

Course Name: Watershed Hydrology

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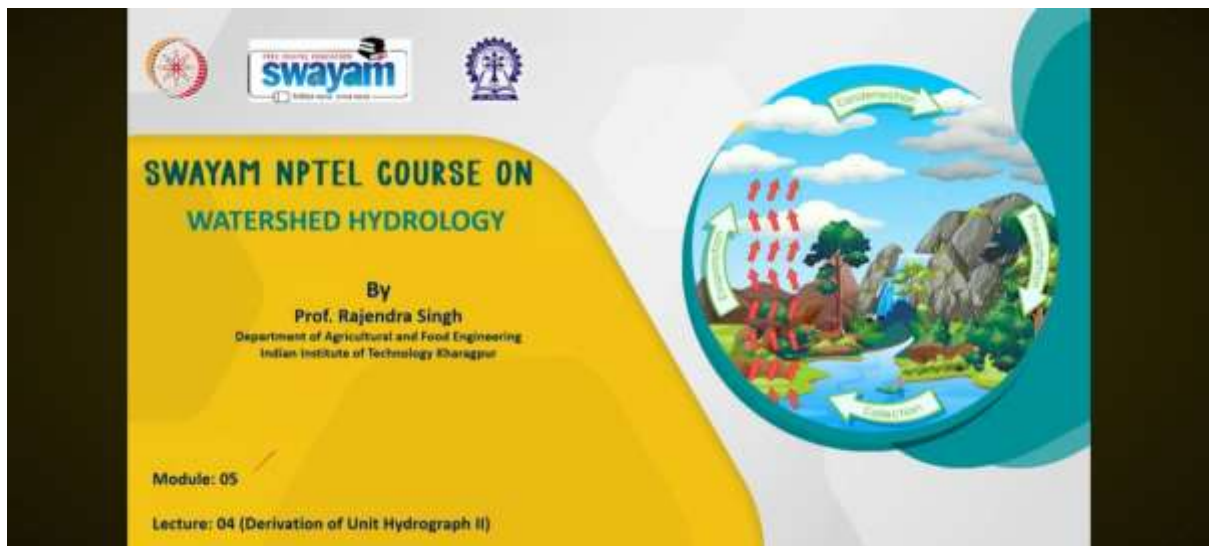
Department Name: Agricultural and Food Engineering

Institute Name: Indian Institute of Technology Kharagpur

Week: 05

Lecture 24: Derivation of Unit Hydrograph II

Hello friends, welcome back to this online certification course on Water State Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are in Module 5, this is Lecture 4, and the topic is the derivation of unit hydrograph. We are currently in Part 2.



In this particular lecture, we will discuss the S hydrograph, also known as the summation hydrograph. We will explore how to derive the S curve or summation hydrograph, and we will delve into the derivation of hydrographs of different durations using the S hydrograph, which we introduced in the previous lecture.

Content- Derivation of Unit Hydrograph II

- S-hydrograph (SH)
- Derivation of S-curve
- Derivation of hydrographs of different durations using the S-hydrograph

To recap, in the previous lecture, we began discussing the derivation of UH (Unit Hydrograph) of different durations. We mentioned that there are two methods, and we also explained that if we have a d -hour unit hydrograph and wish to develop an n - d hour unit hydrograph, the value of n determines which method to employ. If n is an integer, meaning that if we have a 4-hour unit hydrograph and we wish to develop an 8-hour or 12-hour unit hydrograph, then n equals 2 in this case, or n equals 3, or it could be any integer. In such cases, we utilize the method of superposition. However, if n is a fraction, for example, if we have the same 4-hour unit hydrograph but this time we want to develop a 2-hour or 6-hour unit hydrograph, then n is 0.5, or n equals 1.5, or any other fraction. In these instances, we employ the S curve, or summation curve, or S hydrograph. Different names are used for the S curve, such as summation curve or S hydrograph, and today we will develop into this in great detail.

UNIT HYDROGRAPH OF DIFFERENT DURATIONS

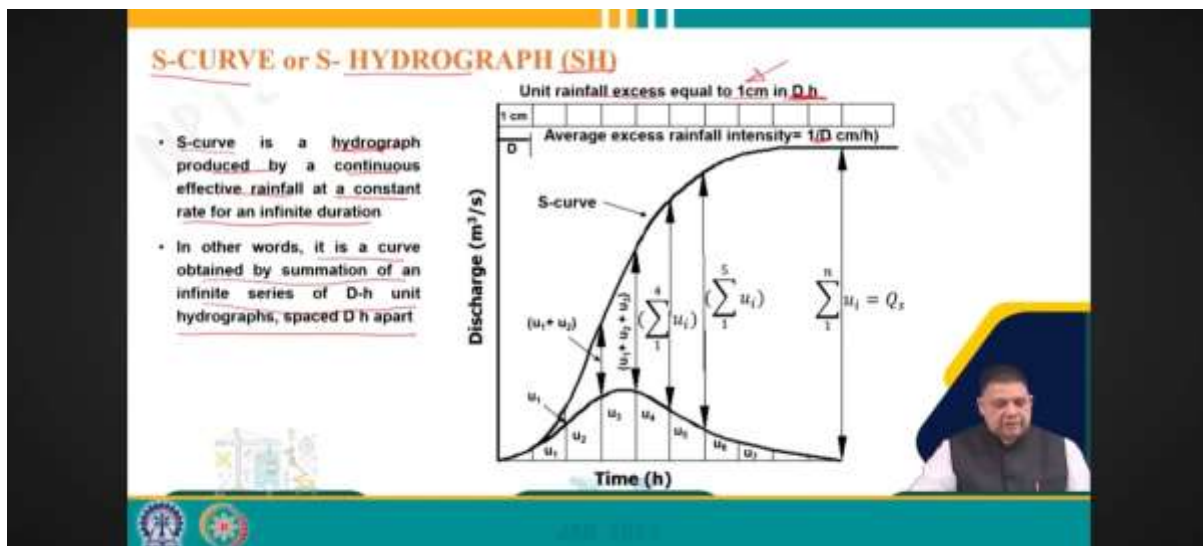
□ For deriving UH of different durations, two methods are normally used:

