Course Name: Watershed Hydrology

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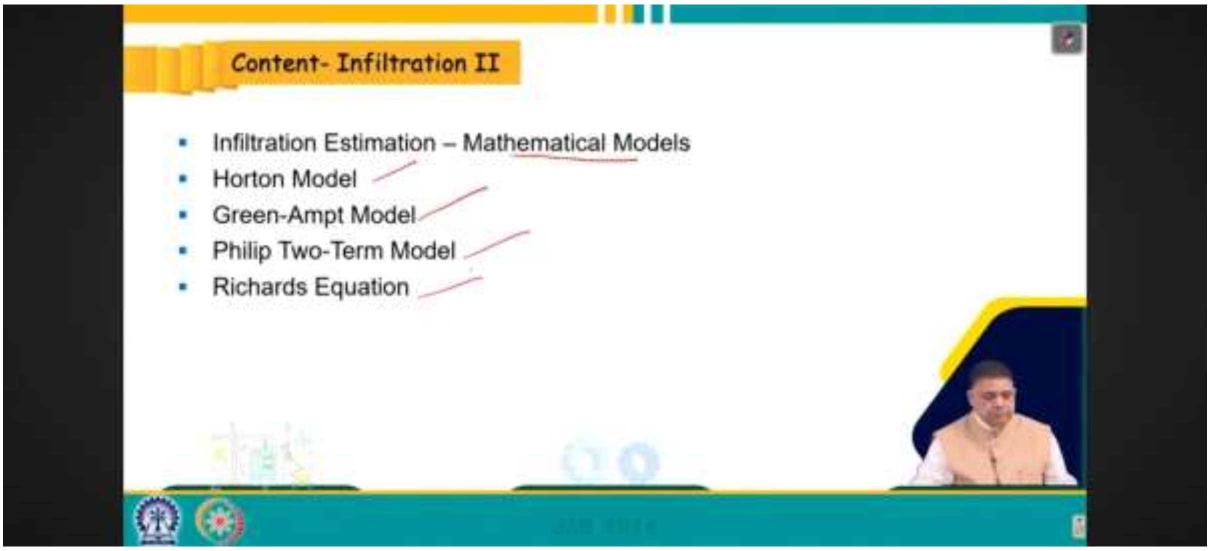
Week: 02

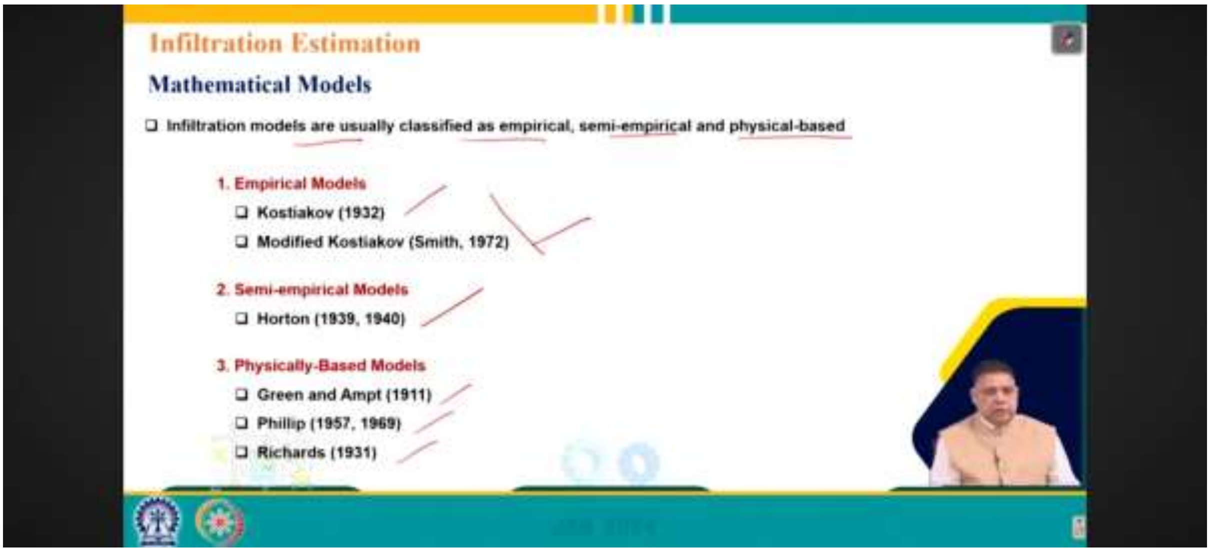
Lecture 10 : Infiltration II

Hello friends, welcome back to this online certification course on watershed hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology Kharagpur. We are currently in Module 2, specifically Lecture 5, where we will delve further into infiltration, focusing on Part 2 of this topic.

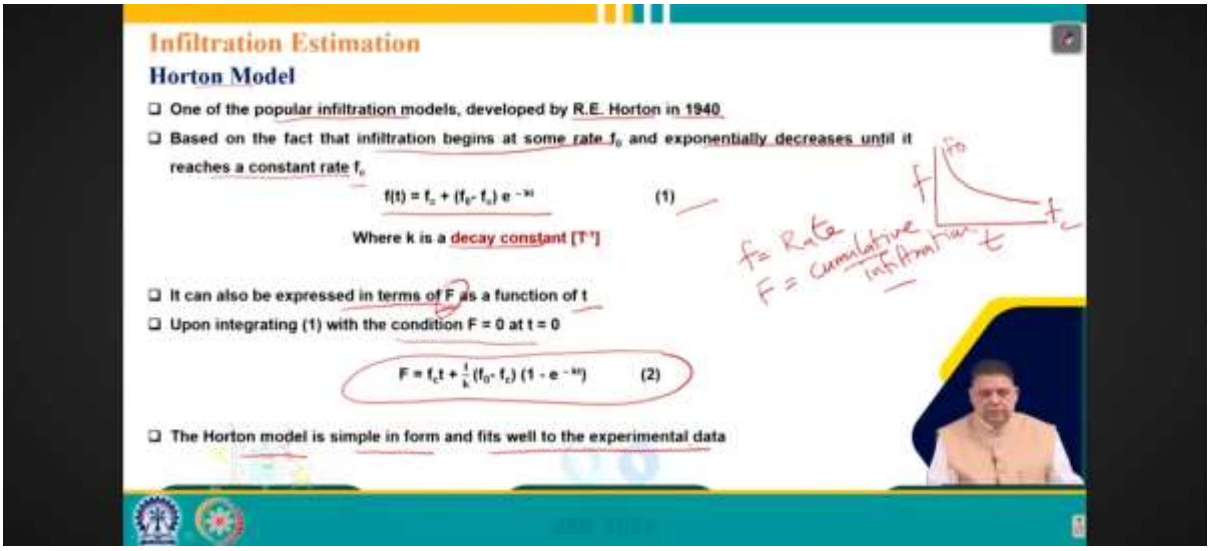
In this lecture, we will build upon what we discussed previously regarding mathematical models for estimating infiltration. We will cover the Horton model, the Green and Ampt model, the Philip two-term model, and the Richards equation. Let's proceed with our exploration of these models.



To provide a recap, in the previous lecture, we discussed that infiltration models are classified as empirical, semi-empirical, and physically based. We introduced two empirical models: the Kostiakov model and the modified Kostiakov model. Today, we will delve into semi-empirical models such as Horton's model, as well as three physically based models: the Green and Ampt model, Philip's model, and the Richards equation.

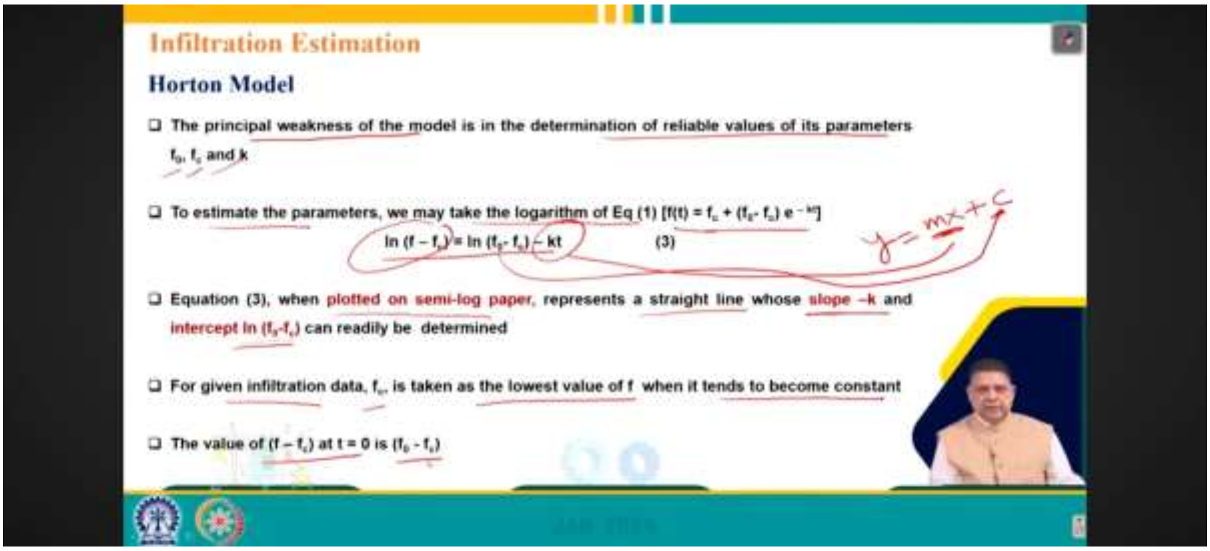


Beginning with the Horton model, it stands as one of the popular infiltration models developed by R. E. Horton in 1940 introduced the Horton model, which is based on the observation that infiltration begins at some rate f0 and exponentially decreases until it reaches a constant rate fc. The infiltration equation proposed by Horton is ft = fc + (f0 – fc)* e^ (-kt) , where k is a decay constant. Essentially, this model is based on the concept we have already discussed: an infiltration curve that starts at a higher rate f0 , then gradually reaches a constant value fc over time. It can also be expressed in terms of F, which represents cumulative infiltration. Remember, we are using F for infiltration rate and cumulative infiltration. Therefore, it's important to keep these two terms distinct. F can also be expressed as a function of time. To obtain the cumulative infiltration, we integrate Equation 1 with the initial condition f0 at (t = 0). This resulting equation will provide us with the cumulative infiltration. The Horton model is simple in form and fits well to the experimental data, which are the advantages of the Horton model.

