

Course Name: Watershed Hydrology

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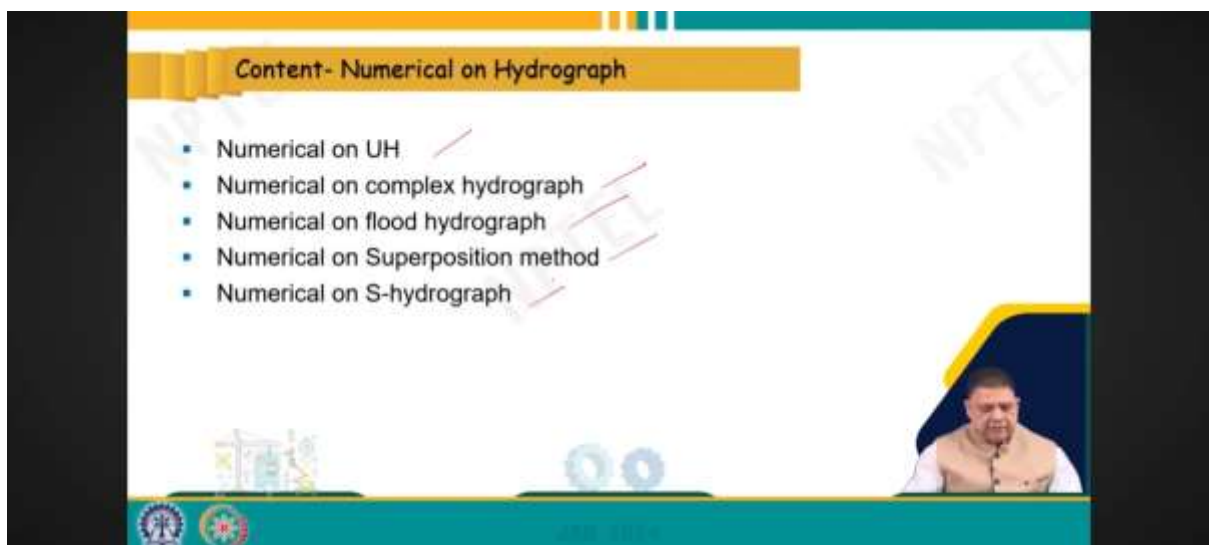
Week: 05

Lecture 25: Numerical on Hydrograph

Hello, friends! Welcome back to this online certification course on Watershed Hydrodynamic Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are currently in Module 5, and this marks the final lecture of this module. Today, we will delve into numerical exercises on hydrographs.



We will cover various aspects, including unit hydrograph, complex hydrograph, flood hydrograph, superposition method, and the summation hydrograph (or S hydrograph) method.



Let's begin with an example extracted from the Gate 2001 examination. The peak of a flood hydrograph resulting from a 6-hour storm is 470 cubic meters per second. The mean depth of rainfall is 8 centimeters. Assuming an average infiltration loss of 0.25 centimeters per hour and a constant base flow of 50 cubic meters per second, estimate the peak discharge of a 6 hour unit hydrograph for this catchment. So, essentially, we are faced with a problem where we have been given a flood hydrograph, and we need to develop a unit hydrograph. So, we know the procedure: if we are given a flood hydrograph, the first thing we will do is deduct the base flow from the flood hydrograph, which will give us the DRH or direct runoff hydrograph. Once we have the direct runoff hydrograph, if we divide the ordinates by the effective rainfall depth (ER), also known as rainfall excess depth (AR), we obtain the UH ordinate. This is the general procedure, and the same steps will be followed here while solving this problem. We have been given the peak of a flood hydrograph as 470 cubic meters per second, the depth of rainfall as 8 centimetre , the duration as 6 hours, the base flow as 15 cubic meter per second, and the infiltration loss in terms of the Φ index as 0.25 centimetre per hour. So, obviously, we need to obtain the value of effective rainfall, which we can calculate using the relationship that the Φ index is equal to $(P - ER)/t_e$. We are given the value of the Φ index as 0.25 centimetre per hour, the depth of rainfall (P) as 8 centimetre, and the effective duration (t_e) as 6 hours. Substituting the known values, we find that the effective rainfall is 6.5 centimetre . Next, for the peak of the DRH of a 6-hour unit hydrograph, we have the peak of the flood hydrograph and the base flow given. Subtracting the base flow (15 cubic meters per second) from the peak of the flood hydrograph (470 cubic meters per second) gives us 455 cubic meters per second, which is the peak of the DRH of the 6-hour hydrograph. Now, to find the peak discharge of the 6-hour hydrograph, we divide the peak of the DRH by the effective rainfall magnitude. So, 455 divided by 6.5 centimetre gives us an answer of 70 cubic meters per second. Therefore, the peak discharge of the 6-hour hydrograph resulting from this peak hydrograph is 70 cubic meters per second. So, a pretty straightforward application of steps for converting the flood hydrograph into a unit hydrograph.