- When ' n ' is an integer
 - e.g., we have a 4-h UH, and we wish to develop an 8-h or 12-h UH
 - Method of Superposition
- When ' n ' is a fraction
 - e.g., we have a 4-h UH, and we wish to develop a 2-h or 6-h UH
 - S-curve (Summation Curve)

Handwritten notes on the slide:
 $n=2$ $n=3$ $D\text{-h UH} \rightarrow nD\text{-UH}$
 $n=0.5$ $n=1.5$

So, starting with the S curve or S hydrograph, SH in short form, is a hydrograph produced by continuous effective rainfall at a constant rate for an infinite duration. We have observed that all flood hydrographs, whether discussing flood hydrographs, direct runoff hydrographs, or unit hydrographs, always involve an ERH slice with a certain duration and magnitude. In the case of a unit hydrograph, the effective rainfall depth is, of course, 1. If it's a DRH, it could be any value other than 1. For example, if a unit rainfall excess equals 1 centimetre over D hours

(though it's not mandatory for it to be 1 centimetre; it could be any value), and if that occurs continuously for D hours, then the average rainfall intensity would be the magnitude divided by D centimetre per hour. The duration D is fixed. If this continuous occurrence happens, then the resulting hydrograph will be the S curve or S hydrograph. Similarly, in this case, we will also apply the same method of superposition. Here, we will have different ordinates; for example, ordinate 1 will be U_1 plus U_2 , and here, U_1 plus U_2 plus U_3 . The ordinates are multiplied because we are considering it as 1 centimetre; otherwise, it would be R_1U_1 , R_2U_2 , depending on the magnitude. Since we are considering 1 centimetre, it is $R_1R_2R_3R_1$. So, that is why we are writing U_1 , U_1 plus U_2 , U_1 plus U_2 plus U_3 , and so on and so forth. Basically, it is a curve obtained by the summation of an infinite series of D-hour intervals in hydrograph space, D hours apart. So, that means, as we saw in the previous lecture, the method of superposition where we lack the unit hydrographs by D-hour duration in order to get the DRH. It's the same concept; we are trying to get a DRH, but in this case, the rainfall is infinite. This means we have to sum up an infinite number of unit hydrographs, and if we do that, then obviously, this forms an S shape, hence it is called an S curve or S hydrograph.



If we superimpose an infinite number of E-hours having a unit depth of 50 rainfall in a 1-hour duration (it could be any duration, any rainfall as I said), then the hydrograph obtained assumes a deformed S shape, and it is termed a summation hydrograph or S hydrograph or S curve. The ordinates of the hydrograph ultimately approach the rate of effective rainfall at the limit or at the point of equilibrium. So, there will come a point where a point of equilibrium is reached, and the flow becomes constant in the case of a summation curve. This occurs because we are using the unit hydrograph to develop this. Therefore, the unit depth of effective rainfall will obviously be one, but the duration could be anything. It could be a D-hour unit hydrograph, which can be superimposed to obtain an S curve. It's not necessarily one hour; we are discussing one hour here, but it could be any D-hour unit hydrograph.

S-HYDROGRAPH (SH)

- If we superimpose an infinite number of ERs having a unit depth of effective rainfall and 1 h duration, then the hydrograph obtained assumes a deformed S-shape and is termed a summation hydrograph or S-hydrograph or S-curve
- The ordinates of the hydrograph ultimately approach the rate of ER in the limit or at the time of equilibrium

If the time base of the unit hydrograph is T hours (where you remember we said the time base is TB, now we are calling it T hour), then continuous rainfall producing one unit of runoff every period would develop a constant outflow at the end of T hours. So, that simply means the equilibrium we just discussed. It is expected that if we superimpose an infinite number of unit hydrographs, then this S curve will reach an equilibrium at the time base of this first unit hydrograph. This is expected. So, that is T hours. Thus, T by D unit hydrographs need to be combined to produce an N curve, which would reach an equilibrium flow Q_E . So, though while defining we say that we need to superimpose an infinite number of unit hydrographs, in practice, if we have a D-hour hydrograph and its time base is T hours, then T by D number of unit hydrographs we superimpose, then we will be getting an S curve which will reach an equilibrium discharge. This equilibrium discharge can be calculated using this formula: $q_e = (2.78 \cdot A) / D$, where A is the area of the basin in square kilometre, D is the D-hour effective rainfall duration in hours, and Q_E represents the maximum rate at which in a year intensity of this can drain out of the catchment. So, it will be an equilibrium discharge, Q, at Q max, basically, this is what we will get.

S-HYDROGRAPH (SH)

- If the time base of the UH is T-h, then a continuous rainfall producing one unit of runoff every period would develop a constant outflow at the end of T-h
- Thus, T/D unit hydrographs need to be combined to produce an S-curve, which would reach the equilibrium flow q_e (equilibrium discharge in cumec)

$$q_e = \frac{2.78A}{D}$$

Where A is in km^2 and D is in h; q_e represents the maximum rate at which an ER intensity of 1/D cm/h can drain out of a catchment of area A

Let us take an example: the ordinates of a 3-hour unit hydrograph are given below. We need to derive an S curve. So, we have the ordinates of 3-hour unit hydrographs given here, and we know this. Obviously, we need to draw several 3-hour units. As we said, we need to superimpose an infinite number of unit hydrographs, each lagging the previous one by D hours.

DERIVATION OF S-CURVE

Example 1

Ordinates of a 3-h unit hydrograph are given below. Derive an S-curve.

