

**Course Name: Watershed Hydrology**

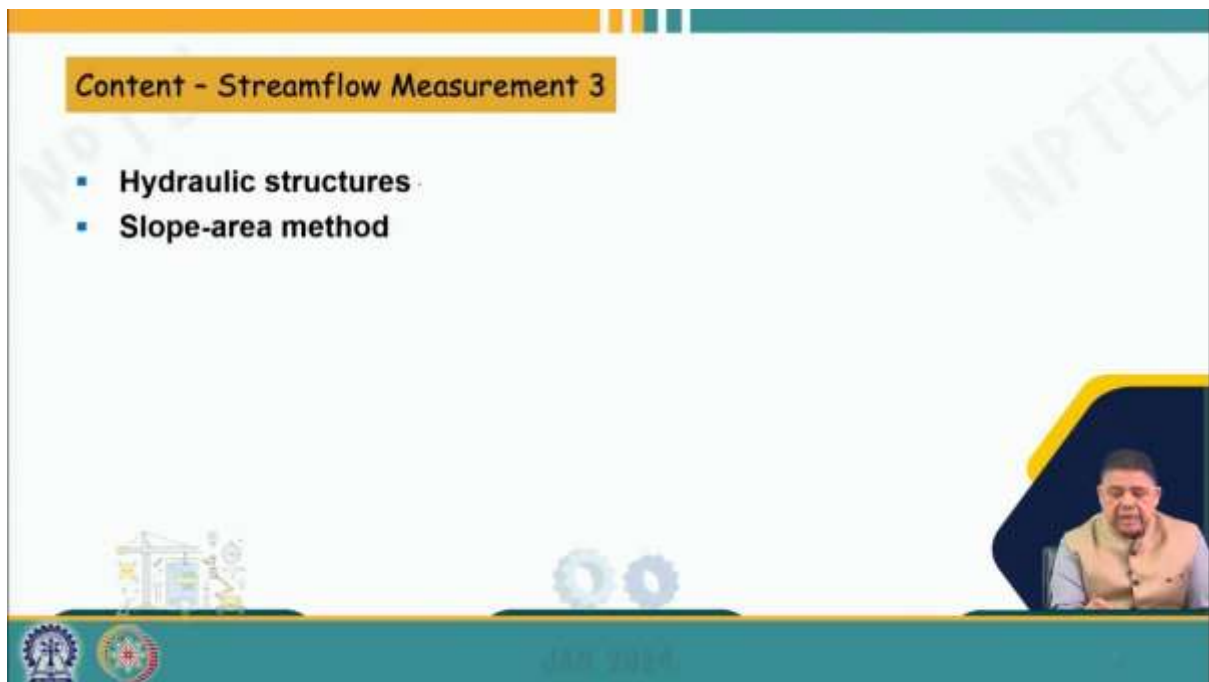
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**Week: 03**

**Lecture 14: Streamflow Measurement 3**



The image shows a slide from an NPTEL lecture. The slide has a white background with a yellow header bar at the top. The title 'Content - Streamflow Measurement 3' is written in black text on a yellow rectangular background. Below the title, there is a bulleted list with two items: 'Hydraulic structures' and 'Slope-area method'. In the bottom right corner, there is a small inset video of a man speaking. The NPTEL logo is visible in the bottom left corner, and the text 'NPTEL' is faintly visible in the background.

Hello friends, welcome back to this online certification course on watershed adaptation. So, today we will be discussing the hydrologic structures and how to determine the stream flow. Now, coming to hydraulic structures, as we know, hydraulic structures like weirs or flumes are commonly used for flow measurement in small streams or channels, which you will often find anywhere you go. These structures offer advantages as they are less influenced by downstream conditions, channel roughness, and backwater compared to the velocity area method. So, while the velocity area method has advantages, there are certain disadvantages that can be taken care of if we use hydraulic structures.

## HYDRAULIC STRUCTURES

- Hydraulic structures, such as weirs or flumes, are commonly used for flow measurement in small streams or channels
- These structures offer advantages as they are less influenced by downstream conditions, channel roughness, and backwater compared to the velocity-area method

### Types of Hydraulic Structures

Hydraulic structures may be divided into the following categories:

- Thin plate weirs
- Broad-crested weirs
- Flumes

Coming to the types of hydraulic structures, they may be divided into three categories: thin plate weirs, broadcasted weirs, and flumes. There are various types within these categories, and we will discuss each one of them one by one.

## HYDRAULIC STRUCTURES

### Rectangular thin plate weir

*Sharp-crested*

- It consists of a flat, rectangular plate placed vertically across the stream, causing water to flow over the top edge of the plate
- The height of the water over the weir, known as the **head or stage**, is then used to calculate the flow rate
- When  $b/B = 1$ , the weir is termed a **full-width rectangular thin plate weir** or a 'suppressed' weir

The equation for discharge for a full-width thin plate weir is

$$Q = \frac{2}{3} C_d \sqrt{2g} b h^{3/2}$$

Where  $Q$  is the discharge ( $m^3 s^{-1}$ );  $g$  is the acceleration due to gravity ( $9.81 m s^{-2}$ );  $C_d$  is the coefficient of discharge;  $b$  is the effective breadth (m); and  $h$  is the gauged head (m)

- When  $b < B$ , the weir is termed a **contracted weir**, and the coefficient of discharge is corrected by multiplying it with a **contraction coefficient**



Now, starting with the thin plate weirs, which are also referred to as sharp-crested weirs, sharp-crested or thin plate weirs. We will first talk about the rectangular thin plate weir or rectangular sharp-crested weir. This rectangular sharp-crested weir consists of a flat rectangular plate placed vertically across the stream, causing water to flow over the top edge of the plate.

