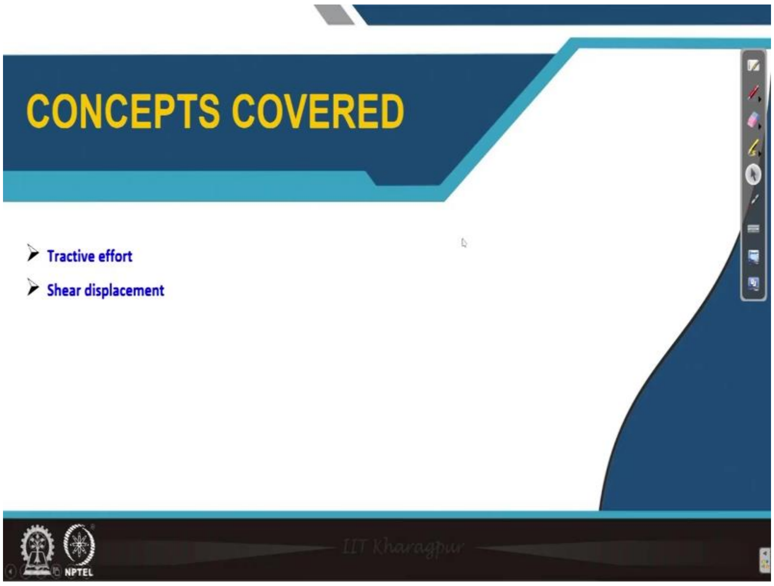
Traction Engineering
Professor Hifjur Raheman
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur
Lecture 17
Tractive Effort and Slip of a Pneumatic Wheel

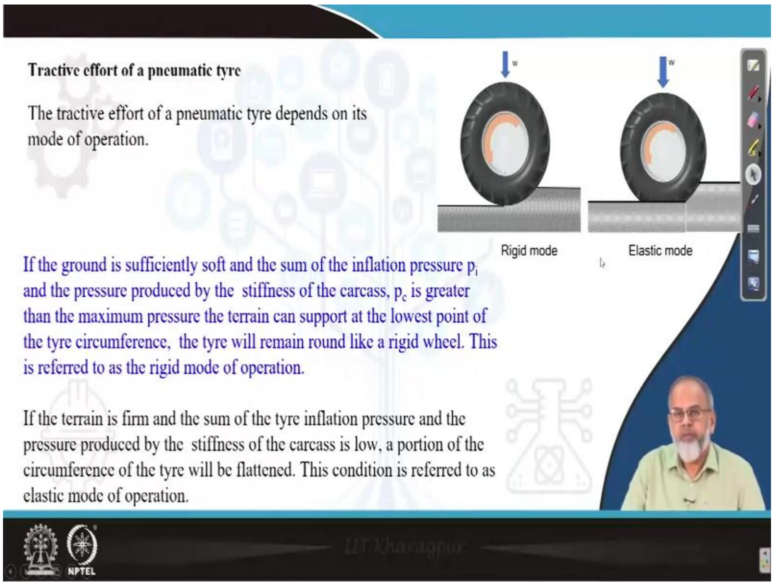
Hi everyone, this is professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur, I welcome you all to this NPTEL online certification course on Traction Engineering. This is lecture 17, where I will try to cover how to find out tractive effort and slip of a pneumatic wheel, when I said it is a pneumatic wheel.

(Refer Slide Time: 0:53)



So, the obvious questions will come to our mind is, whether it is operated in rigid mode or whether it is operated in elastic mode. So, now, the concepts which will be covered will be tractive effort and shear displacement. As you know that tractive effort is related to shear displacement hence, we will cover these two concepts with respect to pneumatic wheel.