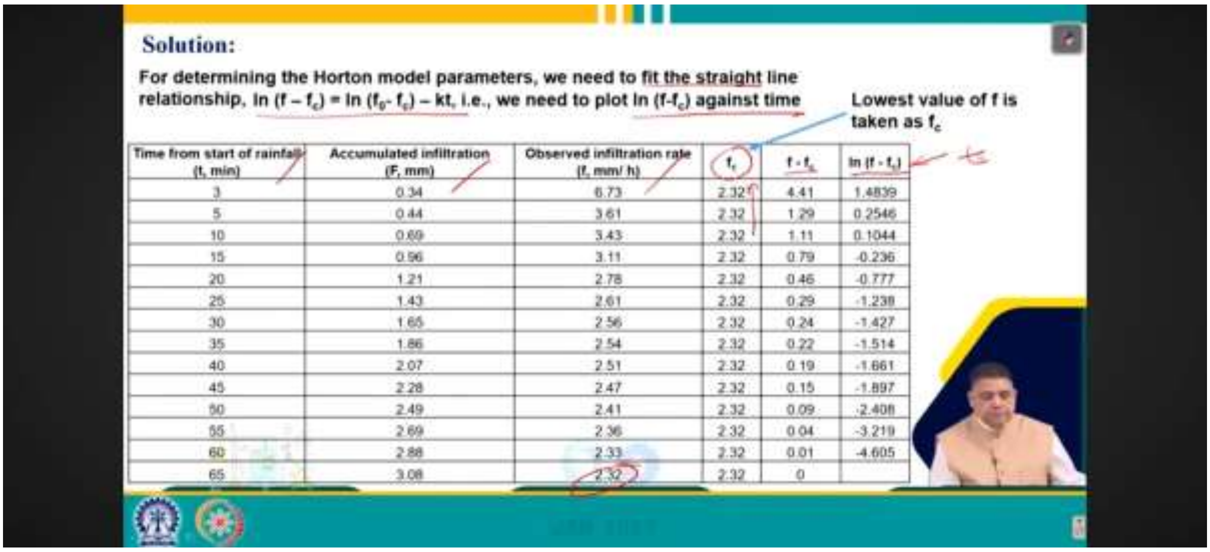


However, the principal weakness of the model lies in determining reliable values for the parameters f0 , fc , and k . If you recall, upon examining the equation, we require these three parameters f0 , fc , and k to determine either infiltration rate or cumulative infiltration.

Hence, the main challenge is obtaining reliable values for these parameters. To estimate these parameters, we utilize Equation 1: ft = fc + (f0 - fc) e^(-kt). If we take the logarithm of this equation, we will obtain this form: ln (f - fc) = ln (f0 - ft) - kt. When Equation 3 is plotted on semi-logarithmic paper, it results in a straight line, following the form y = mx + c . Here, y represents ln (f - fc), m represents the slope k , and c represents the intercept ln (f0 - fc) . Hence, the slope is (-k) and the intercept can be determined. For a given infiltration data, fc is taken as the lowest value of f when it tends to become constant. And the value of (f - fc) at (t = 0) is (f0 - fc), which is the initial value to begin with.

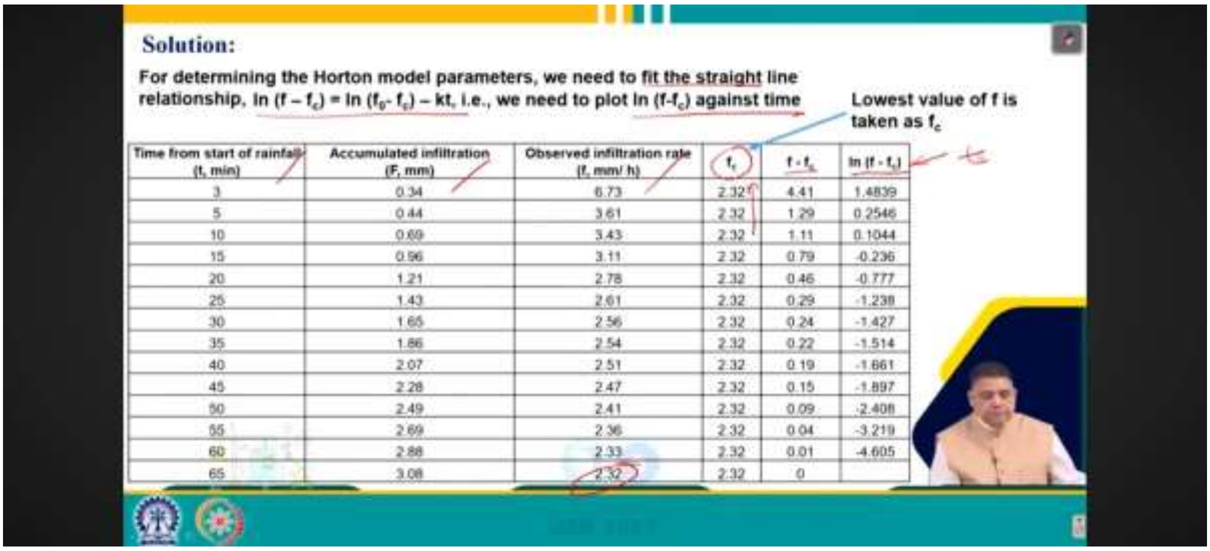


Let's take an example to clarify this further. For an orchard field, the time since the start of the rainfall, accumulated infiltration, and observed infiltration rate are given in the table. We will develop the Horton model for the orchard and plot the observed and estimated infiltration rates. Here's the experimental data provided: the time since the start of the rainfall ranges from 3 minutes to 65 minutes, and we have accumulated infiltration values in millimetre , starting from 0.34 to 3.08 millimetre. We have observed infiltration rates in millimetre per hour initially, it is very high at 6.73 and then decreases to 2.32, which is the lowest value.



Time from start of rainfall (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/ h)	f_c	$f - f_c$	$\ln (f - f_c)$
3	0.34	6.73	2.32	4.41	1.4839
5	0.44	3.61	2.32	1.29	0.2546
10	0.69	3.43	2.32	1.11	0.1044
15	0.96	3.11	2.32	0.79	-0.236
20	1.21	2.78	2.32	0.46	-0.777
25	1.43	2.61	2.32	0.29	-1.238
30	1.65	2.56	2.32	0.24	-1.427
35	1.86	2.54	2.32	0.22	-1.514
40	2.07	2.51	2.32	0.19	-1.661
45	2.28	2.47	2.32	0.15	-1.897
50	2.49	2.41	2.32	0.09	-2.408
55	2.69	2.36	2.32	0.04	-3.219
60	2.88	2.33	2.32	0.01	-4.605
65	3.08	2.32	2.32	0	

Now, to determine the Horton model parameters, as we have discussed earlier, we need to fit the straight-line relationship, which means plotting ln(f – fc) against time. This implies that we need to calculate this relationship. The values provided include time from the start of rainfall or infiltration experimentation, accumulating infiltration, and observed infiltration rate. We take fc as the lowest value of infiltration rate in the experimental data. So, what we're doing here is taking fc equals to 2.32, as listed here. Throughout, fc remains 2.32. Now that we have f and fc , we can calculate the value of (f - fc), and then take the logarithm. This resulting value is what we need to plot against time.



