Course Name: Watershed Hydrology

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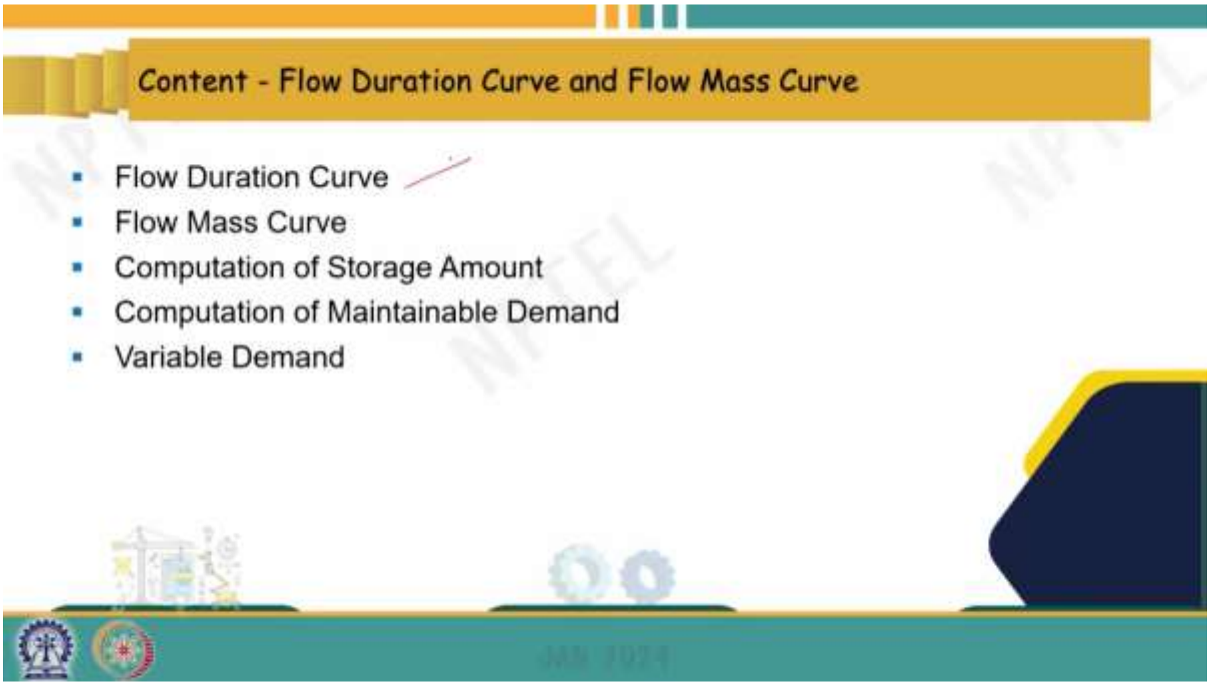
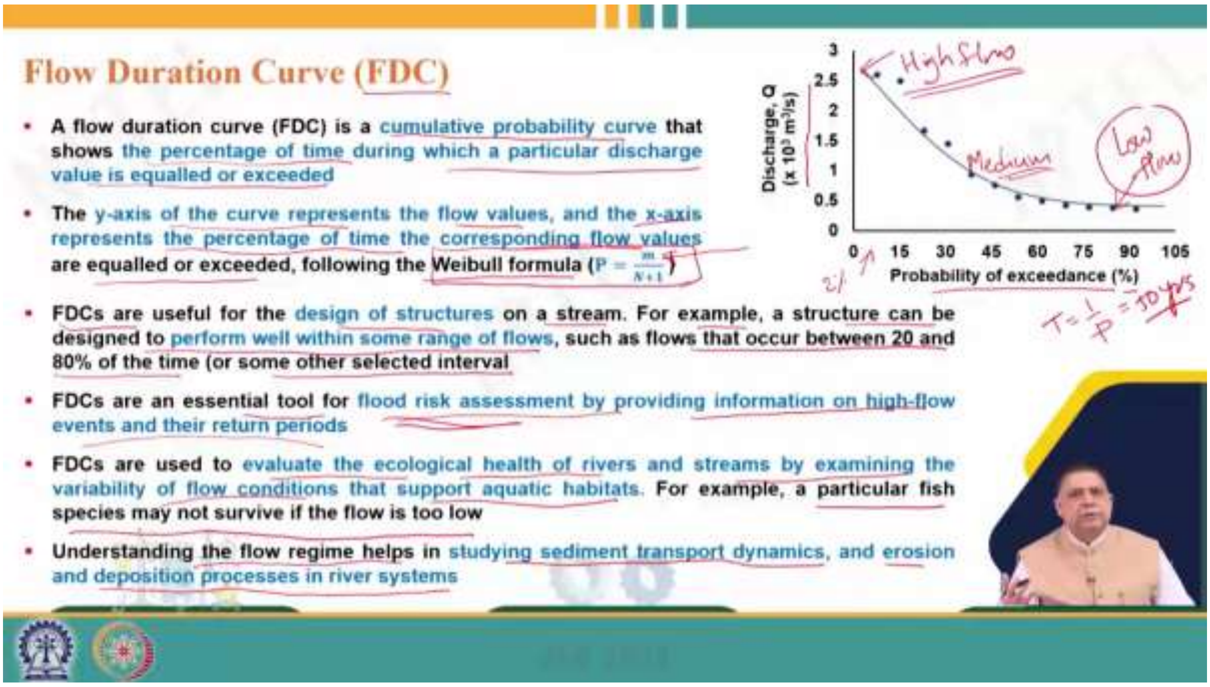
Week: 03

Lecture 15: Flow Duration Curve and Flow Mass Curve

Hello friends, welcome back to this online certification course on Watershed Hydrologic Technology. I am Rajendra Singh, a professor in the Department of Agricultural and Food Engineering at the Indian Institute of Technology Kharagpur. We are in module 3, and this is lecture number 5, and the topic is the flow duration curve and flow mass curve.



In this lecture, we will introduce the flow duration curve, then the flow mass curve, and then we will see how to compute storage amount using a flow mass curve and how to compute the maintainable demand as well as the variable demand. Now, starting with the flow duration curve which is popularly called FDC, it is a flow it is a cumulative probability curve that shows the percentage of time during which a particular discharge value is equal or exceeded. So, this is the curve which you can see on the right.



The y-axis of the curve represents the flow values and the x-axis represents the percentage of time the corresponding flow values are exceeded following the Weibull formula P=M/(N+1). So, as you can see here, this is the discharge plotted against the probability of exceedance N in percent. And if you remember we discussed in the rainfall frequency that the Weibull formula is one of the very popular formulas for frequency law carrying out frequency analysis. And what is done here is that whether discharge series we get we arrange that in the descending order of magnitude and a sign rank number M is a rank number.