So, as you can see here, this is the channel or stream in which we are measuring the flow direction. This is where this thin plate rectangular weir is installed at the end of the channel. Basically, the edge of the weir is sharp, with an angle of 45 degrees, so that the flow does not

touch the surface. This allows for free flow, and the equations used are developed for free flow conditions. That is why a sharp edge is provided here.

The height of water over the weir, known as head or stage, is used to calculate the flow rate. So, as you can see, this is the head, this is the weir crest, which we are calling the crest here. Then, the water level or stage above this is  $h$ , as shown here. Typically,  $h$  is measured around 3 to 4 times the distance far away from this particular stream. So, in this direction, we will maintain a distance of 3 to 4  $h$ , where  $h$  is measured, and when  $b$  by  $B$  is 1.  $B$  is the width of the rectangular weir, and capital  $B$  is the width of the channel. When  $b$  by  $B$  equals 1, the weir is termed as a full width rectangular thin plate weir, meaning that these edges are up to this point.

**HYDRAULIC STRUCTURES**

**Rectangular thin plate weir** *Sharp-crested*


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- When  $b < B$ , the weir is termed a **contracted weir**, and the coefficient of discharge is corrected by multiplying it with a **contraction coefficient**



The diagram shows a 3D perspective of a rectangular weir with water flowing over its crest. A vertical line indicates the head  $h$ . Below it, a 2D cross-section shows a contracted weir with sloped sides at a minimum angle of 45 degrees. An arrow labeled 'Direction of flow' points towards the weir.

The width of the rectangular channel and the width of this rectangular thin plate weir are the same, then it is referred to as a full width rectangular thin plate weir or it is also referred to as a suppressed weir.

If it is a full width thin plate weir, then the discharge can be calculated using this relationship where  $Q$  is the discharge in cubic meters per second,  $g$  is the acceleration due to gravity,  $C$  is the coefficient of discharge,  $b$  is the effective width. If it is  $b$  by  $B$ , then  $b$  is equal to  $B$ , and  $h$  is the gauged head.

So, we have to gauge the head here. If we discussed this, gauges gauge stage is or either head is gauged at a distance about 3 to 4 times of  $h$  itself. And when  $b$  is less than  $B$ , that means, when this is the condition which we are seeing where the width of the rectangular thin plate weir is less than the channel bed capital  $B$  channel bed, then it is referred to as a contracted weir, and the coefficient of discharge is corrected by multiplying with the contraction coefficient. So, besides the coefficient of discharge width, it takes care of the manufacturing issues and also the measurement conditions or experimental conditions besides that a contraction coefficient also has to be used.

## HYDRAULIC STRUCTURES

### Rectangular thin plate weir

#### Example

Calculate the discharge over a rectangular weir having a notch width of 4 m and an approach channel width of 8 m under a measured head of 1 m. The  $C_d$  value is 0.6, and the contraction coefficient is 0.9.

#### Solution

Known,  $g = 9.81 \text{ m/s}^2$ ;  $b = 4 \text{ m}$ ;  $h = 1 \text{ m}$ ,  $C_d = 0.6$ , and contraction coefficient = 0.9.

$$Q = \frac{2}{3} C_d \sqrt{2g} b h^{3/2}$$
$$= \frac{2}{3} \times (0.6 \times 0.9) \times \sqrt{2 \times 9.81} \times 4 \times 1^{3/2}$$
$$Q = 6.38 \text{ m}^3 \text{ s}^{-1}$$

channel width  
 $B = 8 \text{ m}$   
 $b < B$

Just taking a simple example, calculate the discharge over a rectangular weir having a notch width of 4 meters and the approach channel width of 8 meters under a measured head of 1 meter. The  $C_d$  value is 0.6, and the contraction coefficient is 0.9. So, these are the known values we already know:  $b$  is given,  $h$  is given,  $C_d$  is given, and the contraction coefficient is given. Of course, another thing which is given is the channel width, that is, channel width, which is 8 meters.

So, essentially, due to the fact that " $b$ " is less than " $B$ ," it's necessary to apply the contraction coefficient. That's why in the equation

$$\frac{2}{3} \cdot C_d \cdot \sqrt{2g} \cdot b \cdot h^{2/3}$$

as you pointed out, we incorporate the contraction coefficient alongside " $C_d$ " to adjust for contraction. Specifically, the  $\frac{2}{3}$  multiplied by " $C_d$ " represents the correction for the contracted weir. Then, by substituting the known values of  $g$ ,  $b$  and  $h$  into the equation, we arrive at a discharge of 6.38 cubic meters per second. So, the advantage is that just a single measurement of head gives us the discharge, in this case provided we know the coefficient of discharge. Then, other forms of weirs are, ah, triangular, or which is also referred to as venous thin plate weir and it consists of a flat triangular shape plate. So, earlier, we use a rectangular shape plate, now it is a triangular shape plate vertically across the stream. So, obviously, as you can see here, the rest of the things remain the same. This is the approach channel, the flow is coming in this direction, and where we had this rectangular shape thin plate.

## HYDRAULIC STRUCTURES

### Triangular (V-notch) thin plate weir

- This type of weir consists of a flat, triangular-shaped plate vertically across the stream
- This V-notch weir has a 90° angle, and the width across the top is twice the vertical depth, making it the most commonly used type of Triangular weir.

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} h^{5/2}$$

Where, Q is the discharge ( $m^3 s^{-1}$ ); g is the acceleration due to gravity ( $9.81 m s^{-2}$ );  $C_d$  is the coefficient of discharge; h is the gauged head (m);  $\theta$  = angle of notch.

- This V-notch weir is preferred for low-discharge channels



Now we have a triangular or V-notch thin plate placed as you can see and sharp edges are maintained here in a similar fashion.

