

Course Name: Watershed Hydrology

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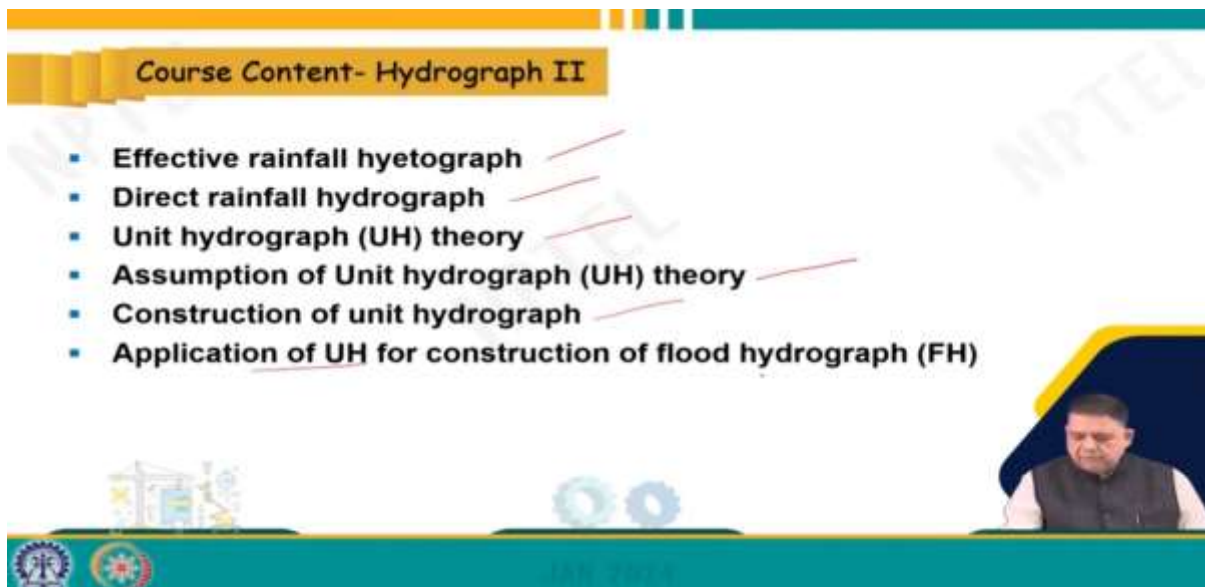
Institute Name: Indian Institute of Technology Kharagpur

Week: 05

Lecture 22: Hydrograph II



Hello friends, welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology Kharagpur. We are in Module 5, and this is Lecture Number 2, focusing on the topic of hydrograph part 2.



In this lecture, we will delve into the effective rainfall hydrograph. Direct runoff hydrograph was introduced in the previous lecture, and we will continue that discussion. Then, we will proceed to the unit hydrograph theory, assumptions of unit hydrograph theory, construction of

unit hydrograph, and how to apply the Unit Hydrograph (UH) for constructing flood hydrograph (FH). So, we'll explore the application of the unit hydrograph in this lecture.

EFFECTIVE RAINFALL HYETOGRAPH

- Effective rainfall or rainfall excess is obtained by subtracting the initial and infiltration losses from the total rainfall
- From a storm hyetograph, the initial and infiltration losses are deducted using the ϕ - index
- The resulting hyetograph is the **Effective Rainfall Hyetograph (ERH)**
- Area under ERH represents the direct runoff volume

Coming to the effective rainfall hydrograph, effective rainfall or rainfall excess is obtained by subtracting the initial and infill losses from the total rainfall, as we discussed in detail while exploring infiltration. In fact, we also discussed the ϕ -index as a part of infiltration indices, where we subtract the initial abstraction and other losses to represent the effective rainfall or rainfall excess (ERH) through the effective rainfall hydrograph. This graph shows only the rainfall hydrograph where rainfall intensity is plotted against time. By deducting the ϕ -index, which represents the average losses, from the total rainfall, we obtain the effective rainfall, depicted in light blue. This represents the effective rainfall hydrograph (ERH). The area under the ERH represents the direct runoff volume, measured in in-depth units.