Time from start of rainfall (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/ h)	f_c	$f - f_c$	$\ln (f - f_c)$
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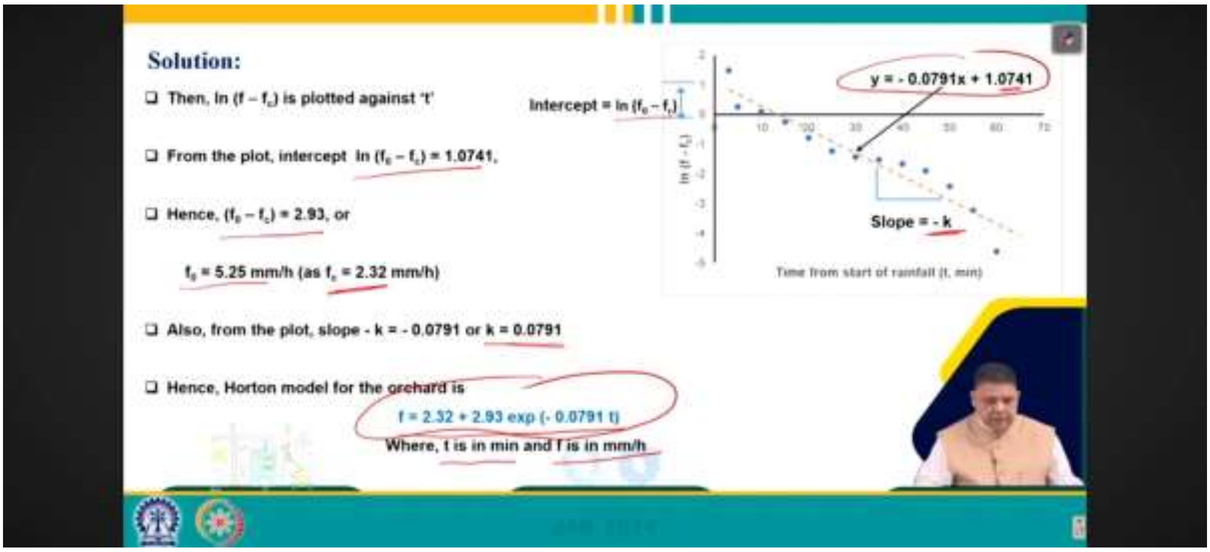
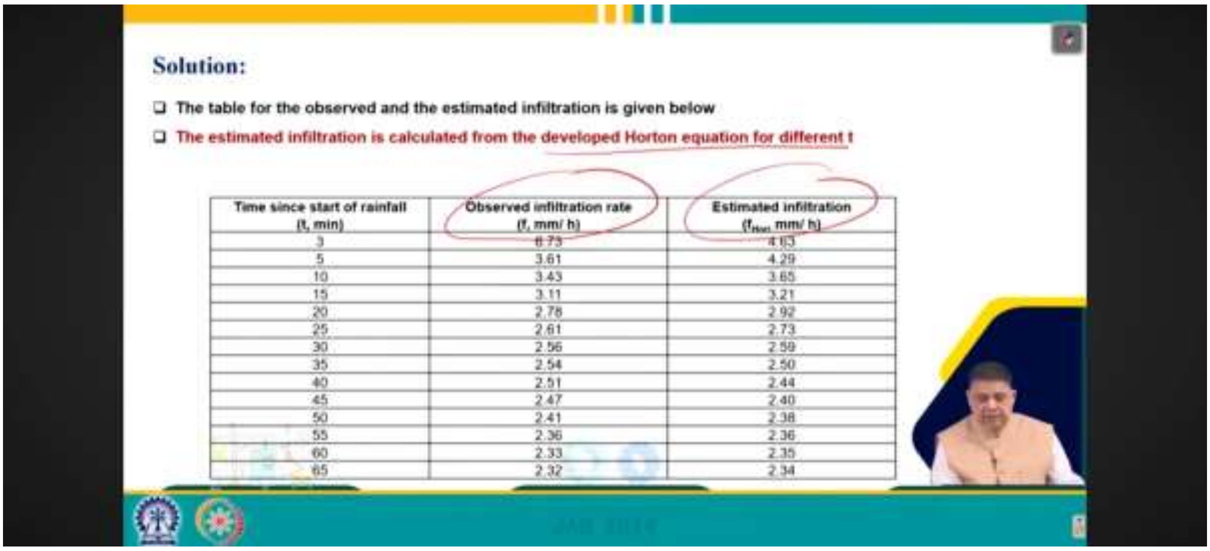
When we plot ln(f - fc) against time, this is the curve we obtain. As we have already discussed, the intercept is ln(f0 - fc) and the slope is -k , derived from this equation. This equation can be fitted using Excel, or one can fit it manually. From the plot, the intercept value we obtain is ln(f0 - fc), which is 1.0741, as shown here. So, from here, (f0 - fc) is 2.93, or f0 value is 5.25 millimetre per hour, because we have taken fc value as 2.32 . From the plot, we observe that -k is approximately -0.0791, meaning k equals 0.0791. So now, we have determined f0 , fc , and k. Therefore, the Horton model for the orchard can be expressed as f = 2.32 + 2.93 exp(-

0.0791t) . Here, k is 0.0791, t is in minutes, and f is in millimetre per hour. This is the Horton model we have derived using the given data for the orchard.

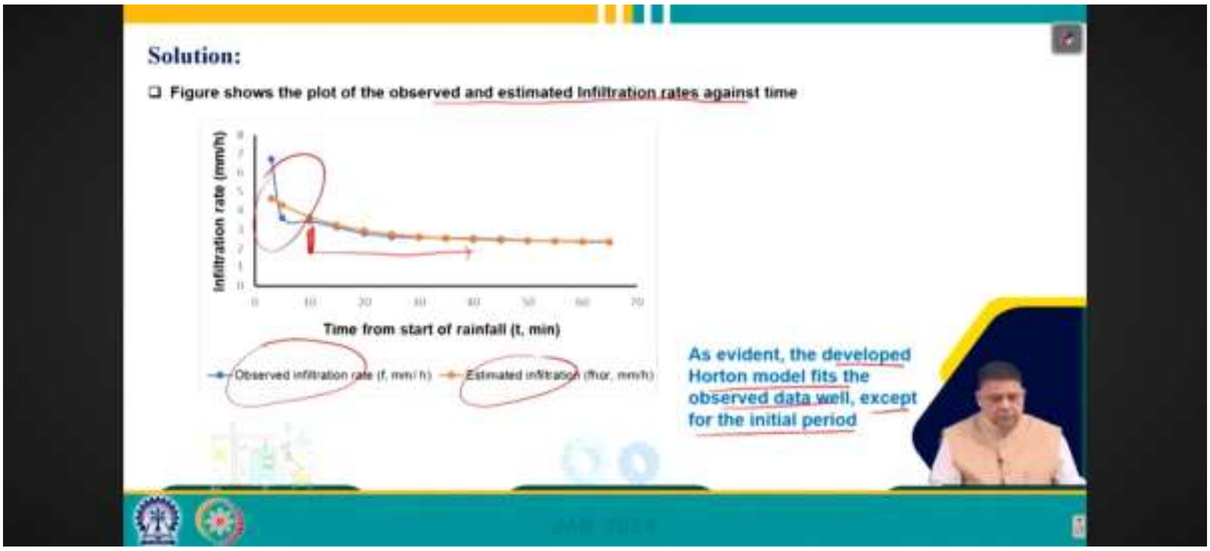


Now, we are also required to plot these values. So, this is the observed infiltration rate, and this is the estimated infiltration rate calculated using the Horton equation we just developed in the previous slide.

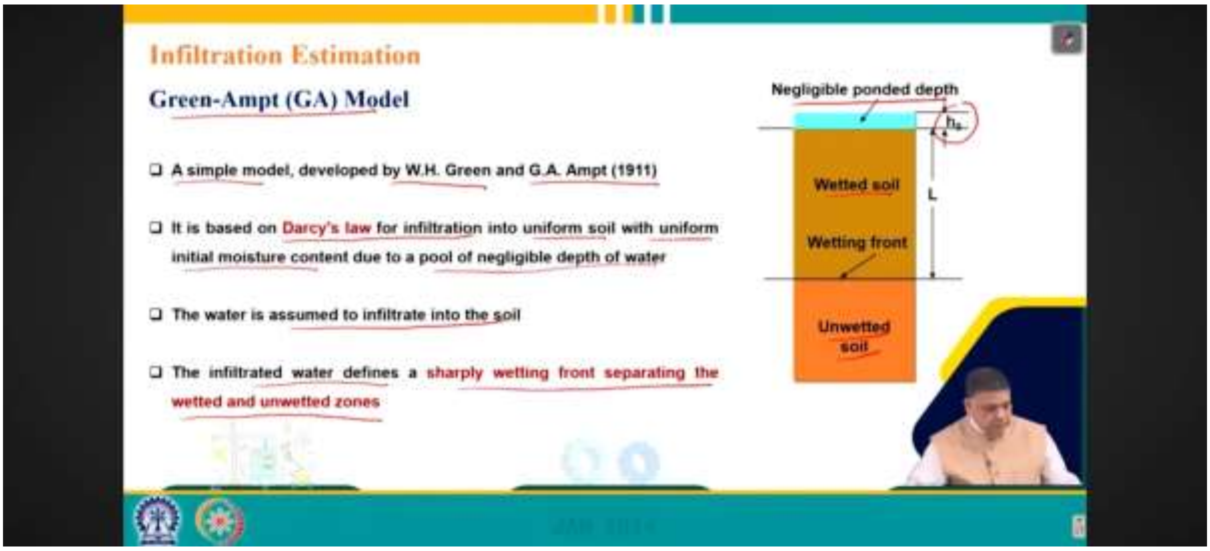


Time since start of rainfall (t, min)	Observed infiltration rate (f, mm/ h)	Estimated infiltration (f_{Hor}, mm/ h)
3	6.73	4.63
5	3.61	4.29
10	3.43	3.65
15	3.11	3.21
20	2.78	2.92
25	2.61	2.73
30	2.56	2.59
35	2.54	2.50
40	2.51	2.44
45	2.47	2.40
50	2.41	2.38
55	2.36	2.36
60	2.33	2.35
65	2.32	2.34

These values are plotted against the measured values, showing the observed and estimated infiltration rates against time. The observed values are represented in blue, while the estimated values are shown in orange. As you can observe, from time t onwards, the data fits very well, but it doesn't perform as well in the beginning. From this, we can infer that the developed Horton model fits the observed data well except for the initial period. As you can see, it doesn't match the observed data well during the initial period.