(Refer Slide Time: 1:25)

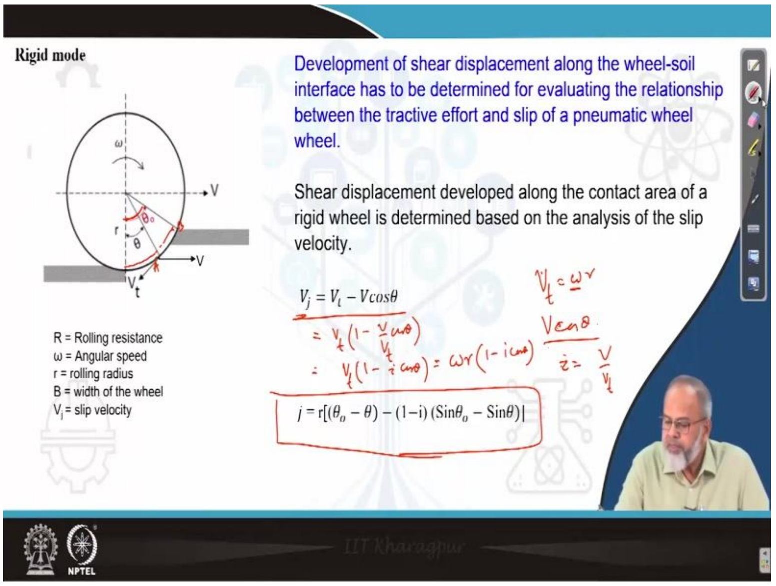


Now, if we look at these two wheels, both are pneumatic wheels, but one is written as rigid mode, the other one is written as elastic mode. The difference is when the pressure due to carcass stiffness and pressure due to inflation of air or the air inflation pressure when the summation is greater than the pressure which the terrain can support at the lowest point of the tyre circumference then the tyre will remain like a rigid wheel. It will remain round like a rigid wheel. So, that means, this will behave like a solid towed wheel or solid powered wheel.

Now, when the wheel is powered, it becomes powered wheel. So, now, our discussion will be confined to powered wheel as we are interested in finding out tractive effort. Now, the second option is elastic mode. Again, the inflation pressure and the pressure due to carcass stiffness, if it is less than the pressure the terrain can support, then the tyre will be flattened at the contact point. So, this condition is called elastic mode. So, these are the two conditions which are shown in the right corner one is rigid, where the circumference of the tyre is perfectly round. Whereas, in the second case, it is not round rather it is flattened at certain portion.

So, if it is in rigid mode, how to find out the tractive effort, if it is in the elastic mode, how to find out the tractive effort that we will discuss separately.

(Refer Slide Time: 3:19)



Now, when it is in rigid mode as I said this will be equivalent to a powered rigid wheel and for powered rigid wheel the main thing is tractive effort which we have already derived. So, we will follow the same procedure. So, when we talk about tractive effort that means, what is the shear stress which is developed? And shear stress is a function of shear displacement, where it is developed? It is developed at the contact patch where the wheel is in contact with the soil. So, this is the face where the shear stress will be developed.

Now, to find out shear stress, we should know what is the shear displacement? And to know the shear displacement, we need to know what is the slip velocity? So, these are all interlinked. So, what is slip velocity? Slip velocity is the difference between the theoretical speed V_t and the actual speed V. So, if you consider a point here as A, then at that point, the theoretical speed will be $V_t = \omega r$, if r is the rolling radius ω is the angular speed, then $V_t = \omega r$ and the actual velocity is V.

Now, if you take a tangential component here at the point A parallel to V_t then V will be, the component will be Vcosθ, because you have considered the point A at an angle theta. So,

$$V_t - V cos\theta = slip\ velocity, V_j$$

So, you have to analyze this slip velocity how it is varying or from there, we will find out what is the corresponding shear displacement and then try to correlate the shear stress with shear displacement. Now, for finding out the shear displacement from the slip velocity, we know that shear displacement will be equal to $x \times V_j$.