DIRECT RUNOFF HYDROGRAPH

- Direct Runoff Hydrograph (DRH) is obtained by separating the baseflow from a flood hydrograph

The ERH gives us the direct runoff volume in-depth units. Then, the direct runoff (DRH), as discussed in the previous lecture, is obtained by separating the base flow from a flood hydrograph. Various methods were discussed for identifying the base flow, and once separated, the resultant graph is referred to as the direct runoff hydrograph. As you can see in the graph, the flood hydrograph is due to effective rainfall. The effective rainfall hydrograph is plotted as a rectangle. The horizontal axis shows the duration of effective rainfall (4 hours in this case), and the vertical scale represents the depth of effective rainfall (2 centimeters).

So, during this 4-hour rainfall event, 2 centimeters of effective rainfall occurred, leading to the generation of the flood hydrograph. Using the methods discussed earlier, we separated the base flow. Subtracting the base flow ordinates from the flood hydrograph ordinates, we obtained the direct runoff ordinates. This is the procedure. An important point to note is that the DRH always touches the x-axis because there is no base flow. So, if you're asked to draw a hydrograph in an examination or viva, remember that the DRH should touch the x-axis, indicating your understanding of the concept.

Both ERH and DRH represent the same runoff volume. We saw that the area under ERH gives us the direct runoff volume in-depth units. If we multiply this by the catchment area, we get the volume, which will be the same as the area under DRH. Therefore, ERH and DRH represent the same volume, a crucial point to remember.

DIRECT RAINFALL HYDROGRAPH

Example 1

The flood hydrograph ordinates at the outlet of a catchment due to a storm are given in the following table. If the area of the catchment is 8791.2 km², determine the depth of the direct runoff. Assume the baseflow as 30 m³/s.

Time (h)	0	6	12	18	24	30	36
Flood Hydrograph Ordinates (m ³ /s)	30	480	2060	4450	6010	6000	5080
Time (h)	42	48	54	60	66	72	78
Flood Hydrograph Ordinates (m ³ /s)	3996	2866	1866	1060	500	170	30

FH
Baseflow
FH - Baseflow = D.

Let's quickly consider an example: the flood hydrograph ordinates at the outlet of a catchment due to a storm are given in the following table.

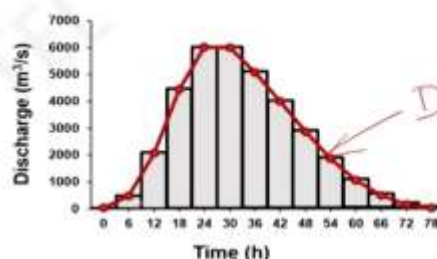
DIRECT RAINFALL HYDROGRAPH

Solution

Time (h)	Flood Hydrograph Ordinates (m ³ /s)	Baseflow (m ³ /s)	DRH Ordinates (m ³ /s)
0	30	30	0
6	480	30	450
12	2080	30	2050
18	4450	30	4420
24	6010	30	5980
30	6000	30	5970
36	5080	30	5050
42	3996	30	3966
48	2866	30	2836
54	1866	30	1836
60	1060	30	1030
66	500	30	470
72	170	30	140
78	30	30	0

DRH ordinates are obtained by subtracting the baseflow from the flood hydrograph ordinates

Next, we need to find the area under the hydrograph



DRH

Time intervals range from 0 to 78 hours, and the flood hydrograph starts at 30, 4, 6, 4, and 18, and so on, up to 30. If the area of the catchment is 8791.2 square kilometers, we can determine the depth of direct runoff, assuming the base flow is 30 cubic meters per second. Conceptually, we understand that the flood hydrograph minus the base flow gives us the DRH. Hence, we can determine the depth of direct runoff using this concept.

