

**Traction Engineering**  
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**Lecture 36: Cornering Properties of tyres**

Hi everyone. This is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL course on Traction Engineering. This is lecture 36, where I will try to cover on cornering properties of tyres. The concepts which will be covered in this class will be slip angle, cornering force, cornering stiffness, cornering coefficient, self-aligning torque, camber thrust, camber stiffness and the equations for estimating total lateral force.

So, before starting, the slip angle, let us see now the axis system which has been defined by society of automotive engineers, which is shown in this figure where there will be three forces, three moments and two angles associated. Three forces, one is in x direction, y direction and z direction. X direction is the longitudinal force, y direction is your lateral force and z direction is your normal weight or the normal force. And the moments which are acting about x axis is overturning moment, about y axis is rolling resistance moment and about z axis is aligning torque. And the two important angles which are associated with this cornering force, now I am going to explain.

The first one is slip angle, is the angle formed between the direction of wheel travel and the line of intersection of wheel plane with the ground or with the road surface. So, which is indicated by an angle  $\alpha$  in this figure. So, this is the angle  $\alpha$  which is denoted as slip angle. And the other important angle is your camber angle, it is the angle between the x-z plane and the wheel plane. So, which is denoted by  $\gamma$ . So, this is shown here. So,  $\gamma$  and  $\alpha$  they are associated with the cornering properties of tyre.

Now, when I said cornering properties that means, when a wheel is moving, there is no applied force at the center of the wheel or neither we are trying to turn the wheel so, it will move in a plane parallel to the wheel plane. But the moment you apply a force at the center which is denoted here by  $F_s$  side force.

So, it will no longer, the wheel will no longer move in the wheel plane, it will move in a plane which is denoted as OA in this figure. OA, which is making an angle alpha in the wheel plane. This is because when you apply a side force, there will be a force developed at the contact patch. So, this force is called a lateral force and we denote it as  $F_{y\alpha}$ . So,  $F_{y\alpha}$  is the lateral force

which is developed at the ground contact patch and when the camber angle is 0, we can call it a cornering force.

So, the lateral force which is developed at the contact patch is also called cornering force. So, then the camber angle is 0. So, what happens exactly, why this wheel is not moving in the wheel plane it is, moving at an angle it is because of the lateral elasticity, lateral elasticity of the tyre. So, when the tyre is moving with uniform speed in the direction of OA, the side force is applied at the wheel center and the cornering force is developed at the ground contact patch. This side force and the cornering force which are developed they are not collinear.

So, you can see in the figure I have indicated, there is a gap between the point at which the side force is applied and the line of action of a lateral force. So, that difference, the distance between these two lines of forces is indicated by  $t_p$ , and  $t_p$  is called the pneumatic trail and because of these, there will be a torque developed. The torque is called aligning torque or self-aligning torque. So, what is its role? Its role is to bring back the wheel into the normal position after you take a turn. So, it is one of the primary restoring moments which help the wheel to or the tyre to return to the original position after negotiating a turn. So, now we will try to find out what are the parameters which are going to affect this self-aligning torque.

So, if you look at the left figure which is the cornering characteristics of a bias ply and a radial ply car tyre. So, you must be wondering why I am discussing about car tyre. Because we do not have many literatures available related to tractor tyres. So, that is why I have taken car tyres and truck tyres, ok. The left figure shows the cornering force versus slip angle. So, what you observe here is, the cornering force increases linearly with increase in slip angle up to a certain value of slip angle after that the rate of increase is decreased and then it rises to a maximum value, ok.

So, same is the case with a radial tyre. It first increases at a faster rate, then the rate of increase is slow down then it becomes almost constant. So, it reaches to a maximum around  $16^\circ$ , beyond  $12^\circ$  it reaches a maximum and the linearity between the cornering force and the slip angle is up to  $4^\circ$ . After that, it is not linear, this is non-linear. So, what we conclude from here is that, the type of tyre which is going to influence the cornering force. Now, we plot cornering force up a normal load with slip angle for a truck tyre. For two different tyres, one is your, the lower one is the bias ply tyre and the upper one is your radial ply tyre.

So, the same trend has been observed, ok. Now, the cornering force, how it is dependent on normal load that we are going to see, at different slip angles. This is for again a car tyre. So, the slip angles are varied from  $2^\circ$ ,  $4^\circ$ ,  $6^\circ$ ,  $8^\circ$ ,  $12^\circ$  and then  $20^\circ$ . And, if you look at the figure, this has been, this study has been carried out at an inflation pressure of 147 kPa and the tyre size is 5.6-13. So, the normal load has been varied from 0 to 5 kN. We can see, this figure clearly indicates that for a given slip angle the cornering force increases, but the rate of increase

is very slow. 2°, 4°, 6°, the rate of increase is very slow. So, it increases with increase in normal load, it increases with increase in slip angle. At a particular load if you draw a vertical line you can see with increase in slip angle the cornering force is increased. That means, now we come to a conclusion that cornering force is a function of normal load.

