Course Name: Watershed Hydrology

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Week: 02

Lecture 10 : Infiltration II

Hello friends, welcome back to this online certification course on watershed hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology Kharagpur. We are currently in Module 2, specifically Lecture 5, where we will delve further into infiltration, focusing on Part 2 of this topic.



In this lecture, we will build upon what we discussed previously regarding mathematical models for estimating infiltration. We will cover the Horton model, the Green and Ampt model, the Philip two-term model, and the Richards equation. Let's proceed with our exploration of these models.



To provide a recap, in the previous lecture, we discussed that infiltration models are classified as empirical, semi-empirical, and physically based. We introduced two empirical models: the Kostiakov model and the modified Kostiakov model. Today, we will delve into semi-empirical models such as Horton's model, as well as three physically based models: the Green and Ampt model, Philip's model, and the Richards equation.



Beginning with the Horton model, it stands as one of the popular infiltration models developed by R. E. Horton in 1940 introduced the Horton model, which is based on the observation that infiltration begins at some rate f0 and exponentially decreases until it reaches a constant rate fc. The infiltration equation proposed by Horton is $ft = fc + (f0 - fc) * e^{(-kt)}$, where k is a decay constant. Essentially, this model is based on the concept we have already discussed: an infiltration curve that starts at a higher rate f0, then gradually reaches a constant value fc over time. It can also be expressed in terms of F, which represents cumulative infiltration. Remember, we are using F for infiltration rate and cumulative infiltration. Therefore, it's important to keep these two terms distinct. F can also be expressed as a function of time. To obtain the cumulative infiltration, we integrate Equation 1 with the initial condition f0 at (t = 0). This resulting equation will provide us with the cumulative infiltration. The Horton model is simple in form and fits well to the experimental data, which are the advantages of the Horton model.

Horton Model One of the popular infiltration models, developed by R.E. Horton in 1940 Based on the fact that infiltration begins at some rate J ₀ and exponentially decreases until it reaches a constant rate t, f(t) = t_+ + (t_e, t_0) e^{-3t} (1) Where k is a decay constant [T [*]] It can also be expressed in terms of F is a function of t	
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reaches a constant rate f_e $f(t) = f_a + (f_e, f_a) e^{-xt}$ (1) Where k is a decay constant [T [*]] It can also be expressed in terms of F as a function of t F = Current et three terms of F as a function of t	
$f(t) = f_{a} + (f_{a}, f_{a}) e^{-st} $ (1) Where k is a decay constant [T [*]] It can also be expressed in terms of F as a function of t It can also be expressed in terms of F as a function of t	
Where k is a decay constant [T [*]] F = Rate the three t	-1
It can also be expressed in terms of Fas a function of t	_1_
Upon integrating (1) with the condition F = 0 at t = 0	
$F = t_{c}t + \frac{1}{k}(t_{0} - t_{c})(1 - e^{-kt}) $ (2)	F
The Horton model is simple in form and fits well to the experimental data	
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However, the principal weakness of the model lies in determining reliable values for the parameters f0, fc, and k. If you recall, upon examining the equation, we require these three parameters f0, fc, and k to determine either infiltration rate or cumulative infiltration.

Hence, the main challenge is obtaining reliable values for these parameters. To estimate these parameters, we utilize Equation 1: $ft = fc + (f0 - fc) e^{(-kt)}$. If we take the logarithm of this equation, we will obtain this form: $\ln (f - fc) = \ln (f0 - ft) - kt$. When Equation 3 is plotted on semi-logarithmic paper, it results in a straight line, following the form y = mx + c. Here, y represents $\ln (f - fc)$, m represents the slope k, and c represents the intercept $\ln (f0 - fc)$. Hence, the slope is (-k) and the intercept can be determined. For a given infiltration data, fc is taken as the lowest value of f when it tends to become constant. And the value of (f - fc) at (t = 0) is (f0 - fc), which is the initial value to begin with.

