Traction Engineering
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Lecture 16
Tractive Effort and Slip of a Track

Hi everyone, this is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you to this NPTEL online certification course on Traction Engineering. This is lecture 16, where I will try to cover tractive effort and slip for a track.

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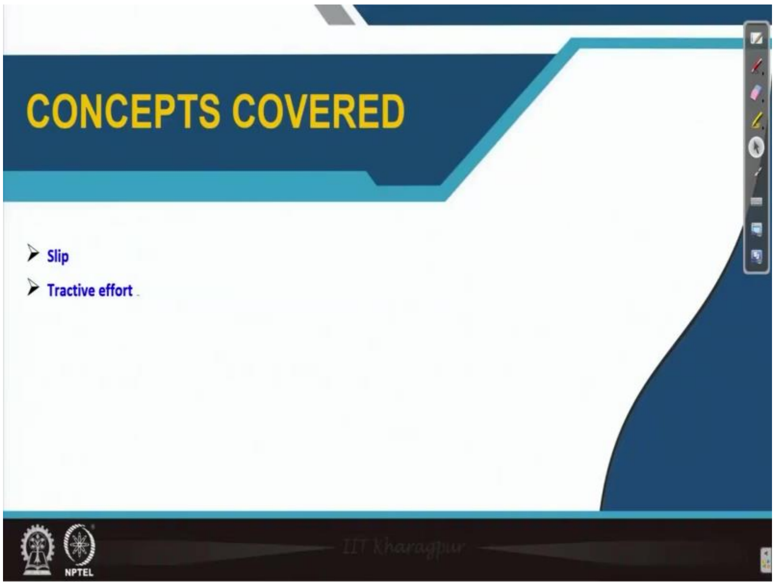

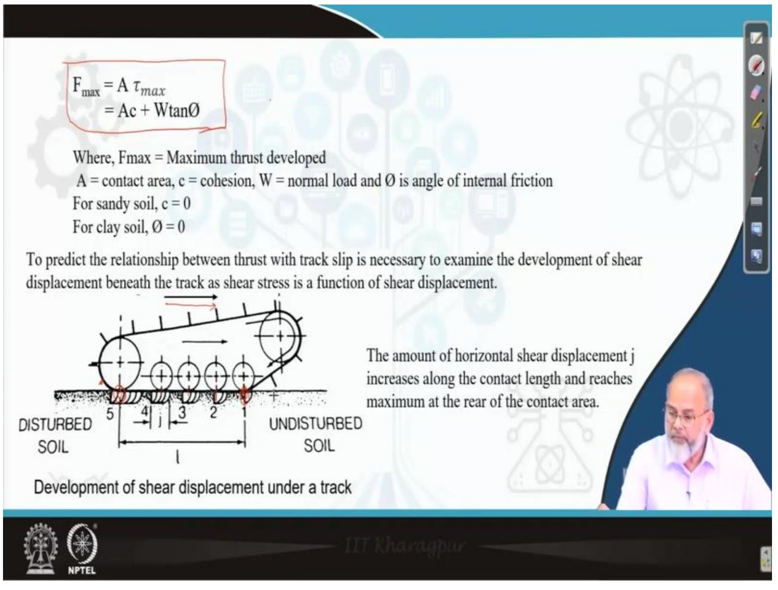

The concepts covered will be again slip as well as tractive effort. As you know, a track has a bigger contact area. If you look at the figure, which is shown in this slide, it has a bigger contact area. So, the maximum tractive force which is developed, is denoted by F max which is equal to shear stress, maximum shear stress into area.

Now, when I say tractive effort developed that means, it is a component of shear stress, shear force which is developed by the contact patch and shear force is nothing but the shear stress which is developed. And if you know this relationship between shear stress and shear displacement then knowing the displacement, you can find out, what is the shear stress which is developed and knowing the contact area and shear stress, you can calculate, what is the shear force which is developed in the contact patch.

And the equation which is given here, the maximum thrust which is or tractive effort which is developed, which will be equal to contact area A into shear stress tau max. So, in other words, I can write as $A \times c + W \tan\phi$. So, here A is the contact area, c is the cohesion and W is the weight acting on the track and ϕ is the angle of internal friction. So, if we look at this equation when a track is operated in sandy soil, pure sandy soil, there is no cohesion. So, area has no role, it is the W which plays or it is the weight which plays the most of the things that means it controls the maximum thrust which is developed.

