Course Name: Watershed Hydrology

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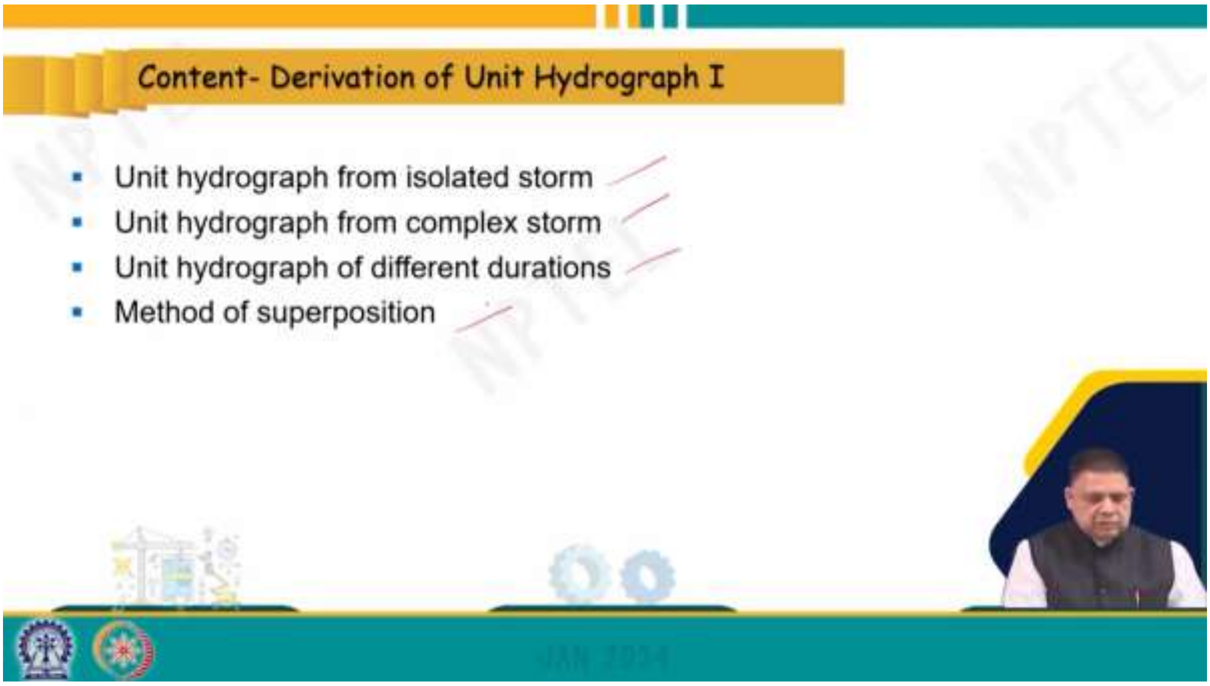
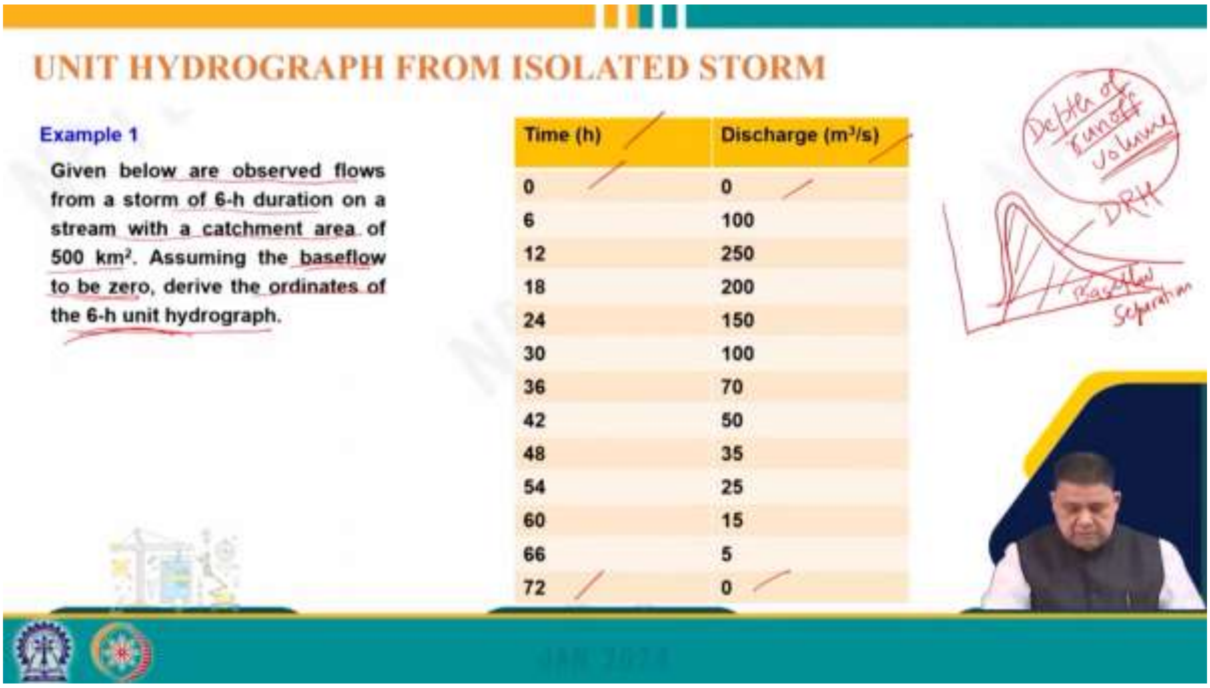
Week: 05

Lecture 23: Derivation of Unit Hydrograph I

Hello friends, welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the department of agriculture and food engineering at the Indian Institute of Technology Kharagpur. We are in module 5, and today we are having lecture number 3, and the topic is the derivation of unit hydrograph, this is part 1.



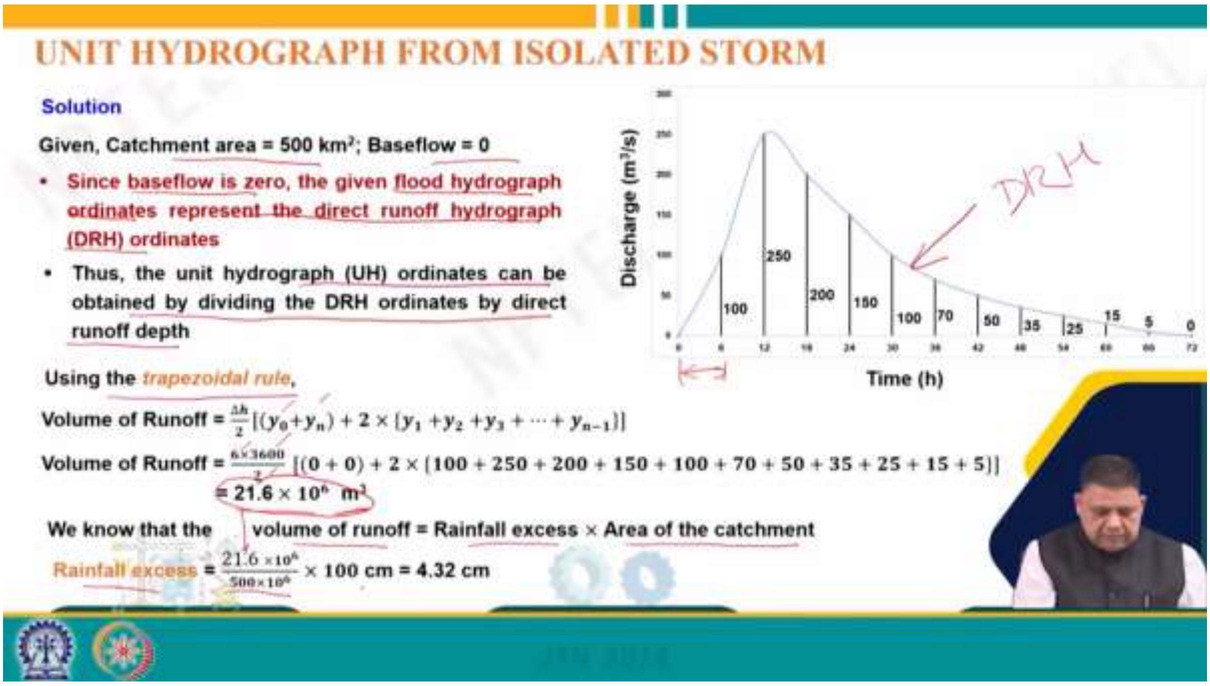
Now, in this particular lecture, we will discuss how to isolate and develop a unit hydrograph from isolated storms, then a unit hydrograph from complex storms, then a unit hydrograph of different durations, and the method of superposition. So, these are the things we are going to discuss in this lecture.



Now, before we move forward, I would like to just take a problem so that we are in tune with what we discussed in the previous class, so that it will be easier for us to move forward.

Let us take an example: given below are observed flows from a storm of 6-hour duration on a stream with a catchment area of 500 square kilometres. Assuming the base flow to be 0, derive the ordinates of the 6-hour unit hydrograph. So, we have been given a time from 0 to 72 hours, and we have been given discharge in cubic meters per second, 0 at 0 and 0 at 72 hours. So, we are being told that this is a flood hydrograph, but the base flow is 0.

So, just to recall what we discussed yesterday, once we have a flood hydrograph given, the first thing we do is base flow separation. And then we get the resulting curve called the direct runoff hydrograph (DRH). It will start from the x-axis, and then what we do is we try to find out the depth of runoff volume by finding the area under the DRH. So, this is the DRH. We try to find out the depth of runoff volume, and then that is in depth units. So, we divide the ordinates of the DRH by the depth of runoff volume in order to get the unit hydrograph ordinates. So, that is the process we follow, and this is what we will discuss.



Here we have been given that the catchment area is 500 square kilometres and the base flow is 0. Since the base flow is 0, the given flood hydrograph ordinates basically represent the direct runoff hydrograph because we deduct the base flow, but since the base flow is not there (it is 0), whatever is given to us is a direct runoff hydrograph, and this is what we have plotted here.

