Traction Engineering
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Lecture 11
Rolling resistance of a rigid towed wheel

Hi everyone, this is Professor H Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL course Traction Engineering. This is the lecture 11 where I will try to cover Rolling resistance of a rigid towed wheel.

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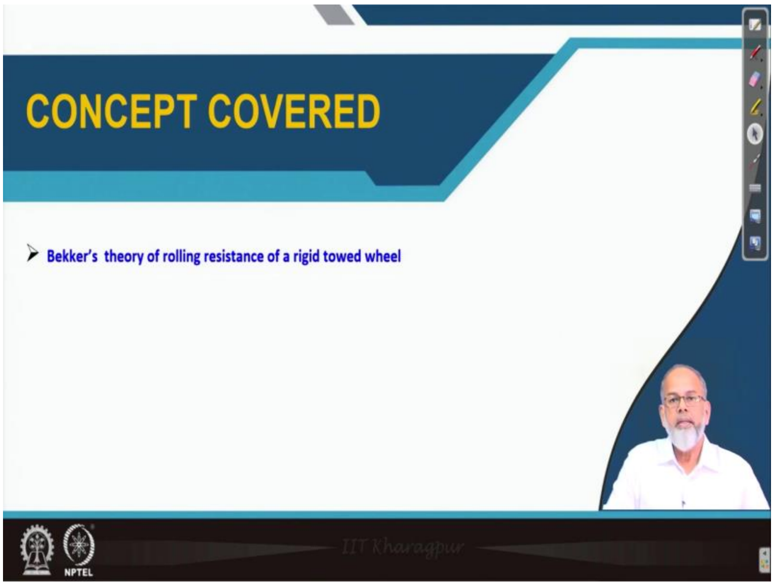

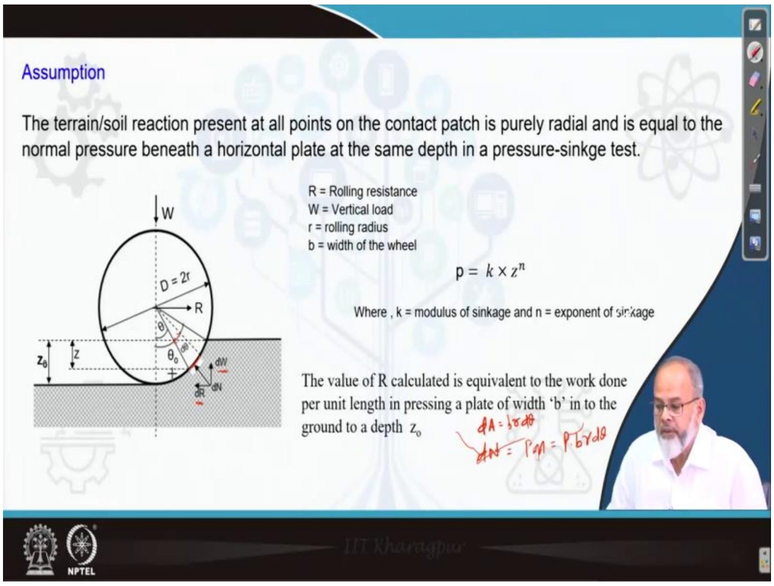

As you know, towed wheel means, the concept that is Bekker's theory of rolling resistance will be applied to find out the rolling resistance of a rigid towed wheel. The towed wheel is nothing but where torque is not acting. Simply, it is used to support the weight or to carry the weight. Now the figure which I am showing here is the towed wheel. Since torque is absent, so there is no shear stress acting at the soil and wheel interface, only the radial stresses are acting. So that is why I have indicated radial stresses along the periphery.

So dN is a radial force, it is not stress. Actually the radial force which is acting corresponding to an angle d theta that means we are considering a soil element with angular interval of d theta, so this is the arc of a circle, so that becomes the length will be r d theta and b is the width of the wheel. So, b into r theta will give you area of this element, this arc, area of this arc.

Now to move this wheel forward, we require to tow it, that means R which is indicated here is the rolling resistance and W is the vertical load which is acting and small r is the rolling radius. As there is no deflection, so rolling radius will be equal to D by 2, diameter by 2 and b will be the width of the wheel. Now as per Bekker's theory, the assumption is the terrain, the terrain or the soil reaction present at all points on the contact patch is purely radial and is equal to the normal pressure beneath a horizontal plate at the same depth in a pressure sinkage test.

