Traction Engineering
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Lecture 38: Steering of wheeled vehicles and steady state handling of front wheel steered vehicles

Hi everyone. This is Professor H. Raheman from Agricultural and Food Engineering Department. I welcome you all to this NPTEL online certification course on Traction Engineering. This is lecture 38 where I will try to cover the steering of wheeled vehicles and steady state handling of front wheel steered vehicle. The concepts which will be covered is steering geometry and the steady state handling of front wheel steered vehicles.

Now, when I say handling characteristics, so handling characteristics of a road vehicle refer to its response to steering commands and to environmental inputs. Inputs could be wind gust; it could be road disturbances which will affect the direction of motion. So, there are two basic issues which are associated with vehicle handling. The first one is the control of the direction of motion of the vehicle. The second one is the ability to stabilize direction of motion against external disturbances. So, first I will see the vehicle as a rigid body. It has 6 degrees of freedom, translation along X, Y and Z axis and the rotations about this axis. So, which are clearly indicated in this figure. There will be a translation about x axis, this one is the X axis and Y axis lateral velocity and Z axis is your vertical velocity and the rotations are called roll velocity, yaw velocity and pitch velocity. With respect to X axis, it is roll velocity; with respect to Y axis, it is pitch velocity; with respect to Z axis, it is yaw velocity. So, this is the vehicle axis system on which I have indicated these forces and the moments which are acting. Then during turning what happens, the vehicle body rolls and this roll motion may cause the wheels to steer, thereby it will affect the handling behavior of the vehicle. So, before we move into the steering, let us first see what is the steering geometry of a vehicle.

So, I have indicated, this is a four wheeled vehicle that means the front wheel are steered and if I draw perpendicular to all these wheels, they will meet at a point O that is the center of rotation. And I have connected O with CG of the vehicle. So, that is indicated as R which is nothing but your turning radius and the wheel base is indicated by small b and the track width is indicated by small t, and δ_i that is the steer angle of the inside front wheel. We are taking a turn to the right. So, that is why δ_i is the steer angle of the inside front wheel which is also called major steer angle. Similar to δ_i, there will be an angle δ_o which is called steer angle of the outside front wheel and it is called minor steer angle.

Now if you look at the right-angle triangle OFC, then

$$cot\delta_i = \frac{x}{b}$$

Similarly, if you look at the right-angle triangle OED,

$$cot\delta_o = \frac{t + x}{b}$$

So, now if I take the difference between

$$cot\delta_i - cot\delta_o = \frac{t}{b}$$

This is the perfect steering conditions where the all the four wheels are rolling and this is what is required, but in practice we never achieve this.

Next, we will see what is the steady state handling performance. When I said steady state handling performance, this is concerned with the directional behavior of a vehicle during a turn under non time varying conditions. For example, a steady state turn is a vehicle negotiating a curve with constant radius and at constant forward speed, we are not changing the forward speed, we are not changing the constant radius, so that is called steady state handling. When the vehicle is negotiating a turn at moderate to higher speed, the effect of centrifugal force which is acting at the CG of the vehicle can no longer be neglected. So, what will happen, to balance the centrifugal force, the tyre must develop appropriate cornering force.

This cornering forces are developed at the contact patch of the front wheels as well as the rear wheels. So, because of this, side force will be acting at the tyre and the tyre will produce a slip angle. So, let us now see, this I have taken the, I have represented this figure assuming that both the wheels are developing the same cornering force and I have indicated only one wheel, one on the front, the other one on the rear and both the wheels that is the minor steer angle and the major steer angle are assumed to be same. So, these are some of the assumptions I made to which has been made to simplify, so that you can derive the fundamental equations related to steering or handling characteristics. So, the first assumption is, a vehicle is negotiating a turn at a constant speed and the forces which are acting on the vehicle will be obviously the centrifugal force and the forces acting through the tyres.

And here the major steer angle of the inside front wheel and the minor steer angle of the outside front wheel they are almost the same and so the forces on each of the front and each of the rear wheels are the same. So, then the forces shown acting on these tyres are simply twice the forces which are acting on each of the wheels. So, here I have indicated the centrifugal force which is equal to WV^2/gR and the vehicle is taking a turn at the steering angle, I have indicated as δ_f and the forces which are acting will be the side force F_{sf} which is on the front wheel and the side force F_{sr} which will be on the rear wheel. And the slip angles corresponding to the front wheel is α_f, this angle and the slip angle at the rear wheel will be alpha α_r and the steer angle that is δ_f, the angle by which the vehicle is turned is δ_f. Now, if you look at this diagram, I have indicated the wheel base as L and the distance of CG from the front center of the wheel that will be equal to L_1 and the rear center of the wheel from the cg the distance is L_2.

