

Course Name: Watershed Hydrology

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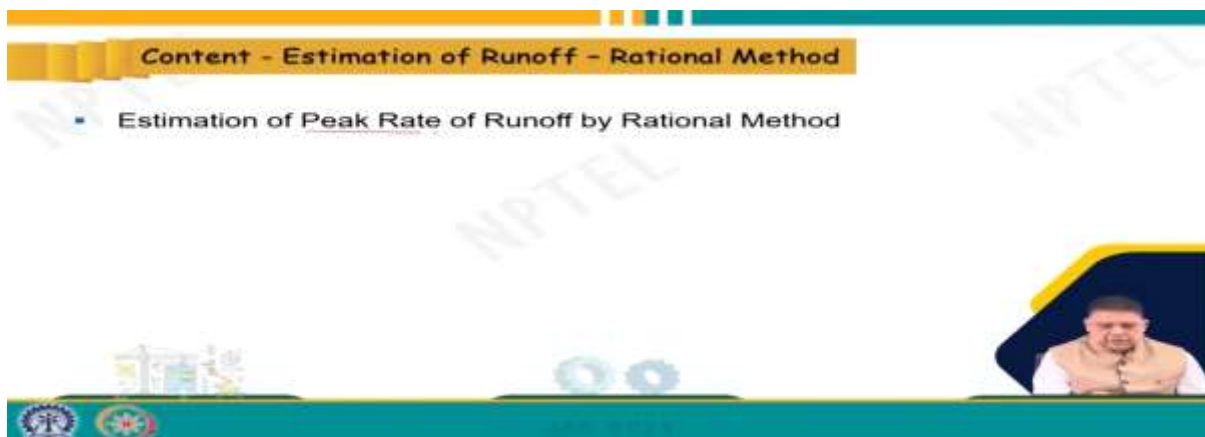
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Lecture 18: Estimation of Runoff – Rational Method



Hello friends, welcome back to this online certification course on Watershed Hydrogenic Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are in Module 4, this is Lecture Number 3, and we will continue with the estimation of runoff, focusing today on the rational method, a very popular method of estimating runoff.



As already mentioned, the only topic we will cover today is the peak rate of runoff by the rational method.

Estimation of Peak Rate of Runoff

Rational Method

It is the most popular method of estimating peak runoff rate for designing soil conservation structures

The peak runoff rate is expressed as,

$$Q_p = CiA$$

(9)

$$Q_p = \frac{CiA}{36}$$

(Handwritten notes: CiA/h, ha)

Where Q_p = Peak runoff rate in m^3/s

C = Runoff coefficient

i = Rainfall intensity in m/s for a duration equal to the time of concentration of the watershed and for a desired return period

A = Area of the watershed in m^2

The method is applicable for watersheds having area less than 1300 ha



The rational method is one of the most popular methods of estimating peak runoff rates for designing soil conservation structures. Of course, we have already discussed runoff, and we know that while designing any kind of soil conservation structures, be it discharge structures or detention structures, we require the runoff volume, and this method gives us the peak runoff rate. So, it is more important for any kind of conservation structure.

This relationship expresses the peak runoff rate in this method: $Q_p=CiA$, Equation number 9. Remember, we are continuing with the numbering for the entire estimation of peak runoff rate. So, that is why this is Equation number 9 because we have used other numbers in the previous lecture. Here, Q_p is the peak runoff rate in cubic meters per second, C is the runoff coefficient, and i is rainfall intensity in meters per second for a duration equal to the time of concentration of the watershed and a desired return period.

That is very important: the intensity definition is very specific and it must be always kept in mind while using the rational method, that is, this rainfall intensity should have a duration equal to the time of concentration of the watershed. So, that is an important condition and, of course, for the desired frequency or desired return period, which we discussed, depending upon the life of the structure, we decide what frequency we must use or what probability we have to use. Remember, we discussed the probability or return period. So, what return period we will use, that decision has to be made, of course. And A is the area of the watershed in square meters.