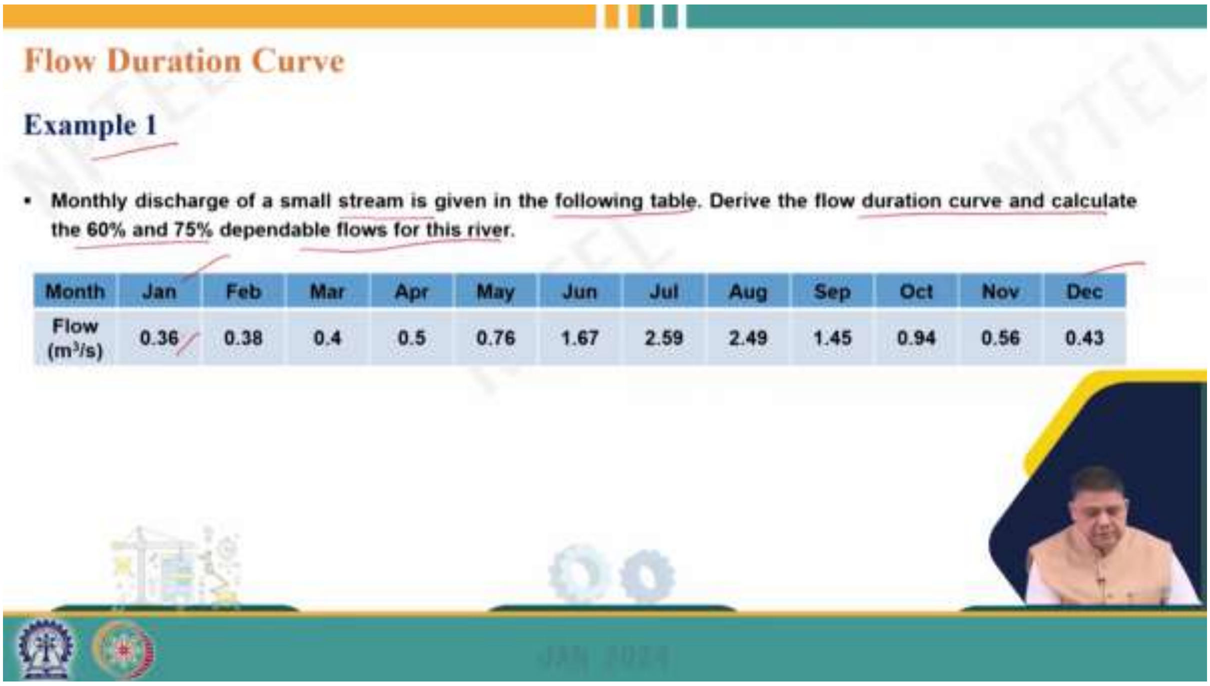
So, rank number 1 is given to the highest value M=1 and M=2 to the second highest value, and M is equal to N for the last value in the series. So, N is the number of data points in a series. So, that is how the Weibull formula we calculate the probability of exceedance, and then we plot this curve. They are FDCs that are useful for the design of structures on a stream. So, it is used using stream data this curve is plotted.

For example, a structure can be designed to perform well within some range of flows such as flow that occurs between 20 and 80 percent of the time or some other selected interval. So, here if you see the probability of exceedance, obviously, at the right-most corner when the probability of exceedance is 90 or 100 then this is the low flow area. Wherein on the other extreme when the probability is low 1 percent 2 percent then this is the high flow area and in between this is the medium flow. So, we have 3 distinct flow regimes here high flow medium flow, and low flow. So, if we know that a particular structure can perform within a certain range then we can find out what is the probability of exceedance of that range.

And this is, of course, used for drinking water supply and for designing the hydropower plants because the hydropower plants they work mostly in the medium range of flow. So, that is how we decide what should be the capacity and all based on this flow duration curve. They are an essential tool for flood risk assessment by providing information on high-flow events and their return period. So, we know that if it is saying 2 percent if we talk about say 2 percent then we know that t equals to 1 by P. So, the t will be 50 years.

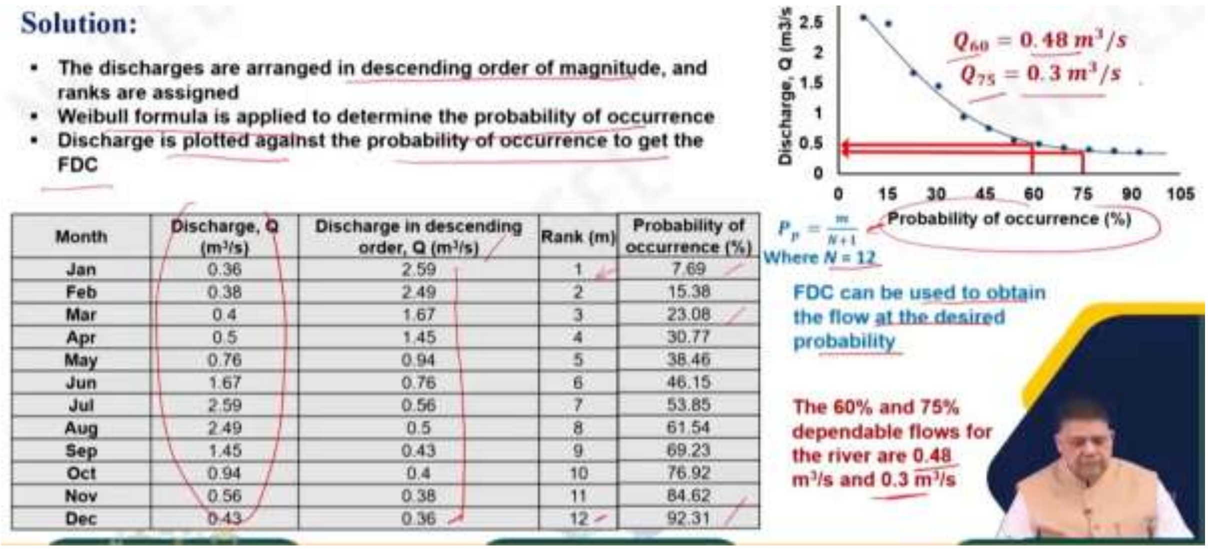
So, from here we can find out what is the 2 percent magnitude of flow, and of course, for floods, when we talk about floods, we are interested in the high flow. So, knowing that also, we can find out what is the possibility of getting a particular magnitude, and then what will be the risk, and then what will be its return period, and then accordingly, preparedness could be taken care of. Then, to evaluate, they are also used to evaluate the ecological health of rivers and streams by examining the variety of flow conditions that support aquatic habitat. So, that is talking more about the low flow conditions. For example, a particular fish species may not survive if the flow is too low.

So, we must find out what is the low flow magnitude so that we can provide a typical type of fish, or if a particular fish survives in a particular stream, then we should ensure that a certain minimum flow is always available there. So, that is how this flow duration curve also helps us in deciding the environmental flow that is required in a particular place. And understanding the flow regime helps in studying sediment transport dynamics, then erosion, and deposition process in the river stream systems. Of course, because sediment transport is directly connected to flow also because we saw that a gradation or degradation can mar the stream flow. So, obviously, for sediment transport dynamics also, the flow duration curve can be used.



Let us take an example here: the monthly discharge of a small stream is given in the following table; derive the flow duration curve and calculate the 60 percent and 75 percent given dependable flow for this river.

And here you see for different months of the year starting from January to December, the flow magnitudes are given, starting at 0.36 in January and 2.4 in December. Of course, the maximum will be somewhere in July or August. So, it is here you can see in July the highest flow is there.



