Traction Engineering

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Lecture 12

Rolling resistance of a pneumatic wheel

Hi everyone, this is Professor H Raheman from Agricultural and Food Engineering Department from IIT Kharagpur, I welcome you all to this NPTEL course on Traction Engineering. Today I will try to cover Rolling resistance of a pneumatic wheel. As you know that wheel and track, they are the two traction devices which are commonly used in a tractor. So that is why we are considering wheel and in subsequent class we will consider how to find out rolling resistance of a track.

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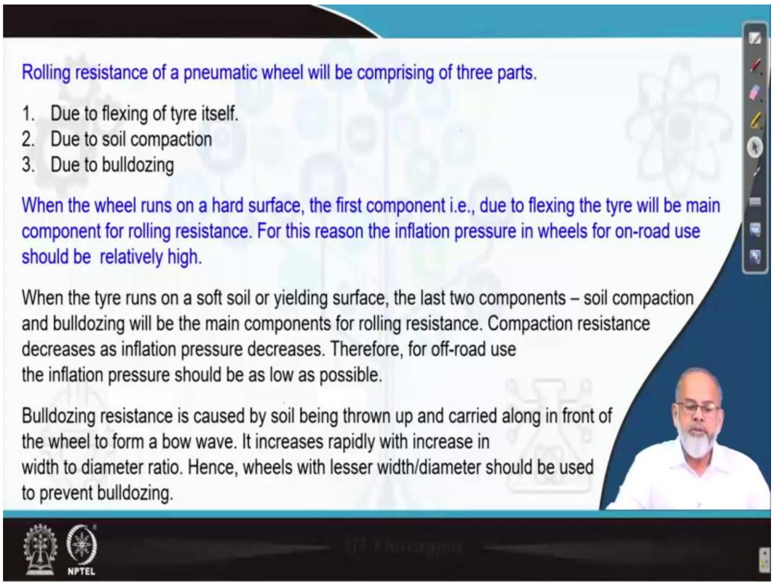

Now, here the concept will be how to find out rolling resistance of pneumatic wheel. So when you consider wheel, rolling resistance it will be because of three things, one is due to flexing of tyre, the other one is due to soil compaction and the rolling resistance could be due to bulldozing. So either all three are present or any one of them will be present so both will, that will contribute towards the rolling resistance.

So when the wheel runs on the hard surface, the first component that is your tyre flexing will be the main component for rolling resistance. So, for this reason the inflation pressure in wheels for on-road vehicles are kept relatively high. When the tyre runs on a soft soil or on yielding surface the last two components that is your soil compaction and bulldozing these are the two components that will contribute towards the rolling resistance. And compaction resistance decreases as inflation pressure decreases why, because when you decrease the inflation pressure, the contact area is increased so ground pressure is decreased hence compaction resistance is decreased.

So therefore, for off-road condition the inflation pressure should be as low as possible and coming to the bulldozing resistance this is basically caused by soil being thrown up and carried along in front of the wheels which will form a bow wave. So it increases rapidly with increase in width to diameter ratio so wheels with lesser width diameter should be used to prevent bulldozing. So this is the background of rolling resistance of a pneumatic wheel.

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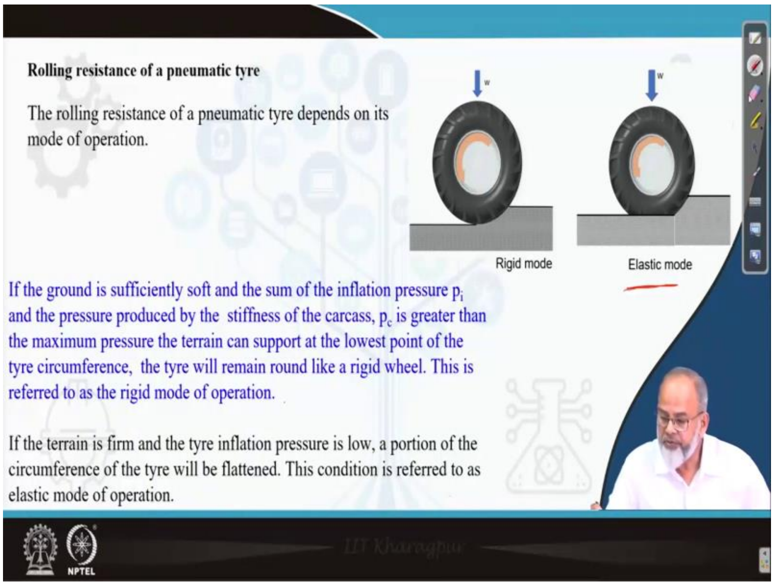