NUMERICAL ON UNIT HYDROGRAPH

Example 1

The peak of a flood hydrograph due to a 6-h storm is 470 m³/s. The mean depth of rainfall is 8.0 cm. Assuming an average infiltration loss of 0.25 cm/h and a constant base flow of 15 m³/s, estimate the peak discharge of a 6-h unit hydrograph for this catchment. (GATE 2001)

Solution

Given: Peak of a flood hydrograph = 470 m³/s; Depth of rainfall = 8 cm; Duration = 6 h; Baseflow = 15 m³/s; Infiltration loss = ϕ - index = 0.25 cm/h

We know that $\phi - \text{index} = \frac{P - ER}{t_e}$

Thus, putting the known values $0.25 = \frac{8 - ER}{6}$

Effective rainfall (ER) = 6.5 cm

Also, Peak of the DRH of 6-h hydrograph = Peak of a flood hydrograph - Baseflow = 470 - 15 m³/s = 455 m³/s

Thus, peak discharge of the 6-h UH = Peak of the DRH/ER = 455/6.5 = 70 m³/s

Handwritten notes:
 Flood Hydrograph = Baseflow + DRH
 UH = DRH/ER

Then, we move on to the next problem, which is taken from the Gate 2015 examination. A catchment area of 720 hectares has a 25-year mean rainfall intensity of 100 millimetre per hour, occurring for a duration equal to the time of concentration. During a storm event, the catchment received a total of 7.5 centimetre of design rainfall over 6 hours. Assuming Φ index of 25

centimetre per hour and a runoff coefficient of 0.6 . Let's calculate the peak ordinate of the 6-hour unit hydrograph. Now, looking at this problem, as you can see from the information provided, if you recall, we have previously discussed terms like time of concentration and runoff coefficient. Essentially, it's asking us to utilize the rational formula to determine the peak discharge initially. The information given includes a runoff coefficient (c) of 0.6, rainfall intensity for duration equal to the time of concentration, which is 100 millimetre per hour, converted into meters per second, giving us 2.78×10^{-5} meters per second. The area of the catchment is given as 720 hectares, which we convert into square meters, resulting in 720×10^4 square meters. Now, we can find the peak of the DRH using the formula $c * i * A$, as per the rational formula. Given the value of c as 0.6, we've found the values of i and A as 2.78×10^{-5} meters per second and 720×10^4 square meters, respectively. So, by inputting these values and solving, we find that the peak of the DRH is 120 cubic meters per second. Additionally, we have been given a rainfall and a phi index value. The Φ index, equal to $(P - ER)/t_e$, is the formula we use to find out effective rainfall. This formula is applicable because we know the phi index, the precipitation value which is given as 7.5 centimetre, and the duration of the effective rainfall given as 6 hours. From there, we determine the effective rainfall value to be 6 centimetre . As we saw in the previous problem, to obtain the peak of the 6-hour unit hydrograph, we divide the peak of the direct runoff hydrograph by the effective rainfall which we have obtained. So, that is 120 divided by 6, which equals 20 cubic meters per second. However, if you observe closely, this problem combines the rational formula with concepts of unit hydrograph and flood hydrograph simultaneously. Hence, it presents a slightly tricky challenge, as seen in the Gate 2015 examination.

NUMERICAL ON UNIT HYDROGRAPH

Example 2

A catchment of 720 ha area has 25-year mean rainfall intensity of 100 mm/h occurring for a duration equal to its time of concentration. During a storm event, the catchment received a total of 7.5 cm design rainfall for 6 hours. Assuming ϕ -index of 0.25 cm/h and runoff coefficient of 0.6, calculate the peak ordinate of the 6-h unit hydrograph. (GATE 2015)

Solution

Given, runoff coefficient, $C = 0.6$; rainfall intensity for a duration equal to the time of concentration = 100 mm/h = 2.78×10^{-5} m/s; area of the catchment = 720 ha = 720×10^4 m²

Peak of DRH (Q_p) = $CIA = 0.6 \times 2.78 \times 10^{-5} \times 720 \times 10^4 = 120 \text{ m}^3/\text{s}$

We know that $\phi - \text{index} = \frac{P - ER}{t_e}$

Thus, putting the known values $0.25 = \frac{7.5 - ER}{6}$

Effective rainfall (ER) = 6.0 cm

Thus, the peak ordinate of the 6-h unit hydrograph = $\frac{\text{Peak of DRH}}{\text{Effective rainfall}} = \frac{120}{6} = 20 \text{ m}^3/\text{s}$

Moving on to the next problem, example number 3, taken from the Gate 2019 examination. This problem states that the peak of a flood hydrograph due to a 5-hour storm is 670 cubic meters per second. The total depth of rainfall is 9 centimetre . Assuming an average infiltration loss of 0.2 centimetre per hour and a constant base flow of 30 cubic meters per second, calculate the peak discharge of a 5-hour unit hydrograph for this catchment. So, this problem sounds very similar to problem number 1, where we were given a flood hydrograph. Knowing the base flow, which is provided here as well, we can obtain the DRH ordinates by subtracting the base flow. Once we have the DRH ordinates, we can also determine the effective rainfall (ER). The UH ordinate is obtained by dividing the DRH by the ER. This concept follows the procedure