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Infiltration Estim	ation		
Horton Model			
The principal weakness	of the model is in the determinatio	n of reliable values of its pa	rameters
fo, f, and k			
To estimate the parameter	ers, we may take the logarithm of Ec	$f_{1}(1) [f(t) = f_{1} + (f_{1} - f_{2}) e^{-kt}]$	mt+++
($\ln(t-t_{1}) = \ln(t_{0}-t_{1}) - kt$	(3)	8-5)
Equation (3), when plot	ted on semi-log paper, represents	a straight line whose slop	e -k and
intercept In (f _a -f _e) can rea	adily be determined		
For given infiltration data	a, \mathbf{f}_{e} is taken as the lowest value of	f when it tends to become o	onstant
The value of (f - f _c) at t =	0 is (f ₀ - f _e)		
00		<u></u>	Fit
(2) (2)			

Let's take an example to clarify this further. For an orchard field, the time since the start of the rainfall, accumulated infiltration, and observed infiltration rate are given in the table. We will develop the Horton model for the orchard and plot the observed and estimated infiltration rates. Here's the experimental data provided: the time since the start of the rainfall ranges from 3 minutes to 65 minutes, and we have accumulated infiltration values in millimetre , starting

from 0.34 to 3.08 millimetre. We have observed infiltration rates in millimetre per hour initially, it is very high at 6.73 and then decreases to 2.32, which is the lowest value.

For determining the relationship, In (f – f,	Horton model paramet	ers, we need to fit the s we need to plot in (f-f,) a	straight Igainst I	line ime	Lowes	t value of f is
10220386100708 <u>6070e</u> 0				-	taken a	as f _e
Time from start of rainfall- (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/ h)	0	1.1	in (f - f,)	er te
3	0.34	6.73	2.32	4.41	1.4839	
5	0.44	3,61	2.32	1.29	0.2546	
10	0.69	3.43	2.32	1.11	0.1044	
15	0.96	3.11	2.32	0.79	0.235	
20	1.21	2.78	2.32	0.46	-0.777	
25	1.43	2.61	2.32	0.29	-1.238	
30	1.65	2.56	2.32	0.24	-1.427	6
35	1.86	2.54	2.32	0.22	-1.514	
40	2.07	2.51	2.32	0.19	+1.661	
45	2.28	2.47	2.32	0.15	-1.897	
60	2.49	2.41	2.32	0.09	2.408	JUNE .
65	2.69	2.36	2.32	0.04	-3.210	
60	2.88	2.33	2.32	0.01	-4.605	
65	3.08	122	2.32	0	10000000	A COLORED A

Now, to determine the Horton model parameters, as we have discussed earlier, we need to fit the straight-line relationship, which means plotting $\ln(f - fc)$ against time. This implies that we need to calculate this relationship. The values provided include time from the start of rainfall or infiltration experimentation, accumulating infiltration, and observed infiltration rate. We take fc as the lowest value of infiltration rate in the experimental data. So, what we're doing here is taking fc equals to 2.32, as listed here. Throughout, fc remains 2.32. Now that we have f and fc , we can calculate the value of (f - fc), and then take the logarithm. This resulting value is what we need to plot against time.

For determining the relationship, in (f – f,	Horton model paramet c) = ln (f ₀ - f _c) - kt, i.e., w	ers, we need to fit the s we need to plot in (f-f _c) a	straight Igainst I	line	Lowes	t value of f is
			1	/	taken a	as f _c
Time from start of rainfall (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/ h)	0	1.1	in (f - f,)	en te
3	0.34	6.73	2.32	4.41	1.4839	
5	0.44	3.61	2.32	1.29	0.2546	
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40	2.07	2.51	2.32	0.19	+1.661	
45	2.28	2.47	2.32	0.15	-1.897	
60	2.49	2.41	2.32	0.09	2.408	
55	2.69	2.36	2.32	0.04	-3.219	
60	2.88	2.33	2.32	0.01	-4.605	
65	3.08	2322	2.32	0	1.00.000.002	and the second second