And a V-notch weir has a 90-degree angle. So, the angle, ah, which, ah, this theta, that is, ah, this V-notch, this is the angle which we use. This angle is used at theta 90 degrees, and the width across the top is twice the vertical depth, which will be quite obvious because if you have ah 45 degrees, that means, ah, this width is the same as depth on either side. So, this becomes ah twice ah the depth and makes it the most commonly used type of triangular weir. So, ah, triangular weir or triangular thin plate weir or V-notch having theta equal to 90 degrees is the most commonly used triangular weir. And the formula for calculating discharge by this ah triangular weir is given here The formula you've provided seems to be:

$$Q = \frac{8}{15} \cdot C_D \cdot \sqrt{2g} \cdot \tan\left(\frac{\theta}{2}\right) \cdot H^{5/2}$$

Where:

- Q is the discharge through the weir,
- $C_D$  is the discharge coefficient,
- g is the acceleration due to gravity,
- $\theta$  is the vertex angle of the triangular weir,
- H is the head of water over the weir.

So, C D G H already we know discharge measurement or the stage measurement or head measurement remains the same at the top of this v point here and at a distance of 3 to 4 times H we measure this using a point gauge or hook gauge we can use. And theta is the angle of the notch which we have already discussed. So, the most popular one has theta equal to 90 degrees and it is 45 degrees  $\tan\left(\frac{\theta}{2}\right)$  so  $\tan 45$  degrees which is 1. And V-notch weir is preferred for low discharge channels. So, low discharge channel in the sense because you know that

compared to a triangular rectangular thin plate weir here because the cross-sectional area is less.

So, that is why for the same discharge the value of H will be significantly higher in the case of a triangular weir compared to a rectangular weir. That means, it is much easier to measure H if it is a low discharge channel it is much easier to find out the value of H or measure H if it is a triangular weir compared to a rectangular weir because even with the change of discharge the H value may not change significantly in the case of a rectangular weir. So, that is why whether discharge is low in a channel it is a triangular weir which is preferred. Then in the third type is a trapezoidal weir the third shape which we use a trapezoidal weir and it has a trapezoidal shaped notch through which water flows. So, the rest of the things remain the same only the thin plate which was rectangular or triangular in this case it is trapezoidal.

**HYDRAULIC STRUCTURES**

**Trapezoidal weir**

- This type of weir has a trapezoidal-shaped notch through which water flows
- A trapezoidal notch is a combination of a rectangular notch and two triangular notches, as shown in the figure.
- Discharge over Trapezoidal notch = discharge over rectangular notch + discharge over triangular notch

$$Q = \frac{2}{3} C_d \sqrt{2g} b h^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} h^{5/2}$$

Where, Q is the discharge ( $m^3 s^{-1}$ ); g is the acceleration due to gravity ( $9.81 m s^{-2}$ );  $C_d$  is the coefficient of discharge; b is the breadth (m); h is the gauged head (m); and  $\theta$  = angel of notch

- Cippoletti weir is a special trapezoidal notch, having a side slope of 1:4 (H : V). The weir is free from the contraction effect, i.e., the contraction coefficient is 1.0

So, basically, in this case, you can, as you can see, it is also a combination of a rectangular notch and two triangular half triangular notches basically. So, basically, a sum of a rectangular notch. This portion gives us a rectangular notch, and then from here to here, and here to here, if you see, we have half triangles. So, combined together, we have a triangle. So, it is a combination of one rectangular notch and a triangular notch, and that is how the discharge over a trapezoidal notch is also calculated, like discharge over the rectangular notch plus discharge over the triangular notch, and we have already seen these formulas.

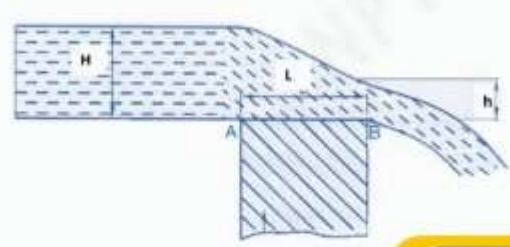
So, the sum of these two formulas gives us the value of the total discharge through a trapezoidal weir, and already these terminologies we have already seen. Now, as far as the trapezoidal weir is concerned, there is a typical type of trapezoidal weir which is known as a septal weir, which is a special trapezoidal notch having a side slope of 1:4, 1 horizontal to 4 verticals. So, the weir is free from contraction effect, that is, a contraction coefficient in this case is 1. So, that is the advantage of this, but as you can see, because we have to maintain a particular side slope 1:4, construction requires a little bit of care in this case, but a septal weir is also quite popularly used in the field conditions.

Then we come to flumes, rather we first go to broadcasted weirs. From thin plate weirs, we go to broadcasted weirs. A broadcasted weir is a hydraulic structure commonly used in open channel flow, and it is a type of weir characterized by a wide flat crest that spans the width of the channel. So, as you can see here, this is the crest; in this case, this is A B, this is the crest and a broadcasted weir is simply put in the channel and it spans the width of the channel. That means, the width is the same as the width of the channel, and they are preferred in flow measurement due to reduced sensitivity to head changes, lower approach velocity, and prevention of submergence. And what happens in this case, they are typically they have a typical design such that ah the flow is obstructed. That means, the ah, the flow here is obstructed in a certain way because it is put in the channel cell.