So, here it is written in plane form CiA , i is in meters per second, and the area is in square meters. So, that is why we get $Q_p=$ cubic meters per second. In some textbooks, you will find this relationship in this form: $Q_p=CiA/36$, but the units there are centimetre per hour, and the area is in hectares. So, obviously, you must remember the units also for using this relationship. To avoid that complication, we can simply use this relationship: i is in meters per second, and A is in square meters. So, that is why we get discharge in cubic meters per second, which is a typical unit we use for discharge, or we say it is "cumec". The method, of course, has certain assumptions which we will see a little later. Its applicability is limited to watersheds having an area of less than 1300 hectares. Of course, this number is also not a very fixed number; in different literature, you will come across different numbers. The idea is that the watershed is because there are certain assumptions. So, the watershed should be of reasonable size to be able to use this method.

Rational Formula

Assumptions

1. Rainfall occurs at uniform intensity for a duration at least equal to the time of concentration, and
2. Rainfall occurs at uniform intensity over the entire area of the watershed

Since, there is hardly any rainfall satisfying both these conditions, the estimation of runoff is only approximate by this method

However, for the design of relatively inexpensive structures where the consequences of failure are limited, this method is considered sufficiently accurate

☐ Use of this method requires three parameters:

- I. Time of concentration (T_c)
- II. Rainfall intensity (i)
- III. Runoff coefficient (C)

$Q = C \cdot i \cdot A$

There are two significant assumptions on which the rational formula or rational method is based. First, rainfall occurs at uniform intensity for a duration at least equal to the time of concentration. So, while defining intensity, we say that intensity should have a duration at least equal to the time of concentration. This assumption states that rainfall occurs at uniform intensity for a duration at least equal to the time of concentration. That means, whatever the time of concentration of the watershed is, for that duration, the intensity of rainfall remains constant.

The other assumption is that rainfall occurs at a uniform intensity over the entire area of the watershed. So, one assumption is about time, and the other is about the aerial distribution of rainfall. The second assumption states that rainfall is uniformly distributed over the entire area of the watershed. That means, the entire watershed area contributes to the runoff because of this rainfall intensity, which has a duration equal to the time of concentration. Since there is hardly any rainfall satisfying both these conditions, the estimation of runoff is only approximate by this method.

It is very difficult to get rainfall that is uniformly distributed over an area, even, say, 100 hectares, forgetting about a larger area or even a few hectares. It is also very difficult to get rainfall where the intensity remains constant or uniform for a certain period, which in this case, is defined as the time of concentration for the watershed. Because it is very difficult to satisfy these conditions or to get rainfall that is constant over the time of concentration or the area, these are the limitations of the rational formula or rational method, and that is why the estimation of runoff is only approximated by this method. However, for the design of relatively inexpensive structures, where the consequences of failures are limited, this method is considered sufficiently accurate.

But, as I said, it is one of the very popular methods for designing soil conservation structures, especially temporary structures or structures having a low life expectancy. In such cases, even if the structure fails, the danger to life or property downstream may not be significant. So, in such cases, this method gives a practical and simple value of the peak runoff rate, and that is why it is quite commonly used.

Now, the application of this method, requires three prerequisites: the time of concentration, because, remember, we say that the time of concentration is important for finding out the duration of rainfall and the rainfall intensity, and that is what we say that rainfall intensity remains constant for a duration equal to the time of concentration. So, i and t_c are linked, and,

of course, the runoff coefficient. If you remember, we said that $Q=CIA$. So, this is the runoff coefficient we are talking about. This parameter also is very important, and we must have a fair idea of what this parameter is about. So, if we know all these things, the area, of course, is very easy to get. Of course, C will require watershed characteristics considerations, and, of course, intensity will require the time of concentration because duration is linked to the time of concentration. That is why this method is vital.