When we plot $\ln(f - fc)$ against time, this is the curve we obtain. As we have already discussed, the intercept is $\ln(f0 - fc)$ and the slope is -k, derived from this equation. This equation can be fitted using Excel, or one can fit it manually. From the plot, the intercept value we obtain is $\ln(f0 - fc)$, which is 1.0741, as shown here. So, from here, (f0 - fc) is 2.93, or f0 value is 5.25 millimetre per hour, because we have taken fc value as 2.32. From the plot, we observe that - k is approximately -0.0791, meaning k equals 0.0791. So now, we have determined f0, fc, and k. Therefore, the Horton model for the orchard can be expressed as f = 2.32 + 2.93 exp(-

0.0791t). Here, k is 0.0791, t is in minutes, and f is in millimetre per hour. This is the Horton model we have derived using the given data for the orchard.

Solution:		² •	N = 0 0791 + 1 0741
Then, In (f – f _e) is plotte	d against 't' 1	$ntercept = in (f_0 - f_i)$	y-constant of
From the plot, intercep	$\ln (f_c - f_c) = 1.0741$	10 47 - 52 64 - 52	10 100 20 40 50 60 72
□ Hence, (f ₀ - f _c) = 2.93, o	r.		Slope = - k
f _s = 5.25 mm/h (as f	= 2.32 mm/h)		* Time from start of rainfall (t, min)
Also, from the plot, slo	oe - k = - 0.0791 or k = 0.0	791	
Hence, Horton model fo	the grehard is f = 2.32 + 2.93 exp Where, t is in min as	(- 0.0791 t) nd T is in mm/h	
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Now, we are also required to plot these values. So, this is the observed infiltration rate, and this is the estimated infiltration rate calculated using the Horton equation we just developed in the previous slide.



These values are plotted against the measured values, showing the observed and estimated infiltration rates against time. The observed values are represented in blue, while the estimated values are shown in orange. As you can observe, from time t onwards, the data fits very well, but it doesn't perform as well in the beginning. From this, we can infer that the developed Horton model fits the observed data well except for the initial period. As you can see, it doesn't match the observed data well during the initial period.

Solution:			
Figure shows the plot of the observed and the plot of the plot of the observed and the plot of the plot of the observed and the plot of t	erved and estimated Infiltration	rates against time	
(unual) are unotertained information rate if, mm	10 40 50 50 20 art of rainfall (t, min)	As evident, the developed Horton model fits the	
abast.		observed data well, except for the initial period	
0 Q			No. 1

Now, let's move on to the next model, the Green-Ampt model, which is a simple model developed by W. H. Green and G. A. Ampt introduced the Green-Ampt model in 1911. Despite its age, it remains a popular model even today. This model is based on Darcy's law for infiltration into uniform soil with uniform initial moisture content and a negligible depth of water pooling on the surface. Several assumptions are made here: the soil is uniform throughout the considered soil column, with a consistent initial moisture content. Additionally, there is minimal ponding depth on the surface. It's assumed that water infiltrates into the soil, resulting in a distinct wetting front that separates the wetted and un-wetted zones. This process aligns with what we observed during the infiltration process discussion, where the soil becomes wetted due to the saturation transmission and the formation of wetting zones. We also noted the presence of a lower boundary wetting front. So, if the wetting front has reached a depth of L, it means that above the wetting front, there is wetted soil, and below it, there is un-wetted or dry soil with the initial moisture content. This implies that the ponded depth h0, is negligible.

Infiltration Estimation		*
Green-Ampt (GA) Model	Negligible ponded depth	
A simple model, developed by W.H. Green and G.A. A	Ampt (1911) Wetled soil	
It is based on Darcy's law for infiltration into uniform initial moisture content due to a pool of negligible de	m soil with uniform epth of water Wetting front	
The water is assumed to infiltrate into the soil	Unwetted	
The infiltrated water defines a sharply wetting fro wetted and unwetted zones	ont separating the	
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The image also illustrates this scenario, where water has pooled to a small depth on the soil surface h0, and the wetting front has penetrated to a depth L since infiltration began. This delineates the wetted zone above the wetting front and the un-wetted soil below it. So, essentially, the pressure gradient causing the flow or downward movement of the wetting front consists of three components. Firstly h0, which represents the depth of ponding water.