we discussed earlier: separating out the base flow from the flood hydrograph to obtain the direct runoff hydrograph and then dividing the direct runoff hydrograph by the effective rainfall magnitude to obtain the unit hydrograph. So, we are using the same concept mathematically in this problem. The peak of the flood hydrograph is given as 670 cubic meters per second, the depth of rainfall is 9 centimetre, the duration of effective rainfall is because of a 5-hour storm, the base flow value is given as 30 cubic meters per second, and the Φ index value is given as 0.2 centimetre per hour. All the values are provided here. Obviously, we need to determine the peak of the DRH; this is the step where we subtract the base flow from the flood hydrograph to obtain the DRH ordinate. So, the peak of the flood hydrograph minus the base flow, which is 670 minus 30, will give us the peak of the direct runoff hydrograph, which comes out to be 640 cubic meters per second. Then, obviously, we need the effective rainfall depth, and for that, we use the standard formula where the Φ index is equal to $P - I_{ER}$ divided by t_e , where we know the value of the Φ index given as 0.2 centimetre per hour. We know I_{ER} , we know the depth of precipitation which is given as 9 centimetre, and we know the duration, t_e , given as 5 hours. So, we have three knowns and only one unknown, which is I_{ER} . So, we can solve this, and it gives us an effective rainfall magnitude of 8 centimetre. Now, once we know the peak of the DRH and once we know the effective rainfall magnitude, we can easily find out the peak of the unit hydrograph by dividing the DRH ordinate by the I_{ER} magnitude. So, 640 divided by 8. The peak of your 5-hour unit hydrograph comes out to be 80 cubic meters per second. Given a flood hydrograph of 5 hours duration for 650 cubic meters per second, by taking the base flow and the x values into account, we obtain the peak of the DRH and the effective rainfall magnitude depth units, which helps us in obtaining the peak discharge of the 5-hour unit hydrograph. So, this example is from GATE 2019

NUMERICAL ON UNIT HYDROGRAPH

Example 3
 The peak of a flood hydrograph due to a 5-h storm is 670 m³/s. The total depth of rainfall is 9 cm. Assuming an average infiltration loss of 0.2 cm/h and a constant baseflow of 30 m³/h, calculate the peak discharge of the 5-h unit hydrograph for this catchment. (GATE 2019)

Solution
 Given, peak of a flood hydrograph = 670 m³/s; Depth of rainfall = 9 cm; Duration = 5 h; Baseflow = 30 m³/s; and ϕ - index = 0.2 cm/h

Peak of the DRH = Peak of a flood hydrograph - Baseflow
 = 670 - 30 m³/s = 640 m³/s

We know that ϕ - index = $\frac{P - I_{ER}}{t_e}$
 Thus, putting the known values 0.2 = $\frac{9 - I_{ER}}{5}$
 Effective rainfall (I_{ER}) = 8.0 cm

Thus, the peak discharge of the 5-h unit hydrograph = $\frac{640}{8} = 80$ m³/s

Handwritten notes in red and blue:
 FH - Baseflow
 DRH
 ER
 UH = DRH / ER

Now, let's consider another example taken from GATE 2018. The 6-hour unit hydrograph of the watershed is represented by an isosceles triangle with a peak of 180 cubic meters per second and a time to peak of 18 hours. The Φ index of this watershed is 3 millimetre per hour, and the constant base flow is 20 cubic meters per second. The accumulated rainfall received in this watershed is 106 millimetre in 12 hours from the start of this storm, respectively. Let's calculate the resulting peak of the flood hydrograph due to this storm. So, this is an inverse problem where we are given the UH ordinate and we need to find out the flood hydrograph. Obviously, if we have the UH ordinate, we need to obtain the DRH, which can be done by multiplying it

by the effective rainfall magnitude and then adding some base flow, which will give us the flood hydrograph. So, it's just the reverse procedure here. The given information includes a total rainfall duration of 12 hours, with a total rainfall of 106 millimetre or 10.6 centimetre . The phi index value is given as 3 millimetre per hour or 0.3 centimetre per hour. Additionally, we have been provided with a 6-hour unit hydrograph, described as an isosceles triangle with all sides equal, and a peak value of 180 cubic meters per second. Since this is a 10-hour duration, it will also be a 10-hour unit hydrograph. So, an isosceles triangle means that all sides are equal. The first thing we need to do is find out the phi index value. Just like in all previous problems, we need to determine the effective rainfall magnitude by knowing the phi index, precipitation (P), and the effective duration (te). The phi index value is given as 0.3 centimetre per hour, rainfall is given as 10.6 centimetre , and the duration is given as 12 hours. By inputting these known values, the effective rainfall depth comes out to be 7 centimetre . Now, the peak of the DRH will be the peak of the UH multiplied by the effective rainfall, which is 180 multiplied by 7. So, obviously, when we had the DRH in earlier problems, we were dividing the DRH ordinates to get the UH ordinate. Now, in this case, we have the UH ordinate. So, we have to multiply the UH ordinate by the effective rainfall depth to get the DRH ordinate. That's what we're doing here: the UH ordinate, which is 180, is being multiplied by the effective rainfall magnitude, which we calculated given the rainfall and effective rainfall duration, and the phi index value. This comes out to be 1260 cubic meters per second. Thus, the peak of the flood hydrograph will be the peak of the DRH plus the base flow. The base flow value is given as 20 cubic meters per second here. So, obviously, 1260 plus 20 equals 1280 cubic meters per second. This problem is different from the previous three problems, where we were given either flood hydrograph information or DRH information and we wanted to develop a unit hydrograph. In this problem, we have been given a unit hydrograph (UH), and from there, we have to develop or obtain the peak of the flood hydrograph. So, it's just the reverse procedure of what we discussed earlier; that's what we have to use, basically.