**HYDRAULIC STRUCTURES**

**Broad-crested weirs**

- A broad-crested weir is a hydraulic structure commonly used in open-channel flow
- It is a type of weir characterised by a wide, flat crest that spans the width of the channel
- Broad-crested weirs are preferred in flow measurement due to reduced sensitivity to head changes, lower approach velocity, and prevention of submergence
- Typically, critical flow occurs on the crest



$$Q = C_d L h \sqrt{2g(H - h)} \text{ or } 1.7 C_d L H^{3/2}$$

Where, Q is the discharge ( $m^3 s^{-1}$ );  $C_d$  is the coefficient of discharge;  $L$  = length of crest (m);  $H$  = height of the head water (m);  $h$  = height of water over crest (m)

So, as the water level rises, the velocity in this portion becomes subcritical. Then, when the crest is reached and beyond it, there is a free-flow condition. It is expected that the velocity will increase beyond the crest, becoming supercritical. This is why a critical velocity is anticipated over the crest. Thus, there will be a critical velocity. This critical depth is essentially measured at the crest. The formula that can be used for critical flow is:

$$Q = 1.7 \cdot C_D \cdot L \cdot h^{3/2}$$

Where  $Q$  is the discharge,  $C_D$  is the coefficient of discharge,  $L$  is the length of the crest and  $h$  is the height of the headwater. Small  $h$  represents the height of the water over the crest. These two measurements, the headwater height and the height of water over the crest are crucial. By obtaining these measurements, we can determine the discharge using the provided equation.

## HYDRAULIC STRUCTURES

### Flume

- ❑ A flume is a flow measurement device which is formed by a constriction in the channel. The constriction can be a narrowing in the channel or a hump, or both
- ❑ Flumes offer advantages over weirs (thin plate), including continuous flow measurement, capacity to transport sediment, no requirement of loss of head, and minimal risk of submergence issues.
- ❑ The popular types of flumes are:
  - Parshall
  - Cut-throat
  - H Flume



Moving on to the third type of hydraulic structures, which are flumes. Flumes serve as flow measurement devices and are created by a constriction in the channel. This constriction can be achieved through channel narrowing, a hump, or both. Therefore, if a channel is present, it can include a hump-like structure or narrowing in its width. This constriction can be achieved either by raising the channel bed with a hump or by narrowing the channel width.

And flumes offer advantages over weirs, thin plate weirs including continuous flow measurement, capacity to transport sediment, no requirement of loss of head, and minimal risk of submergence issues. So, basically, we saw that in the case of thin plate weirs, we had the requirement of free flow, which is not the case in ah in case of flumes, and there are three popular types of flumes which are used: partial flume, cutthroat flume, and edge flume.

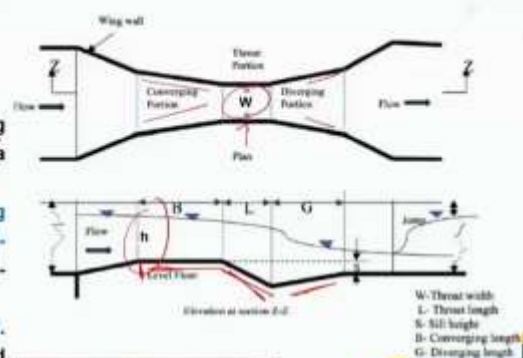
## HYDRAULIC STRUCTURES

### Parshall Flume

- ❑ The Parshall flume operates with a small drop in head, adopting to the venturi principle - Loss of head is only 25% of that for a weir
- ❑ The Parshall flume comprises three main parts: a converging inlet section with a level floor, a throat section with a downward-sloping floor and a diverging outlet section with an upward-sloping floor
- ❑ The size of the flume is determined by the width of its throat. Different sizes of flumes have been calibrated, and formulas and tables developed
- ❑ The general discharge equation is,

$$Q = Kh^2$$

Where K is a dimensional factor which is a function of the throat width W.  
The power n varies between 1.522 and 1.6





Now, let us talk about the partial flume. The partial flume operates with a small drop in head, adopting to the venturi principle. That means, ah we will see that ah there is a narrowing of the path and then that means there is a converging section and there is a diverging section, and because of that venturi principle works here, and the loss of head is only 25 percent that of a weir. It comprises of three main parts: a converging inlet section with a level flow, here you see level flow, and it is here you can see the convergence, and a throat section with a downward sloping flow, this is the throat section here, and as you can see that the slope is in the downward direction, and a diverging outlet section with an upward sloping floor.

So, it is a diverging section as you can see here, and the slope is in the upward direction. So, that means, you can see that there is a conversion, there is a diversion, there is a there is a level floor, there is a there is a downward slope, there is an upward slope. So, the construction, as you can imagine, that construction is a little bit typical, and this is how ah this ah partial flume looks like. And the size of the flume is determined by the width of its throat. So, the width of the throat is  $w$  here, and that basically decides, governs the size of the partial flume, and different sizes of flumes have been calibrated and formulas and tables developed.

So, because all other dimensions here,  $B$ ,  $L$ ,  $G$ , or whatever we use, all these dimensions can be expressed in terms of this  $w$  because that is the governing dimension. So, basically, these flumes can be standardized, and their design can be standardized, and obviously, thus the formula and tables once they are calibrated can be developed for these flumes. And the general discharge equation is given in this form:  $Q$  equals to  $k h u$  where  $k$  is a dimensionless factor which is a function of throat width  $w$ , and  $u$  varies between 1.5 to 2 and 1.6 and  $h$  is, of course, the head which is being measured here just in the beginning of the converging section. So, this is the typical drawing of a partial flume.

## HYDRAULIC STRUCTURES

### Cut-throat Flume

- ❑ The cut-throat flume is an attempt to improve the Parshall flume by simplifying the construction details
- ❑ The cut-throat flume has a flat bottom converging and diverging sections and zero length of throat section.
- ❑ The cut-throat flume can operate either as a free flow or a submerged flow structure
- ❑ The general discharge equation is,
 

$$Q = C_1 h_u^n L$$

Where,  $C_1$  is free flow coefficient,  $h_u$  is upstream depth of flow,  $n$  is the exponent, whose value depends on the flume length,  $L$ .
- ❑ The cut-throat flume is economical and easy to construct as compared to the Parshall flume

Now we come to the cutthroat flume. The name itself suggests the cutthroat flume is an attempt to improve the partial flume by simplifying the construction details because the constructions were a little bit complicated. So, that is why this has been taken care of, and this cutthroat flume has a flat bottom body, converging and diverging sections, and zero length of throat section.