Secondly L, indicating the depth to which the wetting front has penetrated. And finally ψ , which denotes the matrix action or soil suction head, pulling water into the dry soil. Hence, there is a gravitational force due to the head, and additionally, there is a pulling force due to the matrix action. The wetting front moves down into the dry soil due to this matrix action, and the soil texture influences the shape of the wetting front. This concept was elucidated by Chow in 1988. If we examine the left side of the control volume, as mentioned earlier, it consists of uniform soil with uniform initial moisture. If we assume that the initial moisture content was bi throughout the entire depth, then obviously, when the wetting front moves down, there will be a change in the moisture content. Because a constant head is maintained, this change will be negligible. Therefore, in a uniform soil, the moisture content will be uniformly distributed. Let's denote the change in moisture content for the entire profile as $\Delta \theta$. Here, the soil's porosity is represented by θ , and the effective porosity is denoted as θe . Additionally, there is a residual moisture content in the soil. Thus, we have the residual moisture content, the initial moisture content, the change in moisture content, the effective porosity, and the porosity of the soil. So, because of the advancing front, the moisture content of the soil is changing, increasing by the magnitude of $\Delta \theta$ throughout the entire control volume.



The Green-Ampt (GA) model stems from the application of Darcy's law to the wetted zone. Darcy's law states that $f = K^*((h0 + L + \psi) / L)$, where K represents the hydraulic conductivity and I denotes the hydraulic gradient. Similarly, in the GA model, F represents the infiltration rate, K stands for the hydraulic conductivity of the wetted zone, and I signifies the hydraulic gradient, which is the head causing the flow and the flow length. As we previously discussed, there are three components here: h0, L, and ψ . The wetting front has advanced to a depth of L, where L represents the distance from the ground surface to the wetting front, and ψ represents the capillary suction or soil suction at the wetting front, as illustrated in the previous picture. Now, as we mentioned earlier, the ponding depth of water is negligible. So, by neglecting h0, this equation can be written in the form $f = K * ((L + \psi) / L)$. If $\Delta\theta$ represents the change in moisture content as discussed previously, and we also mentioned that the porosity of the soil is θ , which indicates the upper limit of the soil's moisture content. This implies that when the soil is completely saturated, its moisture content will be equal to its porosity. Thus, cumulative infiltration can be calculated as $F = \Delta\theta L$, where L represents the depth, and $\Delta\theta$ is the change in moisture content. Alternatively, we can express L as $L = F / \Delta\theta$. Substituting this

into equation number 2, we get $f = K *((L + \psi) / L)$. By replacing L with $\psi \Delta \theta$ and manipulating, we arrive at the form of the Green-Ampt model.

Green-Ampt (GA) Model
The GA model results from the application of Darcy's Law to the wetted zone as $f = K \frac{h_0 + L + \phi}{L} $ (1)
Where K = Hydraulic conductivity of the wetted zone;
L = Distance from the ground surface to the wetting front;
Ψ = Capillary suction (soil suction) at the wetting front
□ Neglecting h ₀ (negligible ponded depth of water), $f = K \frac{L + \frac{1}{2}}{L}$ (2) □ If $\Delta \Theta$ is the change in the moisture content (Note that the upper limit of $\Delta \Theta$ is porosity, η), then
Cumulative infiltration $F = \Delta \Theta L$ or $L = F/\Delta \Theta$
Substituting L in (2),
$f = K \frac{L+\psi}{L} = K \left[\frac{\psi \Delta \Theta + r}{r} \right] $ (3)

Equation 3 can also be written as f = K + (a K)/F, where a is $\psi \Delta \theta$.

The advantage of this form lies in its usability with available measured infiltration data. The values of 1/F are computed and plotted against time. When plotted, this equation takes the form of y = mx + c, where the intercept c represents K and m represents K as well since we are plotting 1/F. Therefore, the slope represents K. By fitting a straight line through the data points, we can determine the values of K and a from this plot.