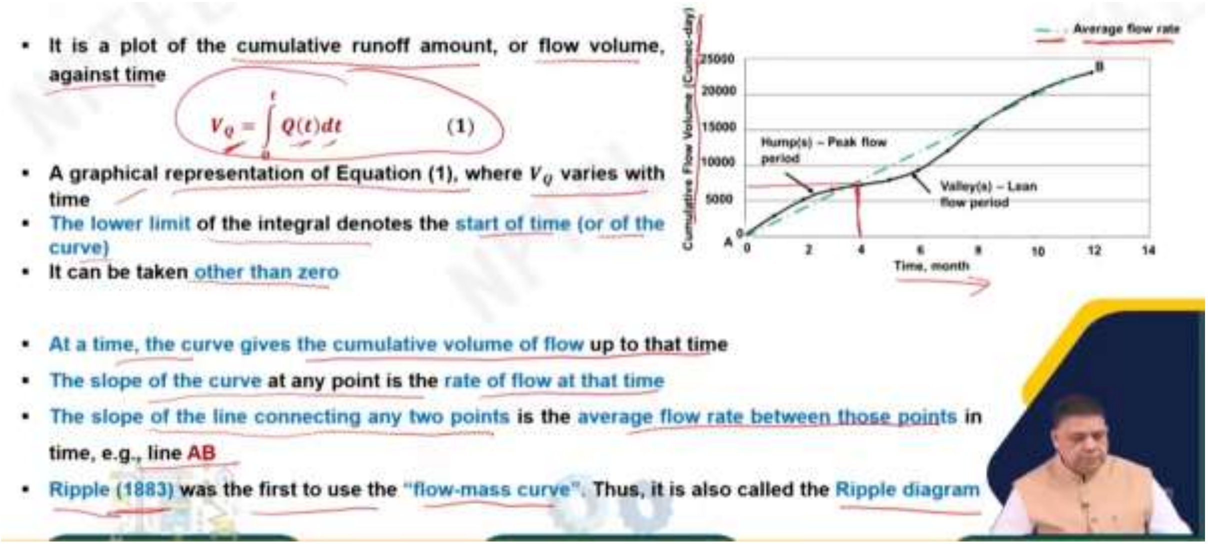
So, as we have already discussed, to draw the flow duration curve, we first need the probability of exceedance for different flow magnitudes.

So, we arrange the data in descending order of magnitude, as you can see here, it is the highest value at the top and the bottom-most value is the lowest, and then we assign rank, rank 1 to the highest value. So, 2.59, which was the highest in July, is assigned rank 1, and 0.36, which was the lowest, is assigned a value of rank 12 because there are 12 monthly data points. So, 12 data points are there, then the formula is applied to determine the probability of occurrence.

And so, we know the Prevails formula M/(N+1), N is 12, M already we have calculated. So, the probability of occurrence we have for different flow magnitudes we know. And of course, then using these two values, the discharge is plotted against the probability of occurrence to get the FDC. So, we have discharge values and we have the probability of occurrence. So, we will plot this curve.

So, either this discharge or this column we can use, and then plot the curve. And then obviously, we can find FDCs that can be used to obtain the flow at the desired probability, and in this case, we require the flow at 60 percent and 80 percent. So, 60 percent, and 80 percent, of the flow values can be read here, and the flows in this dataset are 0.

48 at 0.3, 0.48 at 6.60 percent probability, and 0.3 cubic meters per second at 75 percent probability. So, this is how FDC can be used for finding out fluid at different probabilities.



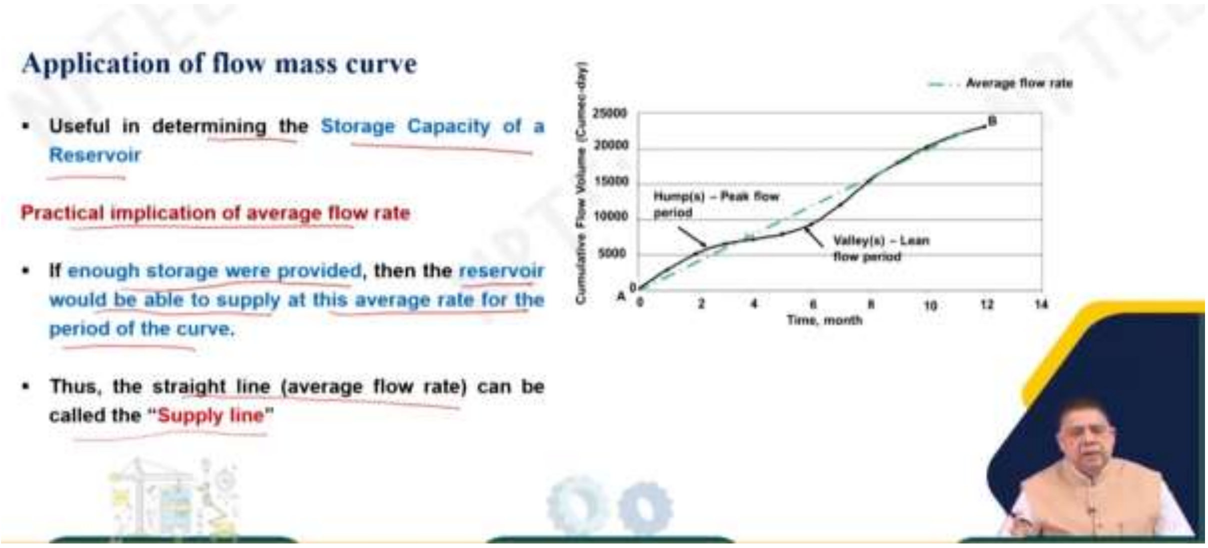
Now, let us come to the flow mass curve, which is a plot of cumulative runoff amount or flow volume against time.

So, basically, it is a graphical representation of equation 1. This is the equation, which is nothing but the integral equation of the Q (t)over time, where V_Q varies with time. So, Q(t), we know that Q is nothing but the stream flow, and we saw different ways of determining or measuring stream flow. So, if we know stream flow values and then integrate them, we will get the cumulative runoff amount or cumulative flow volume. The lower limit of this integral denotes the start time of the curve. So, this value can be other than 0 also.

So, that we can mean, say, suppose we are talking about monthly data. So, we might be interested in, say, April to October. What happens? So, we can start over 4 to 10 and then integrate the value to get the words, and this is how. So, cumulative flow volume in cubic days. We are using a typical, of course, the volume means we can put in cubic meters, but just to keep it simple, we use it cubic day. So, the numbers are reasonable. And of course, here the time could be monthly, weekly, or whatever could be there, and this black line is nothing but the mass curve.

At the time, the curve gives the cumulative volume of flow up to that time. So, suppose I want to find out what is the cumulative flow up to, say, month 4. So, then obviously, I can find out by reading this curve, and the slope of the curve at any point is the rate of flow at that time. So, I can find out the slope of the curve at any given point, and that will tell me the rate of flow at that point. The slope of the line connecting any two points is the average flow rate between those points, say, for example, line AB.

So, if we draw line A between these two points, the first point and the last point, and we draw a straight line joining, this is the green line here, then it is nothing but the average flow rate. So, that is the average flow rate we can expect over the entire period for which the flow mass curve has been drawn. Ripple in 1883 was the first to use the term flow mass curve; that is why this diagram is also called a ripple diagram. So, it is a flow mass curve, flow curve, or ripple diagram.