Then, in case of a pneumatic wheel, it behaves in two different modes during operation, the first mode is in rigid mode, the second mode is in elastic mode. Rigid mode means the rolling resistance, the tyre will simply sink into the soil, it is not flattened.

So, if the ground is sufficiently soft and the sum of the inflation pressure pi and the pressure which is produced by the stiffness of the carcass p_c is greater than the maximum pressure, the terrain can support at the lowest point of the tyre circumference then the tyre will remain like a round like a rigid wheel. So this is referred to as rigid mode of operation.

That means p_i inflation pressure plus pressure which is exerted by the carcass due to carcass stiffness, if sum of these two is greater than the pressure which the terrain can support then it behaves like in a rigid mode. If the terrain is firm and the tyre inflation pressure is low. So what will happen, a portion of the tyre will be flattened which is indicated in this figure. If you look at the right corner, this one, the portion is flattened; this condition is referred as elastic mode of operation. So now we have to see first, whether the wheel is in elastic mode or whether the wheel is in a rigid mode.

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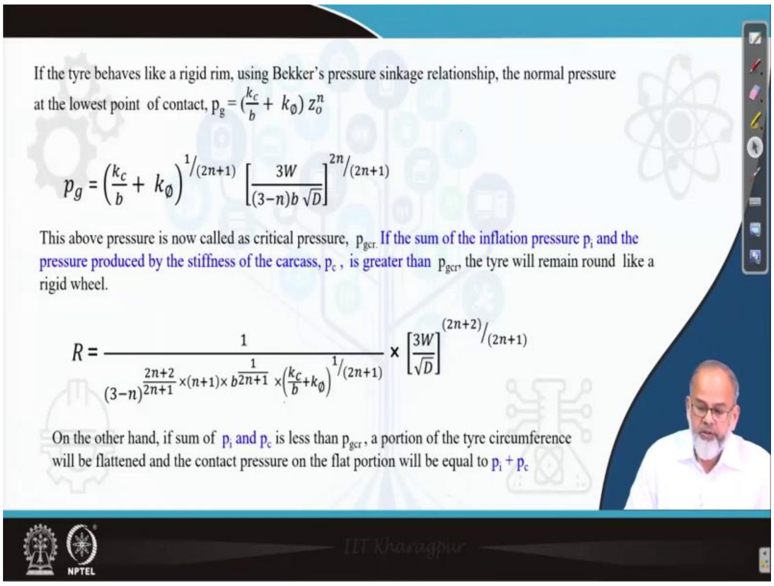

So if the tyre behaves like it is a rigid rim then you have to use the same Bekker's equation what we have utilized in case of a rigid wheel and the normal pressure at the lowest point of the tyre is given by

$$p_g = \left(\frac{k_c}{b} + k_\phi\right) z_0^n$$

Now if you substitute for z_0, just now we have derived for rigid wheels so then it becomes

$$p_g = \left(\frac{k_c}{b} + k_\emptyset\right)^{\frac{1}{2n+1}} \left[\frac{3W}{(3-n)b\sqrt{D}}\right]^{\frac{2n+2}{2n+1}}$$

This pressure is now called critical pressure because this is the pressure which will be utilized to demarcate whether the wheel is in rigid mode of operation or to whether the wheel is in elastic mode of operation. Now that is denoted as p ground critical, ground pressure critical. If the sum of the inflation pressure pi and the pressure which is produced by the stiffness of the carcass p_c is greater than ground pressure critical p_{gcr}, then the tire will remain round like a rigid wheel and the rolling resistance will be equal to just like the rolling resistance of a rigid towed wheel.

$$R = \frac{1}{(3-n)^{\frac{2n+2}{2n+1}} \times (n+1) \times (b)^{\frac{1}{2n+1}} \times \left(\frac{k_c}{b} + k_\emptyset\right)^{\frac{1}{2n+1}}} \left[\frac{3W}{\sqrt{D}}\right]^{\frac{2n+2}{2n+1}}$$

So the behavior will be similar to the behavior of a towed rigid wheel that means rolling resistance expression will be same. Now when sum of the pressure p_i and p_c is lesser than ground critical pressure then a portion of the tyre circumference will be flattened and the contact pressure in the flat portion will be equal to p_i plus p_c.

