)$, rather, $\sinh(m l - x)$, for fins with finite length. To determine the constants c_1 and c_2 obviously, we need two boundary conditions.

So, one at the fin base, and another at the fin tip, and these two boundary conditions are required to be known. Customarily, the temperature at the fin base is known, and this is taken as, at x is equal to 0, this is considered to be known. Then, we can write, the fin base condition is $\theta_0 = T_0 - T_e$, and that is equal to θ_0 , θ at any position 0 is equal to 0, that is, what θ_0 , equals to $T_0 - T_e$, and that is equal to θ_0 , where obviously, T_0 is the fin base temperature. Several different conditions may arise such as, number 1, long fin, that is, the fin is very long, like this, if the base, and if it is the fin. So, it is a long fin, and negligible heat loss from the fin tip, what is that, this is the base, this is the fin. So, the negligible heat loss at the fin tip, this is the fin tip, right and convection at the fin tip, third one is, it could be on a convective condition at the fin tip right.

So, if we take it first, that the long fin. So, for a sufficiently long fin, it can be assumed that the temperature at the tip of the fin approaches the temperature T_e of the surrounding fluid. So, this is the long fin, surrounding fluid is this, it has a heat transfer coefficient of say, h or h_e , whatever you call, and it has a temperature also, as T_e . So, this fin tip, if it is a long fin, will get the temperature of the surrounding T_e , and that is the nature of the long fin, right. Then, the mathematical formulation for one dimensional steady state heat transfer in a long fin that can be written as, $d^2 \theta_x / dx^2 - m^2 \theta_x = 0$, where x is greater than 0.

Therefore, we can write, θ_x , as it tends to 0, is rather tends to 0, at x tends to infinity, right. So, we can write that, m^2 is equal to $h p / A k$ right. So, we take the solution in the form that, $\theta_x = c_1 e^{-m x} + c_2 e^{m x}$

the power $m x$. Then applying the boundary conditions, typically, the second one, first we get c_2 is equal to 0, and applying the first boundary condition, we get c_1 is equal to θ_0 , right. And this we can write, the solution of the differential equation, that can be written as $\theta(x) - \theta_0$, that is, $\frac{T(x) - T_e}{T_0 - T_e}$ equals to $e^{-m x}$.

Though, the heat flow through the fin, that can be determined as, ok. The previous equation, we can write that $\theta(x)$ is equal to $\theta_0 e^{-m x}$. Then, the heat flow through the fin, that can be determined, either by integrating the convective heat transfer curve over the entire fin surface as $Q = \int_0^L h p \theta(x) dx$ by integration of this, or by evaluating the heat flow, at the fin base, as $Q = -A k \frac{d\theta}{dx} \Big|_{x=0}$, is equal to 0 , right. Then, we can obviously, get the results from both these two, they are identical, and since, heat flow through the lateral surface by convection is equal to the heat flow at the fin base by conduction. Assuming the conduction equation and with the help of temperature distribution obtained as $\theta(x)$.

We can write, the heat flow through the fin as $Q = A k \theta_0 m$ and that is equal to $\theta_0 \sqrt{p h k A}$ because m we have already defined as m^2 is equal to $p h / A k$. So, m is $\sqrt{p h / A k}$. Therefore, the second thing is that fins with negligible heat loss at the tip. So, heat transfer area at the fin tip is generally small compared with the lateral area of the fin for heat transfer. Under this conditions or situations, the heat loss from the fin tip is negligible compared with that from the multilateral surfaces. And the boundary condition at the fin tip, characterizing the situation can be taken as $\frac{d\theta}{dx} = 0$ at $x = L$.

The mathematical formulation of the fin problem, this can be written as $\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$, valid between $x = 0$ to $x = L$. So, fins with, we can say, the boundary conditions could be $\theta(0) = T_0 - T_e$ at $x = 0$, equal to θ_0 , and $\frac{d\theta}{dx} = 0$, at $x = L$. So, let us choose the solution in the form, $\theta(x) = c_1 \cosh(m(L-x)) + c_2 \sinh(m(L-x))$. So, by applying the two boundary conditions, we get $c_2 = 0$, and $c_1 = \frac{\theta_0}{\cosh(mL)}$. So, the solution can be rewritten as $\frac{\theta(x) - T_e}{T_0 - T_e} = \frac{\cosh(m(L-x))}{\cosh(mL)}$.