The flood hydrograph ordinates are provided for different times, ranging from 0 to 78 hours, with corresponding values. Subtracting the base flow value of 30, we obtain the DRH ordinates, as listed: 0, 450, 580, and so forth.

So, these are the DRH ordinates we have obtained by subtracting the base flow from the flood hydrograph ordinates. Next, we must find the area under the hydrograph. So, this is the DRH plot, which is essentially the plot of the values we have just obtained. Now, we have to find the area under this curve. The area under the DRH curve can be determined using vertical lines placed at different time intervals.

DIRECT RAINFALL HYDROGRAPH

Solution

Time (h)	DRH Ordinates (m ³ /s)
0	0
6	450
12	2030
18	4420
24	5980
30	5970
36	5050
42	3966
48	2836
54	1836
60	1030
66	470
72	140
78	0

Area under DRH =

$$(6 \times 3600) \left[\frac{1}{2} (450) + \frac{1}{2} (450 + 2030) + \frac{1}{2} (2030 + 4420) + \frac{1}{2} (4420 + 5980) + \frac{1}{2} (5980 + 5970) + \frac{1}{2} (5970 + 5050) + \frac{1}{2} (5050 + 3966) + \frac{1}{2} (3966 + 2836) + \frac{1}{2} (2836 + 1836) + \frac{1}{2} (1836 + 1030) + \frac{1}{2} (1030 + 470) + \frac{1}{2} (470 + 140) + \frac{1}{2} (140) \right]$$

$$= 738244800 \text{ m}^3$$

Direct Runoff Depth = $\frac{\text{Area under DRH}}{\text{Watershed area}} = \frac{738244800}{8791.2 \times 10^6} = 0.084 \text{ m} = 84 \text{ mm}$

Thus, the direct runoff depth in the catchment is 84 mm

Direct Runoff depth may also be determined using the following relationship:

$$\text{Direct Runoff (DR) (in cm)} = 0.36 \times \frac{\sum_{t=1}^N (\text{DRO}_t) \Delta t}{A}$$

Where, DRO = direct runoff ordinates, cumec; Δt = time interval between successive ordinates, h; and A = basin area, km²

We can calculate the area under the DRH using different methods for each segment. For example, the first segment is a triangle, which we can approximate by finding the area of the triangle. Then, for subsequent segments, we can use the trapezoidal rule until we reach the end. For the last two segments, we can use the triangular formula, similar to the slope area method discussed in streamflow measurement. By applying these methods, we can calculate the area under the curve, which comes out to be 738,244,800 cubic meters.

We know that the direct runoff depth is the area under the DRH divided by the watershed area. So, the runoff volume in volumetric units and the watershed area are given. By dividing the runoff volume by the watershed area, we get the direct runoff depth, which is 0.084 meters or 84 millimeters in this case.

Another way to determine the direct runoff depth is by using the following relationship, which is essentially a manipulation of the same rule but written differently for easier calculation. The direct runoff depth (DR) in centimeters can be calculated using the formula 0.36 times the summation of the DRH ordinates (DRO) multiplied by the time interval (ΔT), all divided by the basin area (A) in square kilometers. By summing up the DRH ordinates and applying this formula, we can also obtain the direct runoff depth, which will be the same as previously calculated.

UNIT HYDROGRAPH (UH)

- UH Concept – Sherman (1932)
- UH is the hydrograph of direct runoff resulting from one unit depth (1 cm or mm or inch) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D - h)
- The storm duration is used as a prefix to represent a specific unit hydrograph
 - e.g., 4-h unit hydrograph, or 6-h unit hydrograph

Now, let's delve into the concept of unit hydrograph (UH). The concept of UH was introduced by Sherman in 1932 and is a very useful tool in hydrograph analysis. A unit hydrograph represents the hydrograph of direct runoff resulting from one unit depth of rainfall excess occurring uniformly over the basin at a uniform rate for a specified duration (DH). When drawing a unit hydrograph, the effective rainfall depth must be set to one unit. The resulting graph is called a unit hydrograph, representing the hydrograph of direct runoff resulting from one unit depth of rainfall excess occurring uniformly over the basin at a uniform rate for a specified duration (DH). The storm duration is used as a prefix to represent a specific unit hydrograph, such as a 4-hour unit hydrograph or a 6-hour unit hydrograph.