Estimation of Peak Rate of Runoff

Time of Concentration (T_c)

- ❑ It is the time required for water to flow from the most remote point of the watershed (with respect to flow time) to the outlet
- ❑ The most widely accepted method of computing T_c was developed by Kirpich (1940):

$$T_c = 0.01947L^{0.77}S^{-0.385} \quad (10)$$

Where T_c = Time of concentration of a watershed, min
 L = Maximum length of flow, m (measured along the drainage line)
 S = Slope of the watershed $\left(\frac{\Delta H}{L}\right)$
 ΔH = Elevation difference between the remotest point on the watershed and outlet, m

Runoff hydrograph due to uniform rainfall
(Source: K. Subramanya, Engineering Hydrology)

Now, coming to the time of concentration, it is the time required for the water to flow from the most remote point of the watershed with respect to flow time to the outlet. And if you see this picture here, this is the outlet of the watershed. So, if you locate the remotest point of the watershed, then whatever time the flow takes for traveling from this place to the outlet, of course, through the streams, that is referred to as the time of concentration. The furthest point of the watershed we have located.

Now, why is this significant? The significance of this can be understood from this figure here, which simply shows the runoff and rainfall rate. This is the rainfall rate detailing, and of course, the runoff generation; runoff is here, and on this side, it is time. So, what is happening here is that if rainfall is occurring at a constant intensity over a period, then, obviously, as we know, it will satisfy the abstractions, various kinds of abstractions, including the major abstraction infiltration, and then, of course, the runoff or overland flow will start taking place, and the runoff will be generated. Gradually, with time, runoff increases. It keeps on increasing, and then it peaks, and then it will continue with that peak rate as long as rainfall is continuous.

Now, at this point, rainfall ends, and that is why you see that runoff is there is a recession in the runoff, that is, it is decreasing beyond that point. The interesting thing is the peak value is obtained at a duration equal to the time of concentration. So, that happens because when the water has reached from the farthest point to the outlet, it is expected, and rainfall is continuing, then it is expected that the entire watershed will be contributing to runoff. That means the peak rate of runoff will be when the water starts flowing even from the farthest point of the watershed. So, that is why when the rainfall duration is greater than T_c , then it is expected that the peak value of runoff will be generated. That is why this assumption that i should have a duration at least equal to the time of concentration. Now, finding this type of, that means, in this entire exercise, time of concentration is a very important parameter, and there are several

methods for determining the time of concentration. The most widely accepted method of computing T_c was developed by Kirpich in 1940.

And as per this method, the time of concentration is given by equation number 10, which is $T_c = 0.01947L^{0.077}S^{-0.385}$, where T_c is the time of concentration in minutes because it is a kind of empirical relationship that's why units are already fixed.

So, the time of concentration is in minutes, L is the maximum length of flow in meters, and S is the slope of the watershed. Of course, this length is measured along the drainage line. So, if this is the main drainage line, L is measured on this line, and S is the slope of the watershed, which will come from this relationship: $\Delta H/L$, where ΔH is the elevation difference between the remotest point on the watershed and the outlet. So, if we know the elevation here and the elevation at the outlet, the elevation difference between these two points is ΔH , and if we know the flow path, we know L , and that is why we can find out the slope of the watershed as $\Delta H/L$, which can be used here. So, S is here, and L is here. An important point about this one is 0.01947. Remember India's independence year 1947. So, it is very easy to remember for at least Indian students that it uses the Kirpich formula, which uses a constant of 0.01947. Of course, in some textbooks, you will find that this value has been truncated to 0.0195. So, anyway, if you remember 0.01947 or 0.0195, it does not matter, but this is one of the most popular ways of finding out the time of concentration for a watershed because it just uses the watershed features for determining the time of concentration.