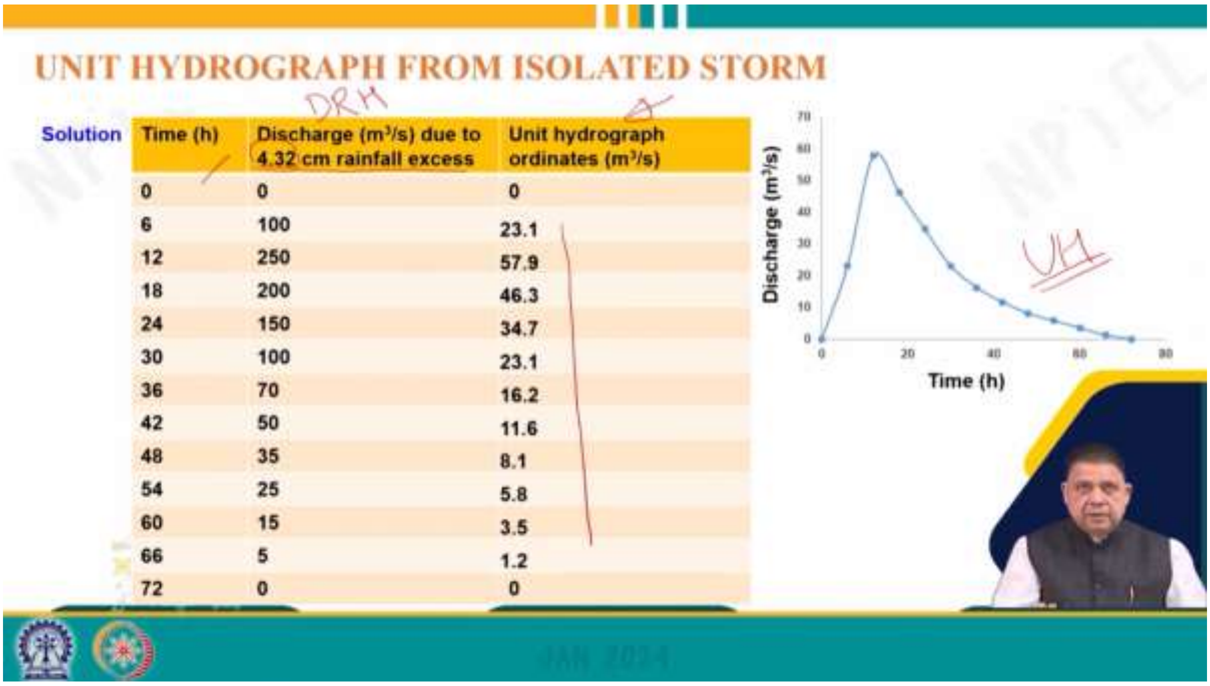
So, this is nothing but the DRH, which stands for direct runoff hydrograph. Unit hydrograph ordinates can be obtained by dividing the DRH ordinates by the direct runoff depth, as we discussed earlier, in depth units. We have seen that the area under the curve can be found out using the trapezoidal rule. That means,

$$volume\ of\ runoff = \frac{\Delta h}{2}[(y_0 + y_n) + 2 \times \{y_1 + y_2 + y_3 + \cdots + y_{n-1}\}]$$

which is the time interval (here it is 6 hours in this case). So, we are using 6 multiplied by 3600 by 2 in seconds, and then for the first and last ordinates, we just take them up because it is a triangle.

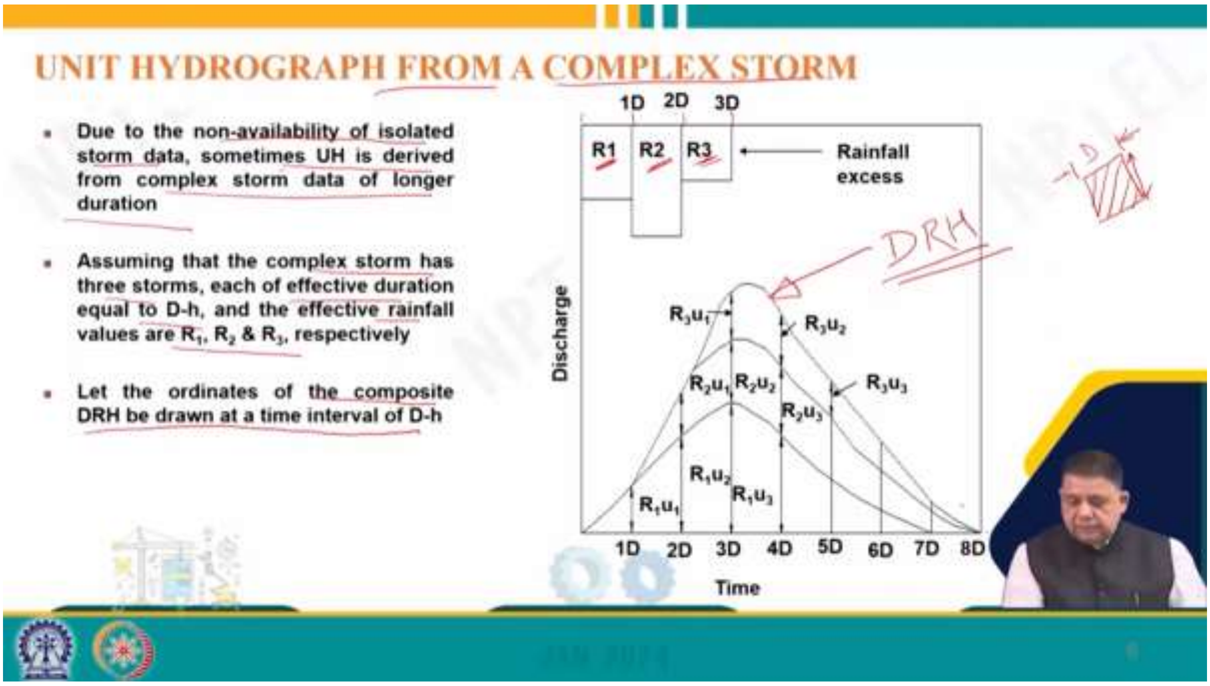
So, half into base into this ordinate will be taken, but for all others, it is the trapezoidal rule. So, basically, this is the generalized formula, and if we put the values here, we will get the value of the volume of runoff as 21.6 multiplied by 10 to the power of 6 cubic meters. And we know that the volume of runoff is equal to the rainfall excess multiplied by the area of the catchment. So, the area of the catchment is already given to us, and the volume of runoff we have already found out.

So, from here, rainfall excess is the volume of runoff, which is 21.6 multiplied by 10 to the power of minus 6 cubic meters, as shown here. The area of the catchment is 500 square kilometres, which we are converting into square meters. Also, we are converting it into depth because typically we use 1 centimetre as the unit of effective rainfall. That is why the rainfall excess we get is 4.32 centimetres in this given case.



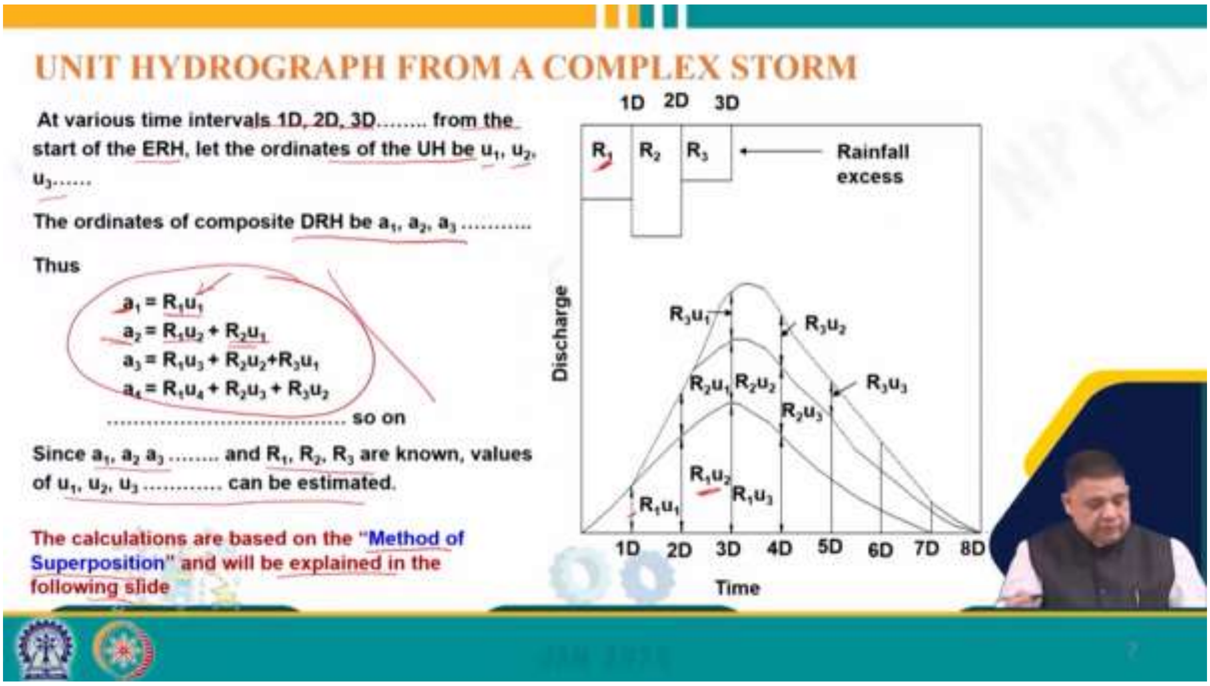
Now, we have time and we have discharge due to the 4.3 centimetres of direct runoff rainfall excess, which is nothing but the DRH. These ordinates are nothing but the DRH ordinates given to us. So, for getting the unit hydrograph ordinates, we have to divide each ordinate by this effective rainfall depth, which is 4.32. That is what it is.

So, these are the unit hydrograph ordinates we get by dividing these ordinates by 4.32, the effective rainfall depth, or volume of direct runoff in depth units, and this is the resultant unit hydrograph plotted here. This is the process, and you have to remember this process because we will be following this process throughout the further discussions as well.

