Traction Engineering
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Lecture 14: Tractive effort and slip of a powered rigid wheel

Hi everyone. This is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL course on Traction Engineering. This is lecture 14, where I will try to cover tractive effort and slip of a powered rigid wheel. When I said it is a powered rigid wheel that means, a rigid wheel which is powered which is moving at an angular speed omega.

So, the concepts which will be covered will be your slip and tractive effort. So, if you look at this figure in the left corner, I have shown a powered wheel with a rolling radius R and it is moving with an angular speed ω and V is the actual speed, which I have indicated here. So, since it is powered that means, it will be developing a shear force along the or shear stress along the contact area. These are, this is the contact area where you can develop the shear stress because of the interaction between wheel and soil.

Now if you want to develop tractive force which is developed by the rigid wheel, then we need to know what is the shear stress developed. And, shear stress is a function of shear displacement. So, we have to find out what is the shear displacement at the contact area, then only you can find out what is the shear stress developed. And, shear stress into the contact area so, that will give you the thrust or the tractive effort which is developed by a rigid wheel. So, shear displacement which is developed along the contact area of a rigid wheel will be decided based on this analysis of slip velocity. Whereas, at slip velocity that means, difference between theoretical speed and actual speed, both are in the same plane, then only you can find out the difference and then that is denoted as V_j.

Now if you consider that wheel is rolling. So, θ_0 represents the angle at which the wheel is in touch with the soil and this is called entry angle. And we will now consider a point 'say A' which is making an angle θ with the vertical. So, at 'A' the peripheral speed will be equal to ωr, ω is the angular speed, r is the rolling radius. So, peripheral speed will be ωr and this will be equal to your V_t which is theoretical speed.

Now, at this point the actual velocity is given as V. Now, if you resolve this into this plane so, this angle will be equal to how much? θ. So, V if is resolved in this plane, then it becomes Vcosθ . So now,

$$V_t - V\cos\theta = V_j$$

So, in other words, you can say slip velocity is nothing, but the tangential component of the absolute velocity at any particular point where you are interested in finding out the slip velocity ok.

Now if I substitute, V in by i, i in the sense we know slip velocity is nothing, but

$$i = 1 - \frac{V}{V_t}$$

where i is the slip, V is the actual velocity and V_t is the theoretical velocity. So, from here,

$$V = (1-i)V_t$$

$$V_j = V_t - (1-i)V_t\cos\theta = V_t[1-(1-i)\cos\theta]$$

you can find out

$$V = (1-i)V_t$$

Now and Vt is nothing, but wr, is the theoretical speeds.

$$V_j = wr[1-(1-i)\cos\theta]$$

So, if you look at this equation, now the slip velocity is a function of angular speed, is a function of rolling radius, is a function of angle theta and is a function of slip. That means, for a given wheel and given operating condition that means, where the angular speed is given, then slip velocity is a function of theta and slip. Now from the slip velocity, you have to find out what is the shear displacement, shear displacement which is occurring at the contact area. So, if j is the shear displacement along the wheel soil interface, then j can be written as: j is the shear displacement, displacement then this can be written as $V_j \times dt$.

So, if you want to find out for the entire contact area, then this has to be integrated from say θ to θ_0, if I am considering from point A ok.

$$j = shear\ displacement = \int_{\theta}^{\theta_0} V_j.dt$$

$$= \int_{\theta}^{\theta_0} wr[1-(1-i)\cos\theta].dt$$

$$\theta = wt$$

$$\frac{d\theta}{dt} = w$$

$$\frac{d\theta}{w} = dt$$

$$= \int_{\theta}^{\theta_0} wr[1-(1-i)\cos\theta].\frac{d\theta}{w}$$

$$= \int_{\theta}^{\theta_0} r[1-(1-i)\cos\theta].d\theta$$

$$j = r[(\theta_0-\theta)-(1-i)(\sin\theta_0-\sin\theta)]$$

The next question is, what is the relationship between shear stress and shear displacement? In nature, from the experimental observation what you observed is, there are three relationships normally existing. So let me show those relationships between shear stress and shear displacement. So, in case of a loose sand or saturated clay or most of the disturbed soil, the relationship between shear stress and shear displacement $\tau(\theta) = \tau_{max}\left(1-e^{-j/k}\right)$; where $\tau_{max}$is written as $c+\sigma(\theta)\tan\emptyset$, σ is the normal pressure.