So, if I take this one

$$V_j = V_t - V cos\theta$$

I take V_t common then it becomes,

$$V_j = V_t\left(1 - \frac{V cos\theta}{V_t}\right)$$

V/V_t is nothing but slip i. So, indirectly I can write that

$$V_j = V_t(1 - icos\theta)$$

$V_t = \omega r$. So,

$$V_j = \omega r(1 - icos\theta)$$

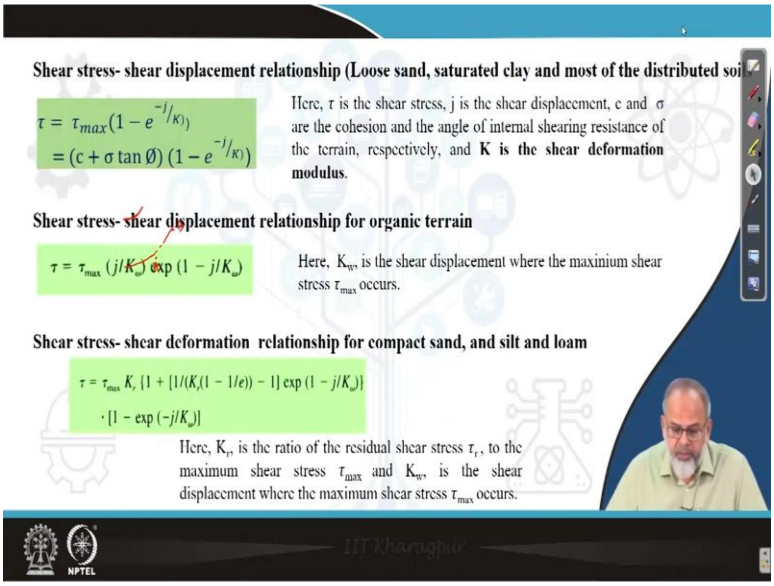
Now, this is at the point A. Now, if you want to find out throughout the contact surface or from the contact surface from A to the point where the wheel is just entering the soil, if you denote it as B then this has to be integrated, okay. This has to be integrated from θ to θ_0.

So, by doing that we will finally develop equation for shear displacement,

$$j = r[(\theta_0 - \theta) - (1 - i)(sin\theta_0 - sin\theta)]$$

That means, displacement is a function of the entry angle θ_0 which is denoted here. Which is shown in this figure and θ_0 is called the entry angle and θ the point where we are interested. Now, once we know the shear displacement next thing is how to correlate this shear displacement with shear stress?

(Refer Slide Time: 7:11)



So, in nature what we have observed is there are three different categories of relationship available are found. One is related to loose soil, the other one loose soil or saturated clay and in disturbed soil and the other one is in organic terrain, then the third one is in compact sand silt and loam. So, these are the three relationships which we have discussed earlier and the relationship in these three conditions like loose sand, saturated clay,

$$\tau = \tau_{max}\left(1 - e^{-j/K}\right)$$

Where, τ is shear stress and τ_{max} is maximum shear stress. The maximum shear stress is described as

$$\tau_{max} = c + \sigma tan\emptyset$$

It is a function of cohesion. It is a function of angle of internal friction of soil and the normal stress which is prevailing. So,

$$\tau = (c + \sigma tan\emptyset)\left(1 - e^{-j/K}\right)$$

Now, in case of organic terrain,

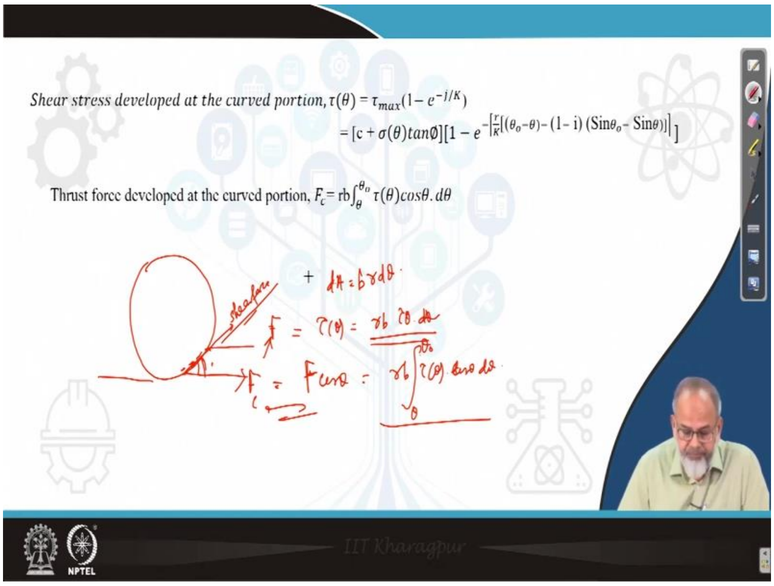
$$\tau = \tau_{max}\left(\frac{j}{K_w}\right)\left(e^{1-j/K_w}\right)$$

So, this K_w is nothing but the shear displacement where the maximum shear stress occurs and the third relationship is,

$$\tau = \tau_{max} K_r \left[1 + \left(\frac{1}{K_r\left(1 - \frac{1}{e}\right)}\right) e^{1-\frac{j}{K_w}}\right] . \left[1 - e^{\frac{-j}{K_w}}\right]$$

So, these are the three relationships available. So, now, depending on our soil condition we have to select one relationship. Then once you select the relationship, next is how to find out shear stress?