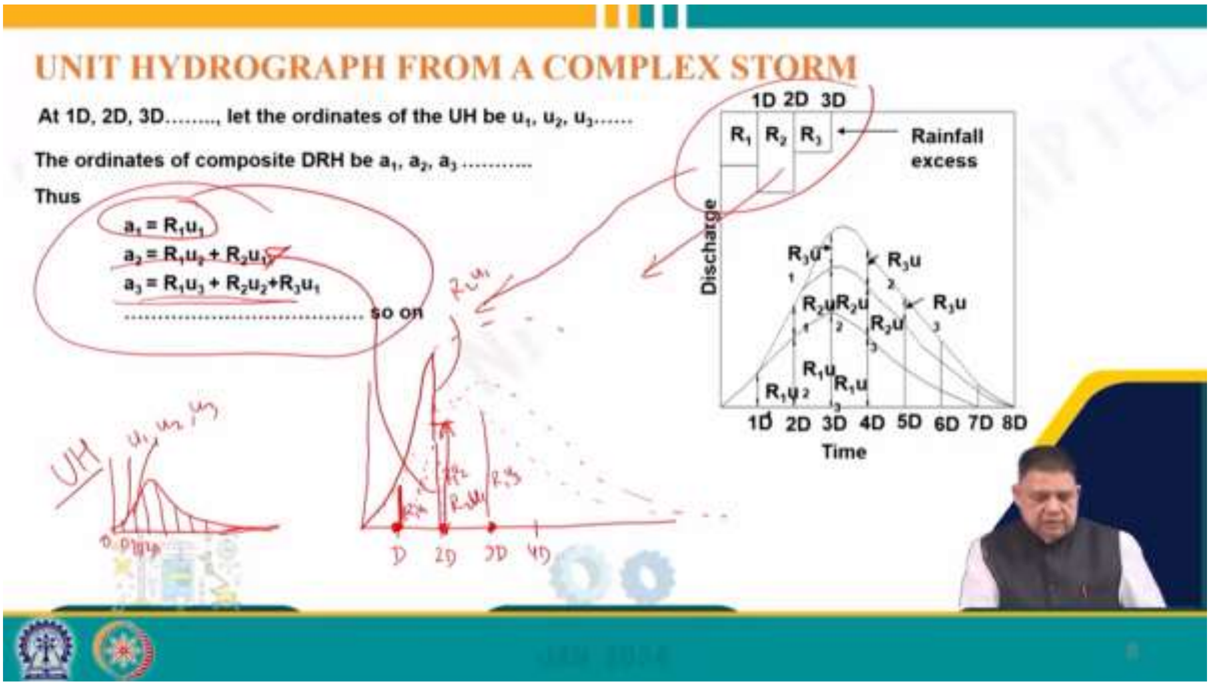


Now, coming to the unit hydrograph from a complex storm. So, when we discussed defining a flood hydrograph, we call it an isolated storm when the duration is, say, certain hours (D) and it has some effective rainfall depth.

So, if a rainfall event is such that it cannot be isolated due to the non-availability of isolated storm data, sometimes the unit hydrograph (UH) is derived from complex storm data of longer duration. So, let us say that here we have a complex storm which has a duration of 3D hours. That means it can be broken into 3 slices or 3 hydrographs, each of D-hour duration but having different effective rainfall magnitudes, that is R1, R2, and R3. This is the rainfall, and we have also been given the result, the direct runoff hydrograph, which is the result of this complex storm. Assuming that the complex storm has 3 storms, each of effective duration of d hours and effective rainfall values of R1, R2, and R3, and let the ordinates of the composite DRH be drawn at time intervals d. So, we are drawing the composite DRH, these are the ordinates which we are drawing.



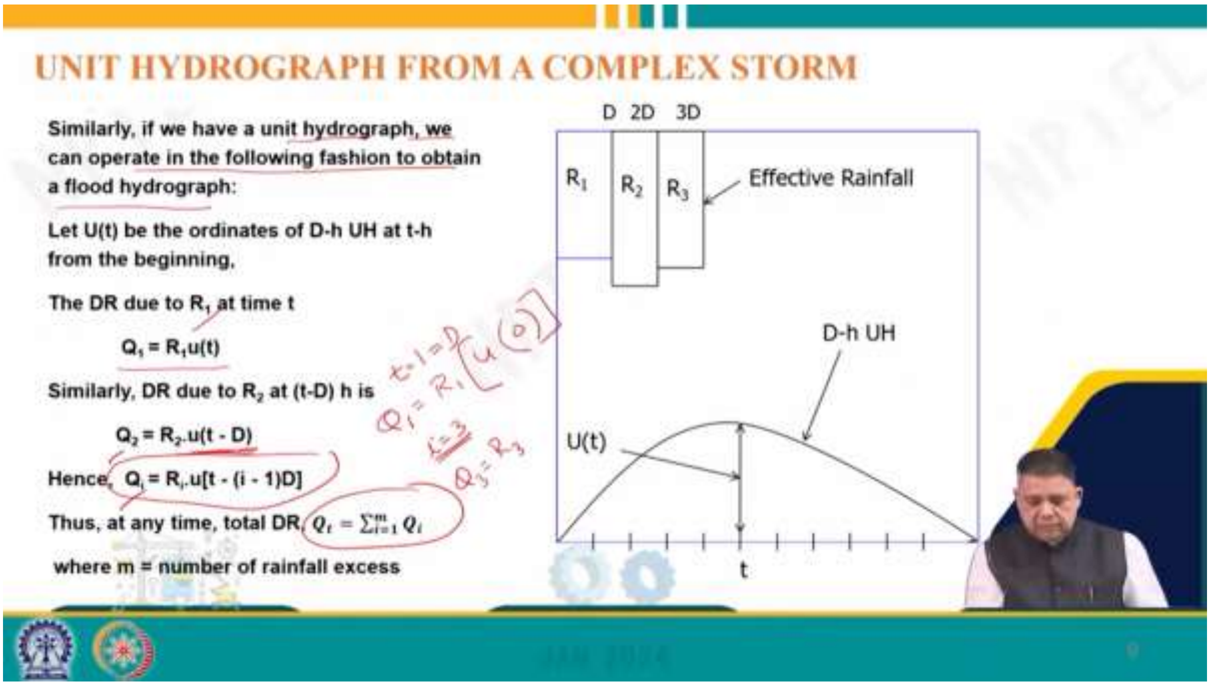
Now, at various time intervals 1D, 2D, and 3D from the start of the ERH, let the ordinates of UH be U1, U2, U3, that is, the UH which we want to find out, the UH ordinates which are U1, U2, U3. The ordinates of the composite DRH will be basically, mathematically, they can be represented like this: a1, which is the first ordinate, is R1U1, where R1 is the effective rainfall of the first duration and U1 is the ordinate at 1D hour. So, this is R1U1, in the second hour, 2D, there will be 2 components, one is R1U2 because of the effective rainfall R1, and there will be a component because of the second effective rainfall, R2U1, and so on. This is how mathematically we will get a system of equations, and here, because a1, a2, a3 are known, as we know the ordinates of the direct runoff hydrograph, and we also know R1, R2, R3, which are the effective rainfall magnitudes. So, obviously, the only unknowns are U1, U2, U3, which are the ordinates of the UH. So, we can solve these and can estimate it. Basically, these calculations that we are seeing here, the system of equations we are seeing, come from the application of the method of superposition. You remember, in one of the assumptions of UH theory was that the method of superposition, the principle of superposition, holds good. So, the method of superposition, and we will see just in the following slide and also later on in the lecture, this method of superposition in great detail.



But let us try to understand this, basically what happens is that suppose you have a unit hydrograph, say U, its ordinates are U1, this is the unit hydrograph. Then its ordinates at different times are U1, U2, U3, U1, U2, U3, these are the ordinates of this unit hydrograph which we are trying to find out. I am just trying to explain it in the converse of the problem we are trying to handle.

So, we have a unit hydrograph already, and then let us say that we have a complex system which is of this nature and we have to operate this on this unit hydrograph. So, what happens is that we basically, in order to get the DRH, I mean as per method of superposition, we superimpose unit hydrographs multiplied by this effective rainfall with each separated from the previous one or each lagging the previous one by D hours. So, basically, if this is our unit hydrograph and this is our effective rainfall, then basically if we want to, in the process of developing the DRH, suppose this is D, this is 2D, this is 3D, this is 4D, and so on, so this is 0D, 2D, 3D, we know the ordinates. So, basically, the first one will be because this hydrograph will be starting at 0. So, basically, here 0 and then here the ordinate which is ordinate of the due to this R1 will be R1U1 and at this point it will be because of the first one it will be like this, let us say we plot something like this and the peak is around 3. So, we also take the peak at 3 and something like this it will be there. So, because of this, this will be R2R1U2 and this will be R1U3 at different times. Then we take up this R2 impact of this R2 and that will come a hydrograph which starts at D actually D hour lag will be there. So, that it starts at D and this will be something like this.