Time (h)	Ordinates of 3-h UH
(1) 0	(2) 0
3	47
6	77
9	162
12	87
15	52
18	32
21	0

So, we draw several 3-hour units lagged by 3 hours from the previous one. This is shown in columns 3 to 9. What you see here is our original UH, the second one lagged by 3 hours, starting at 3 hours, and then the same values are reproduced. The second one starts at 6 hours, and the same values are reproduced. The third one starts at 9 hours, same values, and so on, every one lagging the previous one by 3 hours in this case. In practice, we need to superimpose T_b/D , and T_b in this case is 21 hours. So, T_b/D , that means 7 number of 3-hour unit hydrographs to get an S curve, and here we have superimposed 9 actually, but practically we need to do 7 number only. We have superimposed 8 here; this is a column number, but we are superimposing the 7th number of 8 number unit hydrograph, but practically we need only 7 numbers, and we add the ordinates. So, if we sum up the ordinates, we get the S curve ordinates. This is 0 here; it is the sum of these 2, then the sum of these 3, the sum of these 4, and so on. As you can see, Q_p in this case is 457 cubic meters per second, and it occurs at 18 hours from the beginning of effective rainfall. As you can see here, it approaches a constant value. From here onwards, it approaches a common value; remember, the time base was 21 hours, and from 18 hours onwards, we get a constant value. So, that is why what we said: if we superimpose T_b by D number of unit hydrographs practically, then we will get an S curve which will have a constant equilibrium discharge at the end of the time base of the first hydrograph.

DERIVATION OF S-CURVE

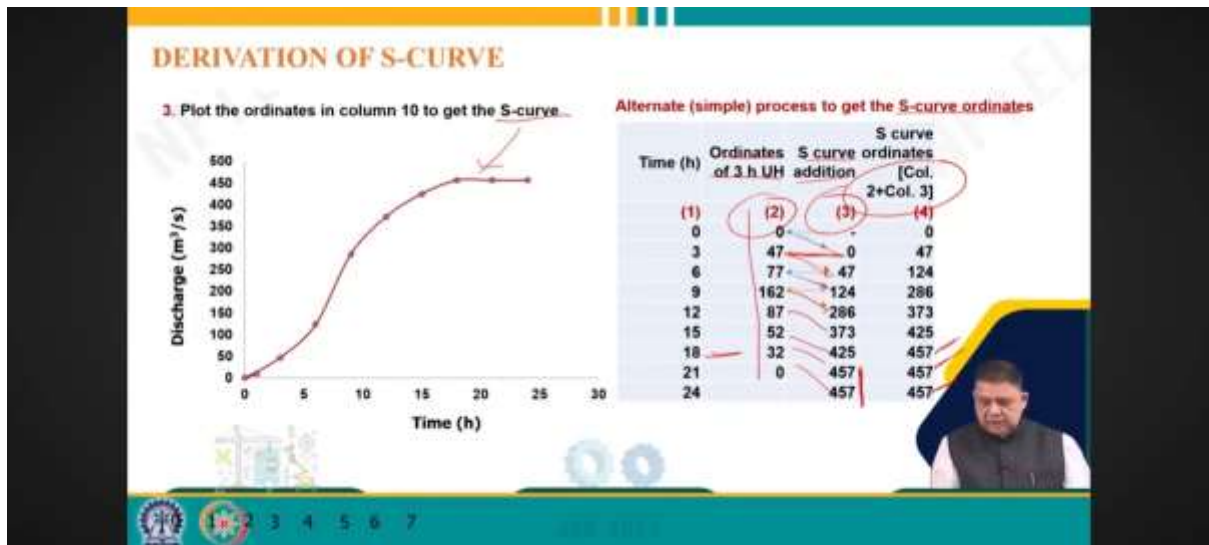
Solution

1. Draw several 3-h UH, lagged by 3 h from the previous one (columns 3 to 9). In practice, we need to superimpose (T_p/D) , i.e., $(21/3)$, or 7 numbers of 3-h UH to get the S-curve
2. Add the ordinates
3. The value of Q_p in this case, is 457 m^3/s and it occurs at 18 h from the beginning of the effective rainfall

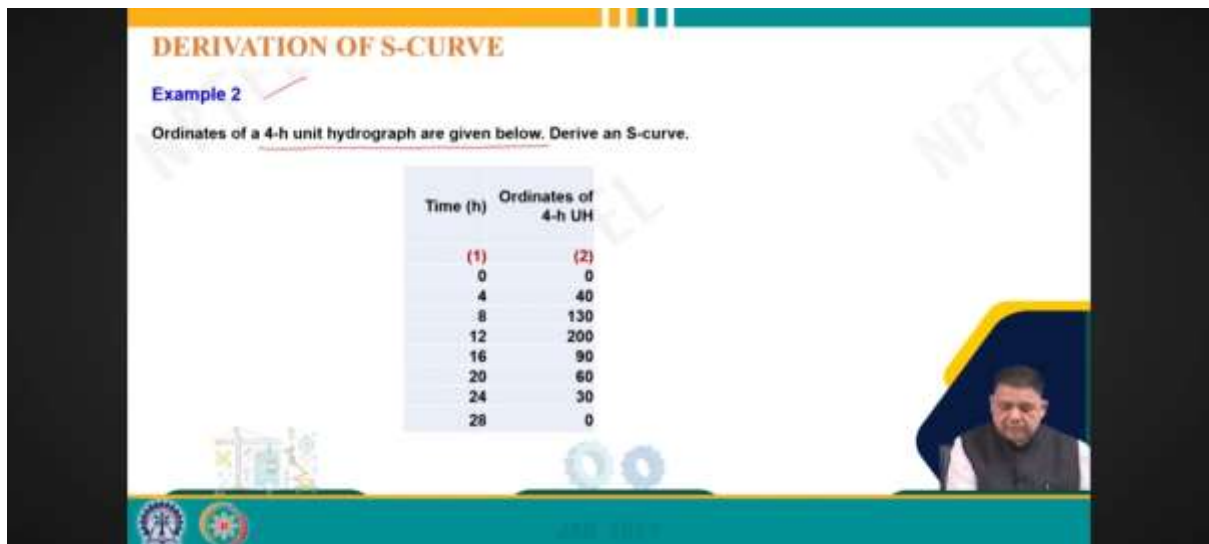
Time (h)	Ordinates of 3 h UH	3 h lagged hydrographs							S-curve ordinates
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	0								0
3	47	0							47
6	77	47	0						124
9	162	77	47	0					286
12	87	162	77	47	0				373
15	52	87	162	77	47	0			425
18	32	52	87	162	77	47	0		457
21	0	32	52	87	162	77	47	0	457
24		0	32	52	87	162	77	47	457