(Refer Slide Time: 9:05)



So, shear stress which will be developed at the curved portion will be equal to τ. I have indicated as τ(θ) here. Because, from which angle we are going to what is the distribution of shear stress that is important. So,

$$\tau(\theta) = \tau_{max}\left(1 - e^{-j/K}\right)$$

Now, again I replaced tau max. I have taken this for the loose sand or the saturated clay.

So, c+σtanϕ. Again σ, ϕ, I have taken as σ(θ), knowing we have to know the pressure distribution, whether it is constant or it is varying. So, all those factors will come into picture. Now, I replaced that displacement with the equation,

$$r[(\theta_0 - \theta) - (1 - i)(sin\theta_0 - sin\theta)]$$

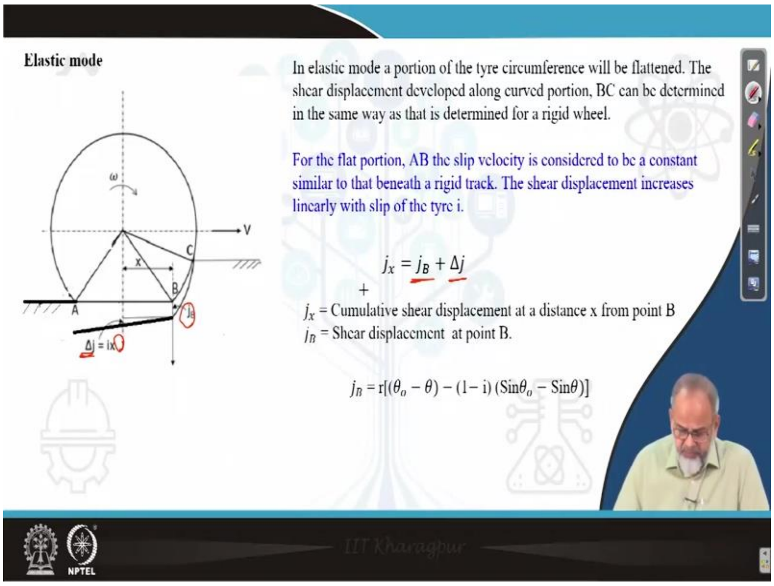
So, this will be the final expression for shear stress. But shear stress is not important what we are interested is to find out the shear force.

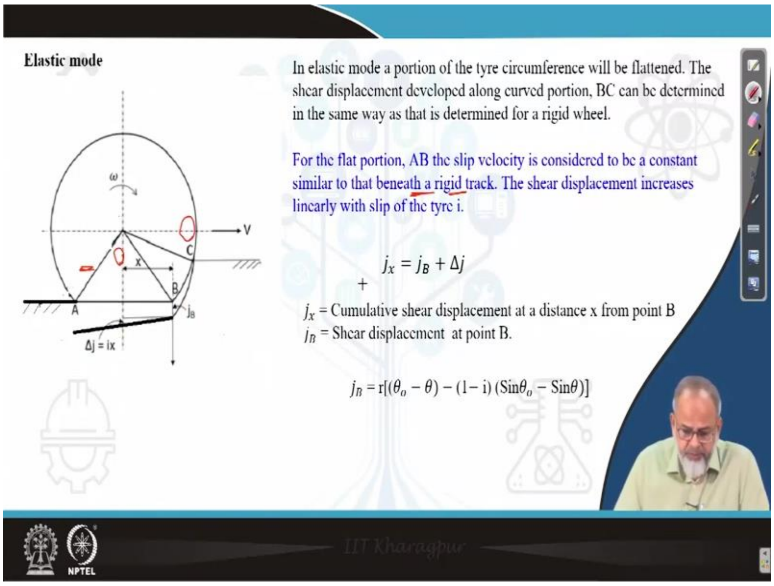
So, in a solid wheel or rigid wheel, shear force will be equal to. So, these are the shear stress. Now, I find out this shear force here. So, we have to multiply the area. So, areas we have considered a point A so area of that elemental area, dA will be equal to if b is the width of the wheel, $dA = b \times r d\theta$.