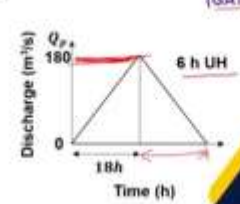
NUMERICAL ON FLOOD HYDROGRAPH

Example 4
 The 6-hour unit hydrograph of the watershed is represented by an isosceles triangle with a peak of 180 m³/s and time to peak of 18 hours. The ϕ -index of this watershed is 3.0 mm/h, and the constant base flow is 20 m³/s. The accumulated rainfall received in this watershed in 12 h from the start of the storm is 106 mm, respectively. Calculate the resulting peak of the flood hydrograph due to this storm event. (GATE 2018)

Solution
 Given,
 Total rainfall duration = 12 h; total rainfall = 106 mm = 10.6 cm
 Also, ϕ - index = 3 mm/h = 0.3 cm/h

Since ϕ - index = $\frac{P-ER}{t_e}$
 Thus, putting the known values $0.3 = \frac{10.6-ER}{12}$
 Effective rainfall (ER) = 7.0 cm

Thus, the peak of the DRH = Peak of UH \times Effective rainfall = 180 \times 7 = 1260 m³/s
 Hence, the peak of the flood hydrograph = Peak of the DRH + Baseflow = 1280 m³/s



Now, let's move on to the next problem, which is taken from the GATE 2012 examination, example number 5. The hourly discharge observations at the mouth of a watershed due to a 2 centimetre excess rainfall during 0 to 1 hour and 3 centimetre excess rainfall during 1 to 2 hours are given in the table below. This is a problem of a complex storm and a complex hydrograph. Essentially, the storm given in this problem is complex, with two slices. The first slice, from 0

to 1 hour, has a magnitude of 2 centimetre . So, from 0 to 1 hour, there is 2 centimetre of effective rainfall, and from 1 to 2 hours, there is 3 centimetre of effective rainfall. These are the ER values already given to us. In all previous problems, we have been calculating the effective rainfall, but here it is directly given. Assume a constant base flow of 1 cubic meter per second. For different times from 0 to 6 hours, we have been given the ordinates of the hydrograph. When we say "hydrograph" or "flood hydrograph," it basically refers to the same thing. So, this is what is given here, and we have been asked to calculate the area of the watershed in square kilometre and to calculate the peak of the 1-hour unit hydrograph and its time of occurrence. These are the two aspects we need to solve for.

NUMERICAL ON COMPLEX HYDROGRAPH

Example 5
 The hourly discharge observations at the mouth of a watershed due to 2 cm excess rainfall during 0 to 1 h and 3 cm excess rainfall during 1 to 2 h are given in the table below. Assume a constant base flow of 1 m³/s.

Time (h)	0	1	2	3	4	5	6
Ordinates of hydrograph (m ³ /s)	1	7	26	37	27	13	1

1. Calculate the area of the watershed in km².
 2. Calculate the peak of 1 h unit hydrograph and its time of occurrence.

(GATE 2012)

Complex Storm
 0-1h 2cm
 1-2h 3cm
 ER

Let's delve into the problem. The ordinates of the flood hydrograph for 0 to 6 hours are provided. Reproducing them, the base flow value is given as 1 cubic meter per second. This simply means that the direct runoff hydrograph can be obtained by subtracting this base flow from the flood hydrograph ordinates. So, 1 minus 1 equals 0, 7 minus 1 equals 6, 26 minus 1 equals 25, 27 minus 1 equals 26, 13 minus 1 equals 12, and 1 minus 1 equals 0. These are our DRH ordinates. Now, when dealing with a complex storm, we need to use the principle of superposition. This means that we assume the 1-hour unit hydrograph (UH) ordinates to be U₀, U₁, U₂, U₃, and so on, in cubic meters per second. As a reminder, in the principle of superposition, we utilize the given UH to develop the DRH due to the two effective rainfall slices. These slices are at 0 to 1 hour with 2 centimetre and 1 to 2 hours with 3 centimetre . So, for the 0 to 1 hour slice, we have 2 centimetre of effective rainfall, and for the 1 to 2 hour slice, we have 3 centimetre . Therefore, what we do is multiply these rainfall amounts with the corresponding unit hydrograph ordinates. So, basically, when converting the direct runoff hydrograph (DRH) and computing the composite DRH, we multiply the effective rainfall magnitude with the unit hydrograph (U) ordinate. These ordinates are denoted as U₀, U₁, U₂, U₃, and so on. Initially, at 0 hours, there is no rainfall, so the ordinate starts at 0. However, at 1 hour, due to the rainfall of 2 centimetre , this ordinate will be multiplied. Therefore, we will have R₁ * U₁ here. At 2 hours, we will have R₁ * U₂, and at 3 hours, we will have R₁ * U₃, and so on. This is how we will plot all the overall R₁ values. Then, due to the additional rainfall of 3 centimetre , we will have another hydrograph, but in this case, it will start at 1 hour. Therefore, at 2 hours, the ordinate will be R₂ * U₁. At the first hour, there is only one component, which is R₁ * U₁. So, at 0 hours, it's 0, but at the first hour, there is only one component, which is 2 * U₁; there is no contribution from R₂. So, R₁ * U₁ equals 0. That simply means 2 * U₁ equals 6. And of course, when we have these 2, then we sum up to get

the composite or DRH. This is basically what we have; this is nothing but a composite DRH ordinates that we have. So, that's how we do this. $2 * U1$ equals 6, so $U1$ comes out to be 3. Now, at 2 hours, as we discussed, there are 2 components. One is because of the first term, that is $R1 * U2$, and the second is because of the second term, which is $R2 * U1$. In this case, $R1$ value is 2, and $R2$ value is 3. So, $3 * U1$ means $2 * U2$ plus $3 * 1$, which is 25. We already know the value of U_n , so by putting the value of U_n , we can get the value of $U2$, and it comes out to be $U3$. Then, subsequently, we will have 2 ordinates: $2 * U3$ and $3 * U2$. So, it's $U1, U2, U3, U4, U5, U6$ here. $U1, U2, U3, U4, U5$, and so on. Obviously, we can solve for all these, and we have to answer what the peak of the unit hydrograph, 1-hour unit hydrograph, is. This 1-hour unit hydrograph has a peak of 8 cubic meters per second, and its time of occurrence is at 2 hours. So, it occurs in 2 hours. So, this is one of the bits we have answered.