So, this term duration is very important because it helps us identify different unit hydrographs. All unit hydrographs have a unit depth, and the only way to differentiate them is through this duration. Hence, we always prefix the duration when referring to unit hydrographs, such as a 4-hour unit hydrograph or a 6-hour unit hydrograph. For instance, a 4-hour unit hydrograph means that 1 unit depth of rainfall excess occurred in 4 hours. Similarly, a 6-hour unit hydrograph represents 1 centimeter of excess rainfall occurring in 6 hours.

UNIT HYDROGRAPH (UH)

ERH and DRH ordinates are divided by the effective rainfall depth - 2 cm (in this case)

Direct Runoff Hydrograph (DRH) due to unit depth of effective rainfall
UNIT HYDROGRAPH (UH)
4-h UH
4-h UH as the ER duration is 4-h

If we look at the effective rainfall hydrograph (ERH), it shows a duration of 4 hours and 2 centimeters. Here, ERH and DRH ordinates are divided by the effective rainfall depth, which is 2 centimeters. By doing this, we convert the effective rainfall magnitude to 1 centimeter, resulting in a unit hydrograph. The direct runoff hydrograph due to a unit depth of effective

rainfall is the unit hydrograph (UH). In this case, it is a 4-hour UH because 1 centimeter of excess rainfall occurred within a 4-hour duration.

We started with the flood hydrograph, separated the base flow, obtained the direct runoff hydrograph, and then converted the effective rainfall magnitude to 1 centimeter to derive the unit hydrograph. We use the duration prefix to represent the unit hydrograph. Flood hydrograph, direct runoff hydrograph, and unit hydrograph can all be obtained from the same graph, provided we know the procedures.

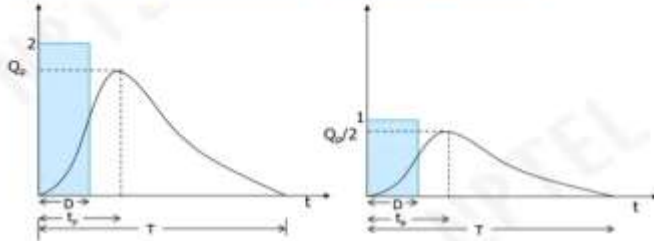
ASSUMPTIONS OF UH THEORY

1. The effective rainfall is uniformly distributed within its duration
2. The effective rainfall is uniformly distributed over the whole area of drainage basin
3. The base time of the direct runoff hydrograph due to an effective rainfall of unit duration is constant
4. The unit hydrograph reflects the basic effects of various physical characteristics of the basin, which do not change in time (time-invariance)
5. The response of the drainage basin is linear. This implies that the principles of proportionality and superposition are applicable

Unit hydrograph theory has certain important principles. Firstly, effective rainfall is assumed to be uniformly distributed within the duration. Secondly, effective rainfall is assumed to be uniformly distributed over the entire area of the drainage basin. These two principles were also discussed earlier when we talked about the rational formula for rainfall runoff. Thirdly, the base time of the direct runoff hydrograph due to an effective rainfall unit duration is constant. This means that the time base remains the same regardless of changes in other factors.

Unit hydrographs reflect the basic effects of various physical characteristics of the basin, which are assumed to be time-invariant. Time invariance is an important assumption, implying that a unit hydrograph developed today can be used in subsequent years unless conditions significantly change. The other two assumptions are related to the linear response of the drainage basin. Linear response implies that the principles of proportionality and superposition apply.