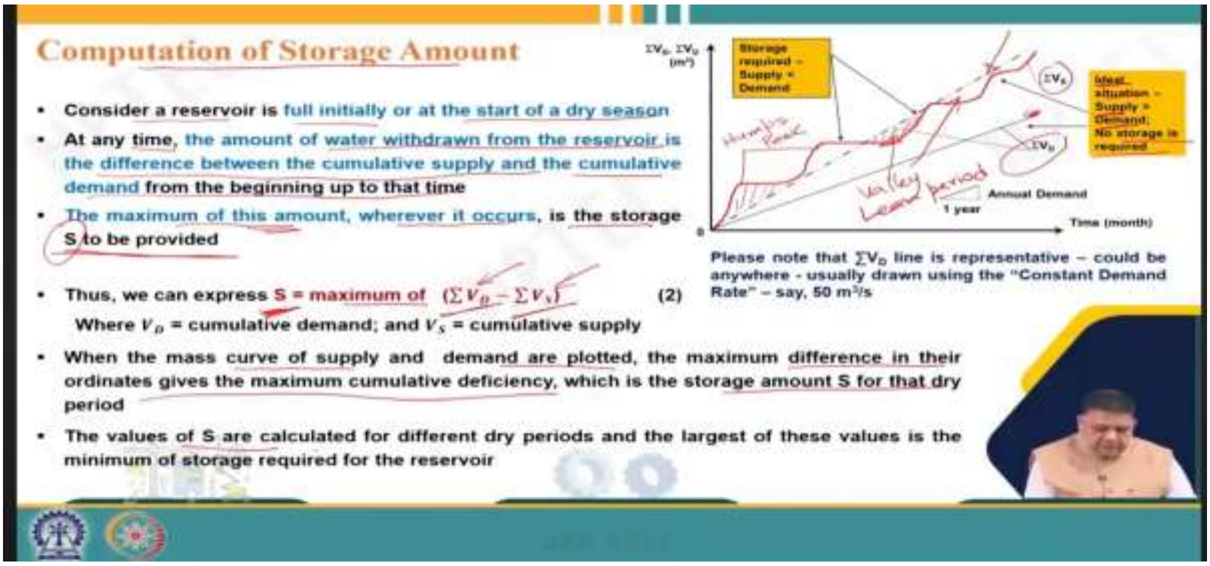
Now, coming to the application of flow mass curves, they have several practical applications; they are useful in determining the storage capacity of a reservoir.

So, the practical, if we look at the average flow rate, if enough storage were provided, then the reservoir would be able to supply at this average rate for the period of the curve. So, that simply means that we have drawn this flow duration curve, and then we have found this average flow rate, which simply gives an idea that during this period, this storage structure, which is the reservoir, would be able to supply us at this average rate; that is the slope of the line will tell us the rate, and that is the average rate. And the straight-line average flow rate can also be called the supply line. So, this is the average flow rate or average supply line, and how much we can supply throughout the curve from this reservoir. So, we can decide on the storage capacity of the curve, if we are interested in supplying a particular rate over a given period, then obviously, we can find out using the flow duration curve what should be the storage capacity to be able to supply that, and we will see that in the next slides.

Before that, just let us say that generally, reservoirs use what is called CSZ or the concept of storage zoning. So, the storage zone is divided into different zones, typically in reservoirs. At the bottom-most, you will find there is daily storage, that is a minimum pool level, and this daily storage is not useful water, it's not the useful storage, but basically, it is provided to tackle the sedimentation throughout the life of the structure. So, throughout life, we expect that sediment will come, and they will be deposited at the bottom of this reservoir. So, to take care of that, daily storage is provided.

Then, for the day-to-day operation of the reservoirs, we provide active storage or minimum useful storage, that is, we define a normal pool level, and of course, we have a minimum pool level. So, whatever stored water is there, is called active storage, and that is used in the day-to-day operation of reservoirs. So, that is the active storage. Above daily storage and active storage, we have flood control storage because reservoirs could be used as flood control structures. So, we do provide flood control structures, and of course, there is surcharge storage also, like if more than expected flood comes, then obviously, we will release that.

And of course, we have spillways provided to spill over the excess flow or the flood. And then we have sluice wedges at different elevations they can be provided depending upon the purpose for supplying the amount of water daily to meet the various kinds of demands. This is a typical dam or reservoir; this is the Kanakabati dam and reservoir, as you can see the dam and the reservoir water you can see it is in Mukutmanipur in the Bankura district of West Bengal.



So, coming back to the use of a flow duration curve in deciding the storage capacity of a reservoir. So, for that, we need to carry out storage amount computation.

So, let us consider a reservoir that is full initially or at the start of the dry season. Typically, reservoirs are designed and operated in such a way that when the dry season comes at the beginning of the dry season, they should be full. So, we have enough water to meet the demand during the dry season. So, at any point, any time, the amount of water withdrawn from the reservoir is the difference between the cumulative supply and cumulative demand from the beginning up to that time. And the maximum of this amount wherever it occurs is the storage capacity we are interested in.

Now, here we see that the red line is the supply line, representing some V_s or the storage. We will also have to have a demand volume, indicating how much demand is there that must be met from this reservoir. Let us hypothetically say that this is one of the demand lines, the bottom one. Now, if we compare the supply line to this demand line, then it is an ideal situation because the supply is consistently greater than the demand. That means we do not require any storage; basically, because we have the time supply greater than demand.

But suppose a picture where our demand line is like this dashed line here, where storage is required at certain periods of time, which could be positive because at times the demand is more than the supply. So, as you can see here, during this period, the supply is more than the demand. So, no problem, but if you look at this period here, the demand is more than the supply. So, you must have water available in your reserve to be able to meet that particular demand, and that is how we decide the storage.

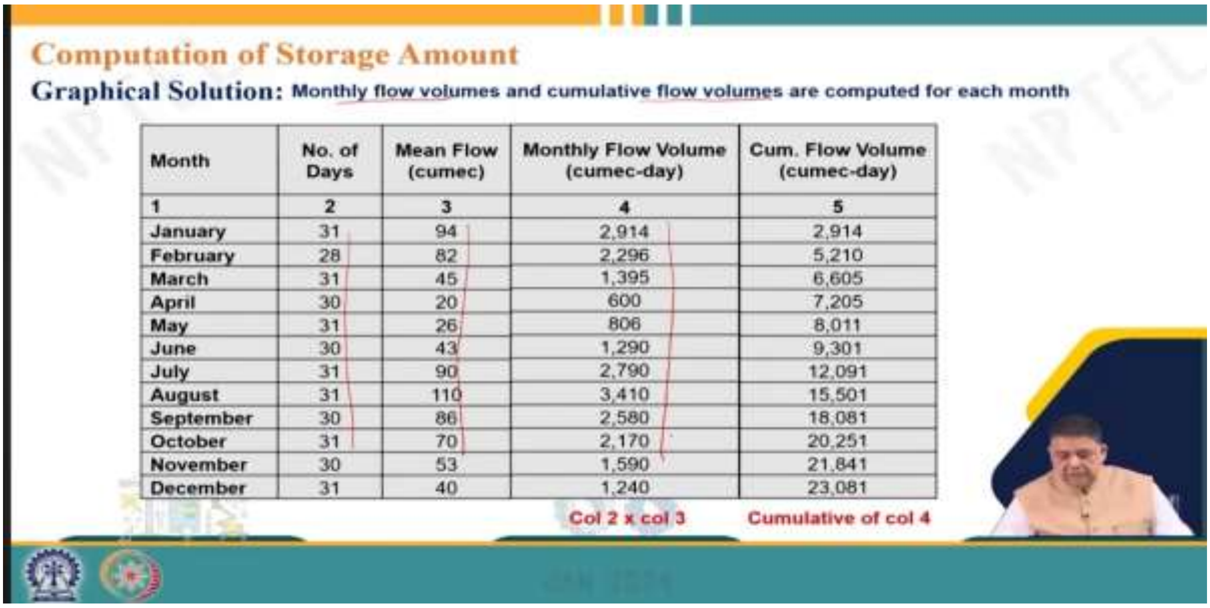
And so, the maximum, I mean, wherever we must keep in mind the lean periods, that is, you see that these are the peak flow which is coming, and these are the lean periods. So, we have humps and valleys. Humps represent peak periods wherein there is a valley here or a valley here, they represent the lean period, meaning when your supply is lower than the demand. So, obviously, for all the lean periods, we must find out what is the storage required to meet a particular demand. And then, wherever this storage requirement is high, that tells us that this storage must be available to us to be able to meet or sustain that demand.