Now, if it is a clay soil, then area comes into picture and W has no role, but the equation cannot be applied for all conditions. It is only giving you maximum thrust, if you are or anybody is interested to find, what is the thrust developed along the contact patch, then we have to know what is the shear displacement under the track at the interface between soil and track. So, to predict the relationship between thrust to tractive slip, or track slip, we have to examine the development of shear displacement beneath the track.

So, it is very simple in the sense because shear stress is a function of shear displacement. So, you have to find out what is the shear displacement at the contact patch. If you look at the figure, there you can see I have indicated 1, 2, 3, 4, 5, 5 points. One is just at the beginning. Because this track is moving in a direction which is given by this arrow and this is the powered wheel which is driving and these are the rollers for being tension. At point 1, point 2, point 3, point 4 and point 5. Point 5 is the endpoint, point 1 is the beginning of shearing.

Now, the shear displacement which is developed at point 1 and the shear displacement which is developed in subsequent points they are not same. Why it is so? Why there is a variation in the shear displacement? Because the link which is shearing at point 1 is just coming in contact with the soil whereas the link which is present, link means link in the track which is present at point 2 or present at point 3 or present at point 4, they are shearing the soil for a considerable period of time.

That is why shear displacement is more as you move towards the rear end of the track that means is the first point is the beginning point, last point 5 is the rear end, front end, rear end. So, as you move towards the rear end, your shear displacement is increasing which is indicated in this figure. And it reaches a maximum value, because after that there is no contact, again the track will be, the links will be not in touch with the soil. So, when the moment it leaves the soil, it comes to this position, there is no shear displacement developed by this link.

So, the maximum which is developed is at this point at the rear end and the minimum which is developed is at the front end. Now, how to find out the shear displacement? So, again we know, what is the slip velocity? What is the slip velocity which is present?

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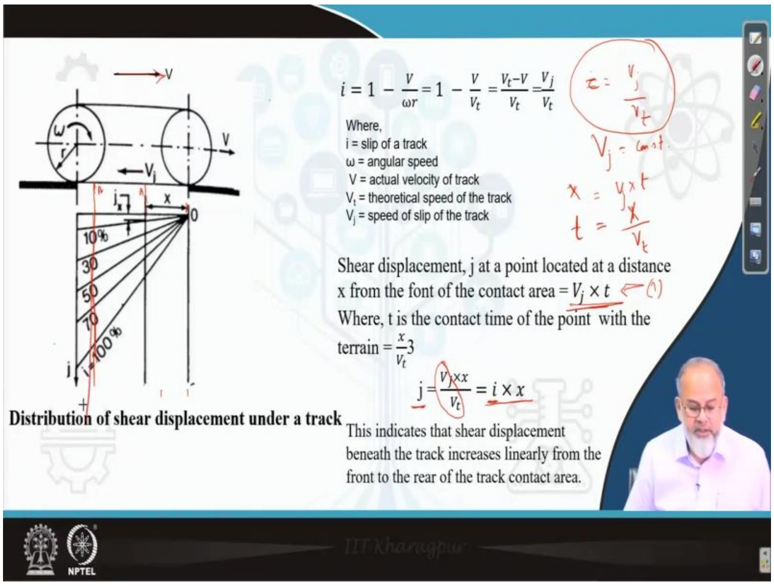

The slip, we can be defined as

$$i = 1 - \frac{V}{\omega r}$$

ω is the angular speed of the sprocket and r is the radius of the sprocket which is driving. So, ωr will give you virtually the theoretical speed which is denoted as V_t. So, I can write as

$$i = 1 - \frac{V}{\omega r} = 1 - \frac{V}{V_t} = \frac{V_t - V}{V_t}$$

So, V is the actual velocity and the difference between theoretical speed and slip speed is denoted as V_j. So, i can written as V_j/V_t; where i is the slip of the track and ω is the angular speed, V is the actual velocity of the track, V_t is the theoretical speed of the track and V_j is the speed of slip of the track. Then suppose I am interested in finding out shear displacement at a point which is at a distance x from the front end of the contact area.

So, before that let us know what is the slip velocity V_j? V_j, when it is acting opposite to the direction of motion, that means the track is slipping, when it is acting along the direction of motion, then the track is skidding. So, I have indicated here V_j direction indicated here the track movement. So, these are in opposite direction, so, the track slips. Now, V_j does not vary along the track length, since link cannot stretch, so, the track cannot stretch. So, that is why we assume that V_j is not varying, it is constant.