ASSUMPTIONS OF UH THEORY



Principle of proportionality

- According to the principle of proportionality, if the duration of ER is fixed but its volume changes, then the duration of DR does not change, but its ordinates do in proportion to the volume of ER

Principle of superposition

- The principle of superposition allows the decomposition of a complex ERH into rectangular blocks or pulses and then superimposing on one another the hydrograph of these rectangular pulses, each of steady intensity, to obtain the total DRH



According to the principle of proportionality, if the duration of effective rainfall is fixed but its volume changes, the duration of the direct runoff does not change. Instead, the ordinates adjust in proportion to the volume of effective rainfall. This principle is crucial for deriving unit hydrographs from direct runoff hydrographs.

So, when we have a D-hour duration effective rainfall with a certain magnitude, such as 2 or 4, we can divide the effective rainfall magnitude by that value to represent one unit depth. Correspondingly, the ordinates will also be divided. As we saw in the previous slide, we had a 4-hour direct runoff hydrograph (DRH), and from that, having 2 centimeters of effective rainfall, we obtained the 4-hour unit hydrograph (UH) by dividing the ordinates of the DRH by 2 to get the unit hydrograph ordinate. This illustrates the principle of proportionality.

The second principle is the principle of superposition, which allows the decomposition of a complex effective rainfall hydrograph (ERH) into rectangular blocks of pulses. These pulses, each of steady intensity, are then superimposed on one another to obtain the total DRH. This principle is applicable when we have a complex storm with different magnitudes in consecutive D hours. By decomposing the ERH into regular pulses and superimposing them, we can obtain the DRH. This principle is crucial for handling complex rainfall events.

UNIT HYDROGRAPH

Limitations

- Since, in practice, assumptions (1) and (2) are never satisfied, these form the limitations of unit hydrograph theory
- Unit hydrograph theory can be applied only for a basin having drainage area between 200 ha to 5,00,000 ha

Applications

- Development of flood hydrograph for extreme rainfall magnitudes for use in the design of hydraulic structures
- Extension of flood-flow records based on rainfall records
- Development of flood forecasting and warning systems based on rainfall

Principle of Proportionality
Principle of Superposition



Now, let's discuss the limitations of the unit hydrograph theory. In practice, assumptions 1 and 2 are rarely satisfied, forming the limitations of the unit hydrograph theory. As we discussed earlier, it's challenging to achieve a uniform distribution of rainfall over both time and area in real-world scenarios. Due to these limitations, the applicability of the unit hydrograph theory is constrained. It is recommended for basins with drainage areas ranging from 12,200 hectares to 5 lakh hectares.

Regarding applications, unit hydrographs are widely used. They are employed in the development of flood hydrographs of extreme rainfall magnitudes for designing hydraulic structures, primarily utilizing the principle of proportionality. By manipulating the ordinates of unit hydrographs through multiplication or division, we can obtain the ordinates of larger events, aiding in the design of hydraulic structures to accommodate anticipated flows. Additionally, unit hydrographs are utilized in extending flood flow records based on rainfall records, leveraging the principle of superposition.

The principle of superposition is crucial because it allows us to extend flood flow records by decomposing a longer rainfall event into slices of uniform distribution. By applying the principle of superposition, we can combine these slices to obtain the total hydrograph, which we'll explore in more detail later. Additionally, unit hydrographs are commonly used in the development of flood forecasting and warning systems, making them essential tools in hydrological analysis.

