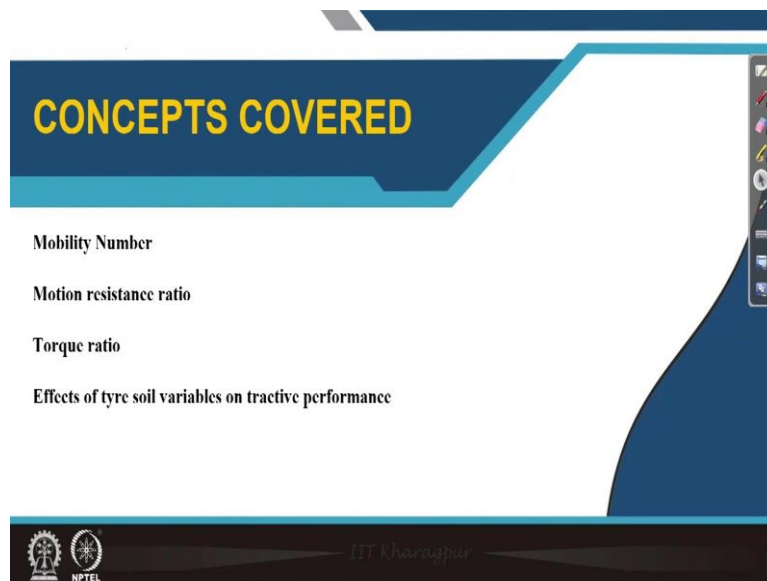


Traction Engineering
Professor Hifjur Raheman
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur
Lecture 19
Tractive Performance Prediction Models – Brixius

Hi everyone, this is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL course on Traction Engineering. This is lecture 19, where I will try to cover tractive performance prediction models developed by Brixius.

As I said, the theoretical derivations which you did for calculating the tractive performance of pneumatic wheels, we need to know the stress distribution, we need to know the shear displacement, we need to know the relationship between shear stress and shear displacement and the distribution of normal pressure. So, these are all theoretically, it seems very sound, but actually it is very difficult to get all those informations. Hence, some equations have been developed.

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So, a set of equations I discussed in my lecture 18. Now, I will discuss the most recently and commonly used equations which have been developed by Brixius. There are plenty of equations available, but these are the two Wismer and Luth and Brixius equations. These are most commonly used one that is why I tried to discuss in detail about those two equations. So, in this lecture, I will try to cover about Brixius equations where I will cover the concepts

of mobility number, concepts of motion resistance ratio, torque ratio and effect of tyre soil variables on tractive performance.

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- Brixius developed equations to predict tractive performance of bias-ply tyre operating in cohesive-frictional soils by taking data obtained from 121 tyre-soil combinations in 2500 field tests.
- Wheel torque, motion resistance, net pull and tractive efficiency are predicted as a function of soil strength, wheel load, tyre size, tyre deflection for pneumatic tyres.
- Considered 11 parameters to describe the soil-wheel system

Parameters	Symbol	Dimensions
Soil:		
Cone Index	CI	FL ⁻²
Tyre:		
Tyre section width	b	L
Overall tyre diameter	d	L
Tyre rolling radius	r	L
Tyre section height	h	L
Tyre deflection	δ	L

System:		
Vertical Load	W	F
Motion resistance	TF	F
Wheel pull	P	F
Wheel torque	Q	FL
Wheel slip	S	-

So Brixius, we have already discussed about Wismer and Luth equation that is why Brixius wanted to modify that equation. So, what he did is, he tried to take data obtained from 121 tyre soil combinations. Look at the numbers 121 tyre soil combinations in 2500 field tests. So, there is a huge amount of data. Based on those data, he tried to develop some equations. So, he tried to identify what are the parameters which are affecting the tractive performance and then he tried to develop some equations with respect to your COT, torque ratio, with respect to motion resistance ratio.