Estimation of Peak Rate of Runoff

Rainfall Intensity (i)

- Rainfall intensity corresponding to a duration T_c and desired return period T (or probability of occurrence, P) is found from the **intensity-duration-frequency (IDF) curve** of the area
- Usually, the relationship is of the form,

$$i = \frac{KT^x}{(T_c + a)^m} \quad (11)$$
 where K , x , a , and m are constants

Kothyari and Garde (1992) relationship for India

Runoff Coefficient (C)

- It represents the integrated effects of the catchment characteristics as a function of surface condition and slope
- The runoff coefficient may be expressed as the ratio of runoff (R) and rainfall (P):

$$C = \frac{R}{P} \quad (12)$$

The slide also features a graph of Average Intensity (mm/h) vs Duration (h) for return periods $T = 100$ yr, $T = 50$ yr, and $T = 25$ yr. The graph shows that average intensity decreases as duration increases and as the return period decreases. A video inset in the bottom right corner shows a man speaking.

Now, we come to rainfall intensity. Rainfall intensity corresponding to a duration T_c can be found from the intensity-duration-frequency curve of the area. For this intensity-duration-frequency curve, we have already discussed the idea of the free curve or the idea of analysis. So, we know that the average intensity for any known duration can be read for different recurrence intervals from this graph. And usually, this relationship is of the form $i = kT^x / (T_c + a)^m$, where k , a , and m are constants. But remember, while discussing the IDF curve, I gave an example of a relationship that was developed by Kothyari and Garde in 1992, which slightly differs from this relationship. But of course, most of the parameters are the same; there is one additional parameter there, R_{24} , which was used in that model.

Then comes the runoff coefficient C , and this runoff coefficient represents the integrated effect of catchment characteristics as a function of surface condition and slope. The runoff coefficient may be expressed as the ratio of runoff to rainfall, that is $C=R/P$. That means, it simply tells us what percentage of rainfall gets converted into runoff. So, if we know rainfall and we know precipitate runoff and we know precipitation, then it tells us what percentage of rainfall gets converted into runoff in simpler terms. So, this can be determined from standard tables, which provide values of C .

Recommended values of the runoff coefficient, C

Types of drainage area		C
Urban Area		
Lawns	Sandy soil, flat, 2%	0.05-0.10
	Sandy soil, steep, 7%	0.15-0.20
	Heavy soil, average, 2-7%	0.18-0.22
Residential area	Single family area	0.30-0.50
	Multiple units	0.60-0.75
Industrial	Light	0.50-0.80
	Heavy	0.60-0.90
Streets		0.70-0.95
Agricultural Area		
Flat	Tight clay, cultivated	0.50
	Tight clay, woodland	0.40
	Sandy loam, cultivated	0.20
	Sandy loam, woodland	0.10
Hilly	Tight clay, cultivated	0.70
	Tight clay, woodland	0.60
	Sandy loam, cultivated	0.40
	Sandy loam, woodland	0.30

Now, we must consider yet another thing, that is, the runoff coefficient for non-homogeneous areas. You remember in the previous table, the value is given for a specific area, but that simply means that initially it was assumed that the watershed has homogeneous land use, soil, and slopes; and that everything is uniform.


Estimation of Peak Rate of Runoff

Runoff Coefficients for Non-homogeneous Areas

- ❑ Rational method assumes that the watershed has homogeneous land use, soil and slope
- ❑ However, if a watershed is non-homogeneous, characterised by highly dispersed areas having different runoff coefficients, a weighted runoff coefficient may be determined as follows:

$$C_w = \frac{\sum_{j=1}^n C_j A_j}{\sum_{j=1}^n A_j} \quad (12)$$

Where, A_j is the area for land cover j ,
 C_j is the runoff coefficient for area j ,
 n is the number of distinct land covers within the watershed, and
 C_w is the weighted runoff coefficient



But practically, because it is not possible, typically watersheds are non-homogeneous, characterized by highly dispersed areas having different runoff coefficients.

