Traction Engineering
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Lecture 15: Tutorial 3 – Computation of rolling resistance of rigid wheel and pneumatic wheel

Hi everyone, this is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL online course on Traction Engineering. This is the lecture 15, where I will try to take a tutorial related to rolling resistance. That means, how to find out rolling resistance of a rigid wheel as well as a pneumatic wheel.

So, the concept will be, how to find out rolling resistance of a rigid wheel on a horizontal as well as inclined surface, how they are different, and then finding out the rolling resistance of a pneumatic wheel. When I say, pneumatic wheel means there will be two modes of operation, one is rigid mode and the other one is elastic mode; and how to find out rolling resistance in these two modes of operation that we are going to discuss with the help of simple problems. Now, if you look at the problem which are given here, if a free rolling rigid wheel of diameter 80 cm, section width 10 cm and the total weight 10 kg. This has to be operated on a horizontal surface with shear deformation modulus 2000 N/m^{n+2} and exponent of sinkage as 0.5. Under these conditions, you have to find out what is the force required to tow the wheel. So, the information which are given as very clear. One is diameter which is given as 80 cm, the other one is section width which is given as 10 cm, b is given as 10 cm and weight which is acting on the wheel is 10 kg. Now, if a wheel is there and we want to find out the rolling resistance. This is a horizontal surface, as I said so, then the first thing is, what is the sinkage and what equation we are going to utilize.

Since the diameter is more than 80 cm. So, we can conveniently apply Bekker's equation for finding out rolling resistance. So, which is nothing,

$$R = \frac{bkZ_o^{n+1}}{n+1}$$

and we are interested in because Z_0 is not known. So, we have to find out what is the value of for Z_0. Now, this is W ok.

So, expression for Z_0 will be, if you look at the Bekker's equation. So, Z_0 is nothing, but this will be equal to

$$Z_o = \left[\frac{3W}{bk\sqrt{2r}(3-n)}\right]^{\frac{2}{2n+1}}$$

Now, in the problem the value of k is given, shear deformation modulus. So, k is given as 2000 N/m^{n+2} and exponent of sinkage n is given as 0.5. So, now, you have to substitute this value. That means, weight is given, k value is given, b value is given, diameter that is 2r is given, 3 minus n, n is given. So, now, you substitute these values and find out what is the, what is the value for sinkage. So, once you know sinkage, you substitute in this equation. Rolling resistance is equal to $(bkZ_0^{n+1})/(n+1)$. So, we will find out the value of rolling resistance. So, rolling resistance is nothing, but the force required to tow the wheel.

So, that is will be the force required to tow the wheel and it comes out to R or the towing force comes out to 71.178 N. Since weight is less, sinkage is less, so obviously, the rolling resistance will be less. So, it is basically the first thing which you look into this, is this rigid wheel, free rolling rigid wheel and there is no power. So, it is a towed wheel.

So, we immediately apply Bekker's equation, but in Bekker's equation, there are certain limitations which are to be looked into. So, limitations is sinkage should be a low and the diameter should not be less than 50 cm. So, so, that way it is satisfying, diameter is 80 cm and sinkage you can verify from here and then apply Bekker's equation to find out what is the rolling resistance. Now, next thing is, if the same wheel is to be towed off on an inclined surface, that means, now the surface is no more horizontal. So, it has been inclined.

So, the new setup will be like this and this inclination is given as 15°. What will be the force required to tow the wheel on the slope? So, for this, you have to draw it. This is the wheel. Now, weight I said 10 kg. So, it will have 2 components. So, this is θ.

So, this component will be $W \cos\theta$ and this component will be $W \sin\theta$. I want to tow the, move the wheel upward against the slope. So, what to do now? We have to find out again, because this wheel is satisfying Bekker's theory that is rolling resistance theory. So, we will use the same equation. R will be equal to

$$R = \frac{bkZ_o^{n+1}}{n+1}$$

but this is only for horizontal surface. Now, this in the surface is inclined and there is a component is acting along the slope $W \sin\theta$ against the direction of travel, opposite to the direction of travel.