And from there, he derived the coefficient of traction and tractive efficiency. So, he identified 11 parameters, and he grouped them into soil, wheel and system parameters. Under soil parameters, he took cone index and under tyre, he took tractor section width, overall tyre diameter, tyre rolling radius, tyre section height, tyre deflection, delta. And here, I have indicated the symbols CI, b, d, r, h and delta and the dimensions. Now, the system parameters are your vertical load which is coming in the wheel, then the motion resistance, towing force, wheel pull, wheel torque and wheel slip.

So, these are the parameters he identified, 11 parameters. He then applied dimensional analysis technique that is Buckingham pi theorem. So, that means 11 minus 2, that should be 9 dimensionless ratio variables.

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- Reduced to 9 dimensionless ratios:

1. Wheel numeric	(Cl bd)/W
2. Width to diameter ratio	b/d
3. Deflection ratio	δ/h
4. Rolling radius to diameter ratio	r/d
5. Section height ratio	h/d
6. Slip	S
7. Torque or gross traction ratio	Q/rW
8. Motion resistance ratio	M/W
9. Pull ratio	P/W

• However, 2 ratios can be derived from other ratios:

$$\frac{P}{W} = \frac{Q}{rW} - \frac{M}{W} \cdot \frac{h}{d} = \frac{1 - 2r}{\delta} \cdot \frac{h}{d}$$

• Rolling radius to diameter ratio is nearly constant for agricultural tractor, hence it can be neglected.

• $\frac{r}{d} \cong 0.475$ i.e., the rolling radius ratio is nearly constant for most agricultural tyres, hence neglected in the soil wheel analysis.

Out of these 9 dimensionless ratio variables, he identified as wheel numeric, $Clbd/W$, width to diameter ratio (b/d), that means section width to wheel diameter, deflection ratio (δ/h), rolling radius to diameter ratio (r/d), section height ratio (h/d), that means section height by overall diameter, slip, torque or gross traction ratio (Q/rW), motion resistance ratio (M/W) and pull ratio as P/W . So, these are the 9 dimensions ratio variables he identified.

Out of these 9 dimensionless ratio variables, he found two ratios, which can be derived from other ratios for example, P/W can be derived if you know Q/rW and if you know M/W . So, the difference of these two will give you P/W . Similarly, h/d , section height ratio that can be derived if you know the rolling radius r and if you know the overall diameter and δ/h ratio. So, these are the two parameters P/W and h/d they are derived from the two other dimensionless ratio variables which you identified.

The other thing is the rolling radius to diameter ratio is nearly constant for agricultural tractor. Hence, he said it can be neglected and that is what the Wismer and Luth did. So, r/d ratio is nearly equivalent to 0.475 for whatever combination of tyre soil he has tried. So, that means, the rolling radius ratio is nearly constant. Hence, he neglected that one. So, out of 9 then you take minus 3. 1, 2 is the dimension ratios which are derived from others. The r/d also he took as constant. So, remaining are 6 dimensionless ratio variables.

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□ Wheel numeric ($CIbd/W$), deflection ratio (δ/h) and width to diameter ratio (b/d) are combined to one dimensionless product called the **mobility number (B_n)**.

$$B_n = \frac{CIbd}{W} \times \frac{\left(1 + 5\frac{\delta}{h}\right)}{\left(1 + 3\frac{b}{d}\right)}$$

So, he defined one terminology B_n , which is called mobility number. So, B_n is mobility number. This is equal to wheel numeric $CIbd/W$ then he has added two things δ/h ratio and b/d ratio. So,

$$B_n = \frac{CIbd}{W} \times \left(\frac{1 + 5\frac{\delta}{h}}{1 + 3\frac{b}{d}} \right)$$

So, that together is called mobility number. Whereas, Wismer and Luth, he defined wheel numeric, only he has taken $CIbd/W$. These two components he has not considered he has taken as a constant. So, this is the advantage we can say. So, we will see what is the advantage we are getting later on.

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Torque or Gross traction ratio:

$$\mu_g = \frac{Q}{rW} = \frac{F}{W} = 0.88 (1 - e^{-0.1B_n}) (1 - e^{-7.5s}) + 0.04$$

Significance of each term:

- 0.88 - Limits maximum torque ratio to 0.92. This is obtained with a large mobility number at high slip
- $e^{-0.1B_n}$ - Mobility number controls the maximum thrust developed which is attained at higher slip.
 - Torque ratio increases with an increase in mobility number.
- $e^{-7.5s}$ - The constant 7.5 is called 'gripping or surface factor' controls the rate of rise of torque ratio with slip.
 - Factor is smaller for wet surface and larger for staked wheels.