Green-Ampt (GA) Mode	el	
From equation (3) we can writ	te	1 ak
	$\begin{array}{c} (1 = K + \frac{aK}{F} & (4) \\ \end{array}$ Where $a = 4i\Delta\Theta$	
If measured infiltration data a	te available, the values of 1/F are computed and plo	tted against f (figure)
By fitting a straight line through	gh the data points, the values of 'K' and 'a' are obtain	ined
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Let's illustrate this with an example. We will apply the Green-Ampt infiltration model to the data provided in Example 1 and plot both the observed and estimated infiltration rates. The data includes time, accumulated infiltration, and observed infiltration rates.

it the Green-Ampt inhitration r	nodel to the data in Example	1. Also, plot the observed and e	estimated
nfiltration rates.			
/	/ /	/	
Time since start of rainfall (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/h)	
3	0.34	6.73	
5	0.44	3.61	
10	0.69	3.43	
15	0.96	3.11	
20	1.25	2.78	
25	1.43	2.61	
30	1.65	2.55	
	1,86	2.54	
40	2.07	2.51	
45	2.28	2.47	200
-50	2.49	2.41	- (B)
55	2.69	2.36	and the second s
60	2.88	2.33	
66	3.00	2 32	All and a second second

In this case, we need to plot F against 1/F. Since F values are already given, we need to calculate the values of 1/F. Once we have 1/F, we plot it against F.

at the attended and	it model, we need to plot	ragainst I/r.		
Hence, estimating 1/F	in the table			
Time from start of rainfall (t, min)	Accumulated infiltration (F, mm)	Observed infiltration rate (f, mm/h) a	(1/F)	
3	0.34	6.73	2.94	
5	0.44	3.61	2.27	
10	0.69	3.43	1.45	
15	0.96	3.11	1.04	
20	1.21	2.78	0.83	
25	1.43	2.61	0.70	
30	1.65	2.56	0.61	1
35	1.86	2.54	0.54	
40	2.07	2.51	0.48	
45	2.28	2.47	0.44	
50	2.49	2.41	0.40	A DOMESTIC OF
55	2,69	2.36	0.37	A DAY
60	2.88	2.33	0.35	(11)
165	3.08	2.32	0.32	1000

From the plot, we can fit a straight line, as shown here, and determine the values of a and K. From the plot, we find that the intercept K is 1. The intercept K is 1.7674, and the slope aK is 1.3363. Therefore, from here, the K value is 1.7674 and the a value is 0.76. Consequently, the fitted Green-Ampt model for the orchard takes this form.

D Plo	otting f against	1/F			tion rate			1	1	645
D Fro	om the plot,				and the second		•	S	ope = aK	
	Intercept, H Slope, al	(= 1.7674 (= 1.3363			80 0.0	Intercept	1.0	13	2.0 2.5	3.0
	Hence,	a = 0.76							1	
C Th	us, the Green-#	Ampt model	for the orchai	d is	\sum				1	
				-						

Additionally, we can utilize the fitted model to obtain estimated infiltration values and then plot both the observed and estimated infiltration rates, as shown here.

		6.2003.5	
The table for the observed and the	estimated infiltration is given t	selow	
The estimated infiltration is calculated	ated from the developed Green-	Ampt equation for different t	
	-		
	X		
Time from start of rainfall	Observed infiltration rate	Estimated infiltration	
(t, min)	(f, mm/b)	(f _{Ga} , mm/h)	-
	6.73	5.70	
5	3,61	4,80	
10	3.43	3.70	
15	3.11	3.16	-
20	2.78	2.87	
25	2.61	2.70	
30	2.56	2.58	
35	2.54	2.49	
40	2.51	2.41	
45	2.47	2.35	
50	2.41	2.30	100
55	2.36	2.26	and the second s
60	2.33	2.23	
10 M	3.99	2.20	and the second sec

In this plot, the observed infiltration rate is represented in blue, while the estimated infiltration rate using the Green-Ampt model is shown in orange. As you can observe, even in this aspect, the model performs better than the Horton model, as it is more physically based, utilizing more physical concepts. As evident, the developed Green-Ampt (GA) model fits the observed data better than the Horton model, even for the initial period. This demonstrates that the Horton model, although physically based, is slightly inferior to the semi-empirical Horton's model we discussed earlier.