So, throat section is eliminated here or cut, and that is why the name cutthroat flume with reference to the partial flume. And it is unlike the partial flume where we had flat diverging and converging sections, but sloping upward diverging section. Here it is flat throughout. The structure under consideration can function in either free flow or submerged flow mode. This versatility allows for flexibility in its operation. Let's denote the throat width as 'w', which signifies the transition point between the convergence and divergence sections. Additionally, 'l' represents the length of the weir. All other dimensions within this structure are expressed relative to 'w' or 'l'.

The general discharge equation takes the form:

$$Q = C_1 \cdot H_a^n \cdot a_H \cdot l$$

In this equation:

- $Q$  represents the discharge.
- $C_1$  is the free flow coefficient.
- $H_a$  denotes the upstream depth of flow, measured at a specific point within the structure.
- $n$  is the exponent, the value of which varies depending on the length of the flume.
- $a_H$  and  $l$  are coefficients representing the area and length of the flow, respectively.

By determining the values of 'w' and 'l', all other relevant dimensions can be derived. This enables the measurement of  $H_a$  in the upstream section, which can then be plugged into the discharge equation to calculate the discharge rate accurately.

### SLOPE-AREA METHOD

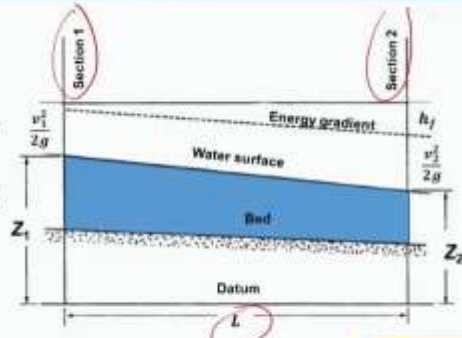
**Concept**

- The Slope-Area Method is a common technique used in hydrology to estimate the discharge or flow rate of a river or stream
- This method consists of estimation of cross sectional area and slope of the energy gradient in the reach
- Energy equation for a reach of non-uniform channel is given by

$$h_f = \left( Z_1 + \frac{v_1^2}{2g} \right) - \left( Z_2 + \frac{v_2^2}{2g} \right)$$

- The energy slope  $S$  can be calculated as

$$S = \frac{(Z_1 - Z_2) + \left( \frac{v_1^2}{2g} - \frac{v_2^2}{2g} \right)}{L}$$



Where  $h_f$  = Energy loss between sections 1 and 2;  
 $Z_1$  = Elevation of water surface at section 1 above a common datum (m);  
 $Z_2$  = Elevation of water surface at section 2 above a common datum (m);  
 $v_1$  = Mean velocity at section 1 (m/s);  
 $v_2$  = Mean velocity at section 2 (m/s);  
 $g$  = Acceleration due to gravity = 9.81 m/s<sup>2</sup>  
 $L$  = Length of channel reach (m)

## SLOPE-AREA METHOD

### Assumptions

- **Steady uniform flow:** The method assumes that the flow conditions (velocity, depth, and discharge) at any given point in the channel do not change over time, and these conditions are constant along the length of a reach
- **Applicability of Manning's equation:** The slope-area method typically employs Manning's equation to relate flow velocity to channel geometry. This assumes that the flow resistance is adequately described by Manning's roughness coefficient, which may not always be the case, especially in channels with complex geometries or vegetation
- **Negligible sediment transport:** The method assumes that sediment transport does not significantly influence the cross-sectional area of flow. In channels with high sediment transport rates, this assumption may not be valid
- **Applicability to open channels:** The slope-area method is primarily applicable to open channels where the water surface is exposed to the atmosphere. It may not be suitable for enclosed conduits or pipes

And it is economical and easy to construct 'H' compared to a partial flume, which is quite obvious because the design is very simple and of course, the cut short section is also taken care taken away. So, that is why it is little bit economical also. Then we approach the slope area method and the slope area method is a common technique used in hydrology to estimate the discharge of flow rate of a river or a stream and it consists of estimation of cross-sectional area and slope of the energy gradient in the reach and basically it is based on the energy principle. So, the energy equation for a reach of non-uniform channel is given by this relationship. So, basically here we select a channel reach that is upstream section that is section 1, upstream section and section 2 downstream section and at this the at a distance of 'l' and then of course, we have to have various measurements in order to be able to use the energy equation.

The energy equation typically used in fluid mechanics, particularly in the context of open channel flow or pipe flow, relates the energy at two different sections along the flow path. It's expressed as:

$$Z_1 + \frac{V_1^2}{2g} = Z_2 + \frac{V_2^2}{2g} + H_f$$

Here,  $Z_1$  is the elevation of the water surface at section 1 above a common datum,  $Z_2$  is the elevation of the water surface at section 2,  $V_1$  is the mean velocity at section 1,  $V_2$  is the mean velocity at section 2,  $g$  is the acceleration due to gravity and  $H_f$  represents the loss of energy or the energy loss between sections 1 and 2, often referred to as the head loss.

The head loss  $H_f$  can be calculated from this equation using known values of  $Z_1$ ,  $V_1$ ,  $Z_2$ ,  $V_2$  and  $g$ . Additionally, the energy slope ( $s$ ) which is the ratio of the head loss  $H_f$  to the length ( $l$ ) between the two sections, can be expressed as:

$$s = \frac{H_f}{l}$$

This expression allows for the calculation of the energy slope given the known values of  $H_f$  and  $l$ .