That means we know that in the pressure sinkage test,

$$p = kz^n$$

So, here z is the sinkage k is the modulus of sinkage and n is the exponent of sinkage. So, to calculate the value of R, what we have to do is, this R will be equivalent to the work done per unit length in pressing a plate of width b into the ground to a depth z_0.

So, suppose the entry angle θ_0 is given as here, this line joining the soil surface with the center line, then at any angle theta we are considering a small element of circumference of the wheel which is subtending an angle dθ. So this is subtending an angle dθ, so dθ will be the arc of that area will be

$$rd\theta \times \mathrm{b}$$

and the radial force which is acting dN,

$$dN = p \times dA$$
$$area = rbd\theta$$
$$dN = pbrd\theta$$

So, that will be the radial force, then this radial force will have two components; one is the vertical component which is denoted as dW, the other one is the horizontal component dR. If you are going to integrate the total horizontal forces acting on this contact patch then we will find out what will be the rolling resistance.

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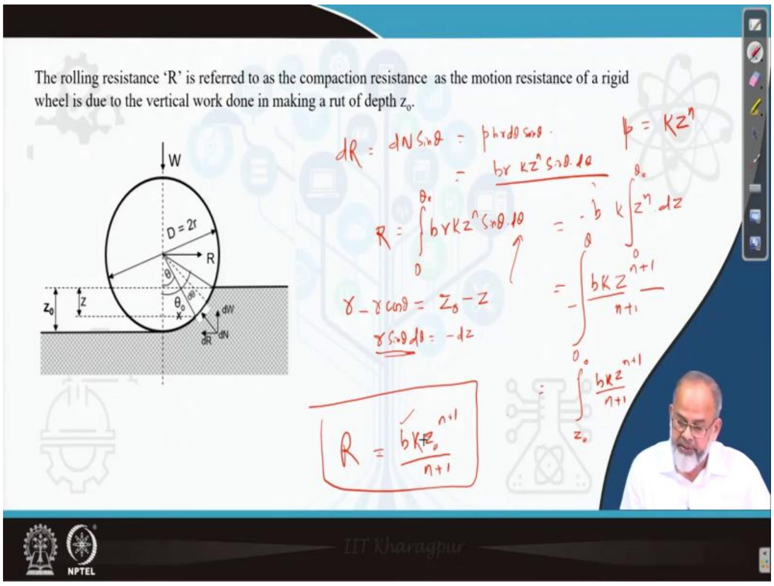