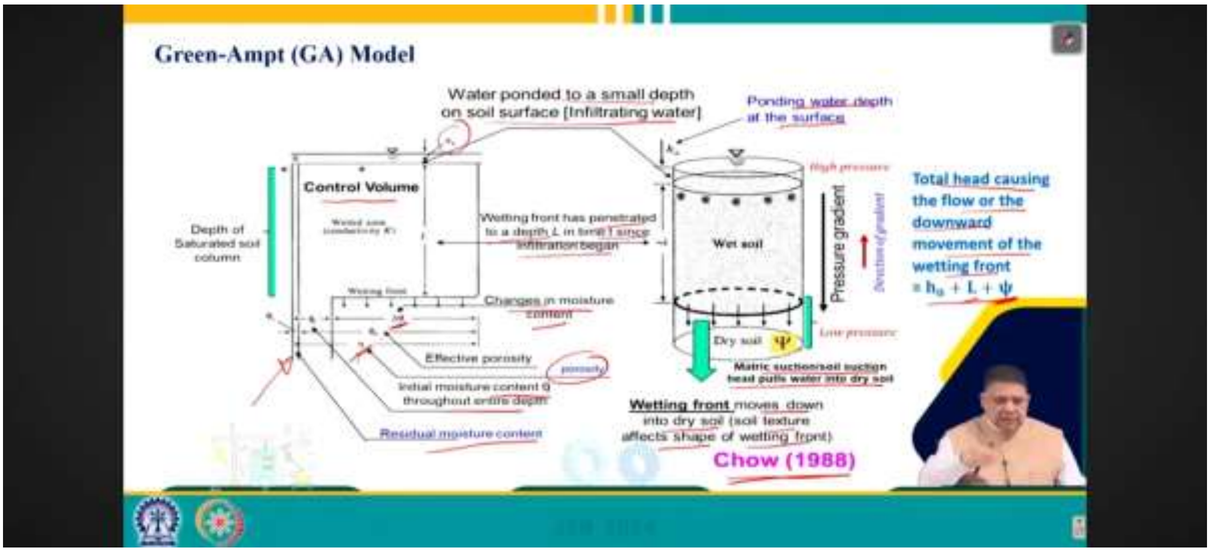


Now, let's move on to the next model, the Green-Ampt model, which is a simple model developed by W. H. Green and G. A. Ampt introduced the Green-Ampt model in 1911. Despite its age, it remains a popular model even today. This model is based on Darcy's law for infiltration into uniform soil with uniform initial moisture content and a negligible depth of water pooling on the surface. Several assumptions are made here: the soil is uniform throughout the considered soil column, with a consistent initial moisture content. Additionally, there is minimal ponding depth on the surface. It's assumed that water infiltrates into the soil, resulting in a distinct wetting front that separates the wetted and un-wetted zones. This process aligns with what we observed during the infiltration process discussion, where the soil becomes wetted due to the saturation transmission and the formation of wetting zones. We also noted the presence of a lower boundary wetting front. So, if the wetting front has reached a depth of L , it means that above the wetting front, there is wetted soil, and below it, there is un-wetted or dry soil with the initial moisture content. This implies that the ponded depth h0 , is negligible.

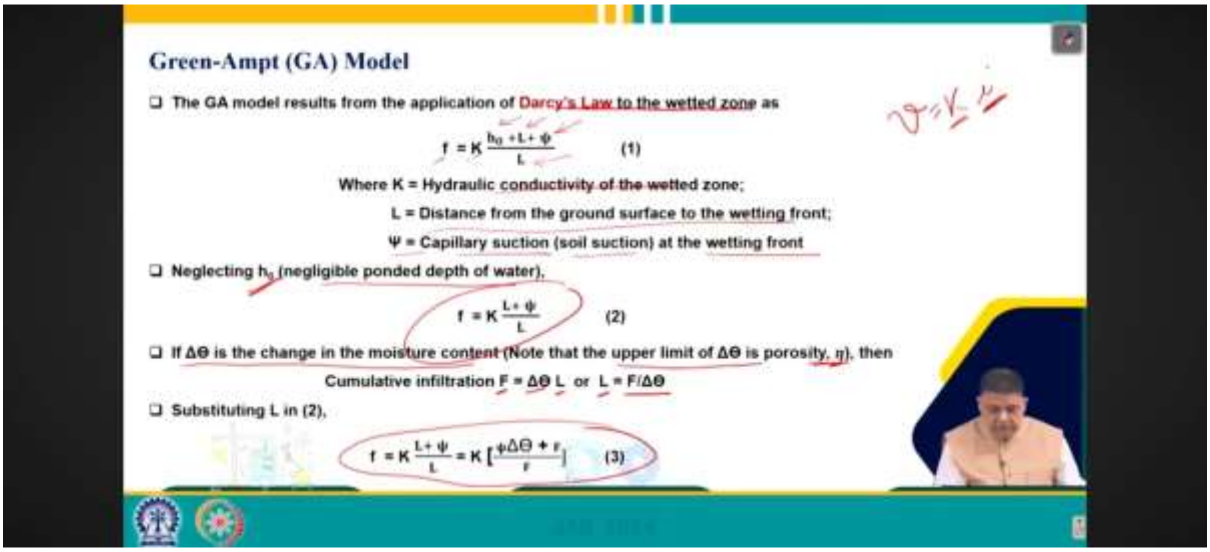


The image also illustrates this scenario, where water has pooled to a small depth on the soil surface h0 , and the wetting front has penetrated to a depth L since infiltration began. This delineates the wetted zone above the wetting front and the un-wetted soil below it. So, essentially, the pressure gradient causing the flow or downward movement of the wetting front consists of three components. Firstly h0 , which represents the depth of ponding water.

Secondly L , indicating the depth to which the wetting front has penetrated. And finally ψ, which denotes the matrix action or soil suction head, pulling water into the dry soil. Hence, there is a gravitational force due to the head, and additionally, there is a pulling force due to the matrix action. The wetting front moves down into the dry soil due to this matrix action, and the soil texture influences the shape of the wetting front. This concept was elucidated by Chow in 1988. If we examine the left side of the control volume, as mentioned earlier, it consists of uniform soil with uniform initial moisture. If we assume that the initial moisture content was θi throughout the entire depth, then obviously, when the wetting front moves down, there will be a change in the moisture content. Because a constant head is maintained, this change will be negligible. Therefore, in a uniform soil, the moisture content will be uniformly distributed. Let's denote the change in moisture content for the entire profile as Δθ . Here, the soil's porosity is represented by θ , and the effective porosity is denoted as θe. Additionally, there is a residual moisture content in the soil. Thus, we have the residual moisture content, the initial moisture content, the change in moisture content, the effective porosity, and the porosity of the soil. So, because of the advancing front, the moisture content of the soil is changing, increasing by the magnitude of Δθ throughout the entire control volume.

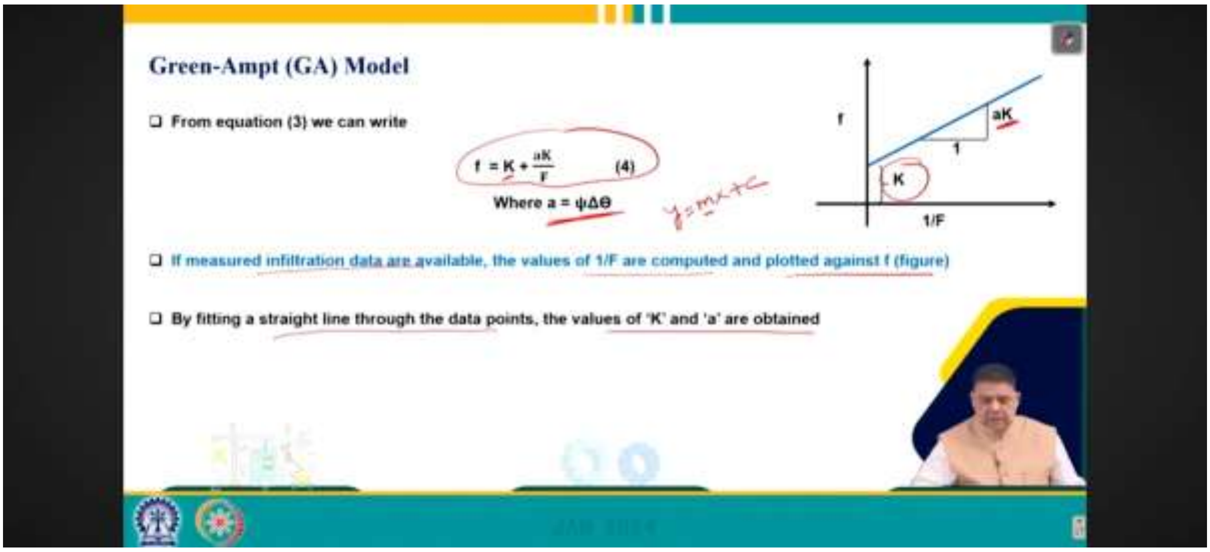


The Green-Ampt (GA) model stems from the application of Darcy's law to the wetted zone. Darcy's law states that f = K*((h0 + L + ψ) / L) , where K represents the hydraulic conductivity and I denotes the hydraulic gradient. Similarly, in the GA model, F represents the infiltration rate, K stands for the hydraulic conductivity of the wetted zone, and I signifies the hydraulic gradient, which is the head causing the flow and the flow length. As we previously discussed, there are three components here: h0 , L , and ψ. The wetting front has advanced to a depth of L , where L represents the distance from the ground surface to the wetting front, and ψ represents the capillary suction or soil suction at the wetting front, as illustrated in the previous picture. Now, as we mentioned earlier, the ponding depth of water is negligible. So, by neglecting h0, this equation can be written in the form f = K *((L + ψ) / L) . If Δθ represents the change in moisture content as discussed previously, and we also mentioned that the porosity of the soil is θ, which indicates the upper limit of the soil's moisture content . This implies that when the soil is completely saturated, its moisture content will be equal to its porosity. Thus, cumulative infiltration can be calculated as F = Δθ L , where L represents the depth, and Δθ is the change in moisture content. Alternatively, we can express L as L = F/ Δθ . Substituting this into equation number 2, we get f = K *((L + ψ) / L) . By replacing L with ψ Δθ and manipulating, we arrive at the form of the Green-Ampt model.