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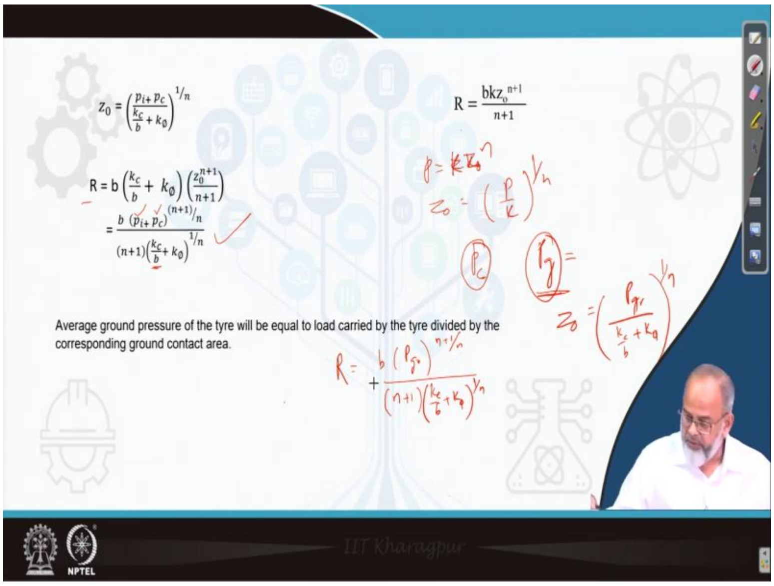

So if contact pressure is p_i+p_c then the sinkage we can calculate,

$$\left(\frac{p_i + p_c}{k}\right)^{1/n}$$

and R we can calculate,

$$R = b(k)\left(\frac{z_0^{n+1}}{n+1}\right)$$

just like in plates sinkage test. Now if I substitute this z_0 so

$$z_0 = \left(\frac{p}{k}\right)^{1/n}$$

So for z_0, if I substitute for p, I have taken p_i+p_c. Now the equation becomes rolling resistance will be equal to

$$R = \frac{b(p_i + p_c)^{\frac{n+1}{n}}}{(n+1)\left(\frac{k_c}{b} + k_\emptyset\right)^{1/n}}$$

That means rolling resistance depends on what is the inflation pressure, it depends what is the pressure stiffness of carcass, the pressure due to stiffness of carcass. Then it depends on the section width of the tyre and the cohesive modulus of sinkage kc and the frictional modulus of sinkage $k_\emptyset$ and exponent of sinkage n. So, average tyre which will be, average ground pressure of the tyre that will be equal to, the problem associated with this p_i+p_c is, difficult to measure p_c because it varies with load, it varies with the type of tyre, it varies with inflation pressure.

So, what Bekker have suggested is, instead of p_i+p_c, it can take ground pressure p_g. This ground pressure value is available from the tyre manufacturer's data So it will be easier for us to calculate. So if we are taking a ground pressure then the equation will be,

$$R = \frac{b(p_g)^{\frac{n+1}{n}}}{(n+1)\left(\frac{k_c}{b} + k_\emptyset\right)^{1/n}}$$

And z_0 also you can similarly do, instead of pi plus pc, I can write

$$z_0 = \left(\frac{p_g}{\frac{k_c}{b} + k_\emptyset}\right)^{1/n}$$

Now how to find out average ground pressure on a flat portion the tyre, the average ground pressure of the tyre will be equal to the load carried by the tyre divided by the corresponding ground contact area. If you know the ground contact area or if you can find out the ground contact area knowing the load, we can find out what will be the ground pressure. Load divided by area that will give you the ground pressure and then utilizing this equation you can find out the rolling resistance.