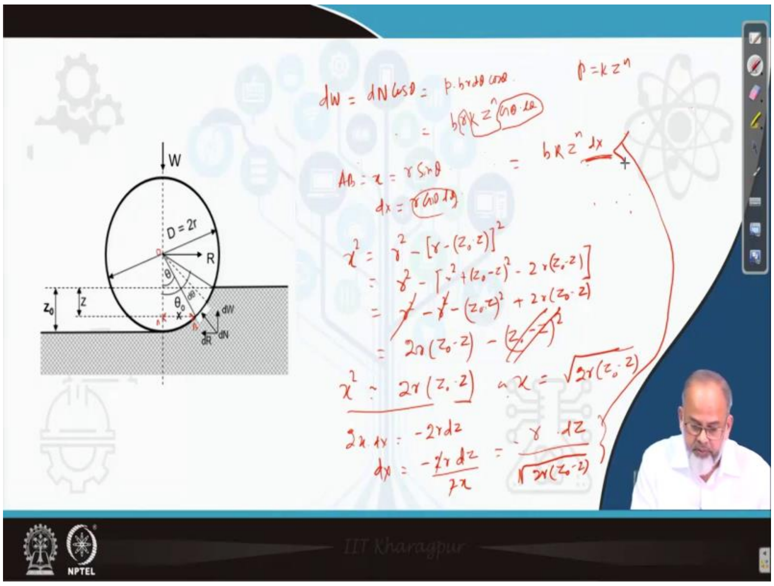

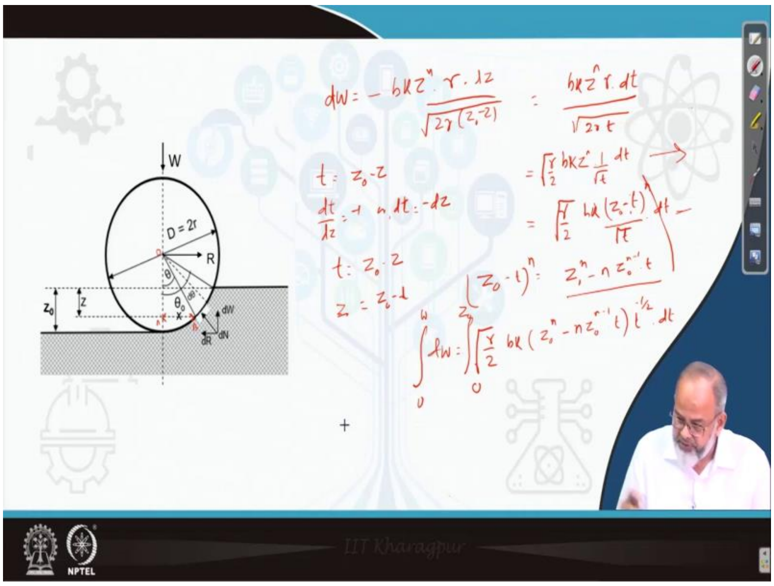

Now looking at this figure, we are going to find out expression for total rolling resistance. Now dR will be how much, here R, what you have considered is the compaction resistance as the motion resistance of rigid wheel is due to the vertical work done in making a rut of depth z_0, so dN have resolved into dR and dW.

$$dR = dN sin\theta \text{ and } dN = pbrd\theta$$

$$dR = (pbrd\theta)sin\theta$$

So now this has to be integrated and p also we know that

$$p = kz^n$$

So now if I substitute for p, so it becomes

$$brkz^n sin\theta d\theta$$

This is the expression for dR.

$$\int_0^{\theta_e} brkz^n sin\theta d\theta$$

Okay, now if you look at this figure r is the rolling radius and

$$r - rcos\theta = z_0 - z$$

$$rsin\theta d\theta = -dz$$

Now if substitute this rsinθdθ in this equation, so this becomes

$$\int_{z_0}^{0} bkz^n(-dz)$$

$$= \frac{bkz_0^{n+1}}{n+1}$$

This is the expression for rolling resistance, that means rolling resistance is a function of tyre width or the wheel width, is a function of exponent of sinkage n, is a function of modulus of sinkage k, is a function of z which is sinkage.

The incremental vertical force dW, if you, the incremental vertical force dW, so that will be equal to the vertical component of dN. So this is nothing but dN cosθ, so dN we have already derived as p×brdθ, so this will be your vertical component of dN. Now if we substitute again for $p = kz^n$

so it becomes

$$brkz^n cos\theta d\theta$$

So, now we know that x which is corresponding to this point, so if I denote it as A and B, so and center as O, so AB is equal to x and x will be equal to rsinθ in the right angle triangle OAB. I can write, as x is equal to rsinθ, now dx/dθ will be equal to rcosθdθ. Now I substitute this rcosθdθ here, so this equation becomes now

$$bkz^n dx$$

Now x square is how much?

$$x^2 = r^2 - [r - (z_0 - z)]^2$$

$$= r^2 - r^2 + 2r\,(z_0 - z) - [z_0 - z]^2$$

Now r^2-r^2 will cancel out, so it becomes

$$2r\,(z_0 - z) - [z_0 - z]^2$$

Now, for soil sinkage when the sinkage is low, z_0 minus z will be very-very low value, so you can neglect this.

$$x^2 = 2r\,(z_0 - z)$$

$$x = \sqrt{2r\,(z_0 - z)}$$

Now if I differentiate again this one, so 2xdx will be equal to -2rdz. Now I have to substitute for dx from here, so from here dx will be equal to -2rdz. This I can write as –r/x. For dx, I can write

$$\frac{-r}{\sqrt{2r\,(z_0 - z)}}dz$$

So now I have to substitute this one here in this equation. So, the assumption for the limit will be that is applicable to soil sinkage, so I can write dW as finally, I can write as

$$dW = \frac{-bkz^n r}{\sqrt{2r\,(z_0 - z)}}dz$$

Now let t = (z_0-z), so

$$\frac{dt}{dz} = -1$$

Or,

$$dt = -dz$$

$$dW = \frac{bkz^n r}{\sqrt{2rt}} dt$$

So, now further if I simplify

$$dW = \sqrt{\frac{r}{2}} \frac{bkz^n}{\sqrt{t}} dt$$

So now for z, if I substitute z we have taken here t = z_0-z, for z if I substitute z = z_0-t or this way, z = z_0-t.