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 - Factor is smaller for wet surface and larger for staked wheels.
- 0.04 - Approximates torque ratio at zero slip for all conditions.

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Then the equation which he developed is torque or gross traction ratio. This is written as Q/rW , Q is the torque and r are the rolling radius, W is the dynamic weight. So, in other words you can write F/W that means tractive force divided by the dynamic load

$$\frac{F}{W} = 0.88 \times (1 - e^{-0.1B_n}) \times (1 - e^{-7.5s}) + 0.04$$

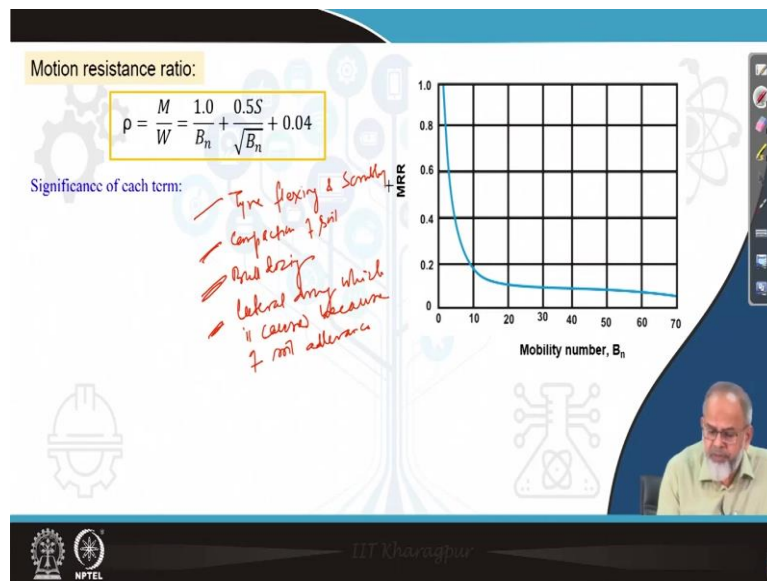
Let us now see, what is the significance of these terms which are included in this equation. The first is 0.88, what does it show?

So, it shows or limits the maximum torque ratio to 0.92. 0.88 and 0.04 if you add that becomes 0.92. That means 95 per cent, sorry 92 per cent of the torque is converted to useful power. And where it is obtained? This is obtained with a large mobility number, at high slip

value. These are the two conditions that means where will you get 0.92, where when there will be large mobility number and when there will be high slip.

Then the second important component is your mobility number $e^{-0.1Bn}$. This mobility number here controls the maximum thrust which is developed and it is developed at a higher slip. The torque ratio increases with increase in mobility number. Torque ratio increases with increase in mobility number. Now, the second important term is this $e^{-7.5s}$. The constant 7.5 is called the gripping or surface factor, it controls the rate of rise of torque ratio curve with slip and the factor is smaller for wet surface and it is larger for staked wheels. Now, he developed another equation. Before that, 0.04, what does it reflects? It reflects the torque ratio at zero slip irrespective of what soil condition. The torque ratio is at zero slip is 0.04.

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So now, let us see what is the equation which is developed for motion resistance ratio, which is denoted as ρ and this is equal to M/W and

$$\rho = \frac{M}{W} = \frac{1.0}{B_n} + \frac{0.5s}{\sqrt{B_n}} + 0.04$$

So, let us now see how the motion resistance ratio is varying and why there is a rolling resistance? First if you know then only can justify this nature of the curve. The motion resistance ratio of a pneumatic wheel is because of so many factors.

So, if you would like to identify those factors, the first is the tyre flexing and scrubbing. Tyre, flexing and scrubbing. The second one is due to compaction of soil, third one is due to bulldozing, then fourth one is due to lateral drag which is caused because of soil adherence to the tyre. So, these are the four reasons for which there will be motion resistance. In a hard soil, basically it is the tyre flexing and scrubbing which will create the motion resistance.