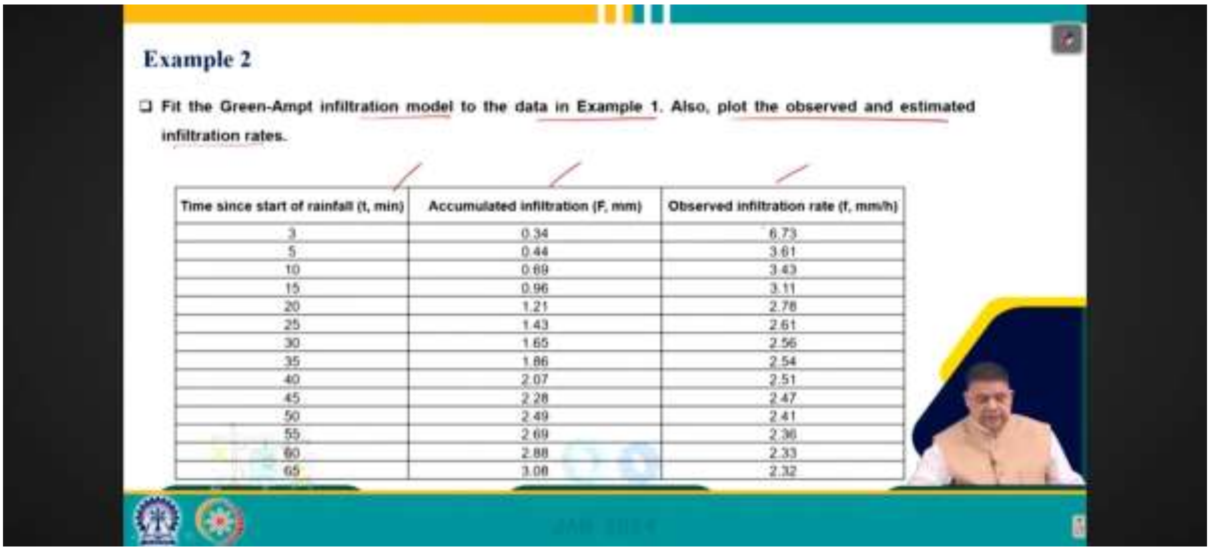


Equation 3 can also be written as f = K + (a K)/F, where a is ψ Δθ .

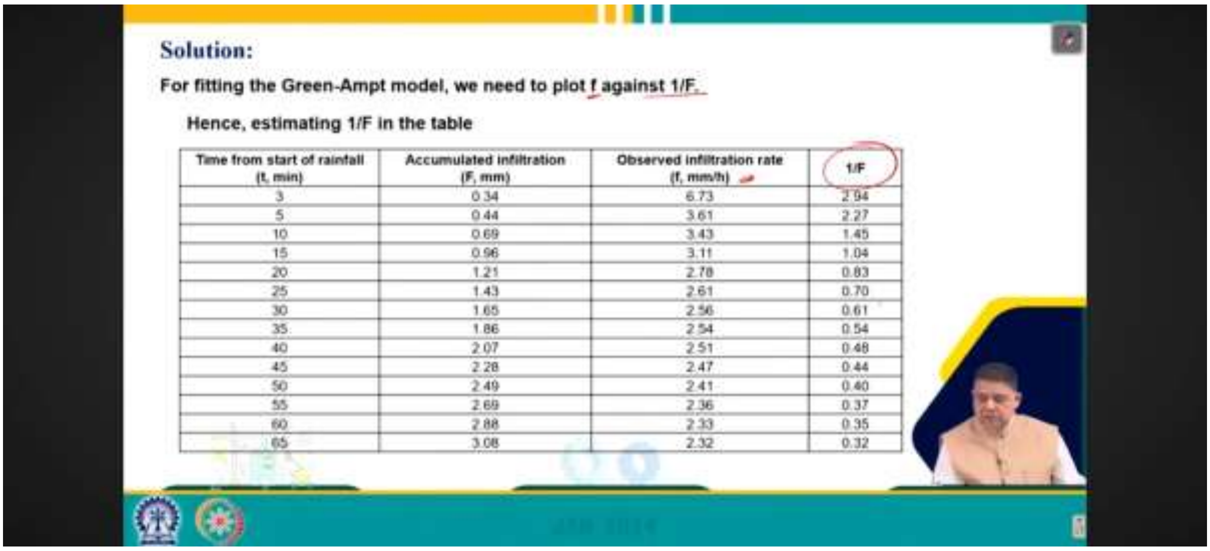
The advantage of this form lies in its usability with available measured infiltration data. The values of 1/F are computed and plotted against time. When plotted, this equation takes the form of y = mx + c , where the intercept c represents K and m represents K as well since we are plotting 1/F . Therefore, the slope represents K . By fitting a straight line through the data points, we can determine the values of K and a from this plot.



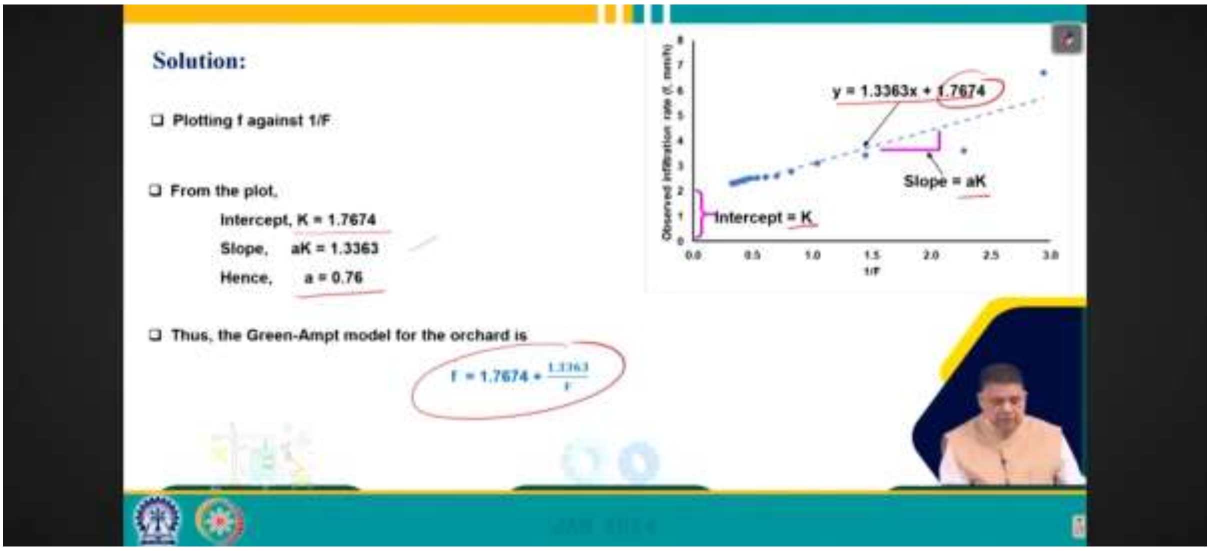
Let's illustrate this with an example. We will apply the Green-Ampt infiltration model to the data provided in Example 1 and plot both the observed and estimated infiltration rates. The data includes time, accumulated infiltration, and observed infiltration rates.



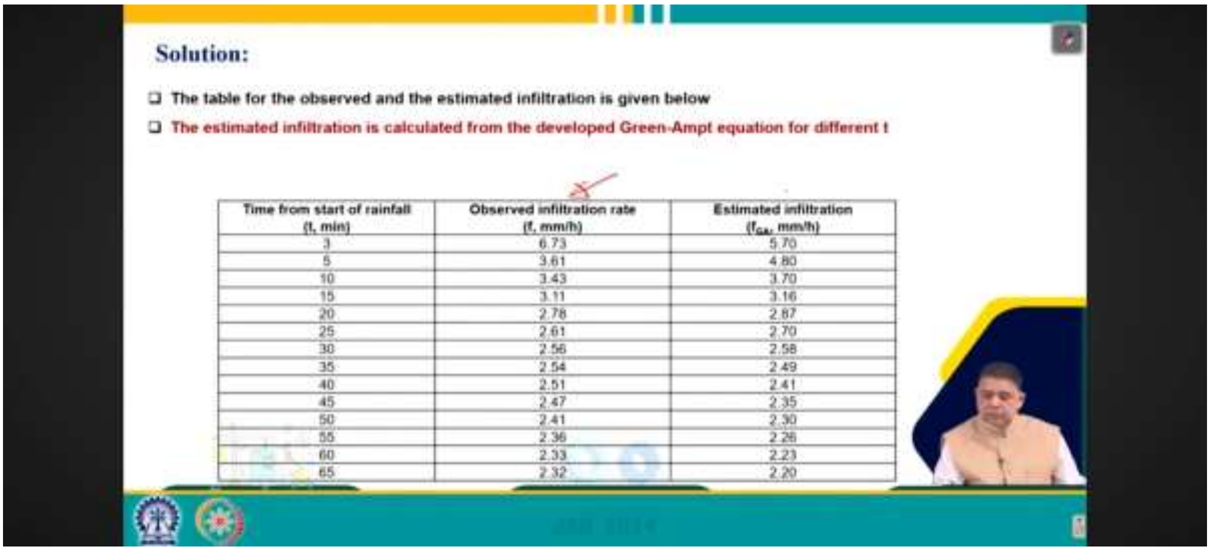
In this case, we need to plot F against 1/F . Since F values are already given, we need to calculate the values of 1/F . Once we have 1/F , we plot it against F .



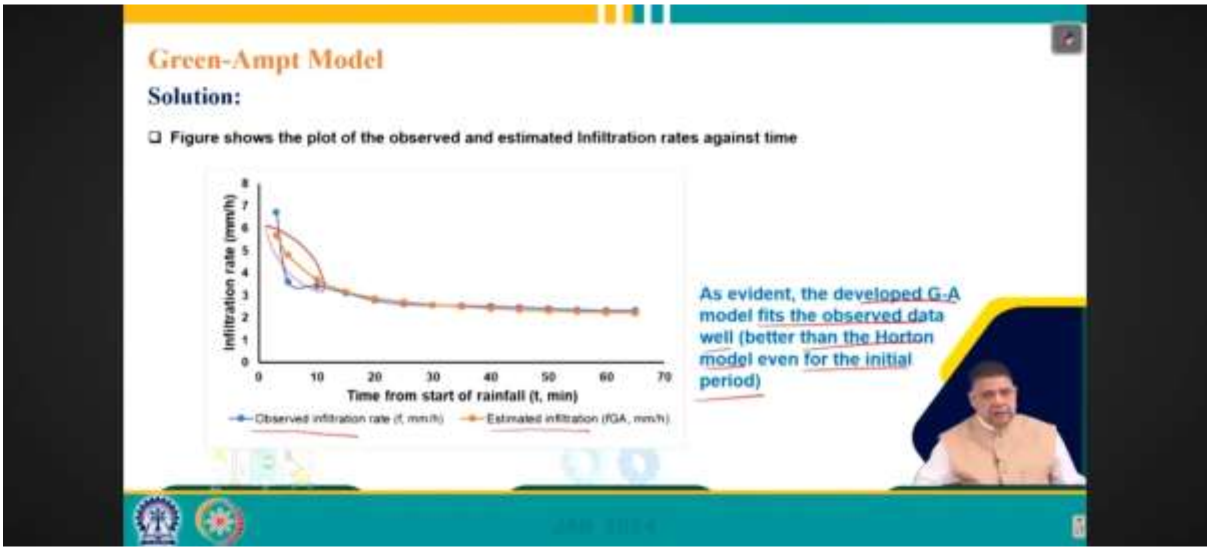
From the plot, we can fit a straight line, as shown here, and determine the values of a and K. From the plot, we find that the intercept K is 1. The intercept K is 1.7674, and the slope aK is 1.3363. Therefore, from here, the K value is 1.7674 and the a value is 0.76. Consequently, the fitted Green-Ampt model for the orchard takes this form.