Now I substitute in this equation,

$$dW = \sqrt{\frac{r}{2}} \frac{bk(z_0 - t)^n}{\sqrt{t}} dt$$

So, this has to be integrated after expanding this term $(z_0\text{-}t)^n$. Now if I expand it and only considering the first two terms and neglecting the rest of the terms, so what I get is

$$(z_0 - t)^n = z_0^n - nz_0^{n-1}\, t$$

This two term I will take and the rest of the terms I will neglect.

Now if I substitute for, so it becomes,

$$\int_0^W dW = \int_0^{z_0} \sqrt{\frac{r}{2}} \frac{bkz_0^n - nz_0^{n-1}\, t}{\sqrt{t}} dt$$

So that will be the final expression. Now this has to be integrated from, so this has to be integrated from 0 to W that means 0 to this has to be integrated from 0 to z_0.

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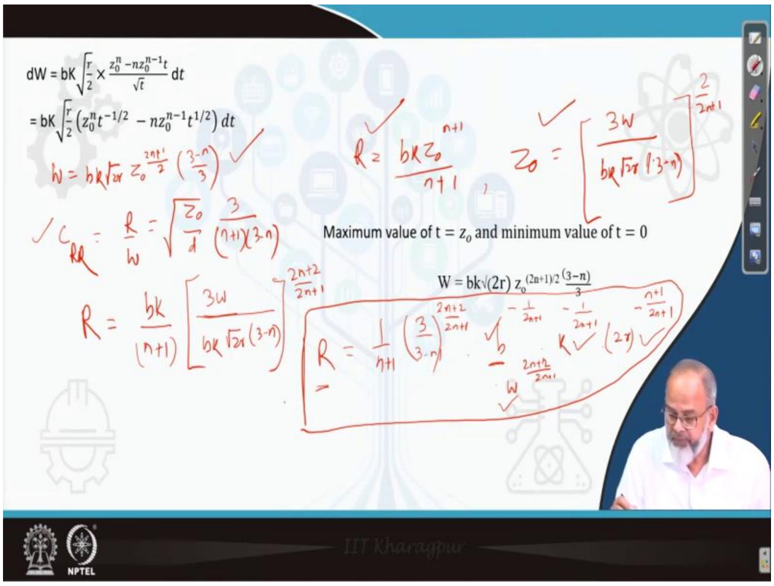

So the final expression for W will be

$$\int_0^W dW = \int_0^{z_0} bk\sqrt{\frac{r}{2}}\left(z_0^n t^{-1/2} - n z_0^{n-1}\, t^{1/2}\right)dt$$

This has to be integrated and final expression will be

$$W = bk\sqrt{2r}z_0^{(2n+1)/2}\left(\frac{3-n}{3}\right)$$

This is the expression for W.

So you have already defined for rolling resistance and so coefficient of rolling resistance will be equal to R/W, so the expression will be

$$\sqrt{\frac{z_0}{d}}\frac{3}{(n+1)(3-n)}$$

Now R expression we know

$$R = \frac{bkz_0^{n+1}}{n+1}$$

Now I substitute for z_0,

$$z_0 = \left[\frac{3W}{br\sqrt{2r}((3-n))}\right]^{\frac{2}{2n+1}}$$

So this is the expression for z_0, this is the expression for R, this is the expression for W and this is the expression for coefficient of rolling resistance. So now finally if I substitute for z_0 in this equation,

$$R = \frac{bk}{(n+1)}\left[\frac{3W}{br\sqrt{2r}\,((3-n))}\right]^{\frac{2n+2}{2n+1}}$$

Now if I want to express individually like what is the effect on b, what is the effect of k like that. So this becomes

$$R = \frac{1}{(n+1)}\left(\frac{3}{3-n}\right)^{\frac{2n+2}{2n+1}}(b)^{\frac{-1}{2n+1}}(k)^{\frac{-1}{2n+1}}(2r)^{\frac{-n}{2n+1}}(W)^{\frac{2n+2}{2n+1}}$$

This is the expression for R.