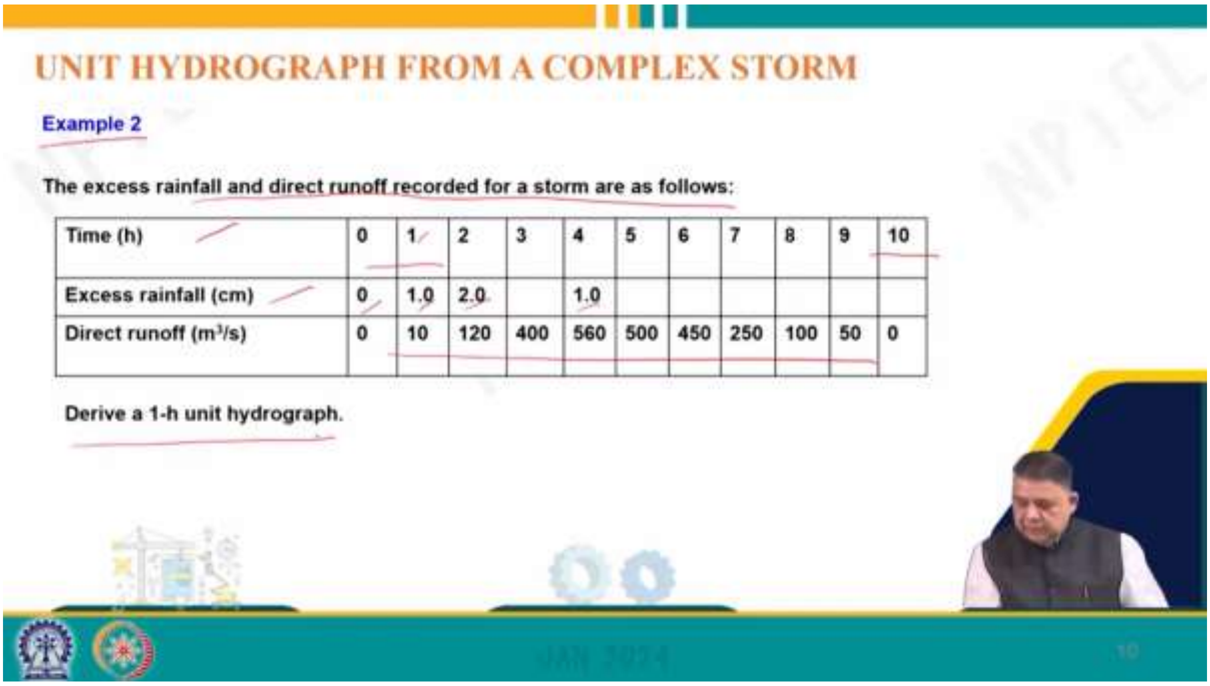
So, at this hour, the ordinate is 0 and that is why at this point only R1 is contributing. So, only R1 is contributing in this hour, but when we come to 2D hours, there are 2 ordinates: one because of this, which is R2U1, that is because of the first at 1 hour, 1D hour, whatever ordinate that will be multiplied, and then we have another one which is here, which is R2U1. So, this is here. So, here basically the sum will come here. So, basically, this direct runoff hydrograph will be like the sum of this, like this. So, this is R1U2 and this is equivalent to R2U1 and so on. So, this is how it will work. So, that is why at 2 hours, 2 to the third, the third one will start from here. So, it so, at 2D, 2D it will be 0, but 3D there will be 3, and then subsequently it will be like that, and that is how the principle of superposition works. And because of that, we are getting this system of equations where R1, R2, R3, and a1, a2, a3 are known; only unknowns are U1 which can we can solve.



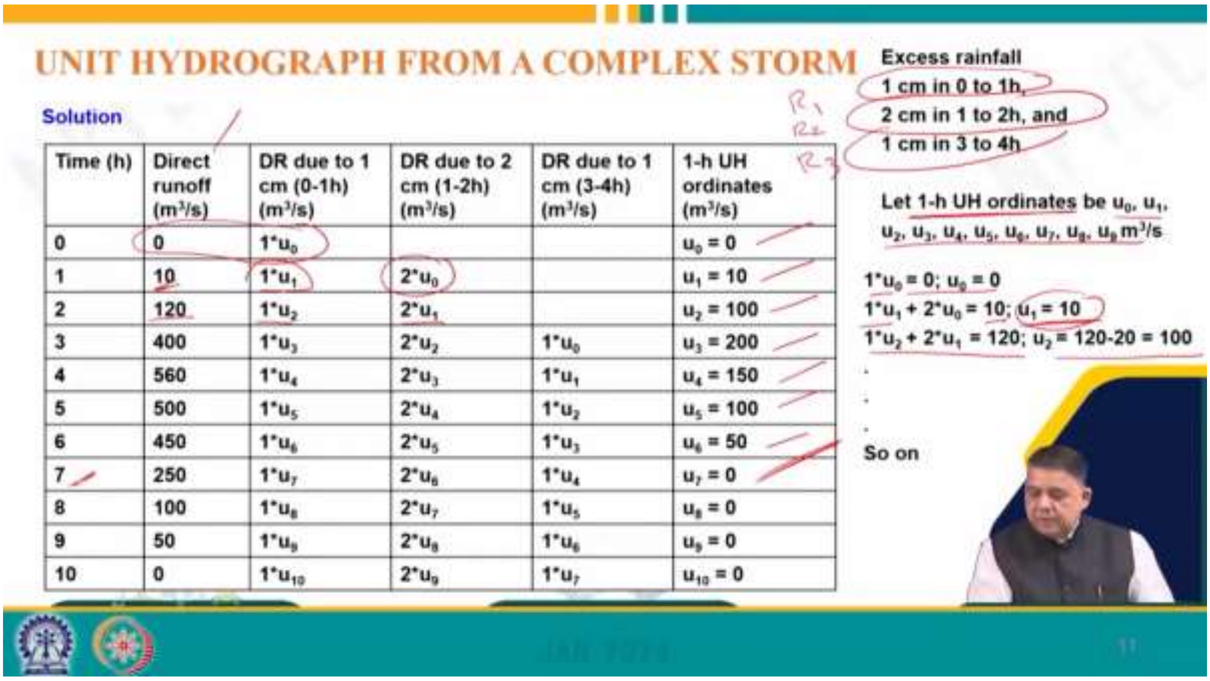
So, the same thing just now we discussed is the same thing that if we have a unit hydrograph and we can operate in the following fashion to obtain a flood hydrograph. So, it is the same thing here, R1 is U1, U2, Q2, if you do it directly because you have only one and we do not want to do. So, this basically the summation will come here and then it will be taken care of. So, because of R2U(t-D) is being done, because there is a lag of D hours and then. So, basically, this is the relationship which is used, which is the same as what we have just now discussed, that is when Q equals to 1, its t equals to 1, or when equal to D.

So, only one will come here. So, $Q_1 = R_1[t-(i-1)D]$

So, it is 0. So, it is only U0, this is like this, and so on. So, that is how we can work out, and then we will be having the sum of. So, if we say 3, I equal to 3, let us say, then Q3 will be R3, basically it will be sum. So, that is why it will be R1R2R3 all will be coming. So, Q1 will be this Q2 and Q3 all 3 sums will be coming actually just now we saw, like we discussed, and we will things will be very clear when we take up the problems.

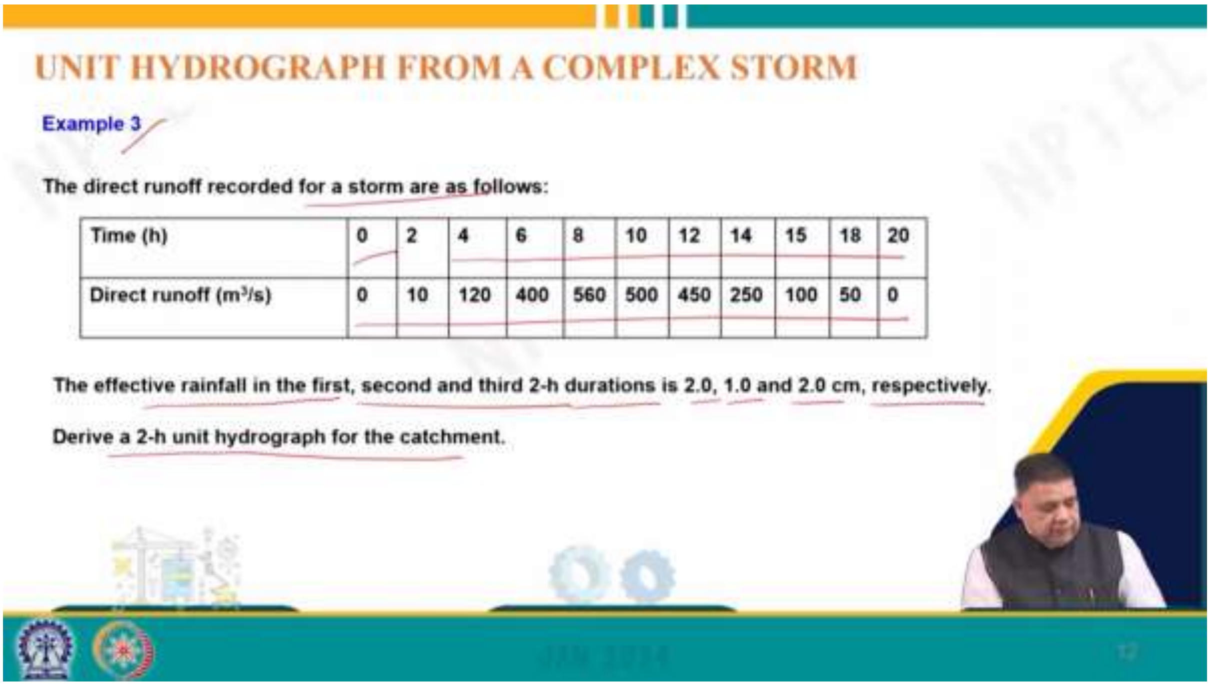


So, let us take example 2, the excess rainfall and direct runoff recorded for a storm is as follows: time is given from 0 to 10 hours, excess rainfall is given here, from 0 to 0 at 1 hour it is 1, at 2 hours it is 2, and at 4 hours it is 1, and direct runoff ordinates are given to us, we have to derive a 1-hour unit hydrograph.