NUMERICAL ON COMPLEX HYDROGRAPH

Solution

Time (h)	Ordinates of hydrograph (m ³ /s)	Baseflow	Direct runoff (m ³ /s)	DR due to 2 cm (0-1h) (m ³ /s)	DR due to 3 cm (1-2h) (m ³ /s)	1-h UH ordinates (m ³ /s)
0	1	1	0	2^*u_0		$u_0 = 0$
1	7	1	6	2^*u_1	3^*u_0	$u_1 = 3$
2	26	1	25	2^*u_2	3^*u_1	$u_2 = 8$
3	37	1	36	2^*u_3	3^*u_2	$u_3 = 6$
4	27	1	26	2^*u_4	3^*u_3	$u_4 = 4$
5	13	1	12	2^*u_5	3^*u_4	$u_5 = 0$
6	1	1	0	2^*u_6	3^*u_5	$u_6 = 0$

Peak of 1 h unit hydrograph = 8 m³/s and its time of occurrence = 2 h

Let 1-h UH ordinates be $u_0, u_1, u_2, u_3, u_4, u_5, u_6$ (m³/s)

$2^*u_0 = 0; u_0 = 0$
 $2^*u_1 + 3^*u_0 = 6; u_1 = 3$
 $2^*u_2 + 3^*u_1 = 25; u_2 = (25-9)/2 = 8$
 $2^*u_3 + 3^*u_2 = 36; u_3 = (36-24)/2 = 6$
 $2^*u_4 + 3^*u_3 = 26; u_4 = (26-18)/2 = 4$
 $2^*u_5 + 3^*u_4 = 12; u_5 = (12-12)/2 = 0$
 $2^*u_6 + 3^*u_5 = 0; u_6 = 0$

The other bit is we have to find the watershed area. For that, obviously, we can find out the area under the curve, that is, the total effective rainfall. It will give the depth of rain runoff volume, and we know that we use the triangular formula for the first and last sections and the trapezoidal rule for the sections in between. So, we know that the area of the unit hydrograph (UH) is excess rainfall multiplied by the watershed area. Therefore, the area under the UH curve is half of this area, multiplied by 1 hour. So, it's 1 multiplied by 3600. Then we have the areas of the trapezoids and finally, we have the area of a triangle. There are 2 triangle areas and 3 trapezoid areas. By solving that, we get the area under the curve as 75600 cubic meters. We know that for the unit hydrograph, excess rainfall is 1 centimetre, which always represents the unit depth of rainfall. So, it's 1 centimetre. We can call it as 0.01. So, that simply means that 0.01 meters multiplied by the watershed area will give us the total runoff volume from the area, which we have calculated as 75600. Using this, we can find out that the watershed area is 75600 divided by 0.01, which is in square meters. Finally, we can say that this equals 7.56 square kilometre. By using the concepts which we have learned, we can find out what the watershed area represented by this unit hydrograph is, and also we found out what was the peak of this ordinate, which is 8 at 2 hours. So, these are the three answers you are looking for in this particular problem.

NUMERICAL ON COMPLEX HYDROGRAPH

Solution

Now, to find the watershed area, we use the fact that

Area under the UH = Excess rainfall × watershed area

$$\text{Area under the UH} = \frac{1}{2} \times (0 + 3) \times 1 \times 3600 + \frac{1}{2} \times (3 + 8) \times 1 \times 3600 + \frac{1}{2} \times (8 + 6) \times 1 \times 3600 + \frac{1}{2} \times (6 + 4) \times 1 \times 3600 + \frac{1}{2} \times (4 + 0) \times 1 \times 3600$$

$$= 75600 \text{ m}^3$$

For UH, excess rainfall = 1 cm = 0.01 m

Thus,

$$0.01 \times \text{watershed area} = 75600$$

$$\text{Watershed area} = 7560000 \text{ m}^2 = 7.56 \text{ km}^2$$

Then we move on to problem number 6, which has been taken from GATE 2003, example 6. It says that the problem is a polyming table that represents the ordinates of a 4-hour unit hydrograph. The average infiltration loss rate and constant base flow of the watershed are 0.5 centimetre per hour and 20 cubic meters per second, respectively. Calculate the peak outflow of the rainfall event of 5 centimetre in 4 hours, calculate the peak of a 12-hour unit hydrograph for the above watershed, and the time and UH ordinates are given here. The 4-hour UH ordinates are given here. So, the depth of rainfall is given, base flow is given, and phi index is given. The duration of rainfall, we know, is 4 hours. Using this, we can find out the relationship using this relationship where phi index is equal to $P - ER$ by t_e , where the phi index value is 0.5 centimetre per hour, effective rainfall is 5 centimetre, and duration is 4 hours. So, by putting the known values, we can solve for excess rainfall, which comes out to be 3 centimetre.

NUMERICAL ON SUPERPOSITION METHOD

Example 6

The following table presents the ordinates of a 4-h unit hydrograph. The average infiltration loss rate and constant baseflow of the watershed are 0.5 cm/h and 20 m³/s, respectively.

- Calculate the peak outflow of a rainfall event of 5 cm in 4 hours.
- Calculate the peak of a 12 h unit hydrograph for the above watershed.