And, the heat flow rate Q through the fin, that can be written, equals to $Q = A k \theta_0 m \tanh(mL)$ is equal to $\theta_0 \sqrt{p h k A} \tanh(mL)$. Now, if mL is sufficiently large, then, $\tanh(mL)$ tends to 1, right. So, as a limiting one, this can be used, right. So, where mL is very high. Now, the third one is the

fin with convection at the tip, most realistic boundary is this one, and the mathematical formulation we can write, $\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$ valid between, $x = 0$ to $x = L$, and $\theta = T - T_0$ at $x = 0$, and $k \frac{d\theta}{dx} + h \theta = 0$ at $x = L$.

Again, if we take the solution, in the form $\theta = c_1 \cosh mL - x + c_2 \sinh mL - x$, then we can write, the application of the boundary condition, that can be written as $\theta = T - T_0$ at $x = 0$ is $c_1 \cosh mL + c_2 \sinh mL = T - T_0$ and $-k c_1 m \sinh mL + h c_1 - k c_2 m \cosh mL + h c_2 = 0$. Now, since $\frac{d\theta}{dx}$ at $x = L$ equals to $-m c_1 \sinh mL - m c_2 \cosh mL$, at $x = L$. So, this is equal to $\theta = T - T_0$. So, by determining the constants c_1 and c_2 and replacing them into this solution, the temperature distribution in the fin that becomes equal to $\frac{\theta - \theta_0}{T - T_0} = \frac{T - T_0 - \theta_0}{T - T_0} \frac{\cosh mL - x + h/k \sinh mL - x}{\cosh mL + h/k \sinh mL}$ and the heat flow rate through the fin is $Q = \theta_0 \sqrt{phkA} \frac{\sinh mL + h/k \cosh mL}{\cosh mL + h/k \sinh mL}$. Now, if $h = 0$, then it corresponds to nothing, but, heat loss, no, heat loss from the tip, right.

So, this is a limiting condition right. So, there are some solutions. I am not going to do this now because fin efficiency, obviously is q_{fin} / q_{ideal} , and that we can again leave it, right. This is how another pictorial view, I would like to show that under different conditions of the fins, how it looks like, and how the surface area of fin heat transfer coefficient and θ_0 that is $T_0 - T_{\infty}$, this is plotted versus $L \sqrt{2h/k}$, right fin efficiency. Another thing, the last thing, one is obviously this, that was for that was, for rectangular, or a pointed fin fins, this is for circular fins, right and we can find out the q , right. Now, one thing is very certain, why we have taken $k^2 k / t^2$ over k rather $2k$ over t over h right.

$$\therefore \eta = \frac{\theta_0 \sqrt{PhkA} \tanh mL}{\theta_0 PLh} = \frac{\tanh mL}{mL}$$

$$\text{where, } mL = L \sqrt{\frac{Ph}{Ak}} = L \sqrt{\frac{2h}{kt}}$$

where $P/A = 2/t$, t being the fin thickness. This is why fin efficiency is plotted against the parameter

$$L \sqrt{\frac{2h}{kt}}$$

Therefore, from the q fin, θ_0 under root $p h A k$ tan hyperbolic $m L$ and q ideal is $p L h \theta_0$, we can write η , that is fin efficiency, equals to θ_0 under root $p h A k$ tan hyperbolic $m L$ over $\theta_0 p L h$ that is, tan hyperbolic $m L$ over $m L$, where, $m L$ is L under root $p h$ over $A k$, and that is for plate, it is $L \sqrt{2 h}$ over $k t$ right $p a$ by a is 2 by T , T being the fin thickness this is why fin efficiency is plotted against L under root $2 h$ over $k T$ right. So, this is how we have shown that why the fin efficiency is plotted against L under root $2 h$ by $k t$, ok. This is the completion of condenser, we will come next with evaporator and expansion device, ok. Thank you so much.