Now, let's move on to the next model in the physically based category, which is the Philip twoterm model. Philip introduced this model in 1957 for uniform soil with uniform soil moisture content and an excess water supply rate at the surface. He found a solution to the flow equation in the form of an infinite series. If we examine the assumptions, we see that the soil is uniform and the moisture content remains uniform. However, unlike in the case of the Green-Ampt model where we assume a negligible ponding depth, here there is an excess water supply at the surface. Under these conditions, water is not a limiting factor, and Philip derived an infinite series to describe the situation. Due to the rapid convergence of the series, Philip considered the first two terms to be sufficient. These two terms constitute the Philip two-term model, hence its name. For this model, the cumulative infiltration F is represented by the equation S t^0.5 + At , while the infiltration rate f is given by $(1/2)*S t^{(-0.5)} + A$. The first equation provides the cumulative infiltration, while the second one describes the infiltration rate. Here, the parameter S represents sorptivity , which is a function of the initial and surface water contents of the soil, as well as the soil water diffusivity. On the other hand, A is a parameter that depends on soil properties. Both A and S are soil properties.

Infiltration Estimation		
		100
Philip Two-Term Model		
D For uniform soil with a uniform soil-mois	ture content and excess water supply ra	te at the
surface, Philip (1957) found a solution to the	flow equation in the form of an infinite series	es.
Because of rapid convergence, the first two	o terms of the series are considered suffi	cient and
constitute the Philip two-term (PTT) model,		
$F = St^{\alpha \beta} + At$	(1)	
$f = \frac{1}{2}St^{n+1} + A$	(2)	
where S = sorptivity, a function of initi	ial and surface water contents of the soil	
A = parameter depending upon soil pr	operties	
-book in		
	0.0	1 1 1
@ @	Second Second	

Now, turning our attention to sorptivity, Philip described it as a measure of the capacity of the medium to absorb or desorb liquid through capillarity alone, with only capillary forces at play. Sorptivity, therefore, represents the amount of liquid, typically water, that a soil medium can

absorb or desorb. Philip proposed a method for determining sorptivity from horizontal infiltration, where water flow is primarily controlled by capillary absorption. He proposed the following relationship for sorptivity: $S = F/\sqrt{t}$. It's important to note that this determination is specifically for horizontal infiltration. This distinction is necessary because, as we discussed during the infiltration process description, vertical infiltration is predominantly influenced by gravitational forces. Therefore, in Philip's definition of sorptivity, capillary forces are the main factor at play. So, this is why we need measurements of horizontal infiltration. When the flow is primarily controlled by capillary absorption, the relationship $S = F/\sqrt{t}$ holds true. For parameter A , Philip in 1974 recommended a value of 0.363k, as it resulted in a realistic estimation of infiltration rate. However, he cautioned against the long-term use of the PTT model. This implies that if you intend to use a model for an extended period, the Philip two-term model might not be suitable. Nonetheless, it performs well for short periods.