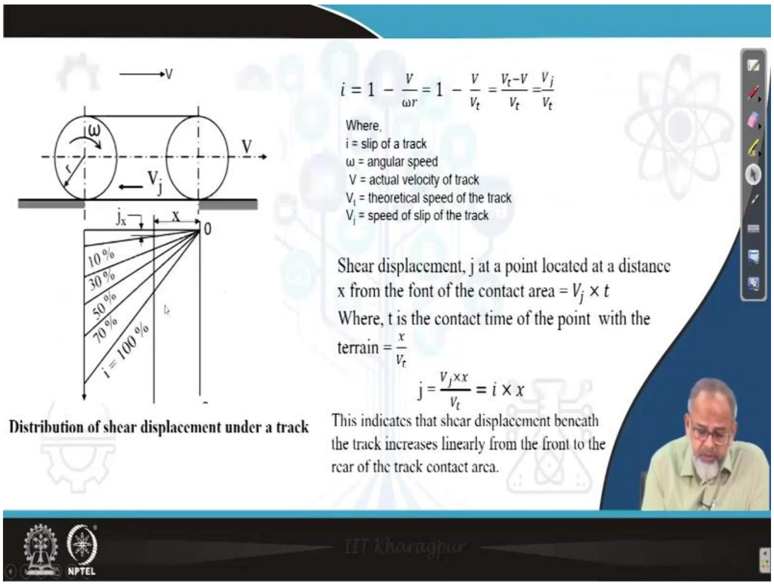
Now, this has to be integrated. So, $\tau \times dA$ will give you thrust force. So, $r \times b\tau\theta d\theta$, this will give the shear force. But we are not interested about the shear force, we are interested in finding out the thrust which is the horizontal component of shear stress, shear force which is developed at the contact patch.

So, this is your F, this is your shear force. So, horizontal component will be $F\cos\theta$, sorry $F_c = F\cos\theta$. So, this has to be multiplied $rb\tau\theta.d\theta$. So, this has to be integrated from theta to theta 0 depending on in which area you are interested to find out the thrust force. So, this will be the expression for finding out thrust in case of pneumatic wheel when it is behaving like a rigid wheel that means, it is moving in a rigid mode.

(Refer Slide Time: 12:23)