In such cases, we have to use a weighted runoff coefficient using this relationship: $C_w = \sum_{j=1}^n C_j A_j / A_j$, where C_j is the runoff coefficient of area j , n is the number of distinct land covers within the watershed, and A_j is the area of the land cover. So, for example, if you have a watershed where you can distinctly see that there are 3 different land characteristics or

features based on land slope and land use, then in that case, our n value becomes 3, and we have to find out the area under each of these, denoted as A_j , and the corresponding C_j . If I know A_j and C_j of all 3, then the sum of $A_j \cdot C_j$ will give me the numerator, and the total area is the denominator. So, I can get the weighted runoff coefficient for a non-homogeneous area.

Estimation of Peak Rate of Runoff

Example 1

An agricultural field has a runoff coefficient of 0.25 and an area of 0.75 sq. km. The slope of the catchment is 0.45% and the maximum length of travel is 1.2 km. The maximum depth of rainfall with a 25-year return period is as follows:

Duration (min)	5	10	20	30	40	60
Depth of Rainfall (mm)	22	31	40	55	62	67

A drop spillway has to be designed for the area for a return period of 25 years. Determine the peak flow rate.

Let's take an example: an agricultural field has a runoff coefficient of 0.25 and an area of 0.75 square kilometers. The slope of the catchment is 0.45 percent, and the maximum length of travel is 1.2 kilometers. The maximum depth of runoff rainfall with a 25-year return period is provided in the table. We have to determine the peak flow rate for designing this structure.

We can use the rational method in this case.

Estimation of Peak Rate of Runoff

Solution:

Given, the maximum length of travel, $L = 1.2 \text{ km} = 1200 \text{ m}$; and the slope of the catchment, $S = 0.45\%$, the Kirpich formula may be used to determine the time of concentration as follows:

$$T_c = 0.01947L^{0.77}S^{-0.385} = 0.01947(1200)^{0.77}(0.0045)^{-0.385} = 36.63 \text{ min}$$

Now from the given table (by interpolation),

Duration (min)	5	10	20	30	40	60
Depth of Rainfall (mm)	22	31	40	55	62	67

Maximum depth of rainfall for 36.63 min duration = $\left((62 - 55) \times \frac{6.63}{20} \right) + 55 = 59.64 \text{ mm}$

Thus, rainfall intensity (i) = $\frac{59.64 \times 60}{36.63} = 97.69 \text{ mm/h}$ or $2.71 \times 10^{-3} \text{ m/s}$

The known values are the maximum length of travel $L=1.2$ kilometers (converted to 1200 meters) and the slope of the catchment $S=0.45$. We may use Kirpich's formula to determine the time of concentration, given by $0.01947L^{0.77}S^{-0.385}$. By substituting the values of L and S , we get $T_c=36.63$ minutes.

Now, we've been provided with a table having different durations and depths of rainfall. So, we need to find the intensity of rainfall for a duration equal to the time of concentration. This means we have to first find out the depth of rainfall for 36.63 minutes and then convert it into intensity to use it in the rational method. We need to interpolate for the duration of 36.63

minutes, which falls between 30 and 40 minutes in the table. Interpolating, we get the maximum depth of rainfall for 36.63 minutes as 59.64 millimeters. Then, we can find the rainfall intensity, which comes out to be 97.69 millimeters per hour or 2.71×10^{-5} meters per second, the unit we require to use.

Estimation of Peak Rate of Runoff
Solution:

- Rainfall intensity (i) = 2.71×10^{-5} m/s
- Runoff coefficient (C) = 0.25
- Area (A) = 0.75 sq. km = 0.75×10^6 m²
- Thus, Peak flow rate

$$Q_p = C i A = 0.25 \times (2.71 \times 10^{-5}) \times (0.75 \times 10^6) = 5.08 \text{ m}^3/\text{s}$$

Thus, the peak flow rate for designing the drop spillway in the area = $5.08 \text{ m}^3/\text{s}$

We have already obtained the rainfall intensity value, and the runoff coefficient is already given as 0.25.