Now in this expression, it clearly indicates that rolling resistance is a function of wheel width, it is a function of modulus of sinkage, it is a function of diameter, it is a function of weight and it is a function of exponent of sinkage. Now if you look at b and k they are all negatively related that means b, k and the diameter that means if diameter we are increasing it is going to reduce rolling resistance, if you are increasing section width that means width of the wheel then it is also going to reduce the rolling resistance. So let us now see how these are affecting the rolling resistance for different n values.

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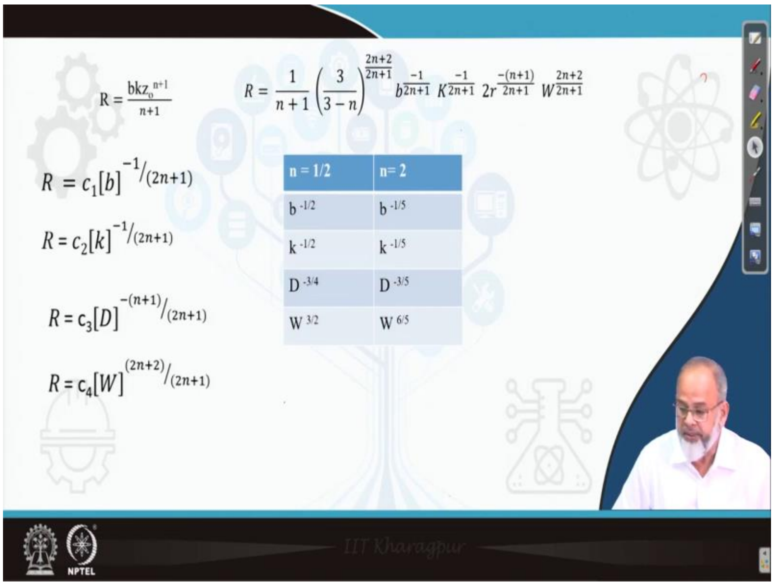

So when we keep everything constant only vary b then

$$R = C_1[b]^{\frac{-1}{2n+1}}$$

When I keep everything constant only vary k then

$$R = C_2[k]^{\frac{-1}{2n+1}}$$

When I keep everything constant only vary diameter then it becomes

$$R = C_3[D]^{\frac{-n}{2n+1}}$$

If I keep everything constant only vary W then

$$R = C_4[W]^{\frac{2n+2}{2n+1}}$$

So now if I vary n from half to 2, so, that itself will show you how b, k, D, and W they are going to influence the rolling resistance. So, what we conclude from here is, increasing b or increasing D because, soil you cannot change, k is related to soil, so b and D if you only consider then that is going to, both of these values are going to reduce the rolling resistance but increasing diameter will have a more effect, will have more effect in reducing the rolling resistance as compared to b.

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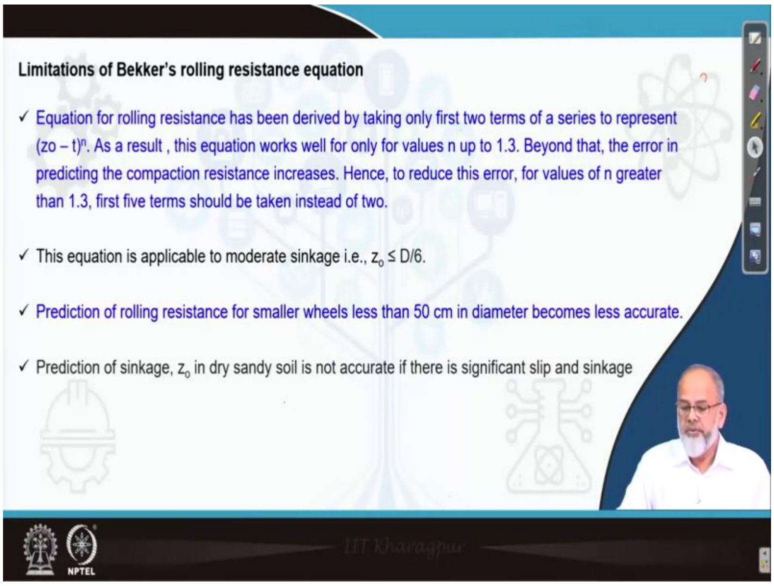

Now there are certain limitations. Bekker has proposed while applying the Bekker's Theory to decide the rolling resistance or to determine the rolling resistance of a towed wheel, the limitations are the equation for rolling resistance which has been derived we have only taken two terms of that binomial expansion, so as a result what happens, this equation works well only when the values of n is within 1.3. If you increase the value of n beyond 1.3 there would be considerable error.