So now, after knowing this, then next thing is what are the terms which are used in this equation motion resistance ratio equation. The first term is $1/B_n$ and so what does it indicate? B_n is in the denominator that means, higher the value of B_n lesser will be the rolling resistance.

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Motion resistance ratio:

$$\rho = \frac{M}{W} = \frac{1.0}{B_n} + \frac{0.5S}{\sqrt{B_n}} + 0.04$$

Significance of each term:

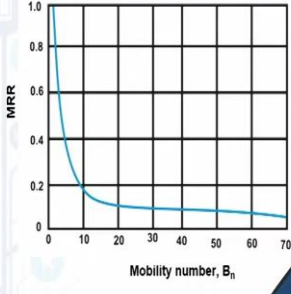
$\frac{1.0}{B_n}$ - As B_n decreases, the MRR increases

Increase in MRR due to

- decrease in soil strength
- decrease in deflection ratio
- increase in width to diameter ratio

0.04 - Value of MRR on hard surface and represents minimum value on any surface. This term is due to tyre flexing and scrubbing.

$\frac{0.5S}{\sqrt{B_n}}$ - MRR increases with slip due to increased tyre sinkage and soil shearing



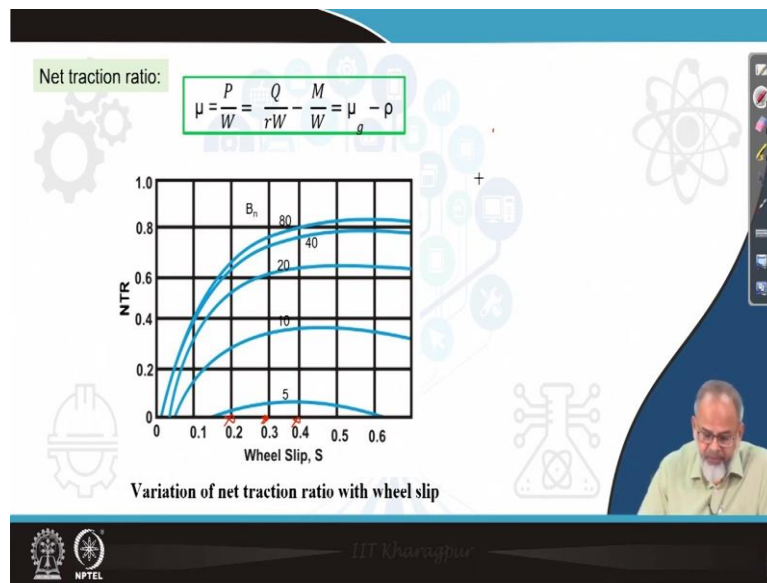
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As B_n decreases, MRR motion resistance ratio increases. So, B_n decreases means B_n could be due to decrease in soil strength, due to decrease in deflection ratio, due to the increase in width to diameter ratio. These are the factors by which you can decrease the B_n value. Since B_n value in the denominator so, that is why $1/B_n$ will give you higher rolling resistance ratio. Now, the second term is $0.5s/(B_n^{0.5})$. If you look at this one, motion resistance ratio increases with slip due to increase in tyre sinkage and soil shearing. So, more the slip more will be the rolling resistance.

Now, the third component is your 0.04 this value represents the motion resistance ratio on a hard surface and it represents the minimum value on any surface. This is basically due to tyre flexing and scrubbing. Now, once we know the equations which are developed for torque ratio, which is nothing but the coefficient of gross traction then we knew the equation which is developed for motion resistance ratio and the nature of the motion resistance ratio curve.

We can see with increase in mobility number we are getting lesser and lesser motion resistance ratio that means, with increase in CI with increase in δ/h ratio, we are getting more mobility number and then you are getting lesser motion resistance ratio.