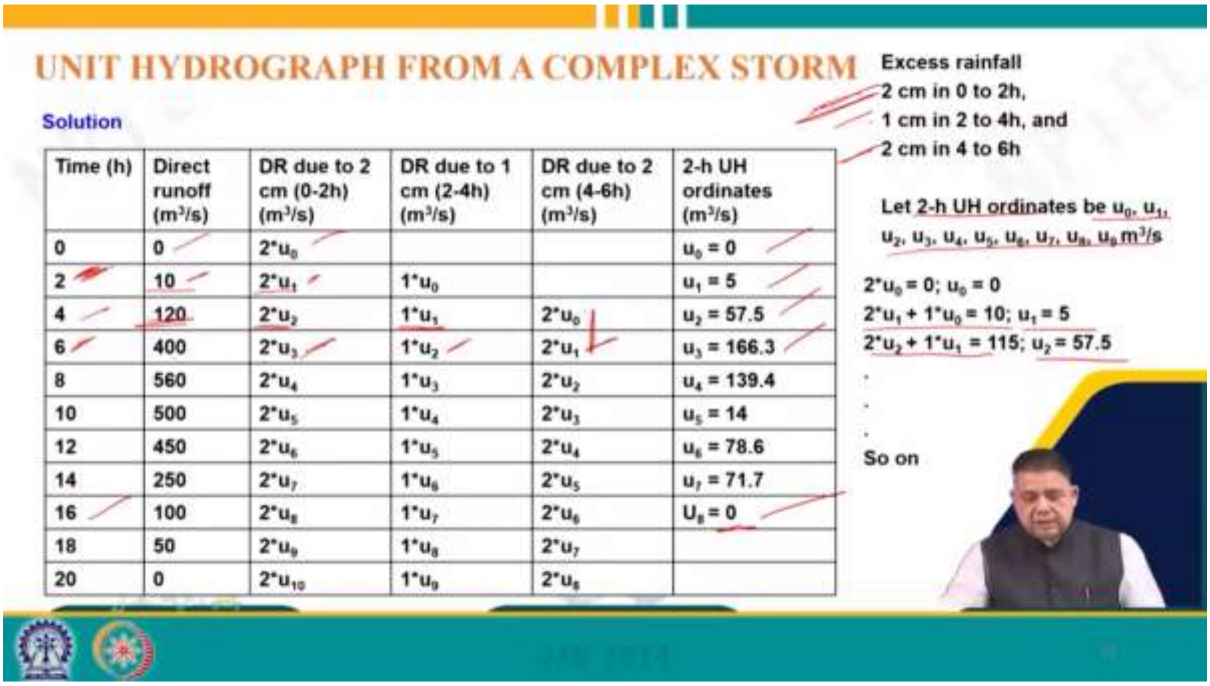


So, direct runoff ordinates are already given here. So, now due to 1 hour, that is this is the formulation we have to remember. So, it is here 1 centimetre, from 0 to 1, 2 centimetres. So, these are your values of R1, R2, and R3, and U1, U2, U3 are the ordinates of the unit hydrograph which we want to derive, 1-hour unit hydrograph because the 1-hour direct runoff hydrograph is given. So, at the first time we discuss, it is only 0. So, nothing will happen, but this one also 1 U1, but 2 of U0 because U0 just starts, we just plotted that at this hour it will start, so it will not have any contribution. So, at this hour, at 1 hour, we will have only ordinate because of this first excess rainfall. So, that is why R1U1, and so, R1U1, this is 0, R1U1 is 10. So, from there we get the value of U1 equal to 10. Then at 2 hours, there will be the sum of 2 ordinates, the first one, second ordinate, and second rainfall, first ordinate, and so, 1U2 plus 2U1 is 120, and we already know the value of Un.

So, U1, so, basically, we can solve this and then you will get a U2 value equal to 100, and that is how we can solve all these equations and can get the values of U0 is 0, U1 is 10, U2 is 100, it is a system of linear equations basically we have solved. U3 is 200, then of course, from this hour it will be all sum of all 3 unless it gets 0. So, it gets 0 here. So, the unit hydrograph is having ordinate only up to 7 hours. So, it is here from here it is 0. So, because of these 3 effective rainfalls operating on this unit hydrograph, if the right runoff we will get is this, and this is because of the method of superposition or principle of superposition, that is how we will get the answer.



We can take yet another example, example 3 let us say, and then the direct runoff recorded for a storm are given. Here, the time is given in 2-hour intervals, direct runoff magnitudes are given, and the effective rainfall in the first, second, and third 2-hour durations are 2, 1, and 2 centimetres. Derive a 2-hour unit hydrograph for the catchment.



UNIT HYDROGRAPH FROM A COMPLEX STORM

Solution

Time (h)	Direct runoff (m^3/s)	DR due to 2 cm (0-2h) (m^3/s)	DR due to 1 cm (2-4h) (m^3/s)	DR due to 2 cm (4-6h) (m^3/s)	2-h UH ordinates (m^3/s)
0	0	2^*u_0			$u_0 = 0$
2	10	2^*u_1	1^*u_0		$u_1 = 5$
4	120	2^*u_2	1^*u_1	2^*u_0	$u_2 = 57.5$
6	400	2^*u_3	1^*u_2	2^*u_1	$u_3 = 166.3$
8	560	2^*u_4	1^*u_3	2^*u_2	$u_4 = 139.4$
10	500	2^*u_5	1^*u_4	2^*u_3	$u_5 = 14$
12	450	2^*u_6	1^*u_5	2^*u_4	$u_6 = 78.6$
14	250	2^*u_7	1^*u_6	2^*u_5	$u_7 = 71.7$
16	100	2^*u_8	1^*u_7	2^*u_6	$U_8 = 0$
18	50	2^*u_9	1^*u_8	2^*u_7	
20	0	2^*u_{10}	1^*u_9	2^*u_8	

Excess rainfall
2 cm in 0 to 2h,
1 cm in 2 to 4h, and
2 cm in 4 to 6h

Let 2-h UH ordinates be u_0, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9 m^3/s

$2^*u_0 = 0$; $u_0 = 0$

$2^*u_1 + 1^*u_0 = 10$; $u_1 = 5$

$2^*u_2 + 1^*u_1 = 115$; $u_2 = 57.5$

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So on

So, it is the same procedure here; we have R1, R2, and R3 values as 2 centimetres, 1 centimetre, and 2 centimetres, and 2-hour UH ordinates we are calling as U0, U1, U2, U3. So, of course, at 0 hours it will be nothing, so it is 0, but at 2 hours, that is D, that is D2 because it is a D-hour unit hydrograph. So, it is a 2-hour unit hydrograph, that is why D equals to 3.

So, in 2 hours, only the first effective rainfall will produce runoff. So, it will be 2*U1 equal to 10. So, 2*U1 equal to 10. So, U1 will come out to 5. At 4 hours, when the direct runoff ordinate is 120, there will be 2 contributions because of the first one, first effective rainfall, and because of the second effective rainfall. As I said, this one starts at 2 hours, so the only first coordinate comes only at 4 hours, at 2 hours it is 0, at 4 hours the first ordinate comes.

So, that means, 2*U2 plus 1*U1 is 120. So, from here, you get U2 equals to 57.5. When it comes to 6 hours, then we will have 3 contributions because of the third ordinate of the unit hydrograph, because of the second ordinate of the unit hydrograph, and because of the first turn it did, because it was started at 4. So, the first contribution will come at 6 hours. So, that means, 2*U3 plus 2*U2 plus 2*U1 is 400, and we know the values of U1, U2. So, putting that value, you can solve, and that is how we can solve the system of linear equations, and we will get U0 is 0, 5, 57.5, 166.3, and so on, and then U8, that is 16 hours, it becomes 0. So, that is our unit hydrograph ordinate which we calculated by solving this system of linear equations, and this we have seen that this comes from the principle of superposition.

UNIT HYDROGRAPH FROM A COMPLEX STORM

Example 4

Ordinates of a 6-h UH are given below.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
6-h UH Ordinates (cumec)	0	50	125	185	160	110	60	36	25	16	8	0

Derive the flood hydrograph for the following storm

Time from the beginning of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

Assume Φ-index as 0.25 cm/h. The baseflow is 15 cumec in the beginning and increases by 2 m^3/s for every 12 h until the end of DRH.

We can take yet another example. So, ordinates of a 6-hour unit hydrograph are given below: at 0 hours to 66 hours, 6-hour UH ordinates are given this time. Derive the flood hydrograph for the following storm time from the beginning of a storm, that is 0, 6, 12, and 18 hours, accumulated rainfall values are given 0, 3.5, 11, and 16.5. The Φ-index value is given as 20.25 centimetres per hour; the base flow is 15 cumec in the beginning and increases by 2 cubic meters per second for every 12 hours until the end of TRS. So, we have given all information and then we have to summarize those first.

UNIT HYDROGRAPH FROM A COMPLEX STORM

Solution

As Φ-index as 0.25 cm/h

Time from the beginning of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5
Rainfall during the 6-h period (cm)	0	3.5	7.5	5.5
Effective rainfall (cm)	0	3.5-6*0.25 = 2	7.5-6*0.25 = 6	5.5-6*0.25 = 4

Rainfall - loss - Φ-index * duration

5.5 - 0.25×6

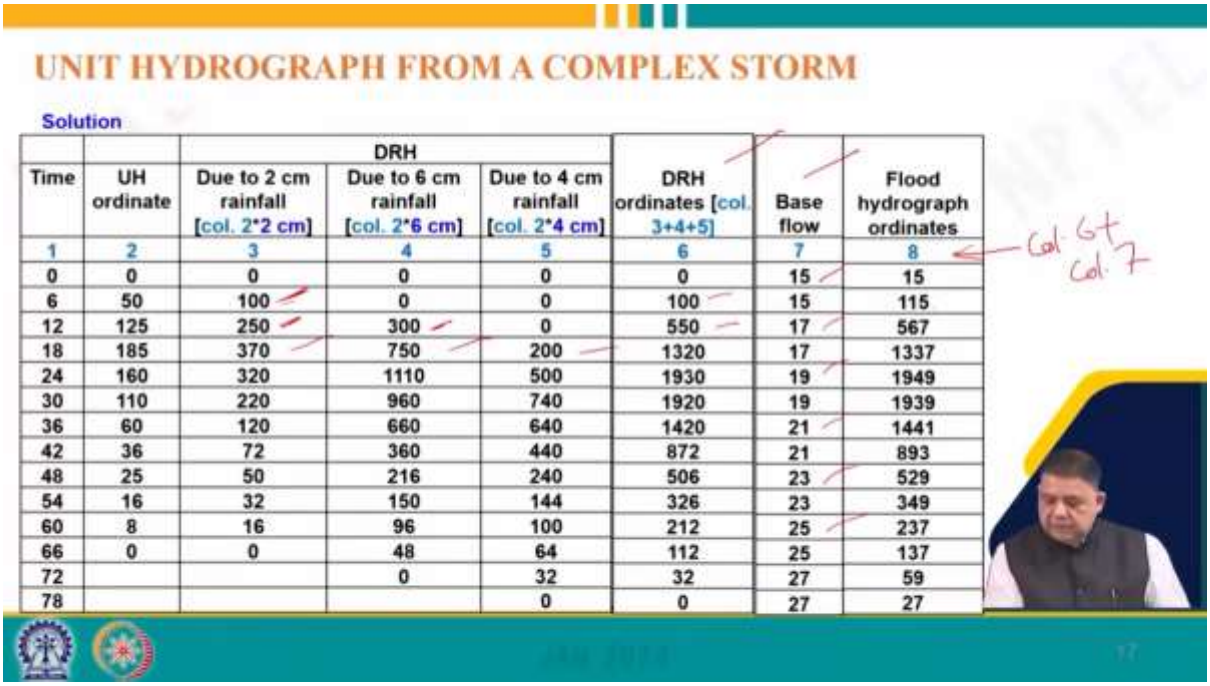
Given the baseflow as 15 cumec in the beginning and increasing by 2 m^3/s for every 12 h until the end of DRH,

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72	78
Base flow (cumec)	15	15	15+2=17	17	19	19	21	21	23	23	25	25	27	27

So, Φ-index values 0.2 centimetres, 25 centimetres per hour, time from beginning 0, 6, 12, 18 hours if accumulated rainfall values are given. So, obviously, we have to first find out what are the rainfall magnitudes during the 6-hour period.