This is the S curve we have obtained, and this is the plot of the table, the S curve plot. This is what we have. So, this is the discharge versus time plotted S curve, and this is the kind of plot we will get. Alternatively, for getting the ordinates of the S curve, we have a procedure where we do S curve addition actually. So, the first column remains the same, ordinates of 3-hour hydrographs are here, and the second column, column 3 in this case, this ordinate column I am talking, time first column is time, second is the original 3-hour ordinate. The third column, what we do is we start lagging by 0, by 3 hours. H_b , as we have done, and if we superimpose another UH, but what we are doing is that we are not superimposing another UH. What we are doing is we are lagging the first one at 0, and then we start summing this value with the left value. So, 0 plus 47 and put it here below, then 47 plus 77 we put it here, 124 and 124, and 162 we put here, then 286 plus 87 we put here, 373 and so on. This is how we will get it. As you see here, we are getting a constant value, but the S curve ordinates will be the sum of column 2 and column 3. So, the original column which is here, and the addition column, if we sum these 2, then we get the S curve ordinates, and as you see that here at 18 hours, we are starting to get a constant value of 457. This is an alternate way. Instead of making multiple columns, you can also do these calculations in a quick way by using this S curve addition. This is an alternate way of getting the S curve ordinates.



Now, let us take an example. Two ordinates of a 4-hour unit hydrograph are given below. We need to derive an S curve. So, these are the 4-hour unit hydrographs .



And we know the procedure: we need to superimpose T_b / D . T_b in this case is 28 hours, so 28 divided by 4 equals 7, which means we need 7 numbers of 4-hour UH, each lagging by 4 hours from the previous one. So, columns 3 to 8. Here, we have superimposed 1, 2, 3, 4, 5, 6, 7, and then we add the ordinates. So, we get the S curve ordinates, and in this case, we see it becomes 550 at 24 hours. So, the value of Q_p in this case is 550 cubic meters per second, and it occurs at 24 hours from the beginning of the effective rainfall. This is how we can get the S curve ordinates.

DERIVATION OF S-CURVE

Solution

1. Superimpose (T_p/D), i.e., (28/4), or 7 numbers of 4-h UH, each lagged by 4 h from the previous one (columns 3 to 8).
2. Add the ordinates
3. The value of Q_p , in this case, is 550 m³/s and it occurs at 24 h from the beginning of the effective rainfall

Time (h)	Ordinates of 4-h UH	3 h lagged hydrographs							S-curve ordinates
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
0	0							0	
4	40	0						40	
8	130	40	0					170	
12	200	130	40	0				370	
16	90	200	130	40	0			460	
20	60	90	200	130	40	0		520	
24	30	60	90	200	130	40	0	550	
28	0	30	60	90	200	130	40	550	

Now, we can also discuss the alternate procedure for the same problem. That is, we have 4-hour ordinates here, then we can use the S curve addition. So, it will start by lagging 0. 0 plus 40 is 40, 40 plus 130 is 170, 170 plus 200 is 370, 370 plus 90 is 460, 460 plus 60 is 520, 520 plus 30 is 550. So, this is where, and then we sum up in the next column, we sum up these 2 values. So, 0, 40, 170, and you see that in the previous procedure, if we discuss about the previous procedure, we got the same ordinates as we got by this method. So, this is a quick method, a much faster method, but one has to be really careful while making the calculation. This is how we can get the S curve ordinates.

DERIVATION OF S-CURVE

Alternate process to get the S-curve ordinates

Time (h)	Ordinates of 4-h UH	S curve addition	S curve ordinates [Col. 2+Col. 3]
(1)	(2)	(3)	(4)
0	0	-	0
4	40	0	40
8	130	40	170
12	200	170	370
16	90	370	460
20	60	460	520
24	30	520	550
28	0	550	550

Now, let us discuss the derivation of hydrographs of other durations. We have already discussed the method of superposition. Now, we want to discuss how to use the S curve for deriving the unit hydrograph of different durations. If you remember, we said that this method is used when we have a D-hour unit hydrograph and we want to develop an n-d hour unit hydrograph, where n is a fraction. In this case, we need to use the S curve technique. Suppose we need to develop a t-hour unit hydrograph using a d-hour unit hydrograph. The procedure remains as discussed earlier: develop the S curve using the D-hour unit hydrograph, as