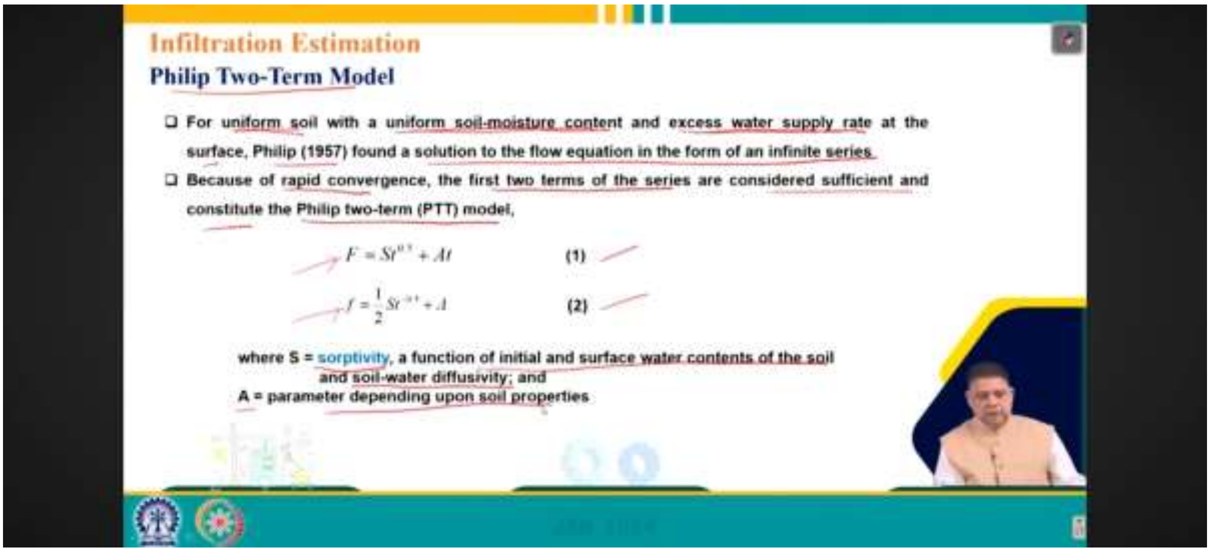
Additionally, we can utilize the fitted model to obtain estimated infiltration values and then plot both the observed and estimated infiltration rates, as shown here.



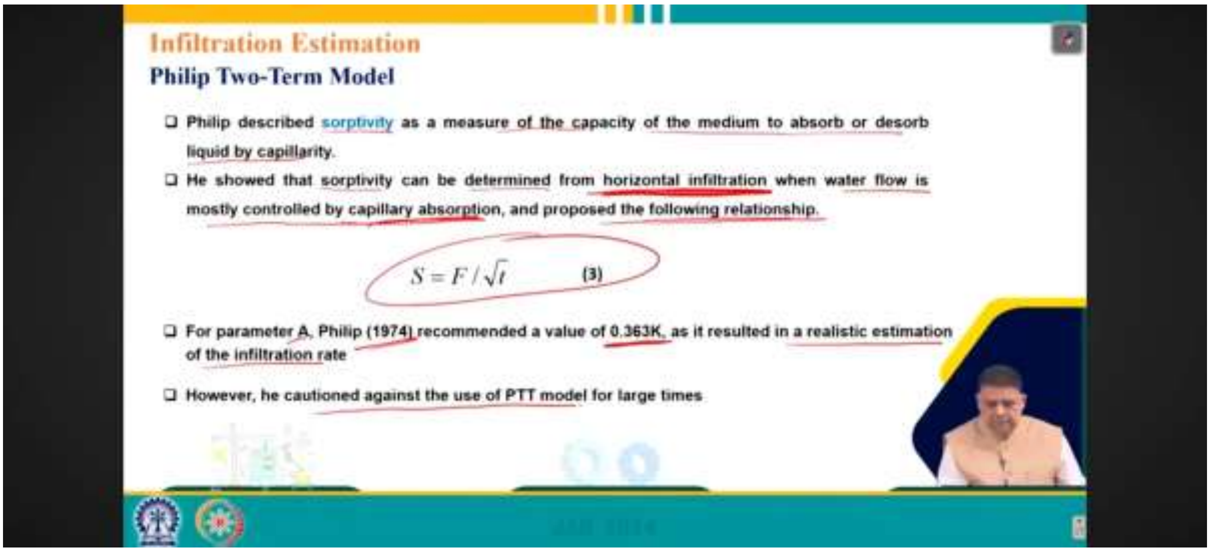
In this plot, the observed infiltration rate is represented in blue, while the estimated infiltration rate using the Green-Ampt model is shown in orange. As you can observe, even in this aspect, the model performs better than the Horton model, as it is more physically based, utilizing more physical concepts. As evident, the developed Green-Ampt (GA) model fits the observed data better than the Horton model, even for the initial period. This demonstrates that the Horton model, although physically based, is slightly inferior to the semi-empirical Horton's model we discussed earlier.



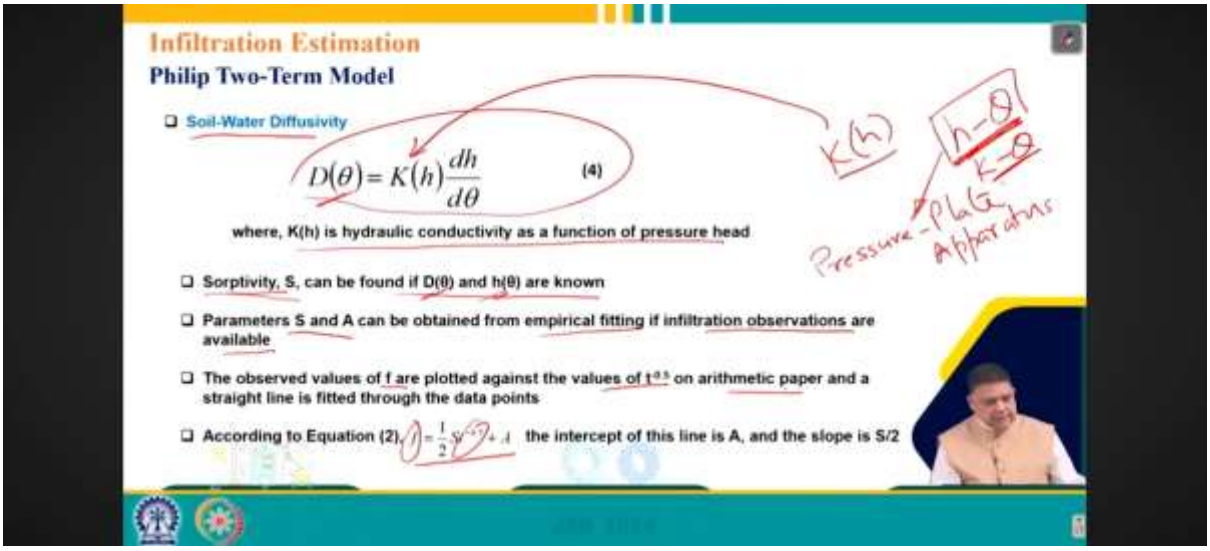
Now, let's move on to the next model in the physically based category, which is the Philip two-term model. Philip introduced this model in 1957 for uniform soil with uniform soil moisture content and an excess water supply rate at the surface. He found a solution to the flow equation in the form of an infinite series. If we examine the assumptions, we see that the soil is uniform and the moisture content remains uniform. However, unlike in the case of the Green-Ampt model where we assume a negligible ponding depth, here there is an excess water supply at the surface. Under these conditions, water is not a limiting factor, and Philip derived an infinite series to describe the situation. Due to the rapid convergence of the series, Philip considered the first two terms to be sufficient. These two terms constitute the Philip two-term model, hence its name. For this model, the cumulative infiltration F is represented by the equation S t^0.5 + At , while the infiltration rate f is given by (1/2)*S t^(-0.5) + A . The first equation provides the cumulative infiltration, while the second one describes the infiltration rate. Here, the parameter S represents sorptivity , which is a function of the initial and surface water contents of the soil, as well as the soil water diffusivity. On the other hand, A is a parameter that depends on soil properties. Both A and S are soil properties.