So, this is 3.5 for the second 6 hours it is 11 minus 3.5, that is 7.5 and 5.5. Then effective rainfall value will be obtained by taking care of the Φ-index in 6 hours. We have losses 8.25 into 6. So, this is what we are doing: 0.25 into 6 for this loss. So, the effective rainfall value we get is 3.2 minus this value. So, 2, 6, and 4 are effective rainfall values. Basically, what we are doing is calculating rainfall minus loss, and the loss is basically, Φ-index value times the duration, the interval 6 hours.

So, that is why from here it is 5.5 minus Φ-index, which is 0.25 centimetres per hour into 6 hours. So, 1.5. So, that is why it comes out to be 4 centimetres, and we have been given that the base flow is 15 cubic in the beginning and increases by 2 cubic meters per second for every 12 hours. So, that is why for different hours, the base flow value we know. For the first up to 12 hours, it will be 15; at 12 hours, it becomes 17; at 24 hours, 19; at 36, 20; and so on. So, it increases by 2 every 12 hours. These are the base flow values throughout the duration of the unit hydrograph.


UNIT HYDROGRAPH FROM A COMPLEX STORM

Solution

Time	UH ordinate	DRH: Due to 2 cm rainfall [col. 2*2 cm]	DRH: Due to 6 cm rainfall [col. 2*6 cm]	DRH: Due to 4 cm rainfall [col. 2*4 cm]	DRH ordinates [col. 3+4+5]	Base flow	Flood hydrograph ordinates
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
6	50	100	0	0	100	15	115
12	125	250	300	0	550	17	567
18	185	370	750	200	1320	17	1337
24	160	320	1110	500	1930	19	1949
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	0	0	48	64	112	25	137
72			0	32	32	27	59
78				0	0	27	27

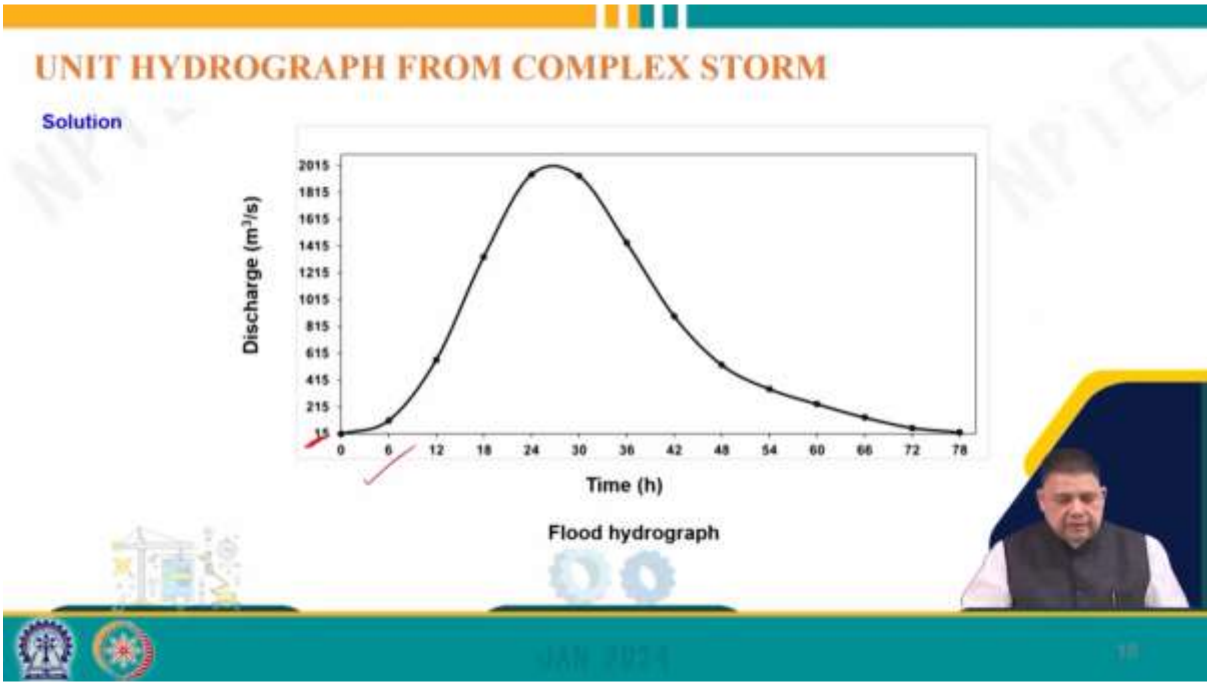
Col. 6 + Col. 7

Now, we have seen that we have 3 effective rainfalls, meaning a complex storm. In the first duration, there were 2 centimetres of effective rainfall; in the second, 6 centimetres; and in the third, 4 centimetres of rainfall, these are the UH ordinates. So, obviously, due to the 2 centimetres, we have to simply multiply because it is the first one. So, obviously, it starts from 0. So, 0 multiplied by 15 is 0; 200, 150 multiplied by 2, 250, and so on. So, simply, we are multiplying all the ordinates by 0. But for the second one, the rainfall starts at D hour and continues to 2D hours, that is, from 6 hours to 12 hours. For that, there will be a lag of 6 hours. So, that means, it will not start at 0, but it will start at 12, 6 hours. So, D hours, 6 hours, and that is why then we will start multiplying. So, 0, 6 times 50 is 300, 6 times 150 is 750. So, there is a lag in multiplication. So, 0 comes here, the multiplication value comes here, and in the case of 4 centimetres, which takes place from 0 to 6 hours, 6 to 12 hours, and 12 to 18 hours.

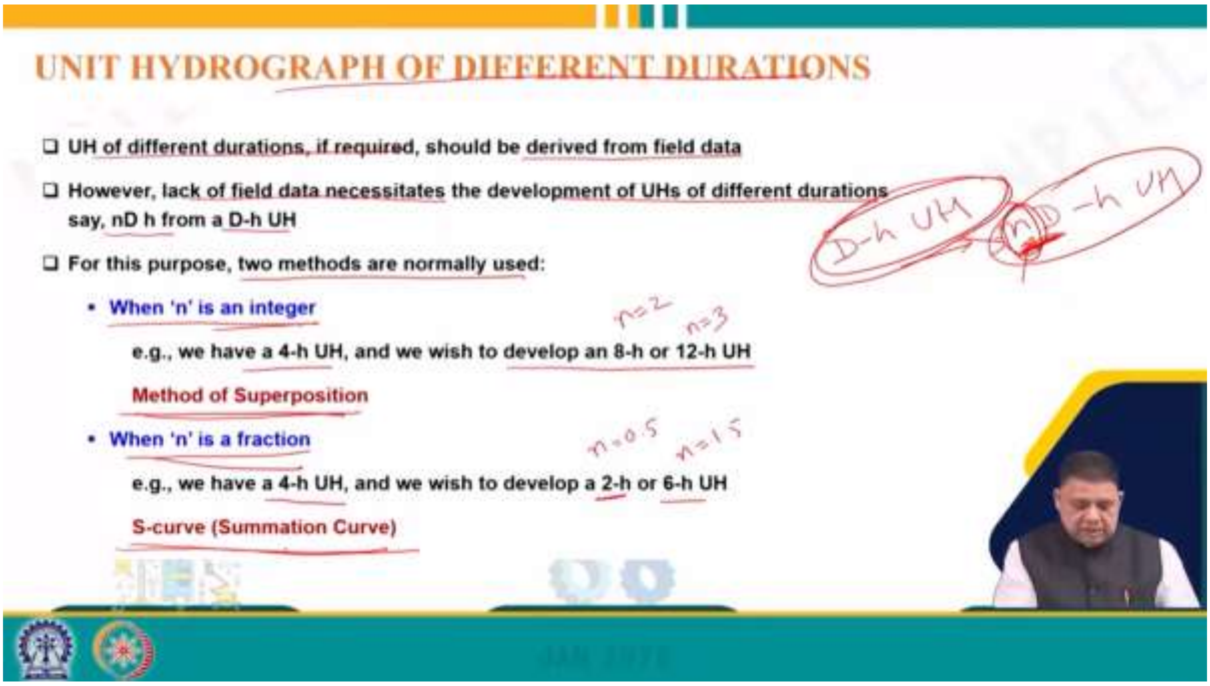
So, obviously, there will be a lag of another one. So, it will start at 12 hours, and because rainfall starts at 12 hours, here you see 0, here it is 50 times 4, 150 times 4, and so on. So, there is a lag every time in subsequent DRH ordinates because of the effective rainfall magnitudes.

And finally, we will get the DRH ordinate by summing up all the ordinates. So, at 0 hour it is 0, but at 6 hours it is 100, that is the contribution only coming from the first one; at 12 hours, the contribution coming from 2, that is 250, and 3, that is 550; and from 18 hours onwards, there will be a contribution from all 3.

So, the sum of these. So, ultimately, we get this DRH ordinate because of this UH ordinate and this effective rainfall. And we know that the base flow has already been calculated: 15, 17, 19, 21 at the gap of 12 hours. So, now, the flood hydrographs will be the sum of DRH ordinates and base flow, that is, the sum of these 2 comes out to this, nothing but column 6 plus column 7 values. So, here 15, 115, 567, and so on. So, these are the values here, and that is how the flood hydrograph can be plotted, and this is our flood hydrograph.