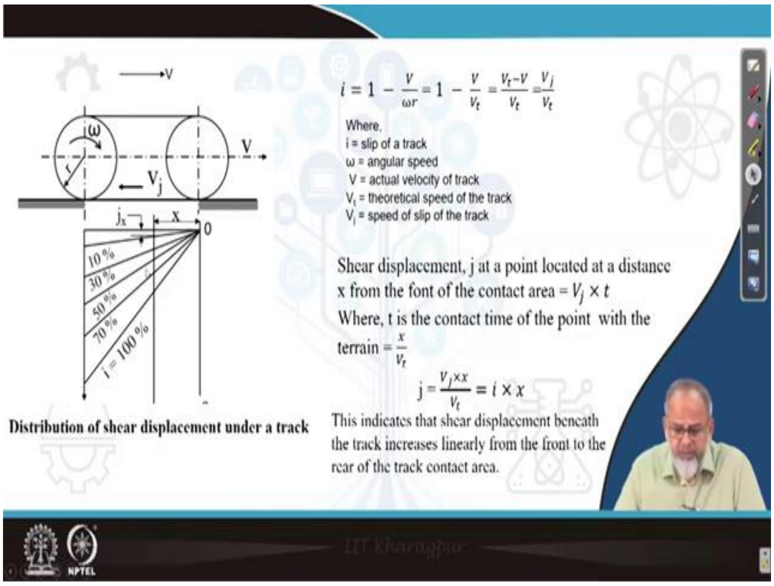


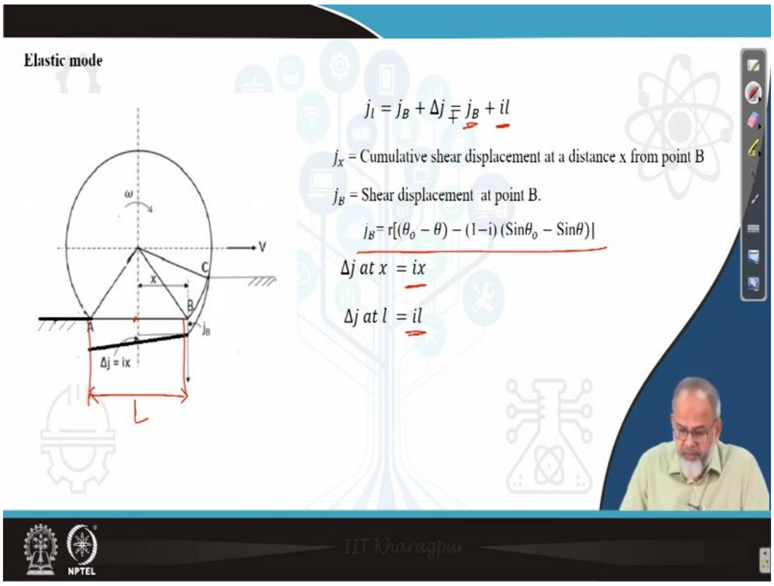
Now the other mode is elastic mode. So, let us see now, what happens to the elastic mode if part of the wheel will be flattened and another component will, which will be curved one. That means, we have to consider the shear displacement along the flat portion, we have to consider the shear displacement and the curved portion. So, I have differentiated or demarcated, you can say the flat portion as AB and the curved portion as BC. Now, to find out the shear displacement or shear stress on the flat portion, we need to know what is the shear displacement in the flat portion?

And similarly, for finding out the shear stress on the curved portion, we have to find out what is the shear displacement in the curved portion. So, if at B the total shear displacement is denoted as j_B and which is denoted a j_B. And at a point suppose x, starting from point B. So, if we are interested in finding out what is the displacement at this point? Then I have denoted that as Δj, Δj.

So, total displacement up to point x will be j_B+Δ j. So, this j_x is nothing but the cumulative shear displacement at a distance x from the point B. Now, we will consider two things one is the curved portion the other one is the flat portion. Let us now see how do we develop this shear stress in curved or flat portion?

(Refer Slide Time: 14:11)





Let us come to the flat portion. So, basically, we want to find out the slip velocity. If the track is moving at a velocity actual velocity V and V_t is the theoretical speed of the track. Then V_t-V will give you, the speed of slip of the track if which is denoted as V_j. If this direction is opposite to the direction of forward movement, then it is positive and then it is called slip. If it is opposite to the theoretical the actual speed or direction of travel then it take as skid.

So, here the theoretical speed is nothing but ωr, where ω is the angular speed of the sprocket or the driver which is driving the track and r is the rolling radius of that driver. Now, shear displacement which is denoted as j_x at a point located at a distance x from the frontal area of the contact area, frontal portion of the contact area. So, that becomes, if you know the slip velocity times t, then that will give you what is the shear displacement during that time. So, $V_j \times t$ and $t = x/V_t$.

So, t is the duration for which the track is in contact with the soil, the link is in contact with the soil. So, now