So, the area of the watershed is 0.75 square kilometers. We can convert that into square meters, which comes out to be 0.75×10^6 square meters. Now, all the requisite parameters are available to us. So, we can use the rational formula $Q_p = C \cdot i \cdot A$, and thus, with all these values, we can calculate and obtain a value of 5.08 cubic meters per second. Using the rational formula, the peak flow rate for designing the drop structure in the area, for a return period of 25 years, comes out to be 5.08 cubic meters per second or 5.08 cumecs. This is how we can estimate the values here.

Rational Method
Example 2
 A 100 ha watershed received rainfall at a rate of 50 mm/h for 2 hours. If the runoff generated by the storm was at the rate of 1 m³/s for 10 hours, then find the runoff coefficient of the watershed?

Solution:

- Intensity of rainfall (i) = 50 mm/h
- Duration of rainfall (D) = 2 hours
- Total depth of rainfall (P) = ($i \times D$) = (50 × 2) mm = 100 mm
- The runoff rate (Q_p) = 1 m³/s
- Total Area of watershed (A) = 100 ha = 10^6 m²
- Total runoff duration (t) = 10 hours = 36000 s
- Total depth of runoff (R) = $\frac{Q_p \cdot t}{A} = \frac{1 \times 36000}{10^6} = 0.036 \text{ m} = 36 \text{ mm}$
- The runoff coefficient of the watershed

$$C = \frac{R}{P} = \frac{36}{100} = 0.36$$

36% of rainfall gets converted into runoff

Handwritten notes:
 $Q = C \cdot i \cdot A$
 $C = \frac{R}{P}$
 Runoff Volume = $Q_p \times \text{Duration} = 1 \text{ m}^3/\text{s} \times 36000 \text{ s} = 36000 \text{ m}^3$
 (GATE 2013)

Now, let's take another example. A 100-hectare watershed received rainfall at a rate of 50 millimeters per hour for 2 hours. If the runoff generated by the storm was at a rate of 1 cubic meter per second for 10 hours, then find the runoff coefficient for the watershed. This question is taken from the GATE 2013 examination. So, in this case, the 100-hectare watershed received an area of 100 millimeters. Therefore, we do not need to use the rational formula in this

problem; it is a simple problem where we just need to determine the value of CC by knowing the runoff and precipitation or rainfall.

The intensity of rainfall is given as 50 millimeters per hour, and the duration of rainfall is given as 2 hours. So, the total depth of rainfall PP is obtained by multiplying the intensity (50 mm/h) by the duration (2 hours), which gives us $50 \times 2 = 100$ millimeters. Now, the numerator RR is given by the runoff rate, which is 1 cubic meter per second, multiplied by the duration of 10 hours. Converting the duration to seconds (10 hours = 36000 seconds) and the watershed area to square meters (100 hectares = 10^6 square meters), we get a total runoff volume of 3600 cubic meters.

Since we need runoff in-depth units, we divide the runoff volume by the watershed area to get the total depth of runoff, which is $3600/10^6 = 0.036$ meters or 36 millimeters. Therefore, the total runoff coefficient for the watershed is $R/P = 36/100 = 0.36$. Alternatively, we can directly calculate C by multiplying Qp and T , then dividing by the area, which yields 0.36.

This means that 36% of the rainfall gets converted into runoff in this problem.

Rational Method

Example 3

Calculate the design peak discharge expected to occur once in 25 years at a site for constructing a drop structure to handle the runoff from a watershed comprising of 20 ha of cultivated land ($C = 0.5$), 15 ha of hilly forest land ($C = 0.3$) and 8 ha of rolling grass land ($C = 0.35$). The maximum length of flow is 2 km and the total fall along the path is 20 m. The maximum rainfall recorded for various durations in 25 years are given below in the table

Duration (min)	5	10	15	20	40	60
Depth of Rainfall (mm)	20	30	40	50	90	120

(GATE 1991)

In this case, we have 20 hectares of cultivated land, 15 hectares of hilly forest land, and 8 hectares of rolling grasslands, with runoff coefficient values of 0.3, 0.4, and 0.35 respectively. The maximum flow length is given as 2 kilometers, with a total fall along the path of 20 meters. The maximum rainfall recorded for various durations over 25 years is provided in the table. This question appeared in the GATE 1991 exam.