So, we can express that storage is the maximum of ($\Sigma V_D - \Sigma V_S$), that is, the demand line minus the supply line. So, wherever the demand is more than the supply, that will tell us what is the total supply required, and V_D and V_S, as we have already seen, are cumulative supply and cumulative demand. So, when the mass curves of supply and demand are plotted, the maximum difference in their ordinates gives the maximum cumulative difference with a storage amount S of that dry period, and the values of S are calculated for different dry periods, and the largest of these values is the minimum storage required for the reservoir.

Suppose we want to estimate the storage requirement, then of course, we draw a demand line of known demand rate, say, for example, our demand rate is 50 cubic meters per second, tangential to various humps in the mass curve. So, this is the mass curve we have plotted, and then we identify the humps, and different humps, and then from there, we draw a line having a slope of 50 cubic meters per second, and of course, we know how to find out the slopes at.

So, I mean, the slope of this line will be vertical minus this vertical divided by this horizontal. So, at any place, we can find out what is the slope, and that slope must be maintained at 50 cubic meters per second. Then find the vertical distance between these demand lines and their respective valley bottoms; these represent the required storage to meet the demand. For example, from the valley bottom to this line is S_1, then again in the second valley, we have S_2, third valley, we have S_3, and we say that this is the storage required to meet that demand in this period. So, the storage required, as far as the reservoir is concerned, is the maximum of S_i wherever it occurs, which gives us what the storage required to sustain this demand.

Let us take a simple example here and compute the minimum reservoir storage required to maintain a demand of 50 cubic meters per second of monthly reservoir flows. The reservoir available data follows. So, these are the different months, January to December, flows are given, and for plotting the mass curve, assume each month has 30 days, and solve the problem both graphically and arithmetically. So, both the problems can be solved both graphically and arithmetically, as we will see. So, this is the supply, and this is the demand we must maintain for that; we must find out the storage.



Computation of Storage Amount

Graphical Solution: Monthly flow volumes and cumulative flow volumes are computed for each month

Month	No. of Days	Mean Flow (cumec)	Monthly Flow Volume (cumec-day)	Cum. Flow Volume (cumec-day)
1	2	3	4	5
January	31	94	2,914	2,914
February	28	82	2,296	5,210
March	31	45	1,395	6,605
April	30	20	600	7,205
May	31	26	806	8,011
June	30	43	1,290	9,301
July	31	90	2,790	12,091
August	31	110	3,410	15,501
September	30	86	2,580	18,081
October	31	70	2,170	20,251
November	30	53	1,590	21,841
December	31	40	1,240	23,081
			Col 2 x col 3	Cumulative of col 4

So, as we have seen, the first thing we must do is put the monthly flow volumes and cumulative flow volumes; we must compute.

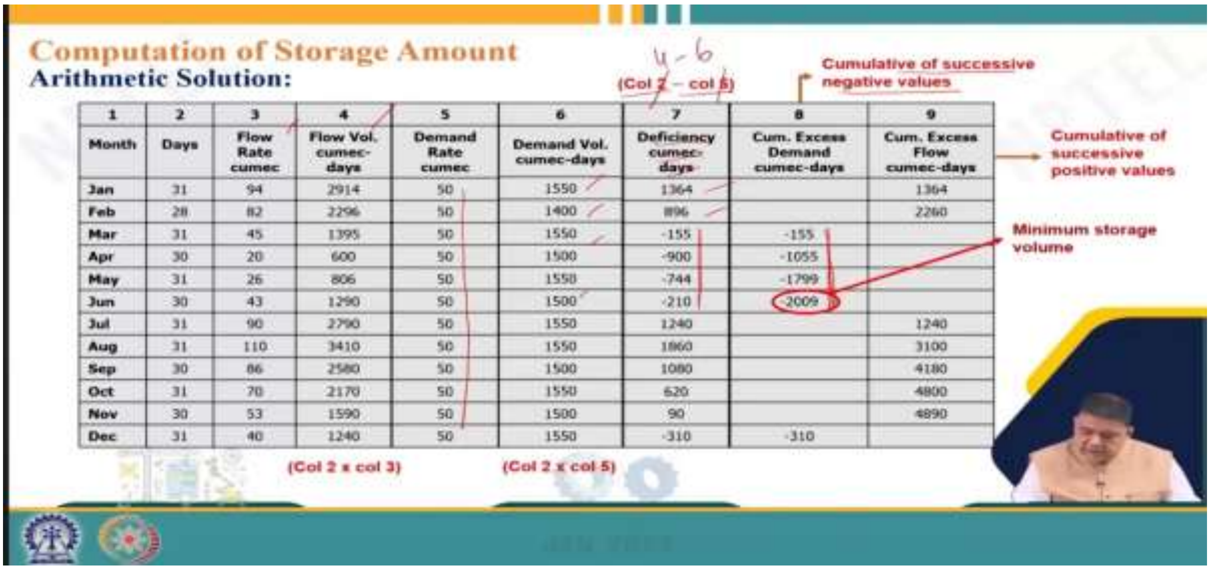
So, we have the number of days in different months and we have mean flow values given. So, the monthly flow volume can be found in column 2, and column 3 will give us the cumulative monthly flow volume. The cumulative of this column, column 4, will give us the cumulative flow volume. So, this is what we have obtained.

Using this cumulative flow volume, we can draw the flow mass curve. Cumulative flow volume is plotted against time to obtain the flow mass curve, and for simplicity of plotting, we have assumed all months are equidistant.

Then, what we do is draw a demand line having a slope of 50 cubic meters per second tangential to the hump at the beginning of the flow mass curve. So, in this example, you see we only have one lean period and that is one hump at A. So, starting from A, we draw a line AB such that the slope of the line is 50 cubic meters per second, which is the demand we must sustain. To compute the required storage, a line parallel to the demand line is drawn tangentially to the valley bottom of the mass curve, say line A_1B_1. This is done just to accurately determine the value of S.

So, what is done is that we identify the valley bottom, and we draw a line having a slope parallel to line AB, which means having the same slope of 50 cubic meters per second. So, this A1B1 is parallel to AB, and the vertical distance between these two parallel lines gives the minimum storage S required to meet the constant demand rate of 50 cubic meters per second.