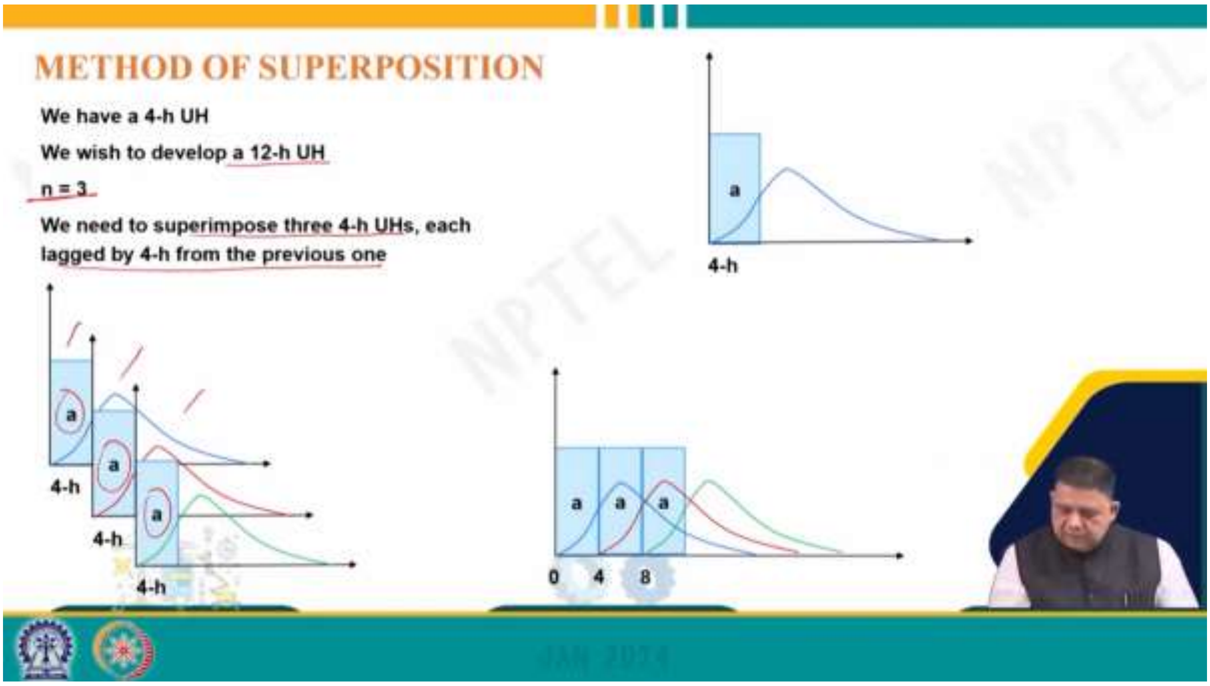


Because it is 15, so it is not starting at 0, but we have started the scale from 15. So, they do not get confused; there is a base flow component of 15 and so on throughout 15, 19, 17, 19, and so on and so forth. So, this is how we can get the unit hydrograph from complex storms.



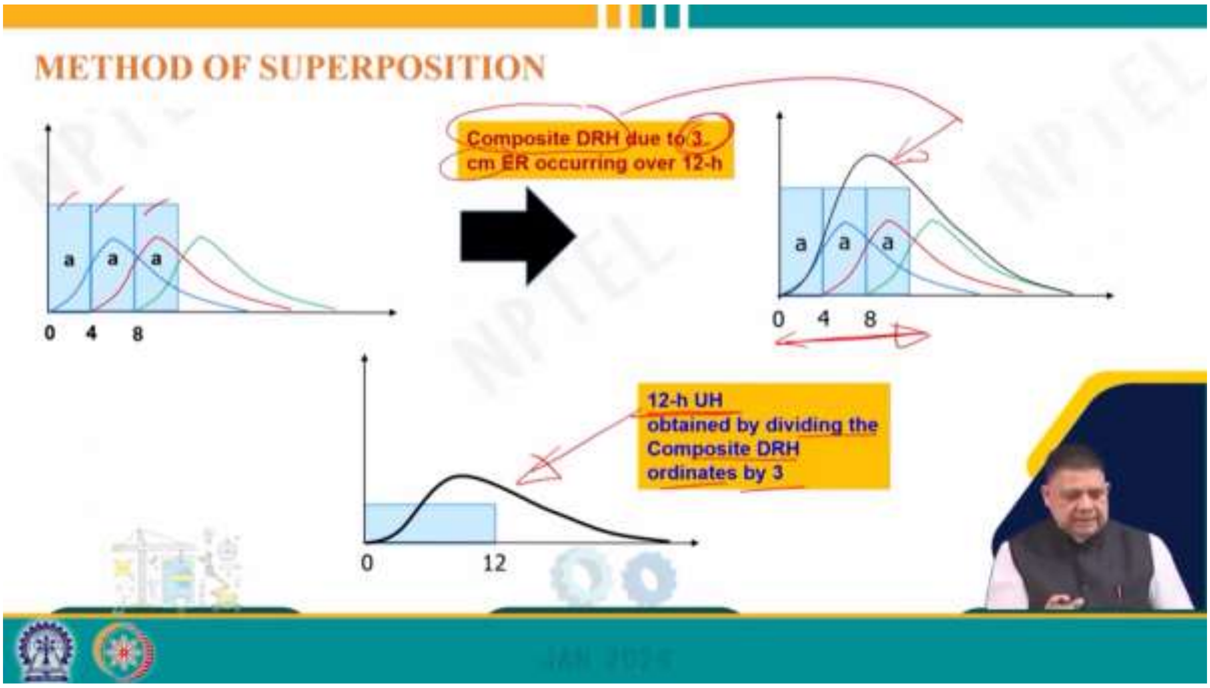
And then we come to the unit hydrograph of different durations. Now, the unit hydrograph of different durations, if required, should be derived from field data. However, due to lack of field data, many times we use a UH of different duration, say nD hour unit hydrograph from a D-hour unit hydrograph. That is, if we have a D-hour unit hydrograph with us, but due to lack of data, we are interested in developing an ND-hour unit hydrograph, and because of the lack of field data, we want to use this as a base to develop this ND-hour unit hydrograph. So, that is called a unit hydrograph of different durations. And for this purpose, we have two different methods available; that is, when n is an integer. So, this ND-hour, which we are interested in, if n is an integer, that means we have a 4-hour unit hydrograph and we wish to develop an 8-hour or 12-hour unit hydrograph, that is an integer multiple. In this case, n is 2, and in this case, n is equal to 3. Integer multiple then in that case we use the method of superposition, but when n is a fraction, it is not an integer. Say, for example, again we have a 4-hour unit hydrograph, but we wish to develop a 2-hour Intergraph, that means n becomes 0.5, or a 6-hour Intergraph where n becomes 1.5. That means n is not an integer, then we have to use a different method which is called an S-curve or summation curve. So, you have to remember if you are given a D-hour unit hydrograph and you want to develop a D-hour nD-hour Intergraph or hydrograph of different duration which has a which is of nD-hour.

So, obviously, the n value will determine which method to use. So, if n is an integer, we can use the method of superposition. When n is a fraction, we have to use the S-curve or summation curve, and obviously, if you have a summation curve, then we can develop a unit hydrograph for even an integer also. So, this is a more generalized method, this is a more specific method, this is what you have to remember.



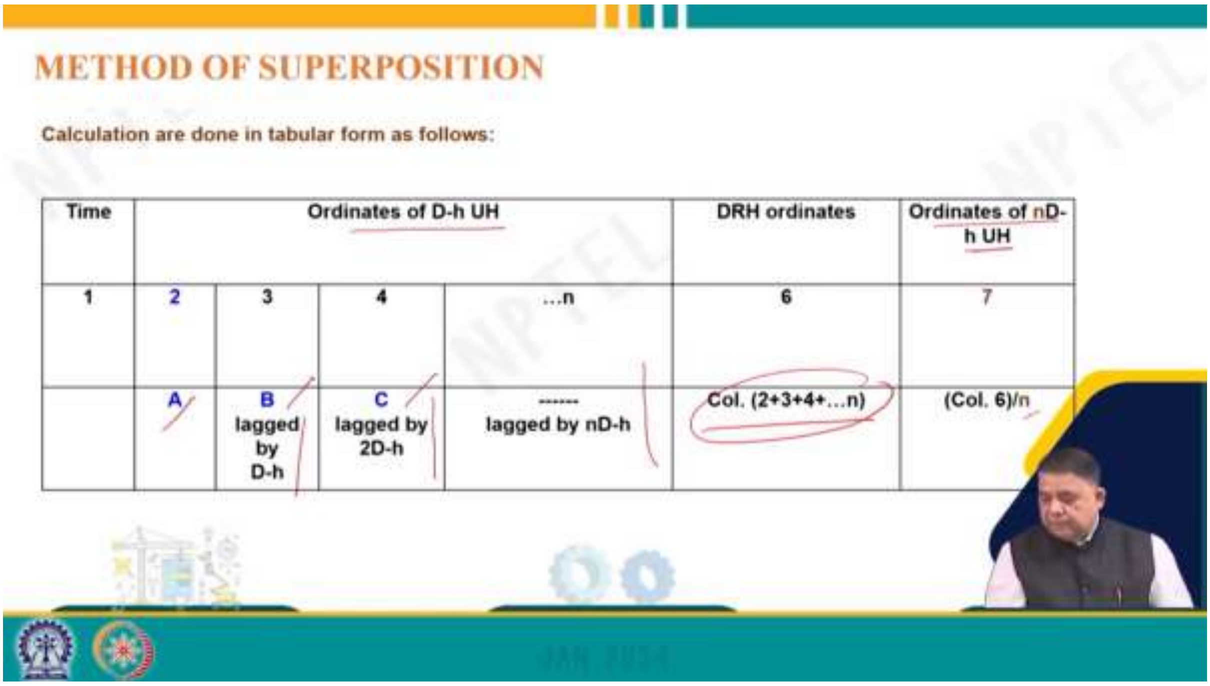
So, coming to the method of superposition, we have a 4-hour unit hydrograph, and we wish to develop a 12-hour Intergraph, that means n is equal to 3, D-hour to ND-hour, we said n equal to 3. So, we need to in that case, we need to superimpose 3 4-hour unit hydrographs each lagged by 4 hours from the previous one.