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Then he derived coefficient of traction or net traction ratio which is

$$\frac{P}{W} = \frac{Q}{rW} - \frac{M}{W}$$

And then he tried to plot NTR versus wheel slip for different B_n values starting from 5 to 80. So, B_n value increase means, your coefficient of traction increases for the same slip. If you look at 0.3 or 0.2 wherever you look at, all the cases we will find that the COT value increases with increase in B_n value that is wheel numeric value.

Why it is so? B_n value increased means the soil is hard. So, there is lesser rolling resistance, hence the power which is supplied to the tractor wheel or to the single wheel that will consume less power to overcome the rolling resistance, hence more power is available as useful power.

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Tractive Efficiency:

$$TE = \frac{\mu}{\mu_g} (1-s) = \frac{P}{\frac{Q}{rW}} (1-s)$$

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$$TE = \frac{\mu}{\mu_g} (1-s) = \frac{P}{\frac{Q}{rW}} (1-s)$$

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Next thing is tractive efficiency. Once you derive motion resistance, coefficient of traction, then we tried to find out what is the expression for tractive efficiency. The expression remains same that means output power by input power. And it is a function of slip So,

$$\text{Tractive efficiency} = \frac{P}{Q/r} \times (1 - s)$$

So, we have divided W to find out this is COT and the other one is your torque ratio. Now, the nature of the curve we can see from here. The variation of TE with wheel slip is shown in this figure. When this wheel slip is less than 5 per cent, what happens? A large portion of the power is required to overcome rolling resistance.

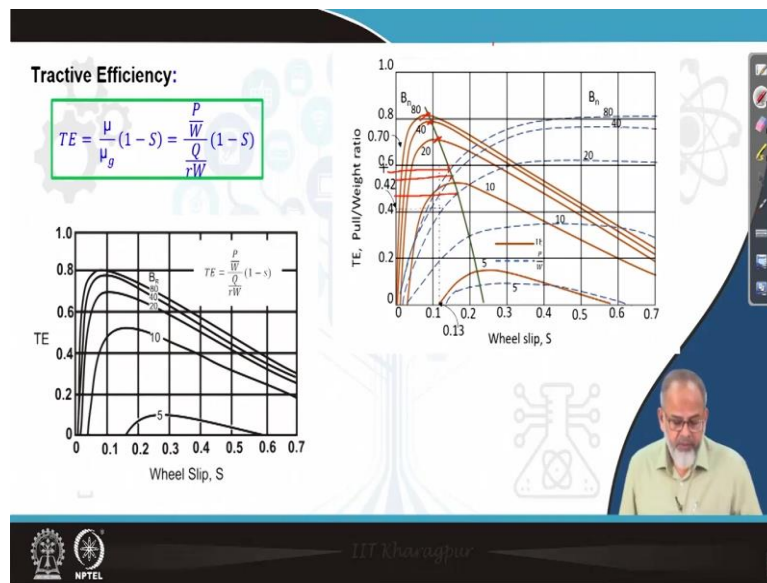
If wheel slip is more than 20 per cent, beyond this again it is going down. Tractive efficiency, if you look at the curve, it is rising to a peak very sharply then it is going down. So, if you are beyond 20 per cent or 25 per cent then there is again a decrease in tractive efficiency. And that decrease in tractive efficiency is because of increasing proportion of increasing percent of input power which is lost due to slippage. If the slip is less than 5 per cent there is loss, most of the power is used for overcoming the rolling resistance. If it is more than 20 per cent, if the slip is more than 20 per cent, most of the power is lost because of the wheel slip. So, we have to operate in between these two so that you can get maximum tractive efficiency. Now, if you try to plot tractive efficiency versus slip and pull versus slip that will give you a better picture. In the sense, the nature of the curves are different. In case of tractive efficiency curve, it reaches to a peak within 10 per cent if you look at when the wheel is hard sorry the mobility number is high from 20 to 80 then you are getting a maximum value within 10 per cent.

Now, if the mobility number is 10 then we are getting maximum value around 15 per cent. The mobility number is 5 then we are getting a maximum value around 25 per cent slip. Whereas, at this slip values if you draw vertical lines, so at this slip values this COT curve that is coefficient of traction curves or the pull to weight ratio curve is not giving the maximum value. So, maximum value you can see, you are getting somewhere here, in case of wheel numeric mobility number 5. In case of mobility number 10 you are getting maximum here, in case of 20 you are getting here sorry, in case of 10 you are getting here somewhere.