So, that means, the rolling distance will be no more the expression which I have given for horizontal condition. So, rolling resistance will be

$$RR = \frac{bkZ_o^{n+1}}{n+1} + W\sin\theta$$

So, this is the additional component which will come into picture. Now, what is Z_0 in this case? How to find out Z_0? For a horizontal soil surface, we calculated Z_0 is

$$Z_o = \left[\frac{3W\cos\theta}{bk\sqrt{2r}(3-n)}\right]^{\frac{2}{2n+1}}$$

So, B is known, k is known, because these are not going to change and exponent of sinkage is not going to change. So, Wsinθ, we know θ. So, you can calculate Wsinθ and then immediately you can find out the Z_0 value and then substituting in this equation, this will be the final value which is required to tow the wheel upward along the slope. Now, if somebody will say, I will move this one downward then the difference will be how much? So, instead of plus W sin theta, because we are rolling it the opposite direction, so, this will be -Wsinθ, ok.

So, that is the difference. This component is acting downwards. So, it will be minus because we are rolling downwards. So, these are the difference we have to take into consideration. I hope, I have clarified in the sense first you have to see whether the whether the conditions which are given are applicable to the Bekker's limitations which are given within that or not. If it is satisfying then you take the help of Bekker's equation and the derivations which are, which we have already done for sinkage that we take and find out and from that sinkage then find out what is the rolling resistance.

And if it is an inclined surface then look at the, first you have to draw it very neatly and then see whether we are moving upward or downward because that W will be now reserved into two components, one is horizontal component, that the other one is vertical component and that vertical component is responsible for developing that sinkage. So, the vertical component will be Wcosθ. So, instead of W, we have to replace with Wcosθ and if you are moving upward then the horizontal component has to be added, and if you are moving downward then the horizontal component W sin theta is to be deducted. So, this is all you have to keep in your mind then only you can solve this problem.

Now, next problem is, a two-wheel drive tractor when it is fitted with a 13.6-28 tyre at the rear axle carrying a load of 15 kN and this has to be operated in soil with a pressure sinkage parameters exponent of sinkage n as 1, frictional modulus of sinkage k_ϕ 1520, 1520 kN/m^{n+2} with both the wheel sharing equal load. Then two inflation pressures 70 and 100 kPa are proposed. Now, the corresponding pressure due to carcass stiffness is 20 and 35 kPa, respectively. So, what is asked is to find out the rolling resistance of rear tyres for both the conditions and then from there you have to suggest which inflation pressure is to be used. So, when I say it is a pneumatic wheel.

So, the first thing which you have to look into is, what is the mode of operation. There is, there are two modes of operation; one is rigid mode, the other one is elastic mode. If it is in rigid mode then you have to follow Bekker's equation, if it is elastic mode you have to follow others, other formulas which I am going to derive which we have already derived in our theory classes. So, to find out the mode of operation, we have to first find out what is the critical ground pressure. What is given, n value is given, n is 1, k_ϕ is given as 1520 kN/m^{n+2}. Since k_c value is not given, that means, k_c is assumed to be 0.

So, k_ϕ value, $K = (k_c/b) + k_\phi$. So, now, since k_c value is not given, we assume as to be 0. So, this component will be 0. Now, k_ϕ will be equal to K. Now, for finding out the critical ground pressure, critical ground pressure, so, we denote it as Pg critical, P_{GRc}.

$$Critical\ ground\ pressure,\ P_{GRc} = k^{1/2n+1}\left[\frac{3W}{(3-n)b\sqrt{D}}\right]^{2n/2n+1} = 100\ kPa$$

So, the axle has to carry a load of 15 kN. So, W will be equal to 15/2 and the other thing which is given is, both the wheels are sharing the same load. That means, equal load. So, that is why, I have divided W as axle weight divided by 2.