So, slip velocity is constant below the track and V_j is opposite to the direction of travel. Now, if you are interested in finding out shear displacement at distance x from the front end of the contact area. So, this is the point say A now, which is at a distance x. Then,

$$x = V_j \times t$$

V_j the slip velocity, t is the time the contact time. So, now, t will be equal to, if t will be equal to, if V_t the theoretical speed, then

$$t = \frac{x}{V_t}$$

Now, if you substitute this expression in this equation, then j that is shear displacement will be equal to,

$$j = \frac{V_j \times x}{V_t}$$

So, that is equal to V_j/V_t we have already expressed that this is equal to the slip. So, instead of V_j/V_t, I have written i. So, j = i×x which indicates that shear displacement is a function of slip, shear displacement is a function of distance or the point under consideration, what is the distance of this point from the front end. So, if you look at this equation again then what you can observe is that, this shear displacement is linearly varying with i. It is varying from front to the rear end of the contact area linearly and with increase in slip, its value is increasing and that is what is reflected in this figure a 10 percent slip, 30 percent slip, 50 percent slip, 70 percent slip, 100 percent slip. So, how this displacement is varying? So, j is on the y axis. So, this figure shows that shear displacement is a function of slip is a function of distance. Now, suppose I increase this distance to this point, so, this is the point B. So, now, this will be the displacement. If I draw a vertical line, this will be the shear displacement.

So, the next question is what we will do with shear displacement? Once you calculate shear displacement what next? So, next is to find out what is the relationship between shear stress and shear displacement which is existing.

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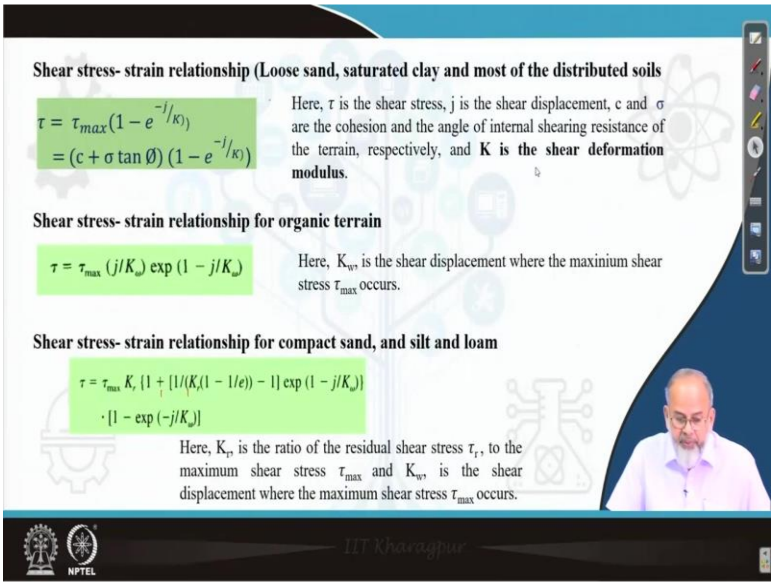