So if you want to reduce the error then you have to take more terms of that expansion, at least five terms should be taken instead of two. Then the other limitation is, this equation is applicable to moderate sinkage that means when z_0 that is sinkage is less than equal to D by 6, one sixth of the wheel diameter. Then the third limitation is the prediction of rolling resistance for smaller wheels that means lesser than 50 cm in diameter is not accurate. Then the last limitation is the prediction of sinkage z0 in dry sandy soil is not accurate because there could be a significant amount of sinkage and slip associated.

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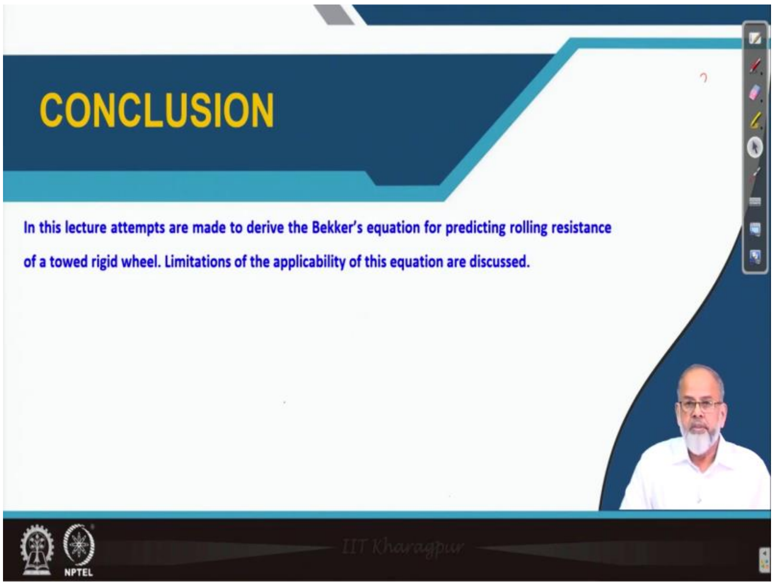

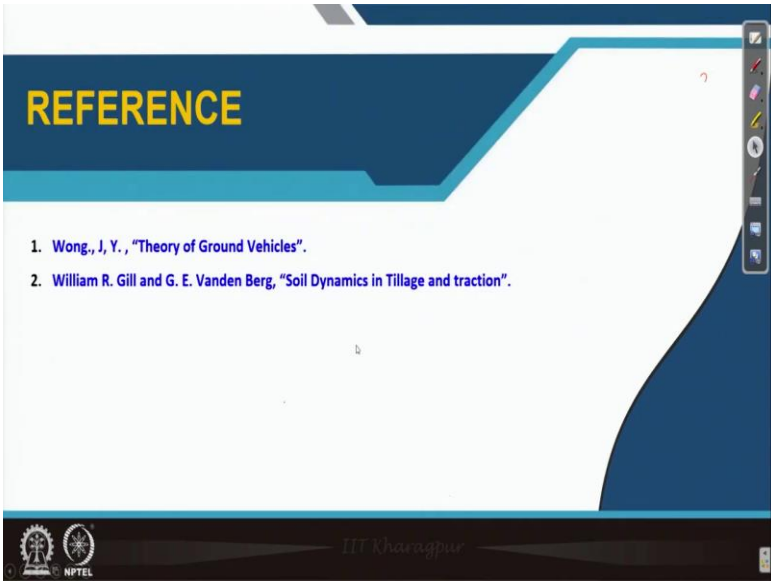

So in brief, we can say in this lecture attempts are made to derive the Bekker's equation for predicting rolling resistance of a towed rigid wheel and we have also discussed the limitations in applying the Bekker's theory for predicting rolling resistance. You can refer to some of the books like Wong J Y Theory of Ground Vehicles and Soil Dynamics in Tillage and traction by William R Gill and Jay Vanden Berg. Thank you.