So, basically, as you can see here, this is nothing but the velocity has  $\frac{v_1^2}{2g}$  which is being considered here. So, basically it is an energy conservation principle which is used between these 2 sections and from there the energy slope can be obtained. There are certain assumptions behind the slope area method such that the flow is steady uniform that is the method assumes that the flow conditions that are velocity depth and discharge at any given point in the channel do not change over time and these conditions are constant along the length of the reach. So, that is an important assumption. Then the next assumption is related to the applicability of Manning's equation that is the slope area method typically employs Manning's equation to relate flow velocity to channel geometry.

So, this assumes that the flow resistance is adequately described by Manning's roughness coefficient, which may not always be the case, especially in channels with complex geometry or vegetation. So, of course, this may not be true, but this is the assumption that this method can be used only in channels where Manning's equation is valid. Then another assumption is negligible sediment transport; the method assumes that sediment transport does not significantly influence the cross-sectional area of flow. In channels with high sediment transport rates, this assumption may not be valid. So, that is one caution that one should not use this method where there is a lot of sediment transport, and of course, this is only applicable to open channels, primarily applicable to open channels where the water surface is exposed to the atmosphere, and this method may not be suitable for enclosed conduits or pipes where the flow is under pressure.

**SLOPE-AREA METHOD**

**Computation of discharge**

- If the channel is straight, comparatively narrow, and the slope of the water surface is essentially uniform, then the discharge may be calculated as

$$Q_{\text{Manning}} = \frac{1}{n} \bar{A} R^{2/3} S^{1/2}$$

Where,  $\bar{A}$  = average cross-sectional area of the channel;  $R$  = hydraulic radius of the channel =  $\frac{\bar{A}}{P}$ ;  
 $P$  = wetted perimeter of the channel;  $S$  = bed slope of the channel =  $\frac{H}{L}$ ;  $H$  = fall of the water surface over the length of the river reach  $L$ ; and  $n$  = the Manning's roughness coefficient

- Here  $\frac{1}{n} \bar{A} R^{2/3}$  is called the 'Conveyance' of the channel
- US Geological Survey has suggested a simple logarithmic model for the computation of discharge ( $Q$ ) in m<sup>3</sup>/s for a uniform channels section area ( $\bar{A}$  in m<sup>2</sup>):

$$\log_{10} Q = 0.191 + 1.33 \log_{10} \bar{A} + 0.05 \log_{10} S - 0.056 (\log_{10} S)^2$$

So, where the flow is under gravity, then only this method is applicable. Now, coming to the computation of the discharge, if the channel is straight, comparatively narrow, and the slope of the water surface is essentially uniform, then the discharge may be calculated using Manning's equation.

1. Manning's Equation:

$$Q = \frac{1}{n} A \cdot R^{2/3} \cdot S^{1/2}$$

Where:

- $Q$  = Discharge (flow rate)
- $A$  = Cross-sectional area of the channel
- $R$  = Hydraulic radius ( $A/PP/P$ , where  $PP$  is the wetted perimeter)
- $S$  = Bed slope ( $H/LH/L$ , where  $HH$  is the fall of the water surface over the length  $LL$  of the river reach)
- $n$  = Manning's roughness coefficient

2. Conveyance:

$$K = \frac{1}{n} A \cdot R^{2/3} \cdot S^{1/2}$$

Conveyance, as mentioned, is also represented as  $KK$  and is derived from Manning's Equation.

3. US Geological Survey Equation for Uniform Channel Section:

$$\log_{10}(Q) = 0.191 + 1.33 \log_{10}(A) + 0.05 \log_{10}(S) + 0.056 \log_{10}(S^2)$$

Where:

- $Q$  = Discharge (flow rate)
- $A$  = Cross-sectional area of the channel
- $S$  = Slope

**SLOPE-AREA METHOD**

**Computation of discharge**

- If the flow is non-uniform, then an average conveyance is estimated as,  
$$K = \sqrt{K_1 K_2}$$
  
Where  $K_1$  and  $K_2$  are conveyance at sections 1 and 2 of the channel reach
- The discharge is  
$$Q = \sqrt{K_1 K_2 S}$$
- When the flow is non-uniform, the discharge estimation includes a trial and error procedure for adjusting the velocity head – the detailed procedure is given through an example

*Handwritten notes:*  $K = \frac{1}{n} R^{2/3} S^{1/2}$

*Video inset:* A man in a light-colored vest speaking.

*Logos:* Several institutional logos are visible at the bottom of the slide.

#### 4. Estimation of Average Conveyance for Non-uniform Flow:

$$K_{avg} = \frac{K_1}{\sqrt{K_1} \times \sqrt{K_2}} \cdot \sqrt{K_1 \times K_2}$$

Where:

- $K_{avg}$  = Average conveyance
- $K_1$  and  $K_2$  are conveyance values for different sections.

So, conveyance 2 sections are calculated and discharge is given by this relationship. So,  $K_1$ ,  $K_2$  times S where S is S is the flow. When the flow is non-uniform, the discharge estimation includes a trial-and-error procedure for adjusting the velocity head and we will see that through an example. Let us take one simple example first that computes the stream discharge through a uniform straight river reach of 200-meter length by using Manning's and USGS equations. The upstream cross-sectional area is equal to the downstream cross-sectional area of the channel which is given as 120 square meters.

**SLOPE-AREA METHOD**

**Example 1**

Compute the stream discharge through a uniform, straight river reach of 200 m length by using Manning's and USGS equations. The upstream cross-sectional area is equal to the downstream cross-sectional area of the channel which is given as 120 m<sup>2</sup>. The fall of the water surface is 0.15 m. The wetted perimeter and the average depth of water flowing through this channel are 60 m and 2 m respectively. Manning's roughness coefficient value is 0.030.