$$shear\ displacement = \frac{V_j}{V_t} \times x$$

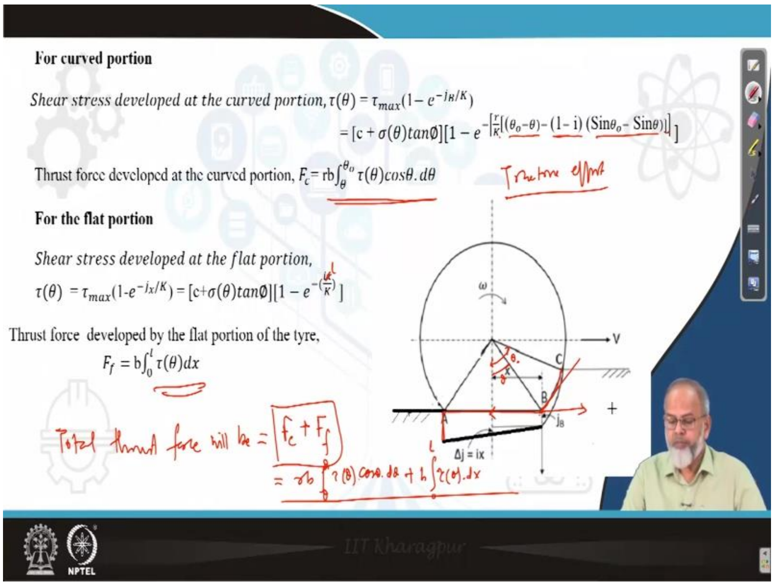
$$\frac{V_j}{V_t} = slip$$

So, it is basically slip into the distance from the front end of the track. Now, if you are in. So, that means, the flat portion will behave like a track and the shear displacement will be calculated or computed as we have followed for computing the shear displacement in case of a track.

So, in a pneumatic wheel, we will consider that shear displacement in this flat portion would be equal to $i \times x$. Now, the total length, if we denote as L which is the contact length. So, if this is the contact length then total displacement will be $i \times L$. If you are interested up to this point, then the distance you have to know. So, at x, it is written as 'ix' and we have already known that the displacement at point B because of the curved portion, this is the displacement.

So, the total displacement will be again summation of $j_B + iL$. Now, next thing is, only shear displacement will not help you, we have to find out what is the shear stress which is prevailing under the flat portion as well as on the curved portion?

(Refer Slide Time: 17:36)



So, for the curved portion, again we have to find out what kind of soil is there? If it is a loose soil, if it is a organic terrain or if it is a compact sand then we have three different kinds of relationship available. So, based on that, we have to find out the relationship between shear stress and shear displacement. So, in this case, I have taken the loose sand where,

$$\tau(\theta) = \tau_{max}\left(1 - e^{-j_B/K}\right)$$

displacement on the curved portion divided by K, shear deformation modulus K.

Now, if you know the normal distribution then τ_{max} I can write as $c+\sigma\tan\phi$ or,

$$\tau(\theta) = (c + \sigma(\theta)tan\emptyset)\left(1 - e^{\left(\frac{r}{K}[(\theta_0-\theta)-(1-i)(sin\theta_0-sin\theta)]\right)}\right)$$

So, θ_0 is entry angle, this is your θ_0 and θ is here. That means corresponding to point B, the angle is θ from the vertical and corresponding to point C, the angle is θ_0. So, now the displacement I have replaced, j_B I have replaced with

$$j_B = r[(\theta_0 - \theta) - (1 - i)(sin\theta_0 - sin\theta)]$$

So, this will be the expression for shear stress.

Now, next thing is what is the shear force which is acting? So, shear force will be again acting tangent to the surface, the contact surface, suppose at point p. Now, the shear force will be equal to whatever shear stress we developed multiply with the area. Area will be equal to rb.dθ. So, this has to be integrated from θ to θ_0, from θ to θ_0. So, that will give you the value of shear force and the cos component of that will be taken to find out the tractive effort. So, tractive effort will be this rb×τ×θ×cosθ.dθ. This has to be integrated from θ to θ_0.

So, that will finally give you what is the tractive effort. That means, now you have find out tractive effort at this point because of the curved portion. Again, for the flat portion, we have to find out the suitable relationship if it is a loose sand then again we take

$$\tau(\theta) = \tau_{max}\left(1 - e^{-j_x/K}\right)$$

If you are interested to find out shear stress up to the point x from the front end of the flat portion, then I replace τ_{max} with c+σ(θ)tanϕ and

$$\tau(\theta) = (c + \sigma(\theta)tan\emptyset)\left(1 - e^{-ix/K}\right)$$

So, I have replaced this j_x with ix. Now, since we are interested for the total length, so, that means, this will be equal to

$$\tau(\theta) = (c + \sigma(\theta)tan\emptyset)\left(1 - e^{-il/K}\right)$$

So, this will be equal to this, instead of x you can take l. Now, the thrust force developed will be, this has to be integrated from this point to the endpoint that means from point B to the point A. So, I have multiplied the area with the shear force. So, that means b×dx the area that has to be integrated from 0 to l. So, this will give you thrust force for the flat portion.