Now for organic terrain, the relationship between shear stress and shear displacement is,

$$\tau = \tau_{max}\left(\frac{j}{K_\omega}\right)exp\left(1-\frac{j}{K_\omega}\right)$$

where K_ω is the shear displacement where the maximum shear stress occurs. Now, if it is a compact sand or silt and loam, then the relationship is

$$\tau = \tau_{max}K_r\left\{1+\left[\frac{1}{K_r(1-1/e)}-1\right]exp\left(1-\frac{j}{K_\omega}\right)\right\}\times[1-\exp(-j/K_\omega)]$$

where K_r is the ratio of residual stress to the maximum shear stress and τ_{max} and K_ω is the shear displacement where the maximum shear stress occurs. You know you can choose any one of these equations and then find out the shear force which is acting. Once you know the shear stress then we will find out what is the shear force acting. So, shear stress will be supposed for example, we take shear displacement in loose sand or in most of the disturbed soil, that is the relationship is,

$$\tau = \tau_{max}\left(1 - e^{-j/k}\right)$$

$$= (c + \sigma tan\emptyset)\left(1 - e^{-j/k}\right)$$

If I take this relationship then the equation will be so, shear stress distribution will be like

$$\tau(\theta) = \tau_{max}\left(1 - e^{-j/k}\right)$$

$$= (c + \sigma(\theta)\tan\emptyset)\left(1 - e^{-\frac{r}{k}[(\theta_0-\theta)-(1-i)(\sin\theta_0-\sin\theta)]}\right)$$

Shear force will be like this. So,shear force will be equal to the shear stress into area. So, I can write shear stress,

$$shear\ force = \tau(\theta).dA$$

$$dA = brd\theta$$

$$shear\ force = br\,\tau(\theta)\,d\theta$$

So, this will be the shear force. Now, for finding out tractive force, you have to resolve into two components. One is the horizontal component and this horizontal component is denoted as tractive effort ok. So, tractive effort will be denoted as F, will be equal to this shear force which you got cos component of that one ok. So, $br\,\tau(\theta)\cos\theta\,d\theta$

So, this has to be integrated from 0 to θ_e. So, this becomes a tractive effort.

$$Tractive\ effort, F = \int_0^{\theta_e} br\,\tau(\theta)\cos\theta\,d\theta$$

Now, while evaluating the performance of a power rigid wheel, we have to find out in addition to tractive effort other parameters like what is the drawbar pull ok, then what is the weight, the expression for weight and then we can find out what is the wheel torque and of course, we can find out other forces like coefficient of traction and coefficient of gross traction CGT. So, for finding out pull, then we have to find out what is the rolling resistance ok. So, if you know thrust, thrust minus rolling resistance will give you the drawbar pull.

Now because of the weight, there will be soil reaction. The radial force will be acting and it will have two components ok. So, the, this horizontal component of radial force, this is radial force, the horizontal component of radial force will be equal to your rolling resistance. So, first we have to calculate the vertical component one vertical component is from the shear force, the other vertical component is from the radial force. So, summation of that will give you, what is the weight. So, weight will be equal to

$$W = rb\left[\int_0^{\theta_e} \sigma(\theta)\cos\theta\, d\theta + \int_0^{\theta_e} \tau(\theta)\sin\theta\, d\theta\right]$$

Now for drawbar pull, this will be equal to P and this is tractive effort which is nothing, but

$$Drawbar\ pull, P = rb\left[\int_0^{\theta_e} \tau(\theta)\cos\theta\, d\theta - \int_0^{\theta_e} \sigma(\theta)\sin\theta\, d\theta\right]$$

for the wheel torque, if you know the tractive effort then tractive effort multiplied with the rolling radius. So, that will give you the wheel torque. So,

$$Wheel\ torque = rb\int_0^{\theta_e} \tau(\theta) d\theta$$

So, these are the expressions which you can calculate. The only thing which we need to know is, what is the distribution of shear stress, what is the distribution of the normal stress. Then only you can find out the shear force, then what is the pull which is developed, what is the vertical load which is acting. Now, one scientist, he found out the shear stress and normal stress distribution and he measured and derived from this formula and then what he observed is very interesting thing, interesting in the sense, this is the measured and predicted shear stress and normal stress in compacted sand at a slip of 22.1 per cent. If you look at this figure, then the procedure which we followed for finding out shear stress and normal stress distribution along the contact area is quite close. We can say, the measured and the predicted values are quite close. Hence, the procedure which we followed is on light. So, in this lecture attempts are made to derive equations for predicting tractive effort for a powered rigid wheel and that the important components are the shear displacement and we have derived an expression for slip velocity and slip velocity from where we tried to find out what is the shear displacement and knowing the shear displacement and shear stress relationship, we can establish what is the shear stress which is prevailing in the contact area. Now, knowing the shear stress then we try to multiply the contact area so that we can find out what is the tractive effort developed.

So, you can refer to the books like the theory of ground vehicles by Wong J Y and soil dynamics in tillage and traction by William R Gill and G Vandenberg. Thank you.