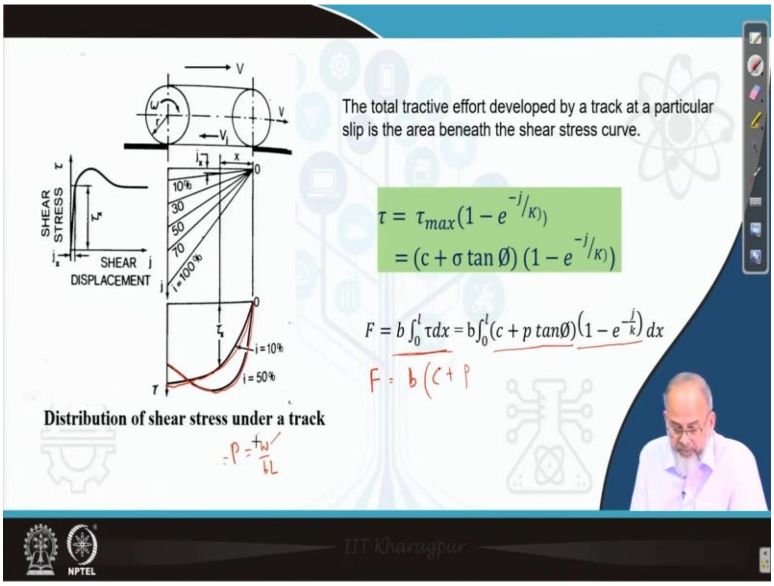

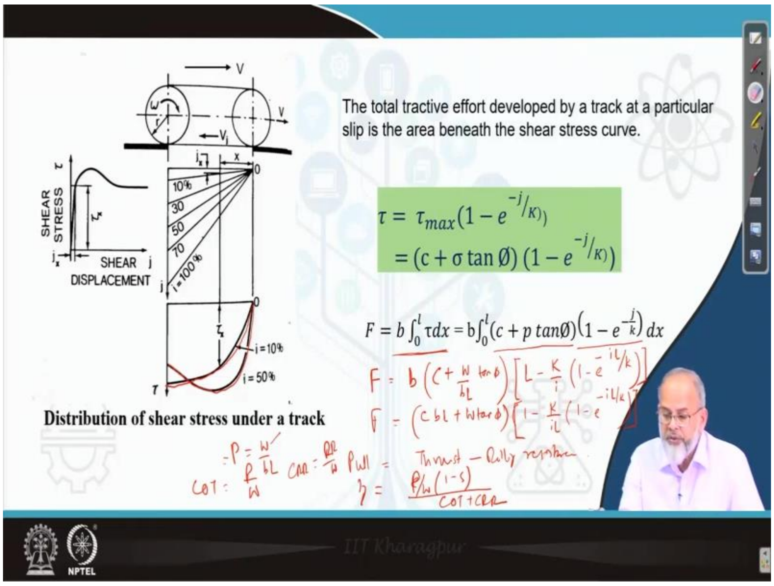

So, in nature what we have observed? There are 3 categories of relationship available, one is for loose sand, saturated clay and most of the disturbed soils and the other one is for organic terrain and the third one is for a compact sand, silt and loam. So, you can take either of these equations which can be utilized to find out, what is the relationship between shear stress and shear displacement. So, once you finalize this, then we have to find out, what is the shear, thrust force developed?

Suppose, for example, we try to find out, we tried to take the relationship that is the in loose and a saturated clay where, relationship,

$$\tau = \tau_{max}\left(1 - e^{-j/K}\right)$$

$$\tau = c + \sigma tan\varphi\left(1 - e^{-j/K}\right)$$

So, here σ we have considered as the normal stress which is acting on the track and that is uniform along the track, it is not changing. So, if that is the case then thrust force will be equal to, how do you find out thrust force? Once you know the relationship for tau.

So, we tried to plot tau versus the displacement at 10 per cent slip, this will be the curve and at 50 percent, this will be the curve. So, like that you will get a series of curves, by varying the slip. Now, if you want to find out thrust force then you have to find out the area under this curve. So, area under this curve means you have to multiply shear stress with the area. So, what I have done is,

$$F = \int_0^l \tau b \times dx$$

Now, if I integrated by putting the relationship between τ and τ_{max} then this is the relationship which you are taking. So, this has to be integrated now. So, what finally you will be getting is, if we integrate. So, you will get an expression for thrust force for a loose soil. For a loose soil what will be the thrust force? Now, the final expression for this will be

$$F = b\int_0^l (c + ptan\emptyset)dx$$

I can write as W since we have assumed p = W/bL, assuming that the pressure is constant beneath the track.

So, W is the weight, b is the width of the track, L is the length of the track. So, W/bL will give you what is the contact pressure. So, instead of p, I can write W/bL. So,

$$F = b\left(c + \frac{W}{bL}tan\emptyset\right)\left[L - \frac{K}{i}\left(1 - e^{-iL/K}\right)\right]$$

Now, if I take L common out then it will be

$$F = (cbL + Wtan\emptyset)\left[1 - \frac{K}{iL}\left(1 - e^{-iL/K}\right)\right]$$

This will be the final expression for thrust. Now, if the rolling resistance, then you can find out what would be the pull developed?

So, pull will be equal to thrust minus rolling resistance. So, if you want to find out the tractive efficiency then pull by weight will give you the tractive efficiency. Pull by weight into 1 minus s. So,

$$Tractive\ efficiency, \eta = \frac{COT}{COT + CRR}(1 - s)$$

COT is P/W by and CRR is RR/W, motion resistance by weight.