**Solution**

- Average cross sectional area of the channel ( $\bar{A}$ ) = 120 m<sup>2</sup>
- Wetted perimeter of the channel ( $P$ ) = 60 m
- Thus, hydraulic radius of the channel,  $R = \frac{\bar{A}}{P} = \frac{120}{60} \text{ m} = 2 \text{ m}$
- Fall of the water surface ( $H$ ) = 0.15 m
- Length of the river reach ( $L$ ) = 200 m
- Thus, the bed Slope of the channel,

$$S = \frac{H}{L} = \frac{0.15}{200} = 0.00075$$

The slide also features a small video inset of a person in the bottom right corner and several logos at the bottom left.

So, it is a uniform section, the fall of water surface is 0.15 meters, the weighted perimeter, and the average depth of water flowing through this channel are 60 meters and 2 meters respectively, and Manning's roughness coefficient value is 0.03. So, it is a uniform cross-section, and that means Manning's equation can be straight away used. So, the average cross-sectional area is given as 120 square meters, the weighted perimeter is given as 60 meters.

So, the hydraulic radius  $R$  a bar by  $P$  comes out to be 2 meters. The fall of water surface  $H$  is given, length of river reach is known. So, bed slope  $S$   $H$  by  $L$  can be calculated as 0.00075 and Manning's roughness coefficient is also given as 0.03. Since the reach is straight and uniform and the cross-sectional area similar, the stream discharge can be computed by Manning's formula that is here using directly Manning's formula.

## SLOPE-AREA METHOD

### Solution

- Manning's roughness coefficient ( $n$ ) = 0.030
- Since the reach is straight and uniform and the cross-sectional areas similar, the stream discharge can be computed by Manning's formula as

$$Q_{\text{Manning}} = \frac{1}{n} \bar{A} R^{2/3} S^{1/2} = \frac{1}{0.030} \times 120 \times (2)^{2/3} \times (0.00075)^{1/2} = 174 \text{ m}^3/\text{s}$$

- By using USGS equation for the slope-area method, the stream discharge can be computed as

$$\log_{10} Q = 0.191 + 1.33 \log_{10} \bar{A} + 0.05 \log_{10} S - 0.056 (\log_{10} S)^2$$

$$\log_{10} Q = 0.191 + 1.33 \log_{10}(120) + 0.05 \log_{10}(0.00075) - 0.056 (\log_{10}(0.00075))^2$$

$$\log_{10} Q = 2.25$$

$$Q_{\text{USGS}} = 10^{2.25} = 177.83 \text{ m}^3/\text{s}$$

We can find out that discharge comes out to be 174 cubic meters per second. But because it is a uniform section, so we also have an option of using the USGS equation for the slope area method by using this logarithmic model. So, here we know A, we know S, both things we have calculated by putting this in the model. We get Q USGS as 177.83 cubic meters per second. As you can see here, by Manning's equation, we got 174; with this model, we got 177.83. So, pretty reasonably close values we get by these two methods.

## SLOPE-AREA METHOD

### Example 2

Compute the stream discharge through a straight river reach of 200 m in length. The upstream and the downstream cross-sectional areas of the channel are 1231 m<sup>2</sup> and 1222 m<sup>2</sup>. The fall of the water surface is 0.3 m. The wetted perimeters for the upstream and downstream of the channel are 320 m and 310 m, respectively. The Manning's roughness coefficient value is 0.030.

### Solution

- Since the channel cross-section is non-uniform, we need to use the iterative procedure for adjusting the velocity head
- Upstream cross-sectional area of the channel ( $A_1$ ) = 1231 m<sup>2</sup>
- Upstream wetted perimeter of the channel ( $P_1$ ) = 320 m
- Upstream hydraulic radius of the channel  $R_1 = \frac{A_1}{P_1} = \frac{1231}{320} \text{ m} = 3.85 \text{ m}$
- Downstream cross-sectional area of the channel ( $A_2$ ) = 1222 m<sup>2</sup>
- Downstream wetted perimeter of the channel ( $P_2$ ) = 310 m
- Downstream hydraulic radius of the channel  $R_2 = \frac{A_2}{P_2} = \frac{1222}{310} \text{ m} = 3.94 \text{ m}$

Then let us take an example, a little bit complicated one: compute the stream discharge through a straight river reach of 200 meters length. The upstream and downstream cross-sectional areas of the channel are 1231 square meters and 1222 square meters. The fall of water surface is 0.3

meters. The weighted perimeter for upstream and downstream of the channel are 320 meters and 310 meters respectively, and the Manning's roughness coefficient is 0.03.

**SLOPE-AREA METHOD**

**Solution:**

- Fall of the water surface ( $H$ ) = 0.3 m
- Length of the river reach ( $L$ ) = 200 m
- Bed Slope of the channel  $S = \frac{H}{L} = \frac{0.3}{200} = 0.0015$
- Manning's roughness coefficient ( $n$ ) = 0.030
- Channel conveyance for the upstream section  $K_1 = \frac{1}{n} A_1 R_1^{2/3} = \frac{1}{0.030} \times 1231 \times (3.85)^{2/3} = 100787$
- Channel conveyance for the downstream section  $K_2 = \frac{1}{n} A_2 R_2^{2/3} = \frac{1}{0.030} \times 1222 \times (3.94)^{2/3} = 101603$
- The discharge can be calculated as  

$$Q = \sqrt{K_1 K_2 S} = \sqrt{100787 \times 101603 \times 0.0015} = 3919 \text{ m}^3/\text{s}$$
- This is a first approximation of discharge since a velocity head adjustment has to be made

Now, because in this case, the channel cross-section is non-uniform, we need to use the iterative procedure for adjusting the velocity head. So, initially, we know  $A_1$ , the cross-sectional area upstream, upstream weighted perimeter, upstream hydraulic radius. So,  $A_1$ ,  $P_1$  is known,  $R_1$  can be calculated. Similarly, downstream cross-sectional area and weighted perimeter is given. So, we can calculate the downstream hydraulic radius also, and now the fall of water surface is all over the reach is also given, and the length of river reach is also known. So, we can calculate the bed slope of the channel, and Manning's roughness coefficient is given.