So, this vertical gap, if calculated, or we find out, then is nothing but the required storage. From the graph, we find that the storage S required to sustain a demand of 50 cubic meters per second is 2300 cubic days. So, this volume of water must be stored in the reservoir to sustain this supply, every supply of 50 cubic meters per second. This is the graphical solution.



1	2	3	4	5	6	7	8	9
Month	Days	Flow Rate cumec	Flow Vol. cumec-days	Demand Rate cumec	Demand Vol. cumec-days	Deficiency cumec-days	Cum. Excess Demand cumec-days	Cum. Excess Flow cumec-days
Jan	31	94	2914	50	1550	1364		1364
Feb	28	82	2296	50	1400	896		2260
Mar	31	45	1395	50	1550	-155	-155	
Apr	30	20	600	50	1500	-900	-1055	
May	31	26	806	50	1550	-744	-1799	
Jun	30	43	1290	50	1500	-210	-2009	
Jul	31	90	2790	50	1550	1240		1240
Aug	31	110	3410	50	1550	1860		3100
Sep	30	86	2580	50	1500	1080		4180
Oct	31	70	2170	50	1550	620		4800
Nov	30	53	1590	50	1500	90		4890
Dec	31	40	1240	50	1550	-310	-310	

Arithmetically, a similar procedure is followed. The flow rate is given, so flow volumes are calculated for different months, and the demand rate is constant at 50. So, the demand volumes are also calculated.

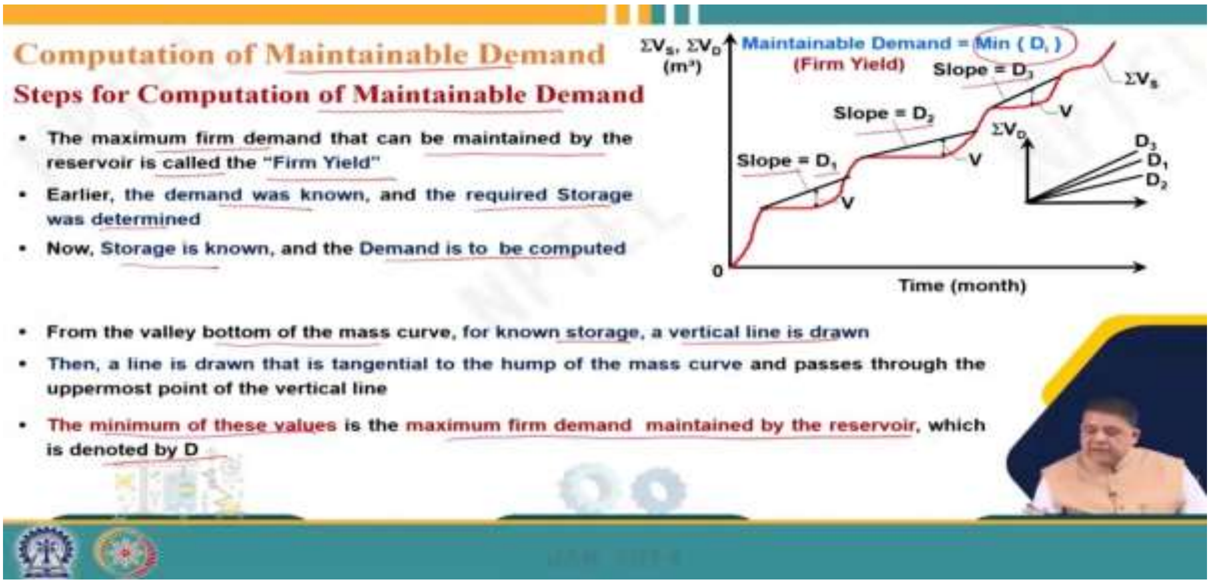
Then, the deficiency in cubic days, which is column 2 minus column 5, is calculated. Column 2 minus 5 deficiency is column 4 minus column 6. This needs correction; it should be column 4 minus column 6, which is the cumulative flow volume minus the cumulative demand volume, giving us the deficiency for different months. Then, we take the cumulative of successive negative values, which represent the deficiency, and these are the four months where negative values or deficiencies are present.

So, we get the cumulative excess demand during this period. Of course, then we have cumulative excess flow for periods other than this period. From here, we find that the total deficiency during the year is 2009 cumec days, and that is the minimum storage volume required.

The maximum cumulative excess demand is in June, and it equals 2009 cubic days, and this is the value of storage required to meet the demand of 50 cubic meters per day.

This value of storage differs from the one obtained graphically; in the graphical solution, we got 2300, while arithmetically, we got 2900. This difference is simply because, in the arithmetical calculation, flow is assumed to vary linearly from one month to another, while in the graphical solution, the mass curvature is curvilinear. So, there is a minor difference in the solutions.

It may be noted that the last column in the table indicates if the reservoir is going to be refilled. If you look at the table, we had 2009. So, right here, in the month of August, this reservoir will be refilled. This is how we can find out the storage requirement.

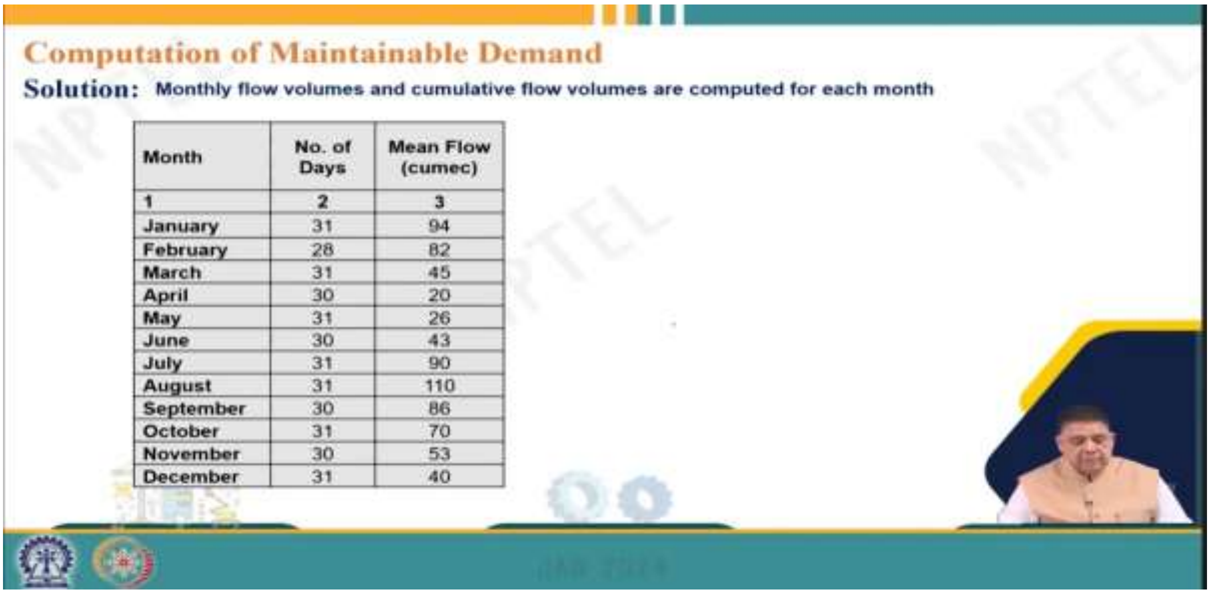


Now, the inverse of the problem is that we already have storage available in a reservoir, and we want to compute what demand this reservoir can sustain. The maximum firm demand that can be maintained by the reservoir is called the firm yield.