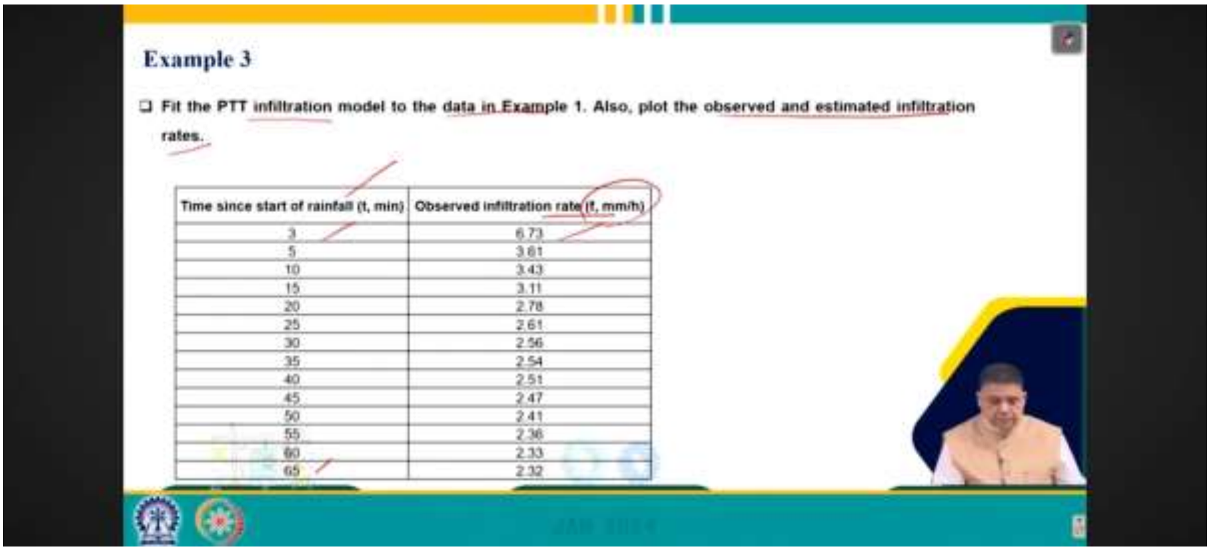
Now, turning our attention to sorptivity, Philip described it as a measure of the capacity of the medium to absorb or desorb liquid through capillarity alone, with only capillary forces at play. Sorptivity, therefore, represents the amount of liquid, typically water, that a soil medium can absorb or desorb. Philip proposed a method for determining sorptivity from horizontal infiltration, where water flow is primarily controlled by capillary absorption. He proposed the following relationship for sorptivity: $S = F/\sqrt{t}$. It's important to note that this determination is specifically for horizontal infiltration. This distinction is necessary because, as we discussed during the infiltration process description, vertical infiltration is predominantly influenced by gravitational forces. Therefore, in Philip's definition of sorptivity, capillary forces are the main factor at play. So, this is why we need measurements of horizontal infiltration. When the flow is primarily controlled by capillary absorption, the relationship $S = F/\sqrt{t}$ holds true. For parameter A , Philip in 1974 recommended a value of 0.363k, as it resulted in a realistic estimation of infiltration rate. However, he cautioned against the long-term use of the PTT model. This implies that if you intend to use a model for an extended period, the Philip two-term model might not be suitable. Nonetheless, it performs well for short periods.



Now, another term that arises is soil water diffusivity, which is defined by the equation $D(\theta) = K(h)*(dh/d\theta)$, where K(h) is the hydraulic conductivity, a function of pressure head, and sorptivity can be found if $D(\theta)$ and $h(\theta)$ are known. If you have studied soil science or conducted experiments using the pressure plate apparatus, you might recall that the moisture soil retention curve obtained from experimentation gives us the soil retention curve or $h(\theta)$ relationship, also known as $h(\psi)$ or $h(\theta)$. This relationship provides the relationship between pressure head and soil moisture content. Once we have the $h(\theta)$ relationship, we can utilize several models, with one very popular model being the van Genuchten model. Using the van Genuchten model, we can develop the $k(\theta)$ relationship. Therefore, the $h(\theta)$ relationship can be obtained from pressure plate experimentation, and based on that, we can fit the data and derive the $K(\theta)$ relationship. Consequently, when we have the $h(\theta)$ and $K(\theta)$ relationships, we can express k in terms of h . So, this is what is being utilized here. With this knowledge, we can determine $D(\theta)$ and $h(\theta)$, and if $D(\theta)$ and $h(\theta)$ are known, S can also be obtained for any given soil from experiments. The parameters S and A can be obtained from empirical fitting if infiltration observations are available, similar to previous equations like Horton and Green-Ampt. The observed values of f are plotted against the values of $t^{-0.5}$, as we previously discussed. This relationship is represented by the equation $f = (1/2)*S\, t^{-0.5} + A$. When we plot f against this, we can observe a linear relationship. The intercept of this line represents A , while the slope is S/2 .

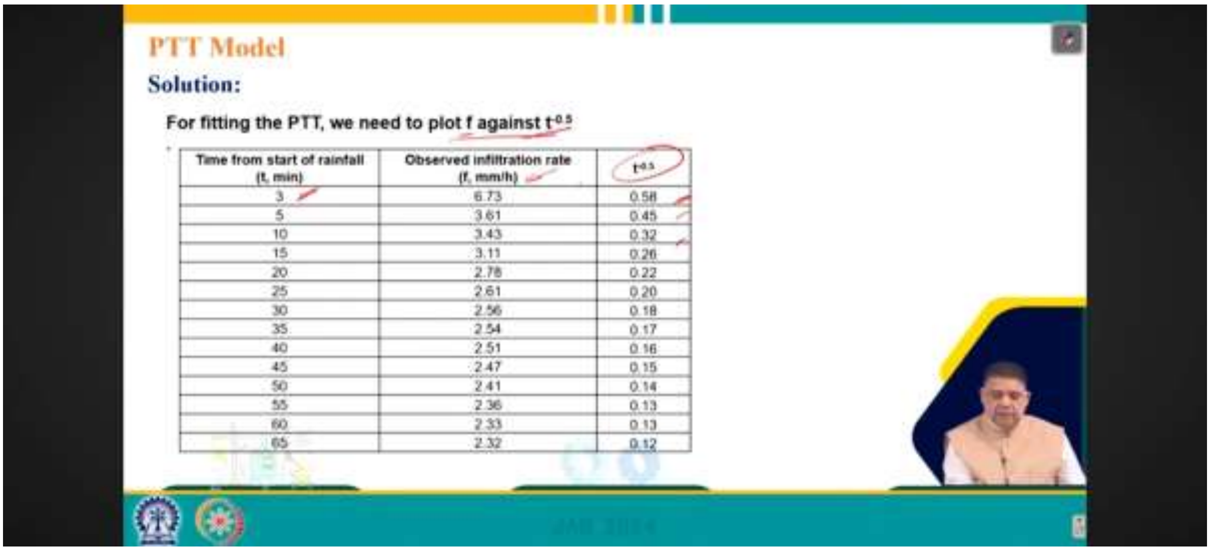


Now, let's consider Example 3, where we fit the PTT infiltration model to the data provided in Example 1 and plot both the observed and estimated infiltration rates. We've already reviewed this data earlier, which includes the time since the start of rainfall (denoted as t in minutes), ranging from 3 to 65, along with the corresponding infiltration rate values in millimetre per hour. Additionally, cumulative infiltration values were provided, but they are not required for the PTT model, so they are not included in this analysis.

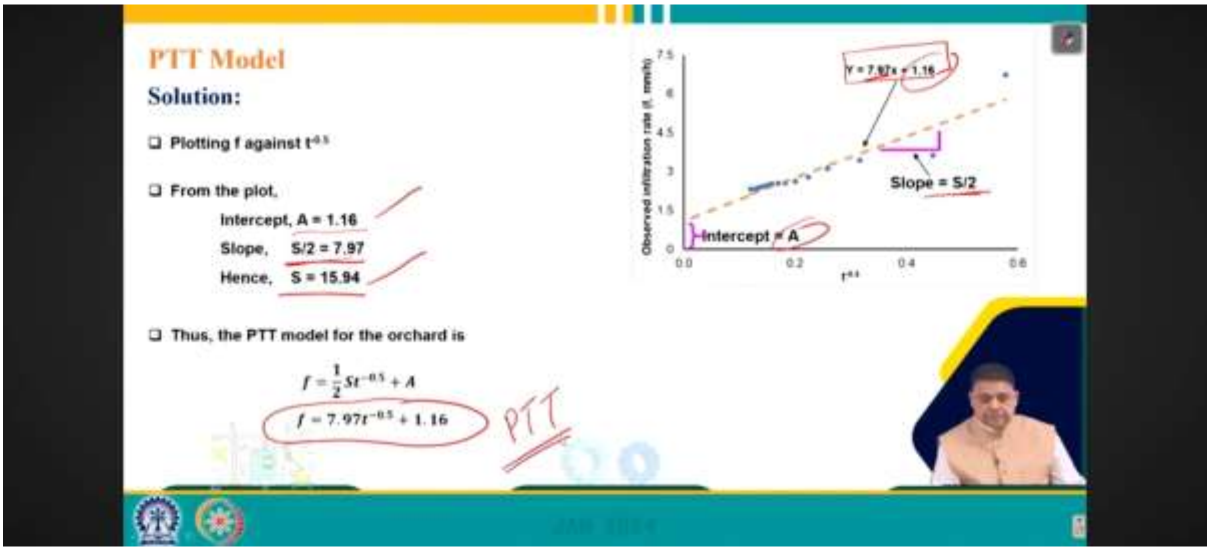


Time since start of rainfall (t, min)	Observed infiltration rate (f, mm/h)
3	6.73
5	3.61
10	3.43
15	3.11
20	2.78
25	2.61
30	2.56
35	2.54
40	2.51
45	2.47
50	2.41
55	2.36
60	2.33
65	2.32