discussed in previous problems. So, if we are given a d-hour unit hydrograph, we can develop an S curve or S hydrograph, and that is what it is. Now, let us discuss the derivation of hydrographs of other durations. We have already discussed the method of superposition. Now, we want to discuss how to use the S curve for deriving the unit hydrograph of different durations. If you remember, we said that this method is used when we have a d-hour unit hydrograph and we want to develop an n-d hour unit hydrograph, where n is a fraction. If n is not an integer, then we need to use this method, the S curve technique. So, suppose we need to develop a t-hour unit hydrograph using a d-hour unit hydrograph. The procedure remains as discussed earlier: develop the S curve using the d-hour unit hydrograph. As discussed in previous problems, if we are given a d-hour unit hydrograph, we can develop an S curve or S hydrograph, and that is what it is. So, let's denote S as curve A here. This is our original unit hydrograph, the first unit S curve developed. Then, because we want to develop a t-hour unit hydrograph, we plot another S curve offset by t hours, let's say S as curve B. Here, the same S curve, which we have developed, is replotted, but with an offset of t hours, representing the duration we are interested in developing. For example, let's consider a scenario where we have a 4-hour unit hydrograph given, and we want to develop a 6-hour unit hydrograph. So, we will develop this S curve using the 4-hour unit hydrograph, and then the same S curve will be lagged by 6 hours. This will represent the 6-hour duration, and this S curve is developed from the 4-hour hydrograph. Then, we will superpose, and in the next step, we obtain the difference in the ordinates of the two S curves, $S_A - S_B$. We can find out these values because for every hour, we know the two ordinates. These values represent the direct runoff hydrograph (DRH) ordinates due to a t-hour effective rainfall having a magnitude of T/D centimetre. So, we divide $S_A - S_B$ by T/D to get the t-hour unit hydrograph. Here, we plot the S curve, then offset plot another S curve, get the difference in ordinates $S_A - S_B$, and then we divide this by T/D . Remember, t is the duration we are interested in, and d is the duration for which we have a unit hydrograph with us. So, we divide by that or multiply by D/T to get the t-hour unit hydrograph. This is how we will get the t-hour unit hydrograph using the S curve method.

DERIVATION OF HYDROGRAPHS OF OTHER DURATIONS

Suppose we need to develop a T-h UH using a D-h UH. We may follow the following steps:

1. Develop the S-curve using the D-h UH as discussed earlier (say, S_A)
2. Plot another S-curve offset by T h (say, S_B)
3. Obtain the difference in the ordinates of the two S-curves ($S_A - S_B$)
4. These are the DRH due to T-h effective rainfall having a magnitude of (T/D) cm
5. Divide the $(S_A - S_B)$ ordinates by (T/D) to get the T-h UH ordinates

Discharge (m^3/s)

Time (h)

S_A

S_B

$(S_A - S_B) D / T$

T-h unit hydrograph

6h

Handwritten notes: D-h UH, D-h UH, n is a fraction, S-curve from 4-h UH, 6-h UH

Let us take an example to make things clear. Here, a 2-hour unit hydrograph for a 200 square kilometre watershed is tabulated below. We need to derive a 1-hour unit hydrograph. These are the times and these are the discharge. Just to be clear, here D equals 2 and T equals 1. So, this is, of course, nD equals to t . So, n is not an integer; it is a fraction. Therefore, we will use the method of S curve in this case. Whenever you encounter a problem of developing hydrographs of different duration or unit hydrographs at different distances, the first thing you have to determine is whether n is an integer or a fraction. If it is an integer, then we can simply use the method of superposition. If it is a fraction, then obviously, we have to use the S curve. That means, we have to develop the S curve using this procedure and this unit hydrograph first.

DERIVATION OF UH OF LESSER DURATION

Example 3

A 2-h UH for a 200 km² watershed is tabulated below. Derive a 1-h UH.

Time (h)	0	1	2	3	4	5	6
Discharge (m ³ /s)	0	90	190	140	90	46	0

*D=2
T=1
n ≠ integer*

So, we have been given the unit hydrograph, which is here, and we need to develop the S curve. Prepare the S curve by superimposing multiple 2-hour unit hydrographs, each lagging by 2 hours from the previous one and summing up the ordinates. Basically, we have to lag several numbers, theoretically 6 by 1, meaning 6 units of hydrographs we have to superimpose, and that is precisely what we have done. Each one is lagged by 2 hours because here it is a 2-hour unit hydrograph, though the ordinates are given at 90-minute intervals. So, just to clarify, it is a 2-hour unit hydrograph, but the ordinates are given every hour. So, this is a confusion you must keep in mind. Ordinates are given every hour, but we have to lag by 2 hours. So, that is why we start at 2 hours, then ordinates at 4 hours, ordinates at 6 hours, ordinates at 8 hours, and so on. And then, obviously, we will sum up to get the ordinates of the S curve. This is the S curve, and as you can see, of course, in this case, it is not reaching a constant value; it is fluctuating from here onwards. You can see 276, 280, 276, 280. So, there is a fluctuation that could happen in some S curves when it approaches equilibrium; it might bubble. So, it might not reach a constant value, but it can also bubble, which is the typical case in this particular problem. Then, we draw another S curve lagging by 1 hour with reference to SA. So, this time, SA has to be lagged by 1 hour. So, here it is 0 hour, 1 hour. That's why we will start lagging from 0 hour. So, it is here, and then we have to find out the difference of these two ordinates, SA minus SB. 0 minus 90 is 90, 190 minus 90 is 100, 230 minus 40 is 190, 280 minus 230 is 50, and 276 minus 280 we are saying 0 because it cannot be negative. So, that is why we are saying 0. This means this is the DRH ordinate due to t by d . So, divide column 10

by t by d , where t is 1 and d is 2. So, 1 by 2 to get the 1-hour unit hydrograph ordinates. So, it is here. We will divide by 1 by 2 or multiply by 2, in other words. So, from 90 to 180, 100 to 200, 40 to 80, 50 to 100, and so on. This is the 1-hour unit hydrograph obtained using the 2-hour unit hydrograph.