So, that is what we must find out. Earlier, the demand was known and the required storage was determined. Now, the storage is known, and we want to find out the demand. So, the problem is just the reverse. From the valley bottom of the mass curve for the known storage, we draw a vertical line. Corresponding to that storage value, we find the different demand slopes. Different demand slopes will be there, and of course, the minimum of these values is the maximum firm demand maintained by the reservoir, denoted by D. So, in the earlier case, we had the maximum of S, and now it is the minimum of D that will tell us the demand, the firm yield, or the maintainable demand which the storage in the reservoir can sustain.

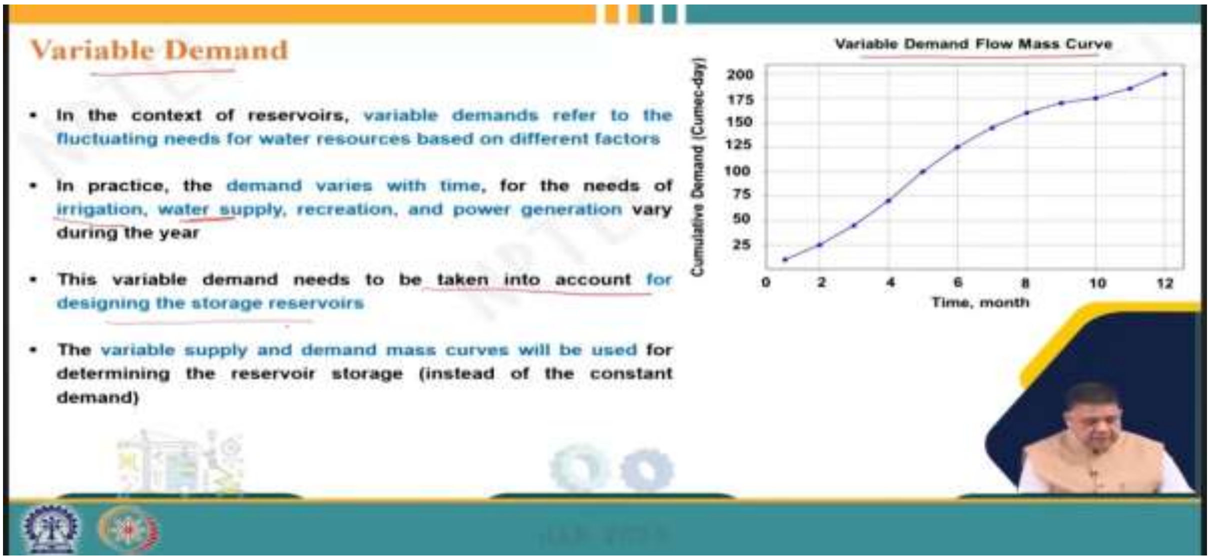
So, we will take an example to compute the maximum uniform demand rate that can be maintained by the reservoir with storage of 3000 cubic days and monthly reservoir flows. Here, the reservoir inflows are given, and we know the storage capacity. Here also, we will assume 30 days for a smooth plotting of the curve.

Monthly flow volume and cumulative flow volumes are calculated as we did earlier, and then of course, we will plot the mass curve. As we said earlier, a vertical line of 3000 cubic meters per day, which is the available storage, is drawn at the valley bottom of the mass curve.



In this case, also, we have only a single valley. So, we will find out the valley bottom and draw a vertical line representing 3000 cubic meters per day times days storage available. Parallel to this line, a tangent is drawn. The line is drawn tangential to the hump of the mass curve and passes through point C. Starting from here, we draw a line tangential to this point M and that crosses point B. Then, parallel to this line, in a similar way, we draw this line M_1N_1, and if the gap is 3000 cubic days, then our slope is fine.

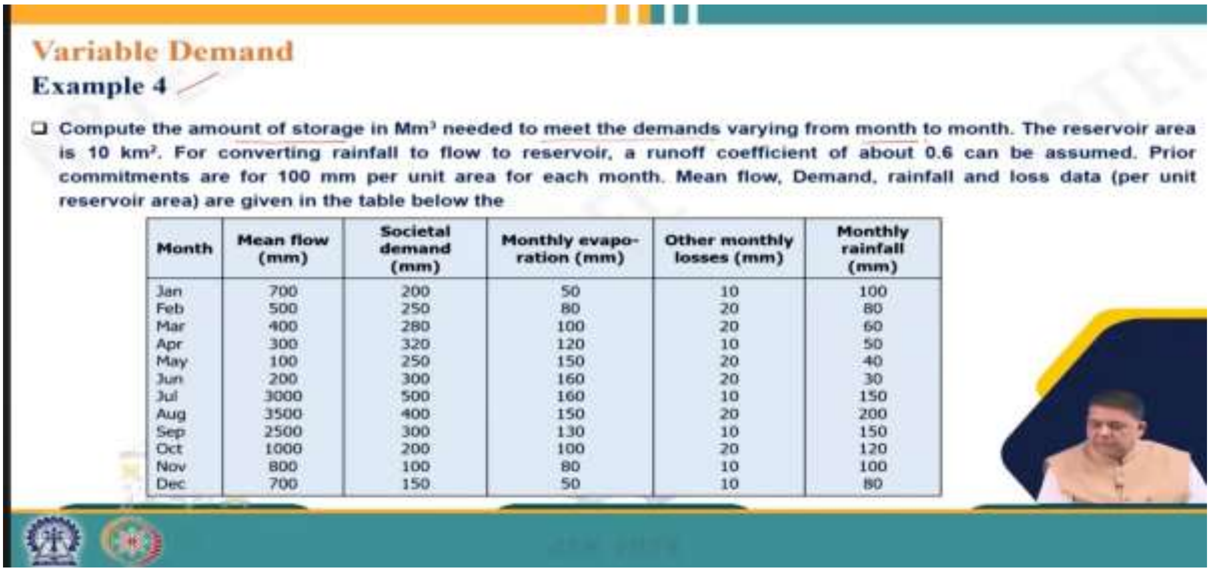
If it is so, then our slope is fine, and then we will find out the slope of the line, and in this case, we find that the slope of line MN comes out to be 63 cubic meters per day. So, that is the demand that can be sustained by this reservoir when the storage available is 3000 cubic days. In the case of a mass curve having more than one valley, the exercise is repeated for each valley, and the lowest of demands is the maintainable demand.



Similar fashion, we can also consider the variable demand, because, in the reservoir, demands could be of various natures and may keep changing.