Time (h)	0	4	8	12	16	20	24	28	32
Ordinates of UH (m ³ /s)	0	180	560	540	260	120	35	8	0

(GATE 2003)

Solution

Given, Depth of rainfall = 5 cm; Baseflow = 20 m³/s; ϕ -index = 0.5 cm/h; Duration = 4 h

$$\phi\text{-index} = \frac{P - ER}{t_e}$$

$$0.5 = \frac{5 - ER}{4}$$

Excess rainfall (ER) = 3 cm

Once we have the 4-hour UH magnitudes at different times, which have been given. For calculating the peak outflow of a rainfall event of 4 centimetre per hour, we first need to calculate the DRH coordinates by multiplying the UH ordinates by ER and then summing the base flow. So, multiplying DRAs due to 3 centimetre of excess rainfall will be coming by multiplying this 4-hour unit hydrograph by 3. So, that comes out to be 180 times 3, which is

540, 560 times is 1680, and so on. This is the DRH ordinate, and then the base flow value is given, which is 20. So, the ordinates of the flood hydrograph will be the sum of these two. So, 0, 560, 1700, 1640, and so on, and we are looking for flood hydrographs. So, this is the peak outflow. That comes out to be 1700 cubic meters per second.

NUMERICAL ON SUPERPOSITION METHOD

Solution

1. For calculating the peak outflow of a rainfall event of 5 cm in 4 hours, we need to calculate the DRH ordinates by multiplying the UH ordinates by ER

Time (h)	4-h UH (m ³ /s)	DRH due to 3 cm ER	Baseflow (m ³ /s)	Ordinates of flood hydrograph (m ³ /s)
0	0	0	20	20
4	180	540	20	560
8	560	1680	20	1700
12	540	1620	20	1640
16	260	780	20	800
20	120	360	20	380
24	35	105	20	125
28	8	24	20	44
32	0	0	20	20

Peak outflow = 1700 m³/s

For calculating the peak of the 12-hour unit hydrograph for the watershed, we need to superimpose 3 4-hour UHs, which have been given, each lagging the previous one by 3 hours because we know that we have to use from a D hour to an nD hour to develop. So here, it is 4 hours to 12 hours. That means n equals to 3, which is an integer. So, the method of superposition can be used. That simply means that in which we have readied the method of superposition, we need to superimpose 3 numbers of D-hour unit hydrograph, which is 4-hour unit hydrographs, each lagging the previous one by 4 hours, and then we can get the combo DRH ordinates. So, this is our original 4-hour unit hydrograph, which is given, and then we have to lag the next one UH by 4 hours. So, that means it starts at 4 hours instead of 0 hours. So, starting from 4 hours, we list the ordinates as it is. We just copy the ordinates, but we start at 0 hour by lagging the previous one by 4 hours. And then we have the third hour, so because we have to superimpose 3, one is original, then this is the second one, the third one will lag by 8 hours. So, here the ordinates will start at 8 hours. So, simply starting from 8 hours, we will copy the values here. And then we will sum up these 3 ordinates to get the ordinates of a composite DRH, which will be because of 3 centimetre of effective rainfall, 1 centimetre each for the 3 hydrographs, 1 centimetre, 1 centimetre, 1 centimetre, so, 3 centimetre . So, this DRH due to this 3-hour centimetre ER will be the sum of the columns 2, 3, and 4. So, the values are 0 here, only one. So, 180 here, these two are there, so, this is 560 plus 180, which is 740. Here, it is 540 plus 560 plus 180, which is 1280. Then, this is how you will get the values of DRH, but we want a 12-hour unit hydrograph ordinates. So, obviously, we have to divide this DRH ordinate by 3, which is the effective rainfall magnitude represented by this composite DRH. So, obviously, we will divide this. So, we have 160, 0, 0, 160 divided by 360, 740 divided by, and so on. And so, the peak of the 12-hour unit hydrograph is 453.3 cubic meters per second, which we obtained by using the principle of superposition. So, that is how we can utilize the principle of superposition and use this method. So, two bits, a little different approaches, but this is how we can do it.

NUMERICAL ON SUPERPOSITION METHOD

Solution

2. For calculating the peak of a 12 h unit hydrograph for the watershed, we need to superimpose three 4-h UH, each lagging the previous one by three hours.

Time (h)	4-h UH (m ³ /s)	Lag by 4 h	Lag by 8 h	DRH du to 3 cm ER (Col. 2+3+4)	12-h UH (Col. 5/3)
(1)	(2)	(3)	(4)	(5)	(6)
0	0	-	-	0	0
4	180	0	-	180	60
8	560	180	0	740	246.7
12	540	560	180	1280	426.7
16	260	540	560	1360	453.3
20	120	260	540	920	306.7
24	35	120	260	415	138.3
28	8	35	120	163	54.3
32	0	8	35	43	14.3
36		0	8	8	2.7
40			0	0	0

Peak of a 12 h unit hydrograph = 453.3 m³/s

*D-h → AD UH
4-h → 12-h UH
7.5*

Then we take the next two problems on S hydrograph or summation hydrograph, and this problem has been taken from GATE 2007. A 4-hour unit hydrograph is used to derive the S hydrograph, the ordinates of the 4-hour UH are given below. So, these are the ordinates of the 4-hour unit hydrograph, 0 to 44 hours, these are the ordinates. Calculate the equilibrium discharge and its time of occurrence for the derived S hydrograph, calculate the area of the watershed. So, we know that we have a 4-hour unit hydrograph, and we have to derive the S hydrograph, and in deriving the S hydrograph, we know that we have to superimpose an infinite number of unit hydrographs. Here, the base time is 44. So, theoretically, we need to divide the total time base, TV, by the duration of the unit hydrograph, D, to determine the number of unit hydrographs we need to superimpose. So, 44 divided by 4 means we need to superimpose 11 unit hydrographs in order to derive the S hydrograph. Alternatively, we can use a simpler and more straightforward approach by employing S curve addition. This method simplifies the process, as creating multiple columns in the table can become cumbersome.