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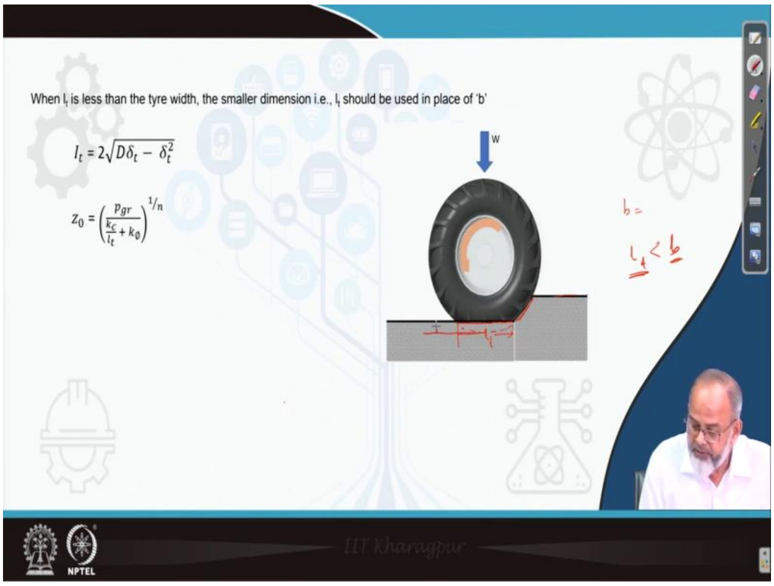