So, just now we earlier we solved a problem, the same thing is here. So, we are superimposing now 3-unit hydrographs here each one lagging by 4 hours, same unit hydrograph, but lagging by 4 hours, the magnitude is the same. In previous cases, we saw the magnitudes were different, the complex storms were there, but in this case, we are using the same unit hydrograph, but the principle remains the same, what we discussed earlier. So, that means, what now we have 3 different, if you plot it like this, so the first one is starting at 0, second is starting at 4 hours, and third is starting at 8 hours.



So, basically, if we sum up the ordinates of these 3-unit hydrographs, because of these 3-unit hydrographs, we get a DRH which is referred to as a composite DRH. The composite DRH is because of 3-unit hydrographs. So, that means, 1 1 1, that is because of 3 centimetres of effective rainfall which is occurring over this 12 hour. Now, the duration has gone to 12 hours and the magnitude has become 3 hours. So, that is why it is a DRH not a unit hydrograph because the effective rainfall magnitude is 3 due to the summing up of these 3 ordinates. Now, we want a unit hydrograph. So, obviously, for getting the 12-hour unit hydrograph, we will divide the composite DRH ordinates by 3.

So, this ordinate, if we divide by effective rainfall 3, then we will get a 12-hour unit hydrograph. So, that is the procedure, the same procedure which we discussed, but this time it is the same unit hydrograph which is being lagged. In previous cases, it was a complex system, so magnitudes were different.



So, calculations can be done in tabular form, just like we saw also. So, the ordinates of the DRH hour, a b lagged by 4 d hour c lagged by 2 d hour and n lagged by n d hour, then the sum of all these ordinates will give us DRH ordinates, and then because it is an n d hour unit hydrograph, ordinates will be this sum divided by the value of n. So, if n was 3, that is why we superimpose 3-unit hydrographs.

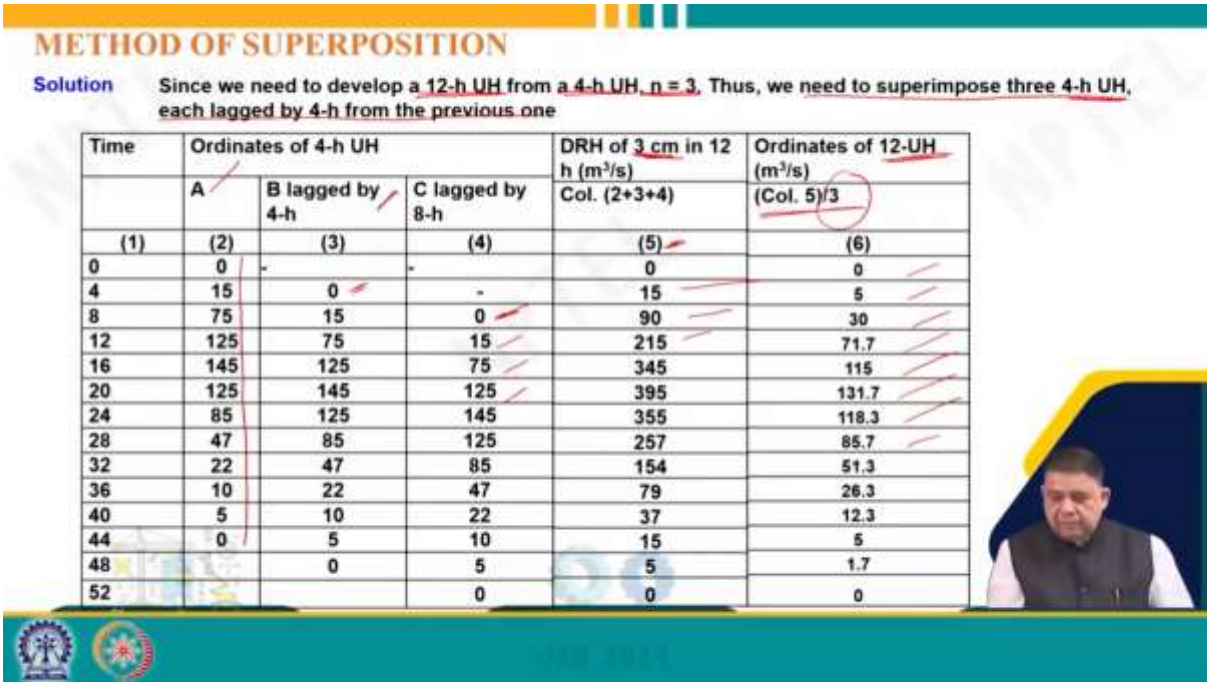
METHOD OF SUPERPOSITION

Example 5

The ordinates of a 4-h unit hydrograph are given below. Derive a 12-h unit hydrograph for the catchment.

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
U H Ordinates (cumec)	0	15	75	125	145	125	85	47	22	10	5	0

Let us take an example now, and the ordinates of a 4-hour unit hydrograph are given below, derived 12-hour unit hydrograph for the catchment. So, these are the UHR ordinates for 3 hours.



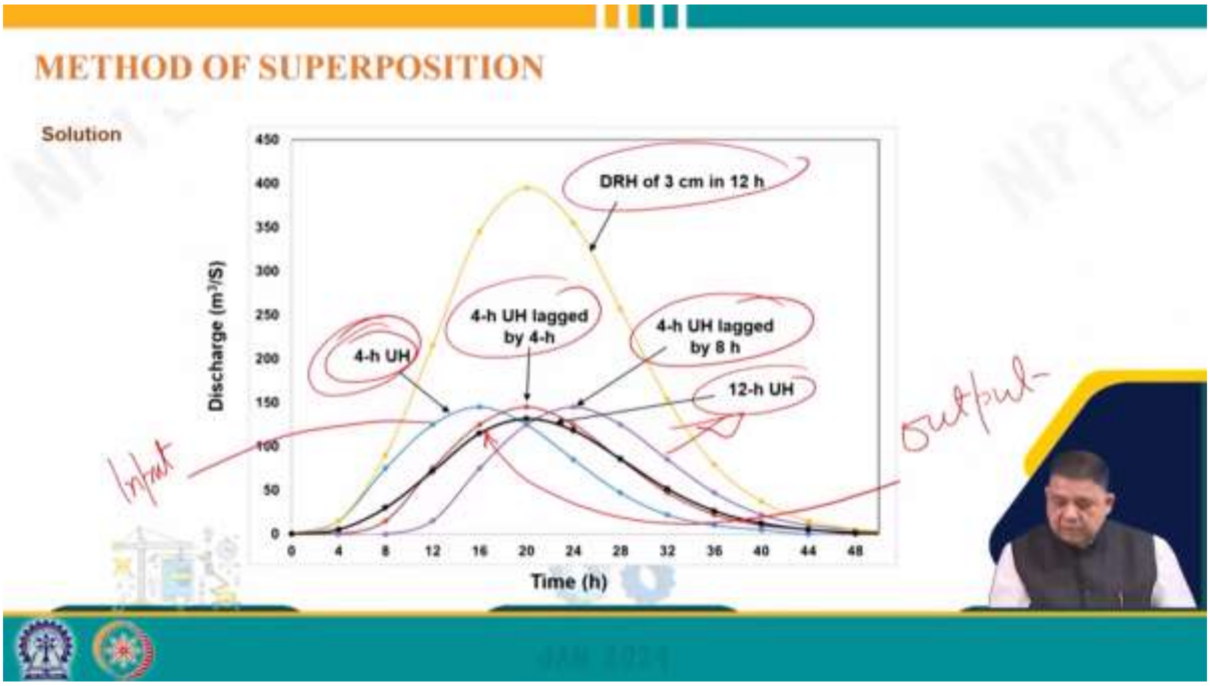
METHOD OF SUPERPOSITION

Solution Since we need to develop a 12-h UH from a 4-h UH, n = 3. Thus, we need to superimpose three 4-h UH, each lagged by 4-h from the previous one

Time	Ordinates of 4-h UH			DRH of 3 cm in 12 h (m^3/s)	Ordinates of 12-UH (m^3/s)
	A	B lagged by 4-h	C lagged by 8-h	Col. (2+3+4)	(Col. 5)/3
(1)	(2)	(3)	(4)	(5)	(6)
0	0	-	-	0	0
4	15	0	-	15	5
8	75	15	0	90	30
12	125	75	15	215	71.7
16	145	125	75	345	115
20	125	145	125	395	131.7
24	85	125	145	355	118.3
28	47	85	125	257	85.7
32	22	47	85	154	51.3
36	10	22	47	79	26.3
40	5	10	22	37	12.3
44	0	5	10	15	5
48		0	5	5	1.7
52			0	0	0

Because we wanted 12-hour unit hydrograph from a 4-hour unit hydrograph, so the value of n is 3. That means, we need to superimpose 3, 4-hour unit hydrographs, each lagged by 4 hours from the previous one. So, the first one will be written as it is, the original coordinates will be written, but the second one will start from 4 hours, it is lagged by 4 hours. So, it starts, same magnitudes, but 0 ,15 75, but starting from 4 hours, and the second one, the third, in fact, the third one, lagged by 8 hours, so that means, it starts at 0, starts at 8. So, 0 15 75. So, this is simply the same unit hydrograph, but being lagged by D hours, which is 4 hours in this case, and then we will sum up the ordinates of these.

Summing up will give us 15, 90, 215, and so on. Because we are superimposing 3-unit hydrographs, this is a composite DRH due to 3 centimetres of effective rainfall in 12 hours. So, the ordinate of the 12-hour unit hydrograph, column 5 by column this column 5 values divided by 3, will give us the ordinates of the 12-hour unit hydrograph. These are the unit ordinates of the 12-hour unit: 0, 5, 30, 71.5, 115, 131.7, 118.3, and so on. This is what we will get actually.



This is the plot of the direct runoff hydrograph. We have a 4-hour unit hydrograph, then a 4-hour unit hydrograph lagged by 4 hours, and a 4-hour hydrograph lagged by 8 hours. In fact, a 4-hour hydrograph lagged by 8 hours and DRH of 3 centimetres, and this is our final 12-hour unit hydrograph. The bold black colour is the 12-hour. So, we started from this 4-hour unit hydrograph, and this black one is our output that we obtained from the 4-hour to 12-hour unit hydrograph using the method of superposition. So, with this, we come to the end of this lecture where we have discussed the unit hydrograph due to complex systems.


THANK YOU