In the earlier slide we come to a conclusion that cornering force is a function of type of tyre. Now, we come to a conclusion that cornering force is a function of normal load, is a function of slip angle. Now when it is dependent on load, when the vehicle is taking a turn, there will be weight transfer, weight transfer from the outer side to inner side or from the inner side to outer side depending on the direction of rotation or direction of movement. So, what will happen? Suppose the inner side gets more load inner tyre or in the inner wheel gets more load and the outer wheel will be getting lesser load. Since cornering force is dependent on the load so, the wheel having more load will develop more force, wheel having less load will develop lesser force.

But if you take individual wheel then, if I take 2 wheels at a time then it will develop a force. If one wheel is developing  $F_y$  then the 2 wheels will develop  $2F_y$ . But now if the wheels are mounted on axle and you are taking a turn so, the side which is getting more load will develop more force. So, I have indicated this one as  $F_{y0}$  and the side which is ah experiencing lesser load will develop lesser cornering force. If you take summation of these two then what will happen? This  $(F_{yi} + F_{yo})/2$  will be lesser than  $F_y$ .

So, for a pair of tyres on an axle to develop the required amount of cornering force to balance a given centrifugal force during a turn the lateral load transfer results in increase in the slip angle of the tyre. So, that means the cornering force developed is less so, we have to slip more to develop more force, lateral force. Now, to compare the cornering behaviour of different tyres, to compare the cornering behaviour of different tyres, we defined a parameter called cornering stiffness. What is it? This is defined as the derivative of cornering force with respect to slip angle at 0 slip angle. So,

$$\left. \frac{\partial F_{y\alpha}}{\partial \alpha} \right|_{\alpha=0} = \text{Cornering stiffness}$$

Now we try to plot cornering stiffness versus normal load for car tyres, for truck tyres of different sizes. So, this bottom 3, these are the car tyres and the upper 3 curves are for the truck tyres where I have indicated RL. So, RL is nothing, but the rated load of the tyre. So, what we observe from this figure is they are carried out at different pressures starting different loads and different same pressure is one the car tyres are kept as 165 kPa and the rated loads are varied for different sizes of tyre. So, what we observe here is for car tyres the cornering stiffness is increasing up to a particular point after that it starts decreasing.

That means, up to rated load it is going to a maximum and then after that when you increase load further the cornering stiffness is decreasing, but same is not the case with the truck tyres, beyond rated load also the cornering stiffness is increasing. So, now we try to plot cornering coefficient that means, we define another parameter since a load, cornering stiffness is dependent on a load. So, we try to define a parameter called cornering coefficient and the cornering coefficient is nothing, but cornering stiffness divided by normal load. And then we try to plot cornering coefficient versus percent rated load. So, what we observe here is 50 percent, 100 percent, 150 percent, 200 percent like that.

So, what we observe the cornering coefficient is going down. That means, it is decreasing with increase in a percentage load on the tyre. Then you try to see what is the effect of different tyres on cornering coefficient when you try to vary the inflation pressure, whether it is a bias ply tyre, whether it is a bias belted tyre or whether it is a radial tyre. We try to verify how these tyres are developing the cornering coefficients at different inflation pressures. The inflation pressures we varied from 110 to 330 kPa and what we observed is shown here, where it is clearly indicated that the radial tyres are giving you better cornering coefficient as compared to your bias tyre or bias belted tyre. If you like to compare bias tyre with bias belted tyre then bias belted tyre is giving you more cornering stiffness as compared to bias tyre.

And for the 3 tyres when you increase the inflation pressure the stiffness or the cornering coefficient is increasing. So, cornering force, then we try to plot with self-aligning torque at different normal loads and at different slip angles. So, what we observe here is the cornering force increases with increase in self-aligning torque or in other words the self-aligning torque is increasing with increase in cornering force because self-aligning torque is nothing, but the product of cornering force into pneumatic trail. So, this has been plotted for different slip angles. So, whether it is  $2^\circ$ ,  $4^\circ$  or  $6^\circ$  or  $8^\circ$ , what we observed is aligning torque is increasing with increase in load.

we varied from 4.45 kN to 6.23 kN. So, cornering force increases with increase in normal load and the self-aligning torque is also increasing and the slip is also increasing. With increase in slip angle you are getting more self-aligning torque.

Now we try to plot self-aligning torque versus normal load for 2 different tyres one is the bias ply truck tyre the other one is radial ply truck tyre. If you look at the bias ply truck tyre what you observe is it is going to a peak with increase in normal load self-aligning torque is increasing then after sometime starts decreasing. Same is the case for all loads.

Now if you look at the slip angle self-aligning torque is decreasing with increase in with increase in, sorry with decrease in normal load. For a given slip angle if you increase normal load then your self-aligning torque is increasing. Same is the case with  $2^\circ$ ,  $4^\circ$ ,  $8^\circ$ . But in all these cases, in all normal load cases, what you observed is, it is reaching to a peak whether it is 8896 N or whether it is 17792 N, it is reaching to a peak and after that it is going down. So, and that peak is achieved roughly around  $6^\circ$  between 4 to  $8^\circ$ . So, you can take a  $6^\circ$ .