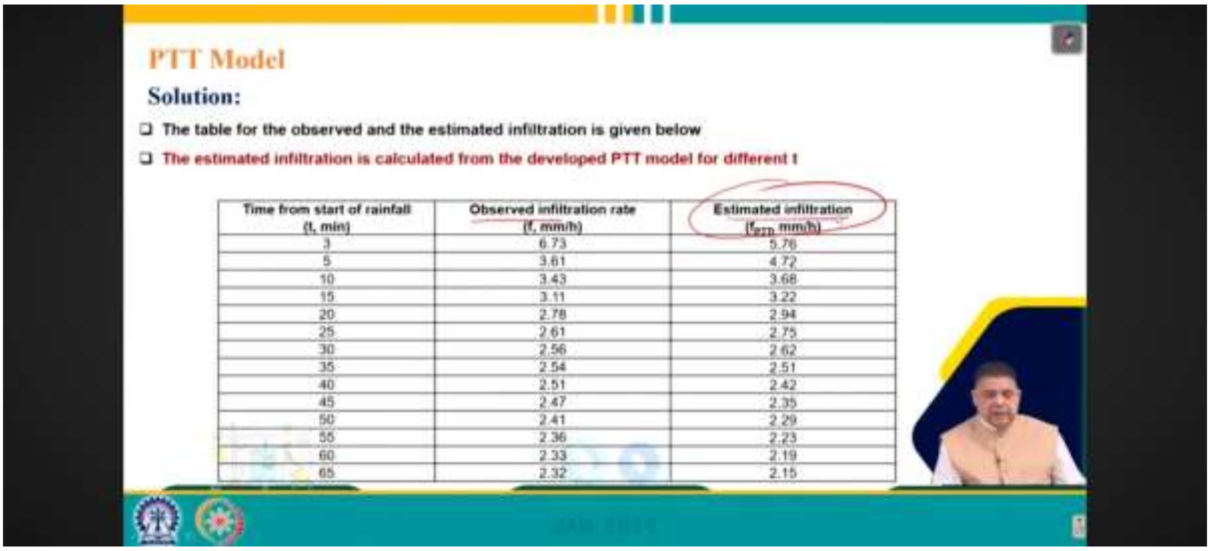
To fit the PTT model, we need to plot f against t^{-0.5} . Although f remains unchanged in this case, we need to calculate t^{-0.5} for each time value. Once we have these values, we can plot f against t^{-0.5} , as shown in the graph.



Upon plotting, we obtain a straight line relationship, represented by the equation y = 7. 97x + 1.16 . The equation obtained through Excel fitting is f = 7.97x +1.16 . As mentioned, Excel was used to generate this equation, but one can also utilize the least square method or other software, or even manually calculate and fit the equation using a calculator. Recalling the equation form f =(1/2)*S t^(-0.5) + A , we know that the slope of the line is S/2 and the intercept is A , with their values already provided. The intercept value is 1.16, indicating that A = 1.16 , and the slope S/2 is 7.97. Hence, the value of S comes out to be 15.94. With these values for A and S , we can plug them into the equation, resulting in f = 7.97t^{-0.5}+1.16 . So, the PTT model for the orchard data yields f = 7.97t^{-0.5}+1.16 .

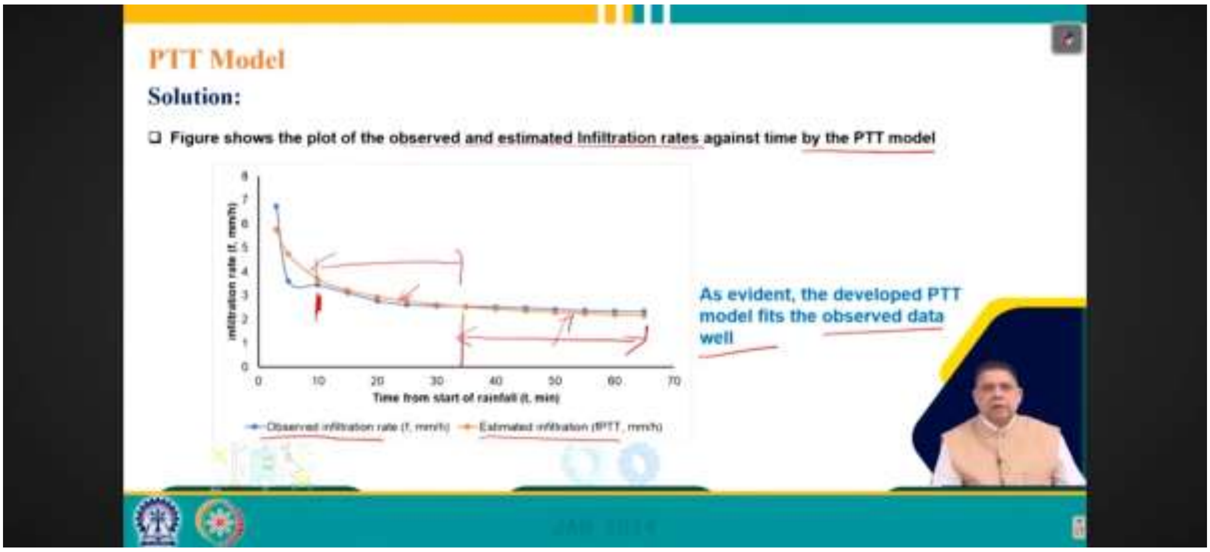


Utilizing this model, we can obtain estimated infiltration values for various times and then plot the observed infiltration rate alongside the estimated infiltration rate, as depicted here.



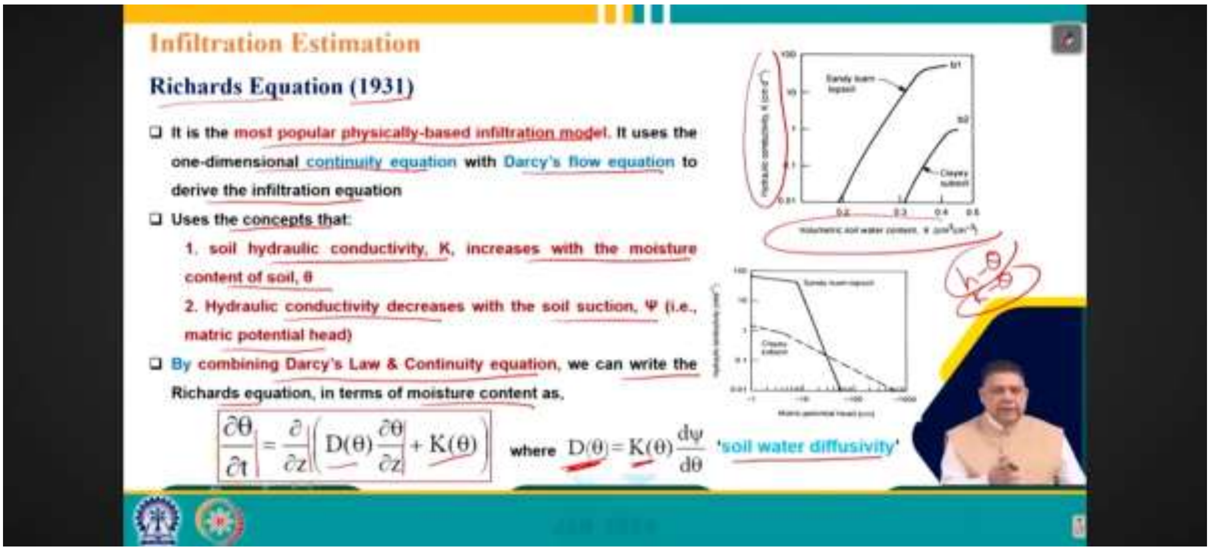
Time from start of rainfall (t, min)	Observed infiltration rate (f, mm/h)	Estimated infiltration (f_{PTT}, mm/h)
3	6.73	5.76
5	3.61	4.72
10	3.43	3.68
15	3.11	3.22
20	2.78	2.94
25	2.61	2.75
30	2.56	2.62
35	2.54	2.51
40	2.51	2.42
45	2.47	2.35
50	2.41	2.29
55	2.36	2.23
60	2.33	2.19
65	2.32	2.15

The figure illustrates the plot of observed and estimated infiltration rates against time using the PTT model. As observed, the blue line represents the observed infiltration rate, while the orange line represents the estimated infiltration. Recall the previous equations, such as the Horton model, where a near-perfect match between the observed infiltration rate and the model's estimation was evident after 10 minutes. Similarly, in the case of the Green-Ampt model, a good match was observed during the initial phases. However, in this instance, as illustrated, the PTT model tends to either underestimate or overestimate infiltration rates even beyond the 10-minute mark. Here, it overestimates in this section and underestimates in that part. So, while it fits the data reasonably well, it doesn't perform as effectively as the previous two models—the Horton model or the Green-Ampt model. This indicates that although it fits the observed data adequately, it doesn't match the performance of the Horton or Green-Ampt models.



Now, let's move on to the final model, Richards equation, which was introduced in 1931. Richards equation is the most widely used physically based infiltration model. It utilizes the one-dimensional continuity equation and Darcy's flow equation to derive the infiltration equation. This model is among the most prominent physically based infiltration models available. It operates on the principle that soil hydraulic conductivity k increases with the moisture content θ. When plotting k against θ for different soils, we observe a characteristic curve. Additionally, hydraulic conductivity decreases with soil suction ψ , which is the matric potential head. As previously mentioned, we can obtain h - θ data from soil moisture retention experiments and then use various models to fit the k - θ data.

So, we can also obtain the values of k versus h and k versus θ . By combining Darcy's law and the continuity equation, we can express the Richards equation in terms of moisture content. This equation is formulated in both moisture content and head, given as $\delta\theta/\delta t = (\delta/\delta z) *(D(\theta)*(\delta\theta/\delta z) + K(\theta)$. Here, $D(\theta)$ represents soil water diffusivity, which we previously discussed while exploring the PTT model. We noted that $D(\theta)$ can be obtained if we have the k- h or k - θ relationship, which can be derived using pressure plate experiments and fitting the k- θ model. By utilizing this method, we can assess infiltration both in the field and through various modeling approaches, whether empirical, semi-empirical, or physically based. With this, we conclude today's lecture .



Thank you all for listening attentively. Please feel free to provide feedback or raise any questions you may have, which can be addressed in our discussion forum. Thank you once again.