So, that way you get 7.5 kN. Now, to find out b and D. So, the tyre is a bias platter and since no aspect ratio is given. So, we assume that aspect ratio as 0.75. So, now, find out the section width b and diameter D. So, section width b will be equal to how much? It is given as 13.6. So, 13.6 × 2.54. So, that will be in centimeter and for finding out D, we have already given that formula in the theory class. It is 1.06 times nominal rim diameter, which is here 28, this is in inches. So, I have to convert it into centimeter or meter then plus 0.75, which is the aspect ratio into 2 × 13.6 into… this is in inches, again multiplied 2.54. So, that way we will get a diameter of 1.27 meter, ok.

So, D is now calculated, b is calculated. Now, substituting here, you have to find out and exponent of sink is given. So, this comes out to be 100 kPa critical ground pressure. Now, two inflation pressures are suggested; one is 70 kPa. So, Pi is 70 and corresponding carcass stiffness is 20 psi. So, P_c corresponding to 70 is 20 kPa.

The other condition, P_i is 100 kPa and P_c is 35 kPa. So, $P_i + P_c$ in the first case is 90 kPa and in the second case $P_i + P_c$ will be equal to 135 kPa.

$$P_i = 70\ kPa, P_c = 20\ kPa$$

$$P_i + P_c = 90\ kPa$$

$$P_i + P_c < P_{GRc} \approx Elastic\ mode\ of\ operation$$

$$Z_0 = \left(\frac{P_i + P_c}{k}\right)^{\frac{1}{n}}, RR = \frac{bkZ_o^{n+1}}{n+1} \times 2$$

So, now the second condition says, $P_i = 100\ kPa, P_c = 35\ kPa$

$$P_i + P_c = 135\ kPa$$

$$P_i + P_c > P_{GRc} \approx rigid\ mode\ of\ operation$$

$$Z_o = \left[\frac{3W}{bk\sqrt{D}(3-n)}\right]^{\frac{2}{2n+1}}, RR = \frac{bkZ_o^{n+1}}{n+1} \times 2$$

Now this is the rolling resistance for a single wheel. If you want to find out rolling resistance for a single wheel, so, that means, you have, this has to be multiplied with 2. Same is the case here, multiplied 2. Now you verify, where the rolling resistance is more than accordingly. We reject that one the, where the rolling resistance is less, that has to be taken and then that, corresponding to that what is the inflation pressure that has to be recommended for continuous operation. So, while calculating the rolling resistance, there are 2 things to be seen; one is what is the mode of operation. Once you decide the mode of operation, that means, we have to first find out what is the critical ground pressure and then knowing the inflation pressure and the pressure due to carcass stiffness, if some of these 2 pressures are lesser than the critical ground pressure then we recommend, we say that the wheel is operating in elastic mode. If it is more than critical ground pressure, we say that this is operating as a rigid wheel. So, if it is in under rigid wheel, then we apply Bekker's equation to find out what is the rolling resistance and if it is in the elastic mode then we first find out Z_0 sinkage, taking into consideration the sum of inflation and carcass pressure and the modulus of sinkage and then from there we find out what is the rolling resistance.

Hope, I have clarified this problem. So, in brief, I can summarize, in this tutorial, I tried to cover the concepts of computing rolling resistance on a horizontal as well as inclined surface, where how to take the component of weight, that is important in inclined surface. So, once you decide that, then it becomes easier for you to calculate the rolling resistance. So, if it is rolling down, then the component, that is sinθ component of W, has to be deducted. If it has to be rolled up, then the cosθ component is to be taken up. Similarly, we tried discuss about the rolling resistance of a pneumatic wheel, where we discussed what is the rigid mode, how to find out rigid mode, then what is the elastic mode and how to decide that the wheel is in under elastic mode. And once you decide that, then following the equations which are given, you can find out what is the rolling resistance. So, the inflation pressure and the carcass stiffness pressure gives to carcass stiffness which gives minimum rolling resistance that has to be recommended for using the using in the track. Thank you.