DERIVATION OF UH OF LESSER DURATION

Solution

Time (h)	Lagged by 2 h						S_A	S_A (S_A Lagged by 1 h)	$(S_A - S_A)$	1-h UH [Col. 10/(T/D)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	0						0	0	0	0
1	90						90	0	90	180
2	190	0					190	90	100	200
3	140	90					230	190	40	80
4	90	190	0				280	230	50	100
5	46	140	90				276	280	0	0
6	0	90	190	0			280	276		
7		46	140	90			276	280		
8		0	90	190	0		280	276		
9			46	140	90		276	280		
10			0	90	190	0	280	276		
11				46	140	90	276	280		
12				0	90	190	280	276		
13							280			

1. Prepare the S-curve (S_A) by superimposing multiple 2-h UHs, each lagged by 2 h from the previous one, and summing up the ordinates (column 8).
2. Draw another S-curve (S_B) lagged by 1-h (with reference to S_A) (column 9).
3. Subtract S_B from S_A (column 10).
4. Divide (column 10) by (T/D) or (1/2) to get the 1-h UH ordinates (column 11).

Now, let's consider another problem: developing a 12-hour unit hydrograph using a 4-hour UH by the S curve method. Times 0, 4, 8, 12, and 44 hours' ordinates are given; 4-hour unit hydrograph ordinates are given as 0, 20, 80, and so on. However, in this case, it's a tricky problem because we have a 4-hour unit hydrograph, but we need to develop a 12-hour unit hydrograph. So, the value of n is 3. So, it is an integer, but still, we could have used the method of superposition. However, we are still bound to use the S curve method because we are directed to do so. But you must remember that given an option, we could have used the method of superposition in this particular problem. However, in this case, because we have been asked to use the S curve, that's why we will first develop the S curve using the 12-hour unit hydrograph and then proceed with the rest of the procedure.

DERIVATION OF HYDROGRAPHS OF OTHER DURATIONS

Example 4
Develop a 12-h UH using a 4-h UH by S-curve method.

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4-h UH (m^3/s)	0	20	80	130	150	130	90	52	27	15	5	0

4-h UH
12-h UH
n=3

Here, we have a 4-hour unit hydrograph, and we are using the alternate method instead of lagging; we are using the addition method. So, the 4-hour lagging will be done. It is 0, and then obviously, we have to sum up: 20, 0, 20, 20 plus 80 is 100, 100 plus 130 is 230, 230 plus 150 is 380, and so on. This is how we get the S curve addition, and S curve ordinates can be obtained by summing up column 2 and column 3. This is basically the S curve ordinates we are getting, and as you can see, here it is reaching a constant value of 699 at 40 hours, though the time base was 44. So, at 40 hours, we are getting a constant value of 699. This is a perfect S curve we will get in this case. Now, we have to develop a 12-hour unit hydrograph from a 4-hour unit hydrograph. So, as you remember, we have to offset the S curve by T hours, as we saw in the generalized explanation. That means, we have to offset this 12-hour S curve, which we have developed using a 4-hour unit hydrograph, by 12 hours. That means, there will be a lagging of 12 hours. So, the same S curve will be reproduced, but it will be lagging the previous one by 12 hours. Starting from 12 hours, we are reproducing the same alternate ordinates, and then we will get the difference between these two, that is SA minus SB. That means, column 4 minus column 5. That means, 0, 20, 100, 230, of course, because up till now there was only the first one there. Now, it will be subtraction. We start with 380 minus 20, which is 360, 510 minus 140, which is 600 minus 230, which is 70, and so on. So, this is the DRH ordinate because of T/D centimetre of effective rainfall, which means 12 by 4 equals to 3 centimetre of effective rainfall, which is written here. So, that simply means that if we divide the ordinates of this DRH by 3 centimetre, we will get a 12-hour unit hydrograph. So, column 6 divided by 3. These are the values: 0, 20 divided by 3, which is 6.7, 100 divided by 3, which is 33.3, and so on. This is our 12-hour unit hydrograph, which we have developed using the 4-hour unit hydrograph given to us.

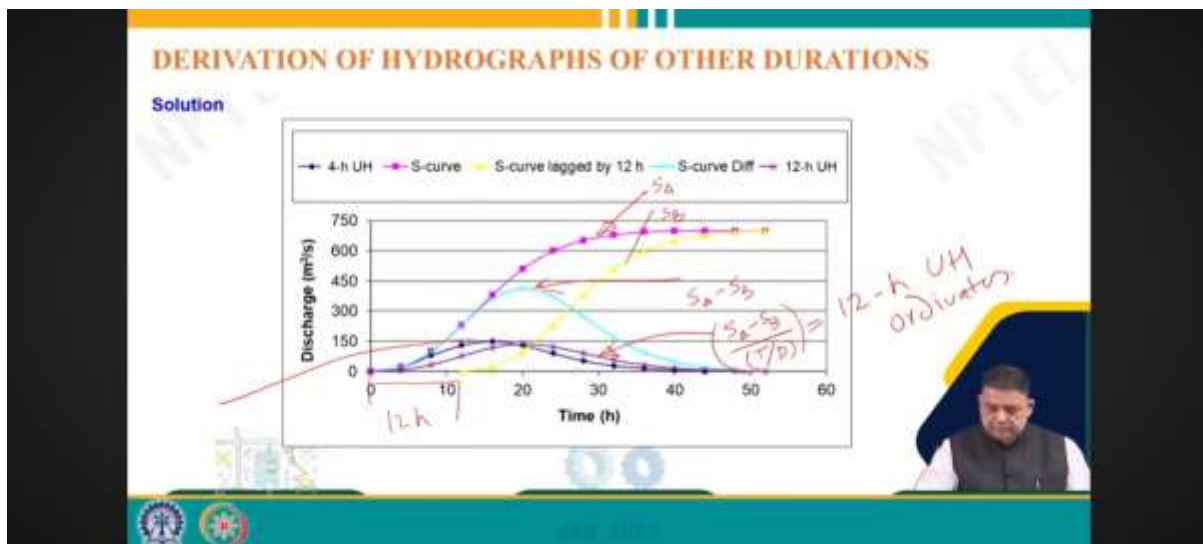
DERIVATION OF HYDROGRAPHS OF OTHER DURATIONS