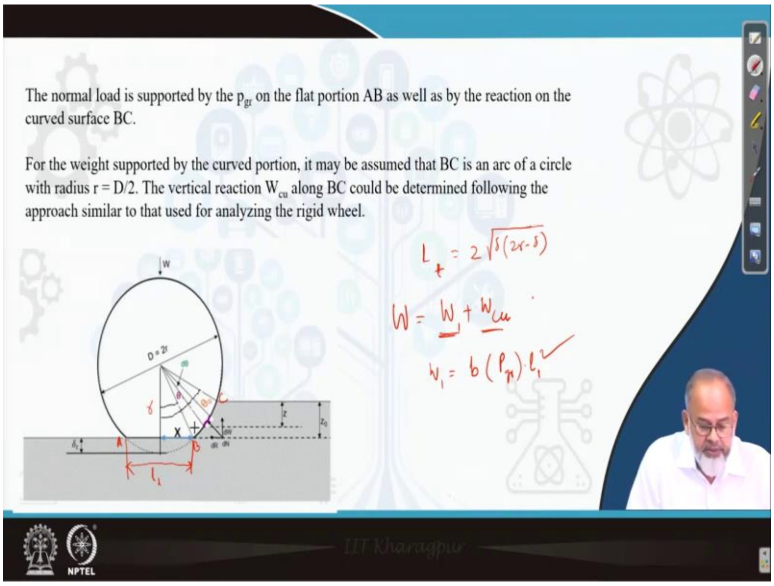

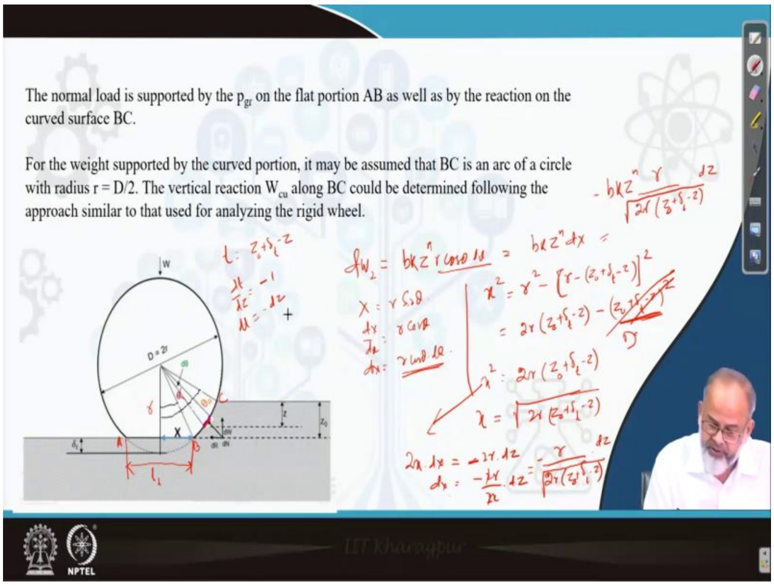

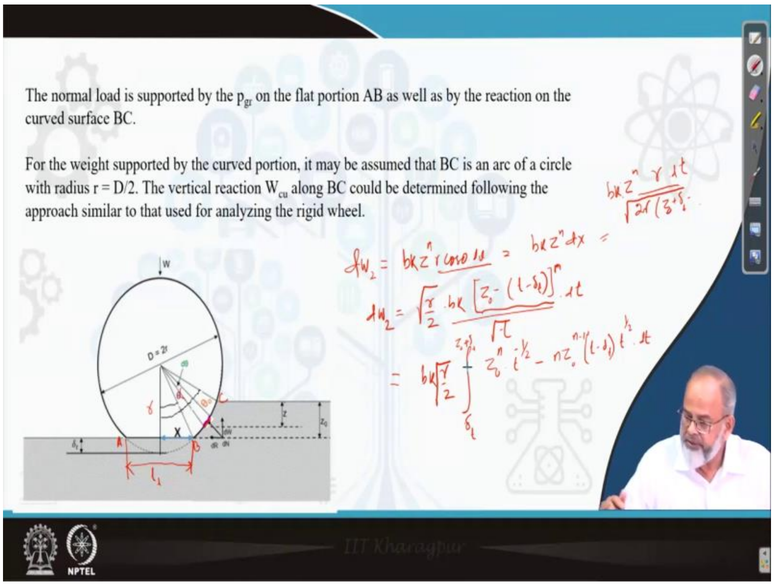

Now if you look at, this is the potion which is flattened, this is the portion which is remaining in curved shape. So entire thing is entering into the soil so that means there is a flat portion, there is a curved portion and the width we have taken as b. Section width of the tyre we have taken as b. Now the contact patch if you denote it as lt. For a wider tyres like terra tyres l_t may be lesser than b because if these are wider tyres. So section width is greater than the contact patch. So, if that is the situation then we have to take l_t instead of b for all those equations which you have derived for sinkage or for rolling resistance. So I have derived expression for l_t so

$$l_t = 2\sqrt{D\delta_t - \delta_t^2}$$

If I take as this is the deflection δ_t, so and this is the radius. So this is your δ_t and this is your radius r so now expression for l_t. So l_t will be this distance. This is your l_t, so

$$l_t = 2\sqrt{\delta_t(D - \delta_t)}$$

So you have to take either b or l to l_t depending on which one is lesser. So, if l_t is lesser then you have to take the value of l_t instead of b in z_0 as well as while calculating the rolling resistance along the flattened portion the pressure will be equal to p_i+p_c which you have taken as ground pressure. And the W which is applied, that is carried by the wheel. So that will be carried or supported by two components. Now so W will be equal to W_1+W_{cu} that means a portion of weight will be supported by the flattened portion and the rest will be supported by the curved portion. Now if I denote this flatted portion as AB and the curved portion as BC so flattened portion W_1 will be equal to if b is the width and if you know the ground pressure so into your l_1 so this is area into pressure that will give you directly the value of W_1. Now for finding out the weight which is supported by the curved portion, we have to proceed in a similar way as you have done for the rigid wheel. That means, in the contact patch, the curved contact patch which is indicated as BC the radial force acting will be dN. So it will have a component dW; it will have a component dR. dR is related to your rolling resistance and dW would be related to the vertical load which is supported. Now dW will be equal to, if I take a small element subtending angle dθ which I have indicated in this. So that area whatever force is acting so that is indicated as dW_2 and

$$dW_2 = bkz^n r cos\theta d\theta$$

Now x is equal to, 'x' I have indicated here, x will be equal to rsinθ Now,

$$\frac{dx}{d\theta} = rcos\theta$$

$$dx = rcos\theta d\theta$$

Now we substitute for this in here, so it becomes

$$dW_2 = bkz^n dx$$

Now x square will be how much.