CONSTRUCTION OF UNIT HYDROGRAPH

1. From the past records, select an isolated storm, uniformly distributed over the basin.
2. Plot the storm hydrograph for the storm.
3. Separate the baseflow from the direct runoff.
4. Determine the direct runoff ordinates by subtracting the ordinates of baseflow from the storm hydrograph ordinates.
5. Calculate the direct runoff depth (DR, in cm) as follows:

$$\text{Direct Runoff (DR)} = 0.36 \times \frac{\sum_{i=1}^N (DRO_i) \Delta t}{A}$$
 Where DRO = direct runoff ordinates, cumec; Δt = time interval between successive ordinates, h; and A = basin area, km²
6. Calculate the UH ordinates by dividing the DRH ordinates by the direct runoff depth as

$$UHO_i = \frac{DRO_i}{DR}$$
 Where UHO = unit hydrograph ordinates

Next, let's discuss the construction of a unit hydrograph. From past records, we need to select an isolated storm that was uniformly distributed over the basin. It's crucial to ensure that the rainfall event is as uniform as possible both in terms of time and area. Once we've identified such a storm, we plot its hydrograph or flood hydrograph. Then, we separate the base flow from the direct runoff, as discussed earlier. By subtracting the base flow ordinates from the storm hydrograph ordinates, we obtain the direct runoff hydrograph.

Afterward, we calculate the direct runoff depth in centimeters, either using the trapezoidal rule or the formula we discussed earlier. Once we have the direct runoff depth, we can derive the ordinates of the unit hydrograph by dividing the direct runoff ordinates by this depth. This process allows us to obtain the unit hydrograph, which is crucial for further hydrological analysis.

UNIT HYDROGRAPH

Example 2

The observed flows from a storm of 6-h duration on a stream having a catchment area of 500 km² is given in the following table. Assuming baseflow to be 10 m³/s, derive the ordinate of a 6-h unit hydrograph.

Time, (h)	Discharge, (m ³ /s)	Time, (h)	Discharge, (m ³ /s)
0	10	42	50
6	100	48	35
12	250	54	25
18	200	60	20
24	150	66	15
30	100	72	10
36	70		

Let's consider an example to illustrate this process. We have observed flows from a storm of 6-hour duration on a stream with a catchment area of 500 square kilometers.

Solution

Firstly, the baseflow is deducted from the discharge to obtain the DRH ordinates (Table)

Time (h)	Discharge (m ³ /s)	Baseflow (m ³ /s)	Direct Runoff (m ³ /s)	Unit hydrograph (m ³ /s)
0	10	10	0	0
6	100	10	90	23
12	250	10	240	62
18	200	10	190	49
24	150	10	140	36
30	100	10	90	23
36	70	10	60	15
42	50	10	40	10
48	35	10	25	6
54	25	10	15	4
60	20	10	10	3
66	15	10	5	1
72	10	10	0	0

Next, the Direct Runoff depth, DR, is determined as,

$$\text{Direct Runoff (DR)} = 0.36 \times \frac{\sum_{t=1}^N (DRO_t) \Delta t}{A}$$

$$\text{Here, } \Delta t = 6h; A = 500 \text{ km}^2$$

From Table,

$$\sum_{t=1}^N (DRO_t) = 905 \text{ m}^3/\text{s}$$

$$\text{Direct Runoff (DR)} = 0.36 \times \frac{905 \times 6}{500} = 3.9 \text{ cm}$$

Unit hydrograph ordinates (UHO_t) = $\frac{DRO_t}{DR}$

Assuming the base flow to be 10 cubic meters per second, we need to derive the ordinates of a 6-hour hydrograph. By subtracting the base flow from the observed flows, we obtain the direct runoff hydrograph. Then, using the provided formula and the given data, we can calculate the direct runoff depth to be 3.9 centimeters. Finally, by dividing the direct runoff ordinates by this depth, we can obtain the ordinates of the unit hydrograph. This process enables us to characterize the hydrological response of the catchment to the given storm event.

So, this is simply 90 divided by 3.9, which gives us approximately 23.08, but when converted to integers, we have 0, 23, 62. Thus, we can plot the hydrograph. It's a straightforward process: from the flood hydrograph, we subtract the base flow to obtain the direct runoff, calculate the direct runoff depth using the provided relationship, and then divide the direct runoff ordinates by this depth to obtain the unit hydrograph ordinate. This process allows us to generate a unit hydrograph from a given flood hydrograph.