Now if you look at the radial ply tyre, the trend is same it is reaching a peak the, but the peak is not that, the peak is little prominent here as compared to your bias ply tyre. So, the

characteristics or the behaviour of the tyre gives you the self-aligning torque whether it is increasing or decreasing it depends on the normal load and it also depends on the slip angle. But what we observed here is, the peak is reaching between 4 to 8°, somewhat around 6°. So, between 2 to 4 you can see, there is a sharp rise which is not there in case of a bias ply tyre ok. So, the decrease in aligning torque at higher slip angle is mainly due to the sliding of the tyre tread in the trailing part of the contact patch.

As a result, that will shift the point of application of the lateral force which is developed at the contact patch, hence the pneumatic trail is reduced, hence the self-aligning torque is reduced. Let me repeat this one, why there is a decrease in aligning torque with increase in slip angle, this is because there will be sliding of the tread in the trailing part of the contact patch which results in shifting of line of application of the lateral force thereby the pneumatic trail is reduced hence the torque which is the product of lateral force into pneumatic trail that is reduced.

So, what you can conclude from this brief discussion is the longitudinal force which affects the aligning torque significantly. In general, the driving torque increases the aligning torque for any slip angle and the inflation pressure and normal load also have noticeable effects on the aligning torque. So, this we have discussed only about slip angle.

Now, we will discuss about the camber angle. The camber angle as I said it is the angle  $\alpha$  between the exit plane and the wheel plane and this is provided to give steering comfort you can easily rotate it. The load coming on the wheel is divided into two components since it is not vertical. So, that is why the camber angle which is denoted here and the kingpin inclination, which is shown here, they are responsible for an easy steering. So, when a wheel is cambered and we try to rotate it then what will happen. It will try to rotate about a point O about this point.

But if you restrict this instead of rotating about O then what will happen? A lateral force will be developed at the contact patch and that force is called camber thrust and that force is developed in the direction of the camber. So, here the direction is like this. So, force will be developed in the bottom like this and the contact patch and the camber thrust acts ahead of the wheel centre unlike your cornering force. Cornering force always acts behind the wheel centre whereas, camber thrust acts ahead of the wheel centre. Now, we try to see which are the factors which are affecting camber thrust.

Obviously, it is a normal load, it could be your camber angle. So, we try to find out what is the effect of normal load on camber thrust. So, camber thrust has been tried for 0°, 2°, 0° means there is no camber angle, it is vertical. 2°, 4°, 6°, 8° and 10° and the load has been varied from 2.7 kN to 8 kN. So, what we observed, with increase in normal load at the camber thrust is increasing and for a particular or for any normal load we tried the camber thrust is increasing with increase in camber angle.

So, camber thrust is a function of both normal load, is a function of camber angle. So, we try to define a terminology called camber stiffness. What is camber stiffness? Similar to your cornering stiffness that means, derivative of camber thrust with respect to camber angle at camber angle is equal to 0. So, that is called camber stiffness. So, now, we try to plot camber stiffness versus normal load for different tyres, ok.

So, what we observed from here is with increase in normal load from 10,000 to 50,000 N, the camber stiffness is increasing for a 11-22 tyre, for a 12-22 tyre or for a 15-22.5 tyre and all these tyres are inflated to the same inflation pressure. So, now, we know that there will be lateral force developed, but the lateral force which is developed due to slip angle and the lateral force which is developed due to camber angle, they are not of equal magnitude. Always the lateral force which is developed because of the camber is always lesser than the lateral force which is developed because of the slip angle.

So, since there are two lateral forces developed. So, now, question arises how to find out the total lateral force. So, total lateral force is nothing, but the summation of two lateral forces which are developed. One is  $F_{y\alpha}$  which is nothing, but a cornering force where the camber angle is 0 and  $F_{y\gamma}$  which is the lateral force or the camber thrust developed at the contact patch when the slip angle is 0. I have put a plus or minus sign the reason is, they may not be in the same direction. The lateral force which are developed because of the slip angle or because of the camber angle, they may not be in the same direction.

If they are in the same direction we add it and if they are not in the same direction then we deduct it they are opposite to each other then we deduct it. So, that is why I put  $\pm$  symbol. Now, for small angles the as we saw in some of the slides that cornering coefficient  $C_\alpha$  and camber cornering stiffness and camber stiffness  $C_\alpha$  and  $C_\gamma$ , they are linearly related to the slip angle and a camber angle. So, that is why I have written,  $F_{y\alpha} = C_\alpha \times \alpha$ . So, this condition the second equation which I have given is applicable only when the slip angle is small.

So, when slip angle is small, you can express the total lateral force in terms of camber stiffness in terms of cornering stiffness. Knowing the slip angle and a camber angle we can find out what is the total force developed. So, in brief what we have discussed is what is the lateral force developed and how it is developed and what are the parameters which are affecting this forces ah that has been discussed and shown in different slides for tyres which are not related to tractors rather it they are related to cars or trucks. Thank you. This is the reference.