$$x^2 = r^2 - [r - (z_0 + \delta_t - z)]^2$$

Now if I simplify this one, so this will become

$$2r\,(z_0 + \delta_t - z) - [z_0 + \delta_t - z]^2$$

Now for shallow sinkage or for this expression with respect to the diameter is very very low so you can neglect this. So we can approximate as

$$2r\,(z_0 + \delta_t - z)$$

Now x will be how much,

$$x = \sqrt{2r\,(z_0 + \delta_t - z)}$$

Now if I differentiate this expression, so what I get is

$$2xdx = -2rdz$$

Now substitute for dx,

will be equal to minus (2r/2x)×zdz. So 2-2 will be cancel out. Now substitute for x, so it becomes

$$dx = \frac{-r}{\sqrt{2r\,(z_0 + \delta_t - z)}}dz$$

So now you substitute for dx in this expression here. So it becomes

$$dW_2 = \frac{-bkz^n r}{\sqrt{2r\,(z_0 + \delta_t - z)}}dz$$

Next is, if I assume that $t=z_0+\delta_t-z$, then dt/dz = -1 or dt = -dz. So now I substitute for -dz as dt so I can remove this minus sign, so if this becomes

$bkz^n r dt$

So dW_2 will be now finally I can write as, if this is clear then I can directly write as

$$dW_2 = \sqrt{\frac{r}{2}}\frac{bk(z_0 - (t - \delta_t))^n}{\sqrt{t}}dt$$

Now this is the expression. Now this term can be expanded by binomially and only keeping the first two terms and neglecting the other terms, so this can be finally written as

$$W_2 = bk\sqrt{\frac{r}{2}}\int_{t}^{z_0+\delta_t} z_0^n t^{-1/2} - nz_0^{n-1}(t - \delta_t)t^{-1/2}dt$$

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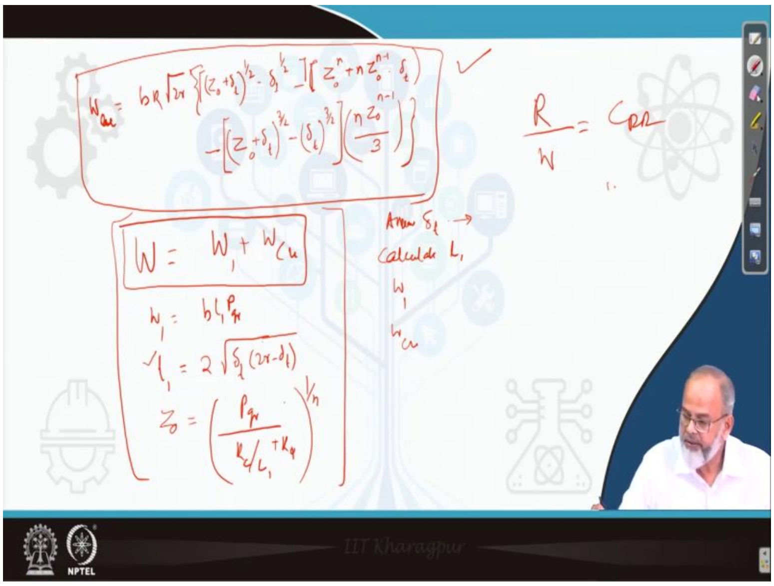

So, what we will be getting is expression, so we will be getting finally

$$W_2 = W_{cu} = bk\sqrt{2r}\left[\left[\left\{(z_0+\delta_t)^{1/2} - \delta_t^{1/2}\right\}\{z_0^n + nz_0^{n-1}\delta_t\}\right] - \left[\left\{(z_0+\delta_t)^{3/2} - \delta_t^{3/2}\right\}\right]\right]$$

So this will be the final expression for the load which is carried by the curved portion.

So total weight will be equal to W_1+W_{cu}. We have expression for W_{cu}. We have expression for W_1

$$W_1 = bl_l p_{gcr}$$

$$l_l = 2\sqrt{\delta_t(D-\delta_t)}$$

$$z_0 = \left(\frac{p_{gr}}{\frac{k_c}{b} + k_\emptyset}\right)^{1/n}$$

So this is a series of equations available.