Now, the angle which is sustained that means, if you draw two lines, perpendicular lines to these two wheels, they will meet at a point O and the distance from O to CG is denoted as R which is the turning radius and the angle OA, this triangle OBA, this is the right angle triangle and the angle substrate at center that is AOB,

$$\angle AOB = \alpha_r + \beta$$

Similarly, the angle on the other right-angle triangle that is OBC. So, angle BOC,

$$\angle BOC = \delta_f - \alpha_f$$

That means, δ_f is the steer angle, α is the slip angle and β is the angle by which the vehicle has turned. So, summation of this will give you the total angle τ. So, now the slip angle of the front and the rear wheels, as I told in the beginning that this will be equal to α_f and α_r.

So, when a vehicle is negotiating a turn at the moderate at higher speed the four tires will develop appropriate slip angles to simplify the analysis the pair of tyres and an axle are represented by a single tyre with double the cornering stiffness. And β I said, is the angle between the direction of motion and the vehicle cg. Motion of the vehicle CG is the direction of, the angle between the direction of motion of the vehicle CG and the vehicle center line. And δ_f is the steer angle of the front wheels and O is the instantaneous center of turn. So, I have indicated those forces.

Now I said τ will be equal to, for small angles you can take, assume to be an arc of the circle. So, that becomes,

$$\tau = \frac{L}{R}$$

L is the wheel base and R is the turning radius. Now this will be equal,

$$\frac{L}{R} = (\alpha_r + \beta) + \delta_f - (\alpha_f + \beta)$$

So, -β, +β plus, they will cancel out. So, ultimately what we are getting is,

$$\delta_f = \frac{L}{R} - \alpha_r + \alpha_f$$

Now we will try to find out, what are the forces and moments which are acting by resolving the forces which are acting on the wheel into the, into two components one is parallel to the radial line the other one is perpendicular to the radial line ok. So, and then taking the algebraic sum of forces and moments we will try to develop the fundamental equations ok. Let us now see, what are the forces which are acting parallel to the radial line. So, the first thing will be WV^2/gR is the centrifugal force. Then this will be, if we look at the front wheel then it will have a component $F_{of}\sin(\delta_f-\beta)$, ok.

$$\frac{WV^2}{gR} + F_{of}\sin(\delta_f - \beta) = F_{sf}\cos(\delta_f - \beta) + F_{or}sin\beta + F_{sr}cos\beta$$

So, F_{sr} is the side force which is acting at the rear wheel and F_{or} is the basically the rolling resistance of the rear wheel and F_{sf} is the side force which is acting on the a front wheel and F_{of} is the rolling resistance of the front wheel. So, this will be the summation of forces or the forces acting parallel to the radial turning radius. Similarly, we have to find out what are the forces acting perpendicular to the turning radius. So, resulting the forces perpendicular to the turning radius, what we get is

$$F_{sr}sin\beta = F_{or}cos\beta + F_{of}\sin(\delta_f - \beta) + F_{sf}\sin(\delta_f - \beta)$$

Now, taking moments about the CG and then equating it to 0, as there is no external moment applied at the CG. So we, what we get is,

$$L_1cos\beta[F_{of}\sin(\delta_f - \beta) - F_{sf}\cos(\delta_f - \beta)] + L_1sin\beta[F_{of}\cos(\delta_f - \beta) + F_{sf}\sin(\delta_f - \beta)] + L_2F_{sr} = 0$$

So, these are the three equations which we got from by resolving the forces parallel to the turning radius, perpendicular to the turning radius and then taking moments about the CG. And the, so, what happens exactly is, this δ_f and β they are not very high angles, they are very small angles. So, you can approximate, is closer to 0.

So, that means, this sin components will be more or less 0 and the cos components will be 1. So, the equation 1 will be resolved as: so, this can be written as

$$\frac{WV^2}{gR} = F_{sf} + F_{sr}$$

This will be equation number 3 and the equation number 2 will be equal to, the sin component, will be 0, ok. So, that means,

$$L_1 F_{sf} = L_2 F_{sr}$$

So, these are the equations which you got from here. When we assume that the α, assume means this δ_f and β, they are small. That is why you have resolved this 1, 2, 3 equations into 3, 4, and 5 ok. Now, from this equation number 5, we can say that

$$F_{sf} = \frac{L_2}{L_1} \times F_{sr}$$

Now, if I substitute this value in equation number 3, so, then it will become,

$$\frac{WV^2}{gR} = F_{sf} + F_{sr}$$

$$\frac{WV^2}{gR} = F_{sr}\left[\frac{L_2}{L_1} + 1\right]$$

Now, from equation 5, what we got is

$$F_{sf} = \frac{L_2}{L_1} \times F_{sr}$$

$$F_{sr} = \frac{L_1}{L_2} \times F_{sf}$$

So, if I substitute in the equation number 3. So,

$$\frac{WV^2}{gR} = F_{sf}\left[\frac{L_1}{L_2} + 1\right]$$

$$F_{sf} = \frac{WL_2}{L_1 + L_2} \times \frac{V^2}{gR}$$

So, now this $W \times L_2/(L_1+L_2)$ is nothing, but what is the weight acting on the front axle, because we have taken F_{sf} is the total weight which are acting on the both the wheels. So, I can write as

$$F_{sf} = \frac{W_f V^2}{2gR}$$

because I assume that W_f is the weight acting on the each of the front wheels ok.