APPLICATION OF UH FOR CONSTRUCTING A FLOOD HYDROGRAPH (FH)

- UH can be used to construct a flood hydrograph resulting from rainfall of same unit duration for which the UH is available.
- UH selected for computing the FH should be corresponding to storm of like duration and pattern (theoretically).
- However, tolerance of as much as 25% of the UH duration can be ordinarily be accepted without any serious error.

3-h UH
 4-h UH
 3-5 h Flood Hydrograph

Calculation procedure for constructing the flood hydrograph due to n cm of rainfall excess using a known UH

Date	Time (h)	UH ordinates (m ³ /s)	Base flow (m ³ /s)	DRH ordinates (m ³ /s)	FH ordinates (m ³ /s)
(1)	(2)	(3)	(4)	(5) = $n \times$ (3)	(6) = (4) + (5)
		Known	Known		



Now, let's discuss the application of the unit hydrograph for constructing a flood hydrograph. This is essentially an inverse problem: instead of starting with a flood hydrograph and deriving a unit hydrograph, we begin with a unit hydrograph and construct a flood hydrograph. The unit hydrograph can be used to construct a flood hydrograph resulting from a rainfall of the same unit duration for which the unit hydrograph is available. It's essential to remember that the unit hydrograph duration should match the duration of the rainfall event, with a tolerance of up to 25 percent of the unit hydrograph duration typically accepted without serious errors. For example, a 4-hour unit hydrograph can be applied for developing flood hydrographs for rainfall events lasting 3 to 5 hours, within this tolerance range.

The calculation procedures are quite straightforward: we have the unit hydrograph ordinates, the effective rainfall depth (in centimeters), and the known base flow values. To develop the direct runoff hydrograph (DRH) ordinates, we multiply the unit hydrograph ordinates by the effective rainfall depth. Once we have the DRH ordinates, we sum them with the base flow to obtain the flood hydrograph ordinates.

APPLICATION OF UH FOR CONSTRUCTING A FLOOD HYDROGRAPH (FH)


Solution

Rainfall-excess of 4 cm

Time (h)	6-h UH ordinates (m ³ /s)	DRH ordinates (m ³ /s)	Baseflow (m ³ /s)	FH ordinates (m ³ /s)
0	0	0 × 4 = 0	25	0 + 25 = 25
6	20	20 × 4 = 80	25	80 + 25 = 105
12	60	240	25	265
18	150	600	25	625
24	120	480	25	505
30	90	360	25	385

Rainfall-excess of 4 cm

Time (h)	6-h UH ordinates (m ³ /s)	DRH ordinates (m ³ /s)	Baseflow (m ³ /s)	FH ordinates (m ³ /s)
36	66	264	25	289
42	50	200	25	225
48	32	128	25	153
54	20	80	25	105
60	10	40	25	65
66	0	0	25	25



Let's consider an example: given the ordinates of a 6-hour unit hydrograph and a rainfall excess of 4 centimeters, along with a base flow of 25 cubic meters per second, we can calculate the flood hydrograph due to a storm. By multiplying the unit hydrograph ordinates by 4 and

summing them with the base flow, we obtain the flood hydrograph ordinates, which complete the hydrological analysis.

In the case of a unit hydrograph, we need to standardize the unit depth to 1. This involves using the principle of proportionality: dividing the direct runoff hydrograph (DRH) ordinates by the direct runoff depth. This yields the unit hydrograph ordinates. The reverse process is equally straightforward: given a unit hydrograph, you multiply the unit hydrograph ordinates to obtain the DRH ordinates, then add the base flow to derive the flood hydrograph ordinates. The procedure is relatively simple, and I trust that after reviewing it, you'll be able to tackle problems confidently. If you have any questions or uncertainties, don't hesitate to ask. We're here to help clarify any doubts you may have. Thank you.