So, now, the total thrust force will be the sum of the thrust force which is developed in the curved portion, sum of the thrust, the thrust force which is developed at the flat portion. So, this is the final expression. So, that means, up to this point we are considering. So, from here to here, then from C to B and B to A. So, finally, the total thrust force will be rb×τ(θ)cosθ.dθ. This is to be integrated from theta to θ_0 plus b× into 0 to lτθdx. This is the final expression.

So, what do we come to know that the important thing in case of a pneumatic wheel is what is the mode of operation? Once you decide the mode of operation based on what is the pressure which is exerted on the ground and the pressure the ground can sustain? Then we decide, what is the mode of operation whether it is in rigid mode or whether it is in elastic mode. So, once it is decided, then it becomes easier for us to find out the thrust force which is developed. So, the procedure is first, if it is a rigid mode than first find out, because the soil will behave like a rigid wheel without any deflection, then we have to follow the same procedure what we followed for rigid wheel.

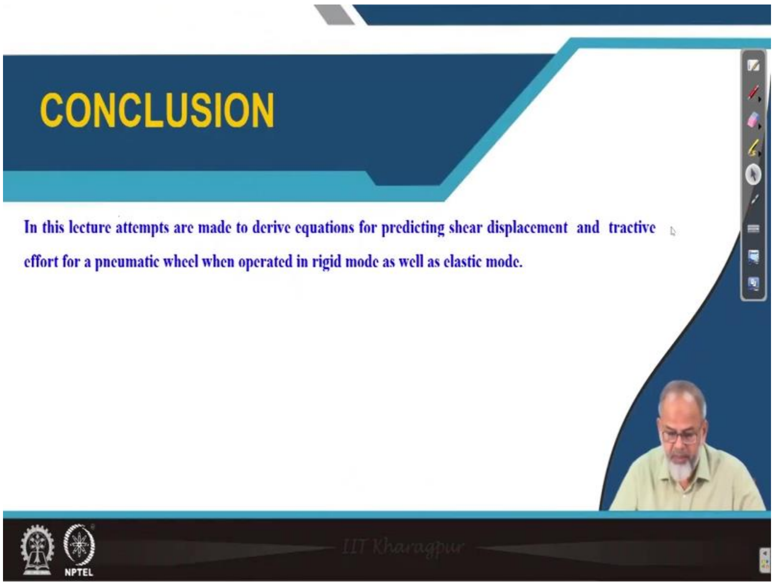
So, you have to find out the shear stress-shear displacement relationship. So, before that we have to find out the shear displacement, taking slip velocity and from there, once you find out the shear displacement then we have to select a suitable relationship depending on which soil conditions the wheel is operated. Now, after getting the suitable relationship, then we find out the shear stress which is developed and then the shear stress has to be multiplied with the contact area knowing the width and radius, we can find out the contact area and this has to be integrated for the portion which is inside the soil. So, that you can find out the thrust force.

If it is in elastic mode, then there will be two sections. Two sections in the sense, we have to find out for two sections, one is the curved section the other one is the flat section. And the curved section will follow the same way as you followed for the rigid wheel. And flat portion we will follow the procedure which we followed for finding out the shear displacement in a track.

Basically, it is linearly related, the shear displacement is linearly related to slip value and the distance from the front end of the track. So, once it is decided the next thing is what is the shear stress shear displacement relationship. Now, taking both shear stress-shear displacement relationships we find out what is the shear stress. And then it has to be multiplied the area to find out the shear force. So, in the curved portion, the shear force is acting at an angle. So, the cos component that means horizontal component has to be found out and then this has to be added with the flat portion to finally get the total tractive force which is developed by a pneumatic wheel in elastic mode.

So, we need to know the relationship. The important things are we need to know the relationship and the pressure distribution beneath the curved portion or beneath the flat portion, then only it will be in a position to carry out this integration.

(Refer Slide Time: 25:53)



I hope I have clarified and I have made an attempt to clarify all these things and you will be in a position to derive equations for predicting shear displacement and from there we can find out what is the tractive effort of a pneumatic wheel. Thank you.