Rational Method

Solution:

- Weighted Runoff coefficient,

$$C_w = \frac{\sum_{j=1}^n C_j A_j}{\sum_{j=1}^n A_j} = \frac{0.5 \times 20 + 0.3 \times 15 + 0.35 \times 8}{20 + 15 + 8} = 0.4$$

- Maximum length of flow (L) = 2 km = 2000 m
- Elevation difference between remotest point on the watershed and outlet (ΔH) = 20 m
- Slope of the watershed (S) = $\frac{\Delta H}{L} = \frac{20}{2000} = 0.01$
- Time of concentration of a watershed can be calculated as

$$T_c = 0.01947 L^{0.77} S^{-0.385} = 0.01947 (2000)^{0.77} (0.01)^{-0.385} = 39.92 \text{ min}$$

Land use	Area (ha)	Runoff coefficient
Cultivated area	20	0.5
Hilly forest land	15	0.3
Rolling grass land	8	0.35

Since we have a watershed with multiple land uses, we need to calculate the weighted runoff coefficient. Given the areas and corresponding coefficients, we can obtain the numerator by summing the products of each coefficient and area, while the denominator is the sum of all areas. By applying these values, we find that the runoff coefficient is 0.4.

The time of concentration for the watershed can be calculated using the Kirpich formula: $0.01947 \times L^{0.77} \times S^{-0.385}$. With L calculated as 2000 meters and S as 0.01, the time of concentration t_c is approximately 39.92 minutes.

Rational Method

Solution:

- Time of concentration $T_c = 39.92 \text{ min}$
- From Table by interpolation:

Duration (min)	5	10	15	20	40	60
Depth of Rainfall (mm)	20	30	40	50	90	120

Maximum depth of rainfall for 39.92 min duration = $\left((90 - 50) \times \frac{39.92}{20} \right) + 50 = 89.84 \text{ mm}$

- Average intensity (i) = $\frac{89.84 \times 60}{39.92} = 135.03 \text{ mm/h}$ or $3.75 \times 10^{-5} \text{ m/s}$
- Total Area (A) = $(20 + 15 + 8) \text{ ha} = 43 \text{ ha} = 43 \times 10^4 \text{ m}^2$
- The design peak discharge

$$Q_p = C I A = 0.4 \times (3.75 \times 10^{-5}) \times (43 \times 10^4) = 6.45 \text{ m}^3/\text{s}$$

Thus, the design peak flow rate for the site = 6.45 m³/s

To determine the rainfall intensity corresponding to this time of concentration, we interpolate between the values given in the table. The resulting runoff depth is 89.84 millimeters in 39.92 minutes, leading to an average intensity of 135.03 millimeters per hour or 3.75×10^{-5} meters per second.

We know the total area is 43 hectares, which equals 43×10^4 square meters. With C , i , and A known, we can use the equation $Q_p = C \cdot i \cdot A$ to calculate the peak runoff rate. Substituting the values, we find Q_p to be 6.45 cubic meters per second. This indicates the design peak runoff rate for constructing the drop structure in this watershed.

In conclusion, the rational method for determining peak runoff rates can be effectively applied under different conditions. To utilize this method, we need to ascertain the time of

concentration, rainfall intensity for the required return period, and the runoff coefficient C , for which tables of values are available for various conditions. Thank you for your attention. Please provide feedback and feel free to ask any questions in the forum. We are here to assist you. Thank you