Now similarly, I will find out

$$F_{sr} = \frac{W_r V^2}{2gR}$$

So, now if I substitute this one in this equation, this equation… So, before that, we have to find out what is the relationship between the F_{sr} and F_{sf} with the slip angle. Usually, F_{sf} is related to the slip angle, F_{sf} is related to the slip angle. That means, the cornering stiffness into slip angle.

Similarly, F_{sr} is the (cornering stiffness of the rear wheel) × α_r. So, I can substitute here for F_{sf}. So, F_{sf} = (cornering stiffness in the front wheel, $C_{\alpha f}$) × α_f,

$$C_{\alpha f} \times \alpha_f = \frac{W_f V^2}{2gR}$$

$$\alpha_f = \frac{W_f V^2}{2C_{\alpha f} gR}$$

Similarly, I will find out expression for α_r which is nothing

$$\alpha_r = \frac{W_r V^2}{2C_{\alpha r} gR}$$

Now, if I substitute this value, this value in this equation, so, finally, what we get is,

$$\delta_f = \frac{L}{R} - \left(\frac{W_R V^2}{C_{\alpha r} gR} - \frac{W_F V^2}{C_{\alpha f} gR}\right)$$

$$\delta_f = \frac{L}{R} - \frac{V^2}{gR}\left(\frac{W_R}{C_{\alpha r}} - \frac{W_F}{C_{\alpha f}}\right)$$

This (W_R/C_{sr} - W_F/C_{sf}), this is nothing, but is called as understeer coefficient.

So, δ_f, that is the steering angle is a function of wheelbase L, is a function of the turning radius R, it is a function of the speed at which is taking a turn, it is a function of the weight distribution, it is a function of the cornering stiffness of rear and front tyre. So, this is the fundamental equation governing the steady state handling behaviour of a vehicle. Now, I will show you another handling behaviour which is nothing, but how the tractor is behaving or what is the stability of tractor when during while moving on a side, side slope. So, we will discuss about side slope stability now. So, the vehicle is moving across a slope. The slope of the surface is β from the horizontal and the wheel has a track width t and the CG height is h from the ground level, ok.

Now $\tan\beta = t/2h$ ok. When the $\tan\beta > t/2h$, t is the track width, h is the height of the CG from the ground, then the vehicle is going to overturn. Now, the other thing which are going to verify, is the whether the tractor will slide down. Why it will slide down? Because the forces which are developed at the contact patch both are the rear wheels as well as the front wheels. They should oppose the weight component which is acting here $W\sin\beta$.

So, that is why I have written $W\sin\beta$ will be greater than $F_{sf} + F_{sr}$. Then what will happen the tractor will become unstable and it will slide down the slope. So, these are the 2 conditions which are to be verified when the tractor is moving across a slope ok. So, when I said F_{sf} means that the total side force acting, when I said F_{sr} means is the total side force acting on the rear wheels. F_{sf} is the total side force which is acting at the front wheels. That means, you have to take side force of the front wheel both the wheels and side force of both the rear wheels. So, now I will try to summarize what exactly we discussed in this class.

We discussed the fundamental equation which is the which governing the steady state handling behaviour of the vehicle and where we found out that the steer angle which is required to negotiate a given curve, how it is dependent on wheel base, how it is dependent on turning radius, forward speed and understeer coefficient of the vehicle. And in fact, it also depends on the weight distribution. Understeer coefficient is nothing, but weight divided by the cornering stiffness. So, that is why in other words we can say, it also depends on the weight distribution on the front and the rear wheels and the steering stiffness of the front and rear tyres. Then we also discuss the handling characteristics of the vehicle, how it is dependent to a greater extent on the relationship between slip angles of the front and rear tyres. Then we also discussed the side slope stability of the tractor and it what we observe that is the side slope stability we have to check two conditions.

The first condition is the overturning and that is dependent on the track width and the height of the CG and the second one is the sliding down of the tractor which is dependent on the side force. That means, the side force which are developed or the lateral force which are developed at the front and the rear tyres that should be sufficient to support the a sin component of the weight of the vehicle. So, you can refer to Wong's book theory of grand vehicles to further improve your knowledge on this. Thank you.