$\frac{T}{D} = \frac{12}{4} = 3 \text{ cm}$

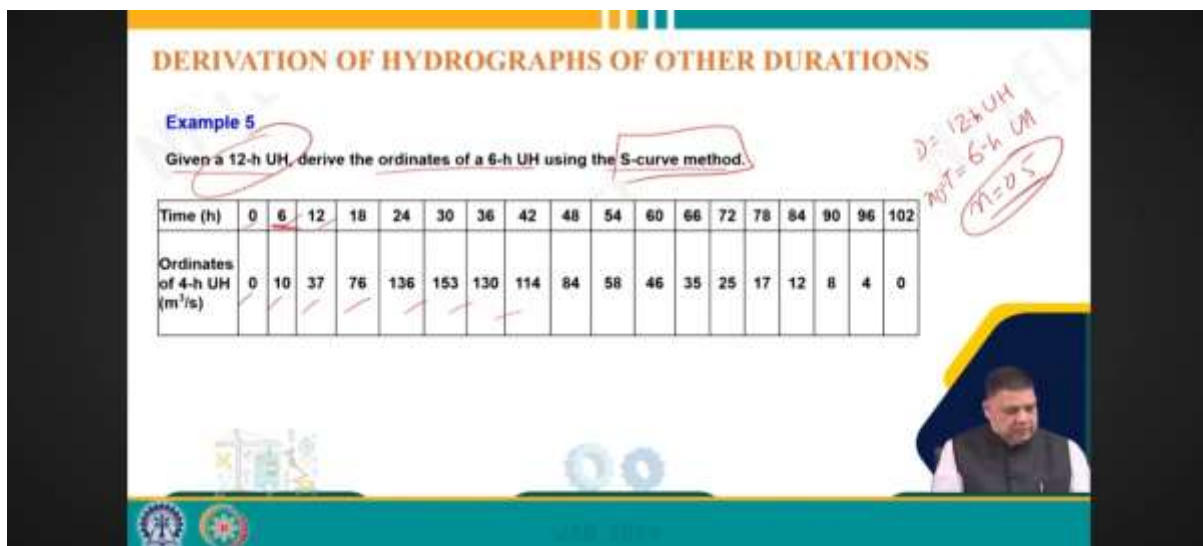
Solution

Time (h)	Ordinate of 4-h UH (m ³ /s)	S-curve addition (m ³ /s)	S-Curve ordinates (m ³ /s) [Col. 2 + Col. 3]	S-Curve lagged by 12 h (m ³ /s)	DRH of 12/4 = 3 cm rainfall excess in 12 h [Col. 4 - Col. 5]	12-h UH Ordinates (m ³ /s) [Col. 6/3]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	0	-	0	-	0	0
4	20	0	20	-	20	6.7
8	80	0+20=20	100	-	100	33.3
12	130	20+80=100	230	0	230	76.7
16	150	100+130=230	380	20	360	120.0
20	130	230+150=380	510	100	410	136.7
24	90	380+130=510	600	230	370	123.3
28	52	510+90=600	652	380	272	90.7
32	27	600+52=652	679	510	169	56.3
36	15	652+27=679	694	600	94	31.3
40	5	694+15=694	699	652	47	15.7
44	0	694+5=699	699	679	20	6.7
48		699+0=699	699	694	5	1.7
52		699	699	699	0	0

And then if you plot it, we can see the difference. So, this one was our original 4-hour unit hydrograph. Using that, we developed this original S curve, SA, which we called. Then we lagged this by 12 hours here and we plotted this S curve once again. This is SA, and this is SB we are calling, and then S curve difference, the odd difference in the ordinates. This is SA - SB, and obviously, then SA - SB divided by T/D, these are the ordinates which we are getting here, which is nothing but 12-hour UH ordinates. So, starting from 4 hours, we can get the 12-hour unit hydrograph, and in this fashion, we can get this here.



let us take up yet another example, that is example 5: given a 12-hour unit hydrograph, derive the ordinates of a 6-hour unit hydrograph using the S curve method. So, at different times 0, 6, 12, at a 6-hour interval, we have been given, of course, remember it is a 12-hour unit hydrograph, but ordinates are given at every 6 hours here, and then it is 0, 10, 37, 76, and so on. And remember here we have a 12-hour UH with us, and we have to develop a 6-hour UH, so that means, this is nothing but nD equals to T , that means, n value is 0.5, that is a fraction. So, obviously, in this case, we are bound to use the S curve method. This is how every time you get a problem of developing a unit hydrograph of different duration, the first thing you should do is find out which method is appropriate. Of course, if the method is specified in the problem, you cannot do anything, but otherwise, you can always handle this by choosing the appropriate method.



Now, in this case, again these are the different times, these are the ordinates of the 12-hour unit hydrograph we have plotted, and then here we could have used the method of superposition, that means, lagging 12-hour unit hydrographs and plotting. This is 102 divided by 12, that means, 102 means actually 12 hours. So, it is basically 96 or it will be like that. So, 9 or 10 units of hydrograph would have been plotted to get this, but instead, we may use the S curve addition. But remember that there will be a lagging of 12 hours, though ordinates are