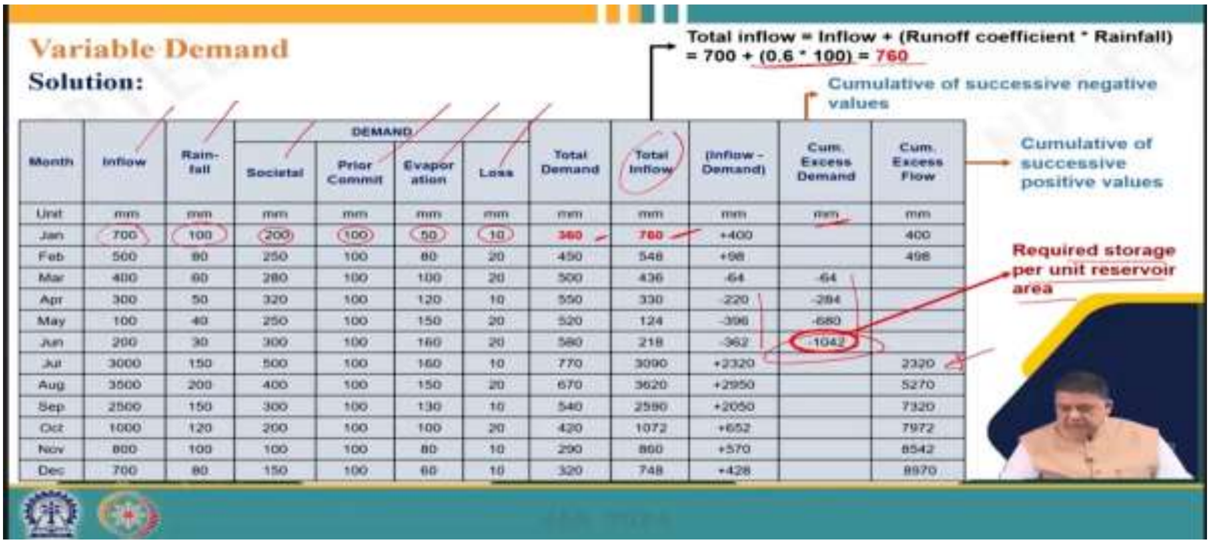
For example, if we consider irrigation, the irrigation supply changes throughout the year; similarly, this could be true for domestic water supply. Of course, that does not fluctuate too much during the year, but irrigation is one that fluctuates. So, this could be a variable demand mass curve. Up until now, we were assuming a constant slope line, but in this case, we can also have a variable demand, and this demand needs to be considered for designing the storage reservoir.

So, obviously, a variable supply and demand mass curve will be used for determining the reservoir storage.



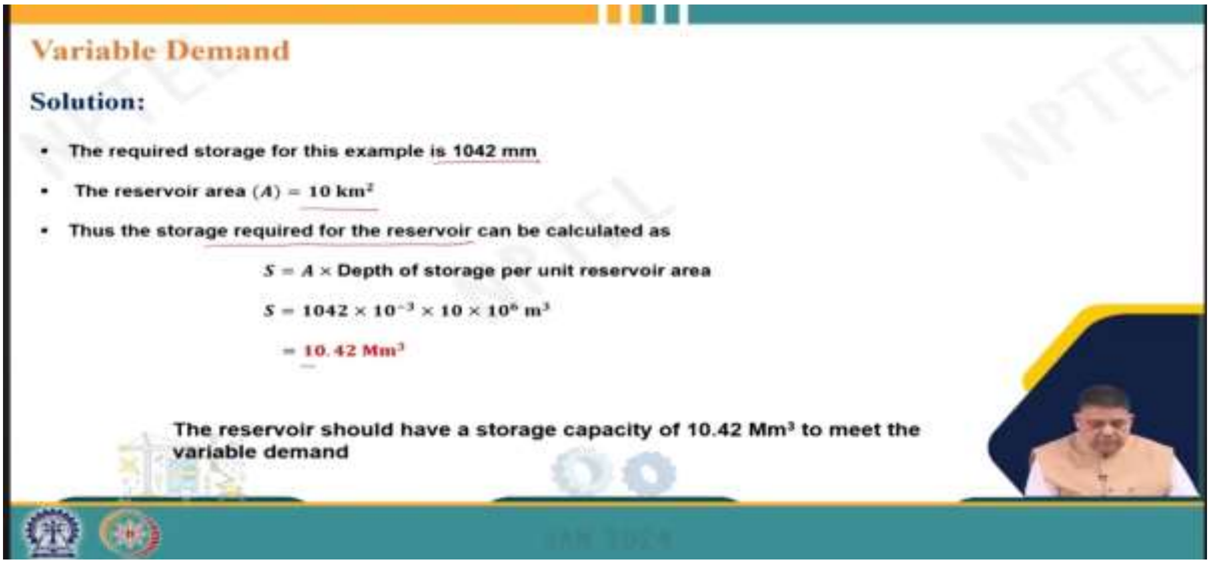
Month	Mean flow (mm)	Societal demand (mm)	Monthly evaporation (mm)	Other monthly losses (mm)	Monthly rainfall (mm)
Jan	700	200	50	10	100
Feb	500	250	80	20	80
Mar	400	280	100	20	60
Apr	300	320	120	10	50
May	100	250	150	20	40
Jun	200	300	160	20	30
Jul	3000	500	160	10	150
Aug	3500	400	150	20	200
Sep	2500	300	130	10	150
Oct	1000	200	100	20	120
Nov	800	100	80	10	100
Dec	700	150	50	10	80

We will take an example to compute the amount of storage needed to meet the demands varying from month to month. The reservoir area is 10 square kilometers. For converting rainfall to flow to reservoir runoff, a coefficient of 0.6 can be assumed. There is a prior commitment of 100 millimeters per unit area for each month. Mean flow demand, rainfall, and loss data per unit reservoir are given.



Month	Inflow	Rainfall	DEMAND: Societal	Prior Commit	Evaporation	Loss	Total Demand	Total Inflow	(Inflow - Demand)	Cum. Excess Demand	Cum. Excess Flow
Unit	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
Jan	700	100	200	100	50	10	360	760	+400		400
Feb	500	80	250	100	80	20	450	548	+98		498
Mar	400	60	280	100	100	20	500	436	-64	-64	
Apr	300	50	320	100	120	10	550	330	-220	-284	
May	100	40	250	100	150	20	520	124	-396	-680	
Jun	200	30	300	100	160	20	580	218	-362	-1042	
Jul	3000	150	500	100	160	10	770	3090	+2320		2320
Aug	3500	200	400	100	150	20	670	3620	+2950		5270
Sep	2500	150	300	100	130	10	540	2590	+2050		7320
Oct	1000	120	200	100	100	20	420	1072	+652		7972
Nov	800	100	100	100	80	10	290	860	+570		8542
Dec	700	80	150	100	60	10	320	748	+428		8970

So, all the data is available. Mean flow, societal demand, monthly evaporation losses, and other monthly losses are given. The monthly rainfall value is also given. So, obviously, we have the inflow, rainfall, and these demands. Like previously, we will find out the deficiency, and the continuous deficiency, and wherever we get, the sum of that will tell us that this is the storage required. In this case, the required storage per unit reservoir area is 1042 millimeters because we are doing all in-depth months, and again, in the very next month, this reservoir will be filled.



So, because the required storage is 1042 millimeters per reservoir area, and the reservoir is 10 square kilometers, the multiplication of that will give us the storage required for the reservoir, which comes out to be 10.42 million cubic meters. So, the reservoir should have a storage capacity of 10.42 million cubic meters to meet the variable demand. As you see, we saw the FDC, flow duration curve, as well as flow mass curve, and we also saw how to use the flow mass curve to design reservoirs or determine the supportable demand of an existing reservoir.

So, please feel free to give your feedback on this discussion and raise your questions or doubts so they can be addressed on the forum. Thank you very much.