Now, if you look at this equation, it is the length that controls the thrust force. If you have same contact area one is having higher length lesser width, the other one is having wider width lesser length than it is the length of the track which controls the thrust force, more the length more will be the thrust force developed.

So, in design you have to keep this in mind. Whenever you got some area try to keep more length. So, that you will get more thrust force developed and since the length is more and contract area will be same for both the cases. So, rolling resistance will be nearly same only the thrust force will be more so, that is why the ultimate pull will be higher in case of lengthy track. Next is, we are talking about the distribution, here very nicely we have assumed that the pressure is uniform. So, we have taken W/bL. If pressure is not uniform, what are the other possibilities? So, let us now discuss, discuss those possibilities.

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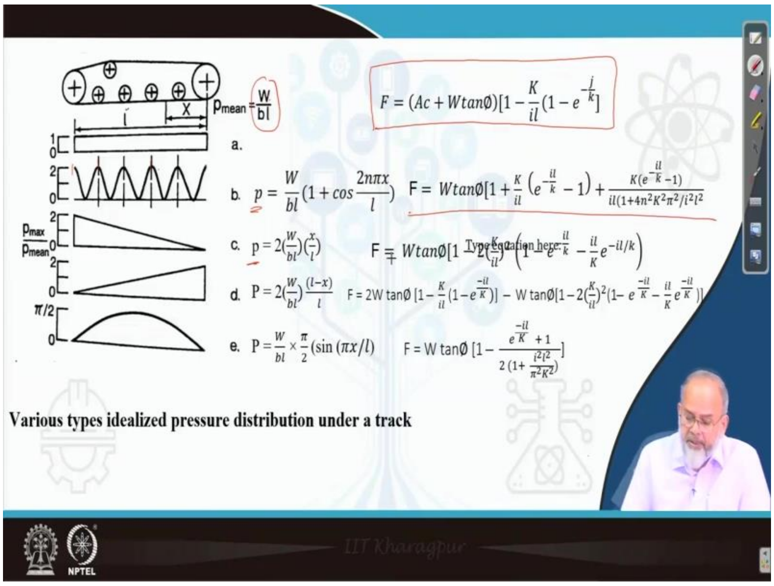

So, there are 1, 2, 3, 4, 5 kinds of pressure distributions. One is uniform pressure distribution where I have indicated p = W/bL and if that is the case then this is the thrust force developed. Now, in the other case, there will be sinusoidal kind of things, now you are getting some peaks at regular intervals, if n is the number of peaks, then p can be the pressure or the normal stress distribution can be written as

$$p = \frac{W}{bL}\left(1 + cos\frac{2\pi n x}{L}\right)$$

Thrust force will be equal to this one.

$$F = Wtan\emptyset\left[1 + \frac{K}{iL}\left(e^{-iL/k} - 1\right) + \frac{K\left(e^{-iL/K} - 1\right)}{iL\left(1 + \frac{4n^2K^2\pi^2}{i^2L^2}\right)}\right]$$

So, it all depends on what is the pressure distribution.

Now, similarly, when the pressure distribution is minimum 0 at the front end of the track, maximum at the rear end of the track, then

$$p = 2\frac{W}{bL}\left(\frac{x}{L}\right)$$

If pressure distribution is maximum at the front-end, minimum at the rear end then

$$p = 2\frac{W}{bL}\left(1 - \frac{x}{L}\right)$$

Similarly, if the pressure distribution is gradually changing, reaching to a peak, starting from the front end and at the rear end, it is again coming to 0 then

$$p = \frac{W}{bL} \times \frac{\pi}{2}\left(sin\frac{\pi x}{L}\right)$$

So, on the right side I have given for all these 5 conditions, what will be the expression for tractive force. These are all derived based on that integration where the pressure distribution is to be substituted by this and then you have to integrate it and finally, the expression is given here. So, what do you conclude from here is thrust which is developed under a track depends on what is the pressure distribution and it depends on what is the shear stress-shear displacement relationship. If you can find out this relationship for different conditions, then we can find out what is the thrust developed by integrating along the contact area and then if you know the shear stress we will try to find out the shear force by multiplying with the area.