available at 6 hours. So, that is why our addition starts. The second or third column, that is, S curve addition, starts at 0 value because it's a 12-hour unit hydrograph. So, there is a bit of a trick here in doing the S curve addition; you have to be very careful. If it is of similar duration, then we have been moving horizontally and straight away, but in this case, it's not the case. We cannot just move horizontally because the values have shifted. So, if horizontally, then the value will come here, 10, then 37. So, the horizontal value, one blank is here, you have to remember this. So, 37, then here, this 10 and 7 is 76, 86, 37, and 136 is 173. There is always a lag of 6 hours because of this. We had to start at 12, but ordinate values are given. If it had been just 0, 37, 136, then life would have been very easy, but in this case, because ordinates are given at every 6 hours. So, you have to be a little careful while adding. Once we have prepared this column by lagging by 12 hours and adding up, then of course, these S curve ordinates can be obtained by summing up column 2 and column 3. These are the ordinates, and as you can see here, it's getting a constant value of 471 cumec at 90 hours. So, that is where it's again a perfect S curve because after that, it gets a constant value and remains. Then, because we have to develop a 6-hour unit hydrograph, basically this is SA, our SB will start at 6 hours lag. So, that is here. The same S curve will now be reproduced at a lag of 6 hours. The same values are being reproduced, and then we will get S A minus S B, that is column 4 minus column 5, which is the difference in ordinates. So, 0, 10, 27, 49, 87, and so on. These are the values we have obtained, and this is because of 0.5 centimetre of rainfall excess. So, obviously, we have to divide by 0.5 or multiply by 2, which means the same thing. So, that's why we are getting the 6-hour ordinates by dividing the values in column 6 by 0.5 or multiplying by 2. So, 10 becomes 20, 27 becomes 54, 98, and so on. These are our 6-hour unit hydrograph ordinates which we have obtained.

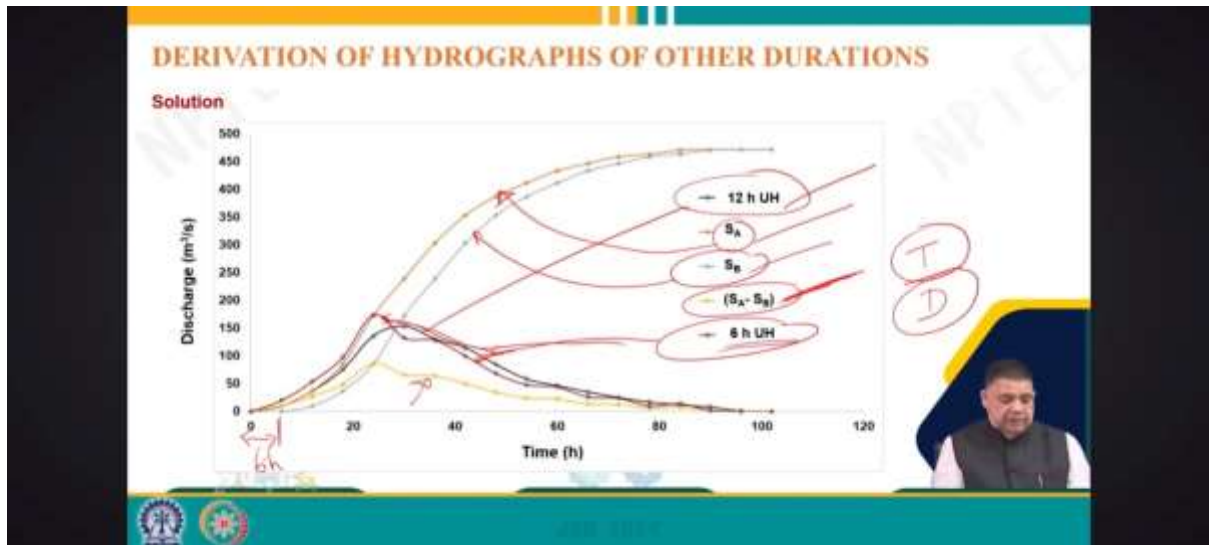
DERIVATION OF HYDROGRAPHS OF OTHER DURATIONS

Solution

Time (h)	Ordinate of 12-h UH (m ³ /s)	S-curve addition (m ³ /s)	S-Curve ordinates (m ³ /s) [Col. 2+Col. 3]	S-Curve lagged by 6-h (m ³ /s)	DRH of 6/12=0.5 cm rainfall excess in 6-h [Col. 4-Col. 5]	6-h UH Ordinates (m ³ /s) [Col. 6/0.5]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	0	-	0	-	0	0
6	10	-	10	0	10	20
12	37	0	37	10	27	54
18	76	10	86	37	49	98
24	136	37	173	86	87	174
30	183	86	239	173	66	132
36	130	173	303	239	64	128
42	114	239	353	303	50	100
48	84	303	387	353	34	68
54	58	353	411	387	24	48
60	46	387	433	411	22	44
66	35	411	448	433	13	26
72	25	433	458	448	12	24
78	17	448	463	458	8	16
84	12	458	470	463	7	14
90	8	463	471	470	1	2
96	1	470	471	471	0	0
102	0	471	471	471	0	0
		471	471	471	0	0
		471	471	471	0	0

So, in the same way as we saw earlier, we started with a 12-hour unit hydrograph in this case. This was our original 12-hour unit hydrograph. Using that, we developed this S curve, which is SA here. Then, we drew the second S curve, which is SB, lagging the previous one by 6 hours. This is 6 hours because we wanted to develop a 6-hour hydrograph given at 12 hours. So, we can then obtain the difference between the ordinates, which is shown in blue and yellow here. Then, we divide the ordinates of SA by that, which is the DRH ordinate, by 0.5, which is the effective rainfall magnitude, to get the 6-hour hydrograph. This is our resultant 6-hour unit hydrograph here, where the peak is located. So, this is our 12-hour hydrograph. Starting

from the 12-hour hydrograph, we first developed the S curve, then we lagged the S curve by the desired number of hours. Next, we got the difference between the ordinates and then divided by the magnitude, which is T by D , where T is the duration we are interested in, and D is the duration for which we have the unit hydrograph. We divided the ordinates of S_A by S_B , which are these ordinates, DRH ordinate, by this magnitude to get the desired 6-hour ordinate.



So, this is how this method works. In this method, we have seen the procedure of first understanding what an S curve is, and then we have seen how to use the S curve to derive unit hydrographs of different durations, typically when we are interested in developing an n - d hour hydrograph from a d -hour hydrograph when n is a fraction. I hope you were able to understand. In case of any doubts or questions, please do write them so that we can address them in the forum. Thank you very much.