So, that means the slip at which you are getting maximum tractive efficiency and the slip at which you are getting maximum pull they are not matching. So, if you want more pull or if you want more tractive efficiency that we have to first find out or you have to make a compromise, so that we will get more pull at the same time more tractive efficiency we cannot say maximum, but we can say we can get more pull or more tractive efficiency.

So, that is why a design curve has been drawn here which is shown by the green line. So, depending on our soil condition, we have to find out which slip should be followed in the field so that you will get both maximum tractive efficiency and more pull.

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So, if you look at this curve, so, you are nearly when the soil is hard that means the mobility number is 80 you can see we are reaching to a peak here, and when the mobility number is 40 we are reaching to a peak here, when the mobility number is 20 we are reaching the peak here.

So, this design curves are basically for higher tractive efficiency. But the pull which are getting is at 20, this is the pull which are getting at 40, this is the pull you are getting, at 80, this is the pull which we are getting, which is less than the maximum pull. So, now this suggests that there should be a design curve taking into account which thing we like more tractive efficiency or we like more pull.

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Soil and wheel parameters	Brixius	Wisner and Luth
CI, kPa	100-5000	-
$\frac{b}{d}$	0.18 - 0.9	0.3
$\frac{\delta}{h}$	0.08 - 0.32	0.2
$\frac{r}{d}$	0.475	0.475

Handwritten notes: 0.92, 0.75

So, these are the sets of equations to which you discuss one is for Wismer and Luth, the other one is for Brixius and let us see now the applicability of Brixius equation. The applicability of Brixius equation is for a cone index starting from 100 to 5000. The very high value 100 to 5000 and the b/d ratio is 0.18 to 0.9 there is the range. δ/h ratio is from 0.08 to 0.32 and r/d ratio is almost constant 0.475. And if you would like to compare Wismer and Luth then there is no indication for cone index value, what is its applicability?

Only the limited value is b/d ratio is only 0.3. If we exceed this value, what will happen, he has not mentioned, neither nobody has tried to find and verify this. And δ/h ratio, if you look at, it is 0.2 and r/d ratio for agriculture tyre it is nearly 0.475. But the advantage which Brixius has done is the maximum value has increased to 0.92 times. Whereas, the tractive effort which is developed by Wismer and Luth is 0.75. There is a difference. And in Brixius equation he has considered both deflection as well as b/d ratio which Wismer and Luth equation has not considered.

The third thing is in case of rolling resistance ratio for which Wismer and Luth equations, there is no difference, you cannot differentiate the rolling resistance of a towed wheel or rolling distance of a powered wheel, because slip factor was not there. Whereas in case of Brixius equations, he has considered slip factor. So, that is why Brixius equations is much better. These equations are much better and it has more applicability as compared to Wismer and Luth's equation. So, one more thing I like to add here is these two equations, these two scientists Brixius as well as Wismer and Luth both have developed equations for bias ply tyre.

Applicability of these equations which are discussed for radial ply tyres is not known. Rather Brixius has indicated that he can change the coefficient values so that it can be applied to radial tyre. Some of the suggestions what he has given is you can change the value of constant 7.5.

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Torque or Gross traction ratio:

$$\mu_g = \frac{Q}{rW} = \frac{F}{W} = 0.88 (1 - e^{-0.1B_n}) (1 - e^{-7.5S}) + 0.04$$

Significance of each term:

- 0.88 - Limits maximum torque ratio to 0.92. This is obtained with a large mobility number at high slip
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Motion resistance ratio:

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Significance of each term:

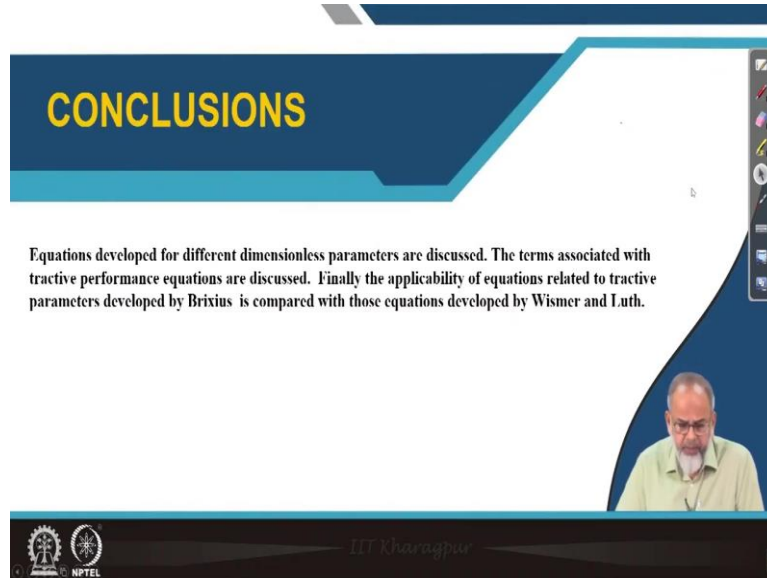
- 0.9 / B_n

The equations, let me go back to the equation there it will be better to show what changes he has suggested if we are interested to find out or apply this equation to radial tyre. The first suggestion he has given is this 7.5. He suggested that instead of 7.5 we can try with 8.5 to 10.5 and the constant 0.04, he said it should be 0.035 or 0.03 that is the range. These are some of the suggestions and in the motion resistance ratio he also suggested a change in the terms of instead of $1/B_n$, he suggested it should be $0.9/B_n$. These are some of the suggestions given by Brixius to apply these equations to radial tyres.

So, nevertheless Brixius equations are better in the sense, it takes into account the slip, it takes into account the δ/h ratio, it takes into account the b/d ratio. So, these are some of the advantages of Brixius equation. So, that is why now, ASABE has recommended these in the

standard, ASABE standard, we find these are the equations which are available for predicting the tractive performance of tyres, pneumatic tyres.

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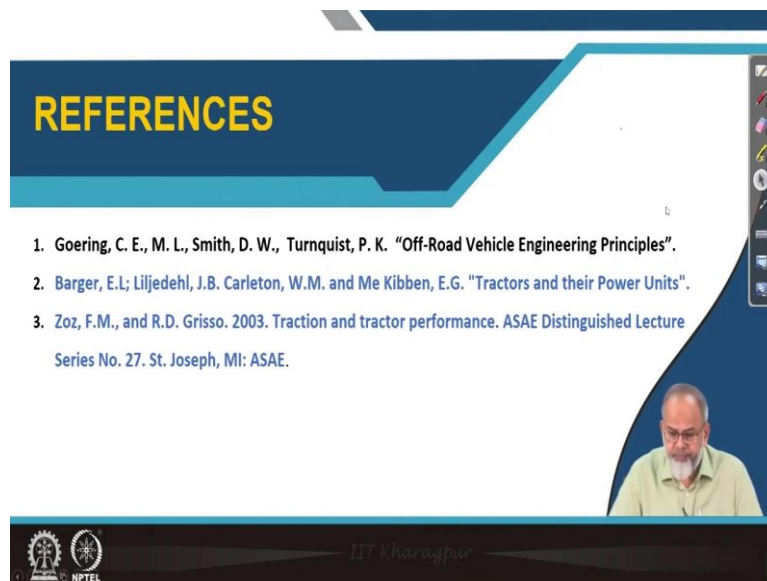
CONCLUSIONS

Equations developed for different dimensionless parameters are discussed. The terms associated with tractive performance equations are discussed. Finally the applicability of equations related to tractive parameters developed by Brixius is compared with those equations developed by Wismer and Luth.

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So, in summary I can say I tried to discuss the equations which are developed and for developing what are the dimensionless ratios, dimensionless ratio variables which are considered those are also discussed. Then, we discussed finally, the applicability of these equations and then compared Brixius's equation with Luth's equation. And what are the modifications, which can be incorporated in the developed equation, so that the equations can be applied to radial ply tyres, That is all. Thank you.