NUMERICAL ON S-HYDROGRAPH

Example 7

A 4-h unit hydrograph (UH) is used to derive S-hydrograph. The ordinates of 4-h UH are given below:

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinates of UH (m ³ /s)	0	20	80	130	150	130	90	52	27	15	5	0

1. Calculate the equilibrium discharge and its time of occurrence for the derived S-hydrograph.
2. Calculate the area of watershed.

(GATE 2007)

*T_b = 44
D = 4
= 11*

Essentially, we are utilizing S curve addition here. First, we calculate the S curve ordinates. These are simply the ordinates of the 4-hour unit hydrograph that are given to us. Then, for S curve addition, we start by lagging it by 4 hours, meaning it starts at 4 hours instead of 0. Next,

we sum up the ordinates from the left column and put them in the next cell. So, let's sum the values in the addition column. It goes like this: 0 plus 20 equals 20, 20 plus 80 equals 100, 100 plus 130 equals 230, 230 plus 120 equals 350, 350 plus 30 equals 380, 380 plus 130 equals 510, 510 plus 90 equals 600, 600 plus 52 equals 652, 652 plus 27 equals 679, 679 plus 15 equals 694, and finally, 694 plus 5 equals 699. This column demonstrates the accumulation of discharge over time. The S curve ordinates can be derived by summing up columns 2 and 3, which means these two columns. So, we simply add the corresponding values: 0, 20, 100, 230, 380. The sum gradually increases to 510, 600, 652, 679, 694, and finally reaches 699. This signifies that it has reached an equilibrium discharge of 699 at 40 hours. From column 4, we can determine the equilibrium discharge, which is the point where the discharge becomes constant. As we previously discussed, when we sum up multiple unit hydrographs, the discharge tends towards a constant equilibrium discharge close to the time base. This discharge is referred to as the equilibrium discharge. Hence, the equilibrium discharge is 699 cubic meters per second, and it occurs 40 hours from the beginning. This information is extracted directly from the table. Now, we need to calculate the area. We can use the relationship $q_e = 2.78A/D$. We already know the value of q_e , which is 699, and we also have the value of D , which was generated from the 4-hour unit hydrograph. Alright, let's tackle the problem. By plugging in the values of q_e and D , which are 699 and 4 respectively, we can calculate the area in square kilometre. The result is 1005.75 square kilometre . So, that's the solution, derived from developing the S hydrograph.

NUMERICAL ON S-HYDROGRAPH

Solution
Let us first calculate the S-curve ordinates.

Time (h)	Ordinate of 4-h UH (m ³ /s)	S-curve addition (m ³ /s)	S-Curve ordinates (m ³ /s) [Col. 2 + Col. 3]
(1)	(2)	(3)	(4)
0	0	-	0
4	20	0	20
8	80	20	100
12	130	100	230
16	150	230	380
20	130	380	510
24	90	510	600
28	52	600	652
32	27	652	679
36	15	679	694
40	5	694	699
44	0	699	699
48		699	699
52		699	699


From Col. 4, the equilibrium discharge (q_e) = 699 m³/s
It occurs at 40 h from the beginning of the effective rainfall

Also,

$$q_e = \frac{2.78A}{D}$$
 (A is in km² and D is in h)

$$699 = \frac{2.78 \times A}{4}$$

 Area (A) = 1005.75 km²



Now, let's move on to the last problem from the Gate 2023 examination. We're given the ordinates of a 6-hour hydrograph for a catchment in the table below. The catchment has a phi index of 0.25 centimetre per hour and a base flow of 710.5 cubic meters per second. Our task is to calculate the peak of the flood hydrograph generated from this catchment due to a storm of 45 centimetre received over 6 hours. These are the ordinates.

NUMERICAL ON S-HYDROGRAPH

Example 8

The ordinates of a 6-h S-hydrograph of a catchment are given in the table below. The catchment has a phi-index of 0.25 cm/h and baseflow of 10.5 m³/s. Calculate the peak of the flood hydrograph generated from this catchment due to a storm of 45 mm received during 6 h.

Time (h)	0	6	12	18	24	30	36	42	48
Ordinates (m ³ /s)	0	30	90	180	252	306	342	360	360

(GATE 2023)

First, we'll use the S hydrograph to develop a 6-hour unit hydrograph. Then, we'll determine the effective rainfall using this relationship, which yields an ER of 3 centimetre. Subsequently, we'll develop DRH ordinate and flood hydrograph ordinates. The ordinates of the S hydrograph are given. So, to develop the 6-hour unit hydrograph, we simply lag it by 6 hours and then subtracting SA from SB will give us the DRH ordinates. So, we start with 0, 30, 60, and so on. Then, for the 6-hour unit hydrograph, we divide by 1 since T and D are the same, resulting in DRH ordinates. Now, we need to find the DRH ordinates due to the effective rainfall of 3 centimetre. This can be obtained by multiplying the 6-hour UH ordinates by 3 centimeters and then adding the base flow, as the value is given here. Essentially, we are using two S hydrographs, SA and SB, to develop a unit hydrograph for different durations. Fortunately, in this case, both T and D were the same. So, the procedure remains the same: lagging, obtaining DRH, and dividing by T by D, which is 1 in this case. So, the peak of the flood hydrograph in this case comes out to be 280.5 cubic meters.