To calculate the discharge for both the upstream and downstream sections, we first determine the conveyance for each section, denoted as  $k_1$  and  $k_2$ , respectively. Utilizing the following equations:

$$k_1 = \frac{1}{n} \cdot \frac{A_1 R_1^2}{3}$$

$$k_2 = \frac{1}{n} \cdot \frac{A_2 R_2^2}{3}$$

where  $n$  represents Manning's roughness coefficient,  $A$  denotes cross-sectional area and  $R$  signifies hydraulic radius. Given values yield

$$k_1 = 10^7.87$$

$$k_2 = 1.1^{16.03}$$

Subsequently, we utilize the relationship  $Q = k_1 \cdot k_2 \cdot S$  to determine the discharge, where  $Q$  stands for discharge and  $S$  represents the slope. With the computed values, the initial discharge is 3919 cubic meters per second.



## SLOPE-AREA METHOD

### Solution:

#### □ Second approximation of $Q$

- Velocity head for upstream section can be calculated as

$$\frac{v_1^2}{2g} = \left(\frac{Q}{A_1}\right)^2 \times \frac{1}{2g} = \left(\frac{3919}{1231}\right)^2 \times \frac{1}{2 \times 9.81} = 0.516 \text{ m}$$

- Velocity head for downstream section can be calculated as

$$\frac{v_2^2}{2g} = \left(\frac{Q}{A_2}\right)^2 \times \frac{1}{2g} = \left(\frac{3919}{1222}\right)^2 \times \frac{1}{2 \times 9.81} = 0.524 \text{ m}$$

- The velocity head difference is  $\approx -0.008 \text{ m}$

- The adjusted fall can be calculated as

$$H_{Adjusted} = (0.3 - 0.008) \text{ m} = 0.292 \text{ m}$$

- New bed Slope of the channel can be calculated as,

$$S_{New} = \frac{H_{Adjusted}}{L} = \frac{0.292}{200} = 0.00146$$

For the second approximation, we incorporate velocity adjustments. We recalculate the velocity head using the newly determined discharge. The velocity heads for the upstream and downstream sections are 0.516 and 0.524 meters, respectively resulting in a difference of 0.008 meters.

Consequently, we compute the adjusted fall. Initially, the fall was 0.3 meters, but with the updated discharge, it is adjusted to 0.292 meters. This yields a new slope of 0.00146.

Finally, utilizing the updated discharge formula, the revised discharge is calculated to be 3869 cubic meters per second.

The third approximation, because we have to continue this trial-and-error procedure unless the two discharges are the same. So, we will now go with this iteration with the new discharge  $Q$  equals to 3869. The procedure remains the same; we will calculate the velocity head, we can calculate the velocity at difference, and the adjusted fall in this case will be 0.3 minus 0.007, which comes out to be 0.293. So, the new slope we have to calculate, and then we have to calculate the new discharge, which comes out to be 3870 cubic meters per second.

## SLOPE-AREA METHOD

### Solution:

- Fourth approximation of  $Q$ : The above procedure is repeated using the new value of  $Q = 3870 \text{ m}^3/\text{s}$

- Velocity head for upstream section can be calculated as

$$\frac{v_1^2}{2g} = \left(\frac{Q}{A_1}\right)^2 \times \frac{1}{2g} = \left(\frac{3870}{1231}\right)^2 \times \frac{1}{2 \times 9.81} = 0.504 \text{ m}$$

- Velocity head for downstream section can be calculated as

$$\frac{v_2^2}{2g} = \left(\frac{Q}{A_2}\right)^2 \times \frac{1}{2g} = \left(\frac{3870}{1222}\right)^2 \times \frac{1}{2 \times 9.81} = 0.511 \text{ m}$$

- The velocity head difference =  $-0.007 \text{ m}$ ,
- The adjusted fall can be calculated as  $H_{Adjusted} = (0.3 - 0.007) \text{ m} = 0.293 \text{ m}$
- New bed Slope of the channel can be calculated as,

$$S_{New} = \frac{H_{Adjusted}}{L} = \frac{0.293}{200} = 0.0014$$

- The updated discharge can be calculated as

$$Q = \sqrt{K_1 K_2 S_{New}} = \sqrt{100787 \times 101603 \times 0.0014} = 3870 \text{ m}^3/\text{s}$$

Since the  $Q$  here is the same as that in the third approximation, the procedure terminates and

$$Q = 3870 \text{ m}^3/\text{s}$$



So, with this, we move to the next approximation again with this  $Q$  equals 3870, and we calculate the velocity head at section 1, velocity at section 2, and the velocity difference, which comes out to be 0.007. We adjust to get the edge adjusted and  $S$  new, which again comes out to be 0.0014, and the discharge comes out to be 3870. So, now if you remember, the  $Q$  here is the same as that in the third approximation. So, the procedure will terminate here with the discharge measurement being 3870 cubic meters per second.

So, with this, we come to the end of stream flow measurement. We have seen all the direct and indirect methods, and ah, thank you very much. Please give your feedback and also ah raise your doubts or questions, which can be answered in the forum.

Thank you very much.