Now, another term that arises is soil water diffusivity, which is defined by the equation $D(\theta) =$ $K(h)^*(dh/d\theta)$, where K(h) is the hydraulic conductivity, a function of pressure head, and sorptivity can be found if $D(\theta)$ and $h(\theta)$ are known. If you have studied soil science or conducted experiments using the pressure plate apparatus, you might recall that the moisture soil retention curve obtained from experimentation gives us the soil retention curve or $h(\theta)$ relationship, also known as $h(\psi)$ or $h(\theta)$. This relationship provides the relationship between pressure head and soil moisture content. Once we have the $h(\theta)$ relationship, we can utilize several models, with one very popular model being the van Genuchten model. Using the van Genuchten model, we can develop the k (θ) relationship. Therefore, the h(θ) relationship can be obtained from pressure plate experimentation, and based on that, we can fit the data and derive the K (θ) relationship. Consequently, when we have the h(θ) and K (θ) relationships, we can express k in terms of h. So, this is what is being utilized here. With this knowledge, we can determine $D(\theta)$ and $h(\theta)$, and if $D(\theta)$ and $h(\theta)$ are known, S can also be obtained for any given soil from experiments. The parameters S and A can be obtained from empirical fitting if infiltration observations are available, similar to previous equations like Horton and Green-Ampt. The observed values of f are plotted against the values of $t^{-0.5}$, as we previously discussed. This relationship is represented by the equation $f = (1/2) * S t^{-0.5} + A$. When we plot f against this, we can observe a linear relationship. The intercept of this line represents A , while the slope is S/2.

Philip Two-Term Model			
• Soil-Water Diffusivity $D(\theta) = K(h) \frac{dh}{dt}$	(<u>1</u>) (4)	LE X	010
where, K(h) is hydraulic conduc	ctivity as a function of pressure	head Quessive P	play of us
Sorptivity, S, can be found if D(0) ar	nd h(8) are known	Sac	
Parameters S and A can be obtained available	d from empirical fitting if infiltra	tion observations are	
The observed values of <u>f</u> are plotted straight line is fitted through the data	d against the values of t ⁰³ on a ita points	rithmetic paper and a	a
According to Equation (2) $\int = \frac{1}{2} \frac{1}{3} \frac{1}{3$	$\frac{9}{2+4}$ the intercept of this line	is A, and the slope is S/2	7
00			PLE

Now, let's consider Example 3, where we fit the PTT infiltration model to the data provided in Example 1 and plot both the observed and estimated infiltration rates. We've already reviewed this data earlier, which includes the time since the start of rainfall (denoted as t in minutes), ranging from 3 to 65, along with the corresponding infiltration rate values in millimetre per hour. Additionally, cumulative infiltration values were provided, but they are not required for the PTT model, so they are not included in this analysis.

Fit the PTT infiltration model to	the data in Example 1. Also, plot the observed and estin	mated infiltration
rates		
Time since start of rainfall (t, min)	Observed infiltration rate (t, mm/h)	
3	6.73	
5	3.61	
10	3.43	
15	3.11	
20	2.78	
25	2,61	
30	2.56	
35	2.54	
40	2.51	
45	2.47	CONT.
	2.41	6)
55	2.36	
60	2.33	
66	2.12	

To fit the PTT model, we need to plot f against t^{-0.5} . Although f remains unchanged in this case, we need to calculate t^{-0.5} for each time value. Once we have these values, we can plot f against t^{-0.5} , as shown in the graph.

- Stations the DYY out an			
Time from start of rainfall (t, min)	Observed infiltration rate	tes	
3	6.73	0.58	
5	3.61	0.45	
10	3.43	0.32	
15	3.11	0.26	
20	2.78	0.22	
25	2.61	0.20	
30	2.56	0.58	
35	2.54	0.17	
40	2.51	0.16	
45	2.47	0.15	
50	2,41	0.14	
55	2.36	0.13	
60	2.33	0.13	
65	2.32	0.12	

Upon plotting, we obtain a straight line relationship, represented by the equation y = 7.97x + 1.16. The equation obtained through Excel fitting is f = 7.97x + 1.16. As mentioned, Excel was used to generate this equation, but one can also utilize the least square method or other software, or even manually calculate and fit the equation using a calculator. Recalling the equation form $f = (1/2)*St^{(-0.5)} + A$, we know that the slope of the line is S/2 and the intercept is A, with their values already provided. The intercept value is 1.16, indicating that A = 1.16, and the slope S/2 is 7.97. Hence, the value of S comes out to be 15.94. With these values for A and S, we can plug them into the equation, resulting in $f = 7.97t^{(-0.5)+1.16}$. So, the PTT model for the orchard data yields $f = 7.97t^{(-0.5)+1.16}$.