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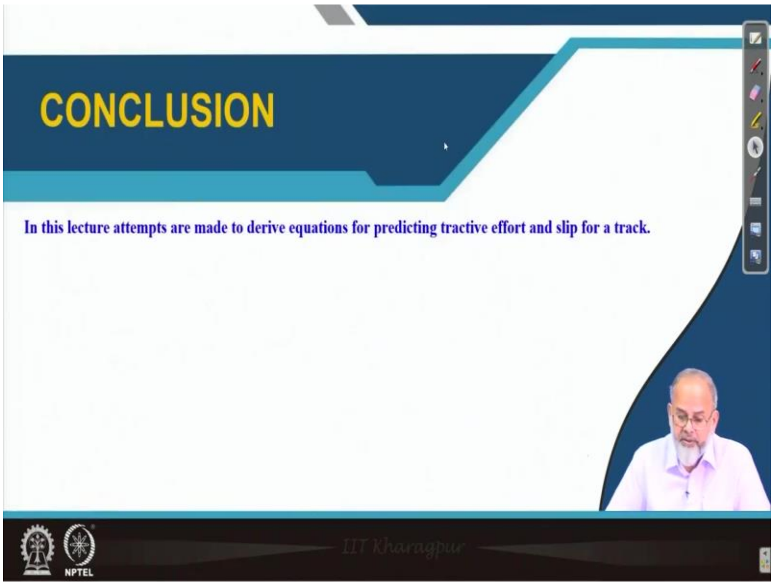

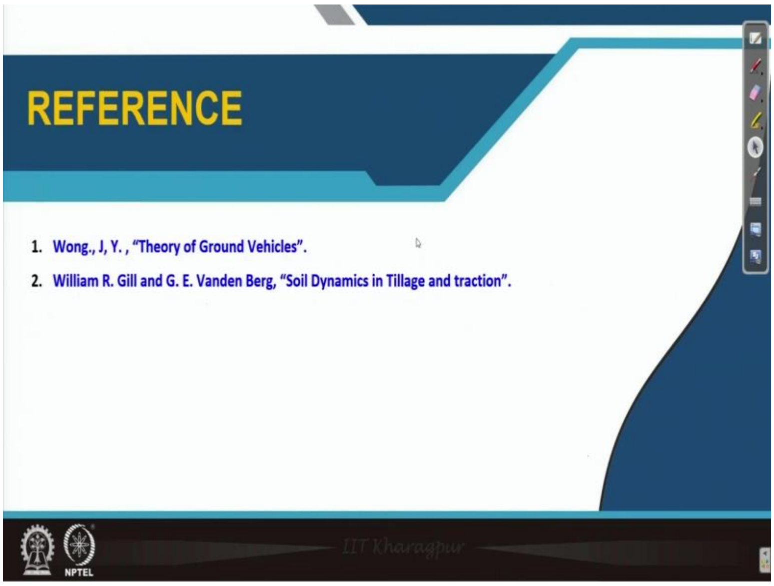

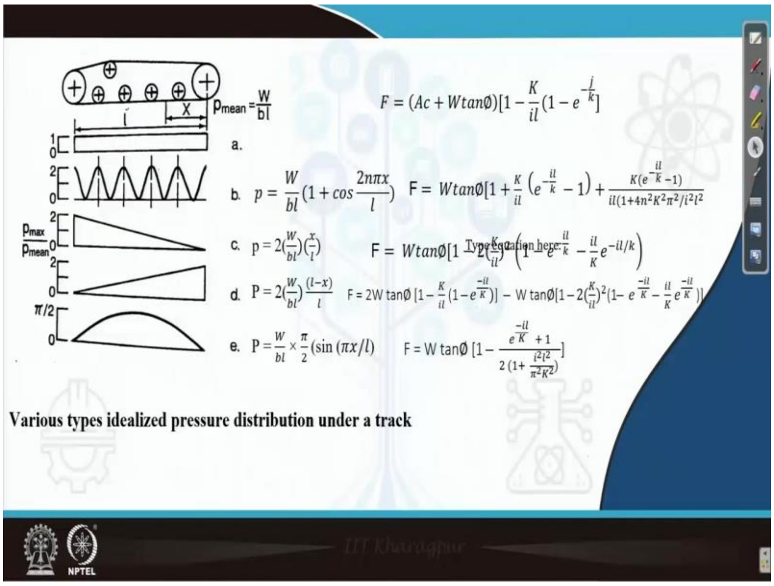

So, this is how we will try to find out the tractive effort for a track. So, only thing is one has to know what is the pressure distribution and what is the shear stress shear displacement relationship then one can find out what is the thrust which will be developed.