So what you have to do is, first you have to assume a value of δ_t which is the deflection part then calculate l_1. So assume a δ_t, calculate l_1, then find out W_1 and W_{cu}. Then summation of this W_1 and W_{cu} should satisfy this condition. If this condition is not satisfied, then you have to change the value of δ_t till we get the convergence.

So this is, this could be an iterative procedure by which we can carry out and find out what is W. Once you know W then you can find out knowing the rolling resistance divided by W that will give you coefficient of rolling resistance.

So this way you can find out the rolling resistance, coefficient of rolling resistance for a pneumatic wheel when it is in rigid mode of operation or when it is in elastic mode of operation.

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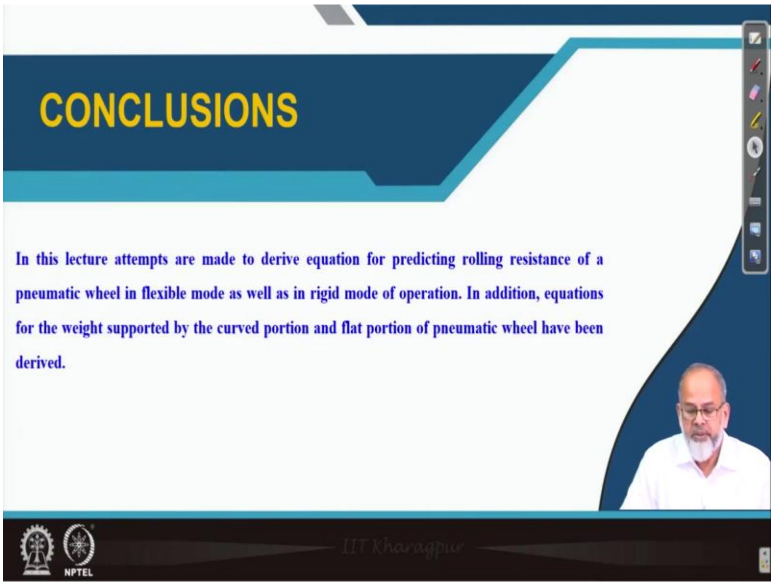

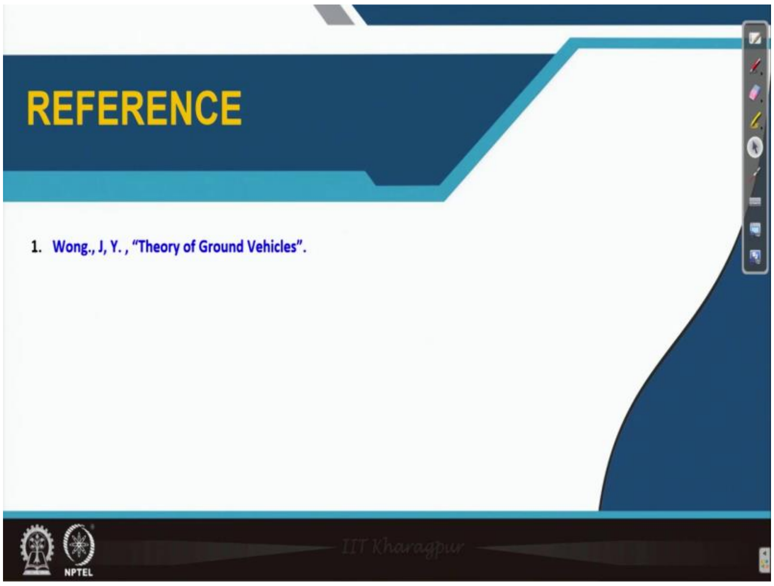

So in this lecture I tried to derive the equation for predicting rolling resistance of a pneumatic wheel both in rigid mode as well as in flexible mode of operation. In addition, equations are being derived for finding out weight carried by the different section of the wheel. Different section means there will be a flat portion, there will be a curved portion.

So what will be the weight which will be carried on the flat portion and what will be the weight which will be carried at the curved portion. You can refer to the book like Theory of Ground Vehicles for further enhancing your knowledge. Thank you.