PTT M	fodel		2 ⁷⁵	Y - 7.87 + 1.18	
Solution	1:		н с. 16- 3	1-	
D Plotting) f against t ^{o.s}		11 4.5 50	1	
From th	ne plot.			Slope = 5	12
h	ntercept, A = 1.16		Lintercent # A	2	
5	Slope, 5/2 = 7.97		8 00 02	0.4	0.0
,	tence, S = 15.94			t _{e1}	
🗆 Thus, th	he PTT model for the orchard	l is			
	$f = \frac{1}{2}St^{-0.5} + A$			<u>/</u>	
	f = 7.97t-0.5 + 1.16	DPTZ			- C
	THE N				7
				1000	

Utilizing this model, we can obtain estimated infiltration values for various times and then plot the observed infiltration rate alongside the estimated infiltration rate, as depicted here.

I Model			
ition:			
		121800	
table for the observed and the a	stimated infiltration is given b	elow	
estimated infiltration is calculat	ed from the developed PTT ma	odel for different 1	
	0.02		
Time from start of minfall	Observed willingtion rate	Rollmand Inditionities	
time from start of raintal	Observed infittration rate	Esumated entitration	
i, ming	6.73	5 76	
5	3.61	472	
10	3.43	3.68	
15	3.11	3.22	
20	2.78	2.94	
25	2.61	2.75	
30	2.56	2.62	
35	2.54	2.51	and the second s
40	2.51	2.42	
45	2.47	2.35	
50	2.41	2,29	
55	2.36	2.23	and the second se
60	2.33	2.19	N B
	2.32	2.15	and the second se

The figure illustrates the plot of observed and estimated infiltration rates against time using the PTT model. As observed, the blue line represents the observed infiltration rate, while the orange line represents the estimated infiltration. Recall the previous equations, such as the Horton model, where a near-perfect match between the observed infiltration rate and the model's estimation was evident after 10 minutes. Similarly, in the case of the Green-Ampt model, a good match was observed during the initial phases. However, in this instance, as illustrated, the PTT model tends to either underestimate or overestimate infiltration rates even beyond the 10-minute mark. Here, it overestimates in this section and underestimates in that part. So, while it fits the data reasonably well, it doesn't perform as effectively as the previous two models—the Horton model or the Green-Ampt model. This indicates that although it fits the observed data adequately, it doesn't match the performance of the Horton or Green-Ampt models.



Now, let's move on to the final model, Richards equation, which was introduced in 1931. Richards equation is the most widely used physically based infiltration model. It utilizes the one-dimensional continuity equation and Darcy's flow equation to derive the infiltration equation. This model is among the most prominent physically based infiltration models available. It operates on the principle that soil hydraulic conductivity k increases with the

moisture content θ . When plotting k against θ for different soils, we observe a characteristic curve. Additionally, hydraulic conductivity decreases with soil suction ψ , which is the matric potential head. As previously mentioned, we can obtain h - θ data from soil moisture retention experiments and then use various models to fit the k - θ data.

So, we can also obtain the values of k versus h and k versus θ . By combining Darcy's law and the continuity equation, we can express the Richards equation in terms of moisture content. This equation is formulated in both moisture content and head, given as $\delta\theta/\delta t = (\delta/\delta z) * (D(\theta)*(\delta\theta/\delta z) + K(\theta))$. Here, $D(\theta)$ represents soil water diffusivity, which we previously discussed while exploring the PTT model. We noted that $D(\theta)$ can be obtained if we have the k-h or k - θ relationship, which can be derived using pressure plate experiments and fitting the k- θ model. By utilizing this method, we can assess infiltration both in the field and through various modeling approaches, whether empirical, semi-empirical, or physically based. With this, we conclude today's lecture .



Thank you all for listening attentively. Please feel free to provide feedback or raise any questions you may have, which can be addressed in our discussion forum. Thank you once again.