NUMERICAL ON S-HYDROGRAPH

Solution

First, the S-hydrograph is used to develop a 6-h unit hydrograph. The effective rainfall is determined using the data given.

$$\phi - \text{index} = \frac{P - ER}{t_c} \text{ or } 0.25 = \frac{45 - ER}{6}, \text{ Thus, } ER = 3 \text{ cm}$$

Subsequently, DRH ordinates and flood hydrograph ordinates are developed.

Time (h)	Ordinates (m ³ /s) S _A	Ordinates (m ³ /s) S _B	DRH of 6/6=1cm (S _A -S _B)	6-h UH	DRH of 3 cm ER	Baseflow (m ³ /s)	Ordinates of flood hydrograph
0	0	-	0	0	0	10.5	10.5
6	30	0	30	30	90	10.5	100.5
12	90	30	60	60	180	10.5	190.5
18	180	90	90	90	270	10.5	280.5
24	252	180	72	72	216	10.5	226.5
30	306	252	54	54	162	10.5	172.5
36	342	306	36	36	108	10.5	118.5
42	360	342	18	18	54	10.5	64.5
48	360	360	0	0	0	10.5	10.5
54	360	360	0	0	0	10.5	10.5

Peak of the flood hydrograph = 280.5 m³/s

D=T=6

Now, let's quickly address the last problem. The successive 3-hour ordinates of a 3-hour storm for a particular watershed are given here. The depth of storm rainfall is 60 mm, assuming a constant base flow of 5 mm per hour. We need to determine the 6-hour ordinates. We have the

ordinate values for the 6-hour storm, the depth of the storm, base flow, Φ index, and duration. Thus, we can calculate the effective rainfall of 4.5 centimetre .

NUMERICAL ON S-HYDROGRAPH

Example 9
 A successive three hourly ordinates of a 3-hour storm for a particular watershed are 5, 47, 122, 107, 95, 65, 47, 35, 23, 14 and 5 cumecs. The depth of storm rainfall is 60 mm. Assuming a constant base flow of 5 cumec and an average storm loss of 5 mm/h, determine the 6-h UH ordinates.

Solution
 Given, the ordinates of a 6-h storm hydrograph,

Time (h)	3	6	9	12	15	18	21	24	27	30	33
Discharge (cumec)	5	47	122	107	95	65	47	35	23	14	5

Also, depth of storm rainfall = 60 mm = 6 cm; Baseflow = 5 cumec; ϕ - index = 0.5 cm/h; Duration = 3 h

Thus, $\phi - \text{index} = \frac{P - ER}{t_p}$
 $0.5 = \frac{6 - ER}{3}$ **Effective rainfall (ER) = 4.5 cm**

Then, we have the flood hydrograph ordinates. By subtracting the base flow (column 2 minus column 3), we obtain the DRH ordinates. Since we have 3-hour UH ordinates, we will develop the 3-hour UH to get the 6-hour unit hydrograph. So, obviously, we have to use the S curve as instructed. That's why we're lagging the S curve and then summing up the values. The S curve ordinates will be the sum of these two, starting from 0 plus 9.3 and so on. Then, we lag this S curve by 6 hours. So, we're lagging it by 6 hours. These are the values we obtain, and then we calculate $S(A) - S(B)$ by dividing by T by D . T is 6, and D is 3, so it's divided by 2. These are the 3-hour unit hydrograph ordinates. I apologize, this is for the 6-hour unit hydrograph, not the 3-hour. So, from the 3-hour to the 6-hour unit hydrograph, this is what we're getting.

NUMERICAL ON S-HYDROGRAPH

Solution The flood hydrograph ordinates, baseflow and ER are used to determine the 3-h UH. The 3-h UH is then used to develop the S-curve. The developed S-curve is subsequently used to derive the 6-h UH.

Time (h)	Discharge (cumec)	Baseflow (cumec)	Ordinates of DRH (Col. 2 - Col. 3)	3-h UH (Col. 4/4.5 cm)	S-curve addition	S-curve ordinates (S_A)	S_A (S_A lag by 6 h)	$S_A - S_B$	6-h UH (Col. 9/(6/3))
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	5	5	0	0	-	0	-	0	0
3	47	5	42	9.33	0	9.33	-	9.33	4.67
6	122	5	117	26	9.33	35.3	0	35.3	17.7
9	107	5	102	22.7	35.3	58	9.33	48.7	24.3
12	95	5	90	20	58	78	35.3	42.7	21.3
15	65	5	60	13.3	78	91.3	58	33.3	16.7
18	47	5	42	9.33	91.3	101	78	22.7	11.3
21	35	5	30	6.67	101	107	91.3	16	8
24	23	5	18	4	107	111	101	10.7	5.33
27	14	5	9	2	111	113	107	6	3
30	5	5	0	0	113	113	111	2	1
33						113	113	0	0

$\frac{T}{D} = \frac{6}{3} = 2$
 Ordinates of 6-h UH

This way, we've found the ordinates for the 6-hour. This method illustrates how we can solve various kinds of problems using the concepts we've learned. Thank you very much. Please give your feedback. If you have any doubts or questions, please do not hesitate to raise them. We'll be happy to address them on the forum. Thank you.



THANK YOU

