Course Name: Watershed Hydrology

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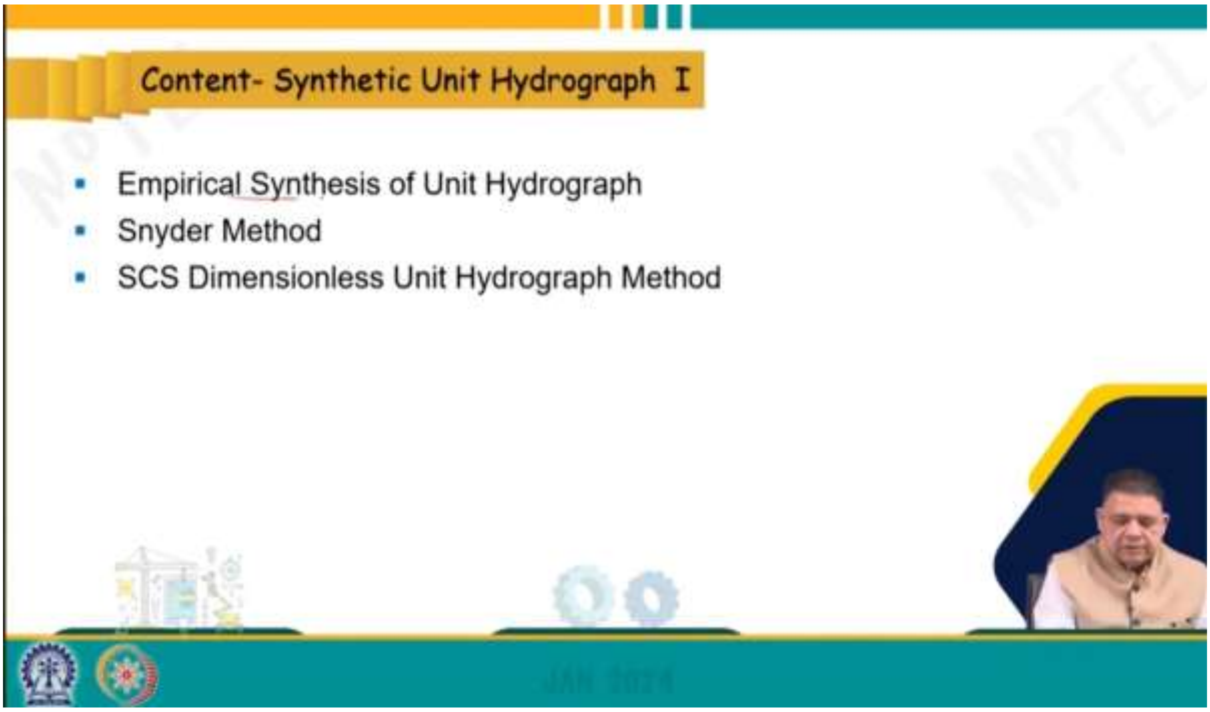
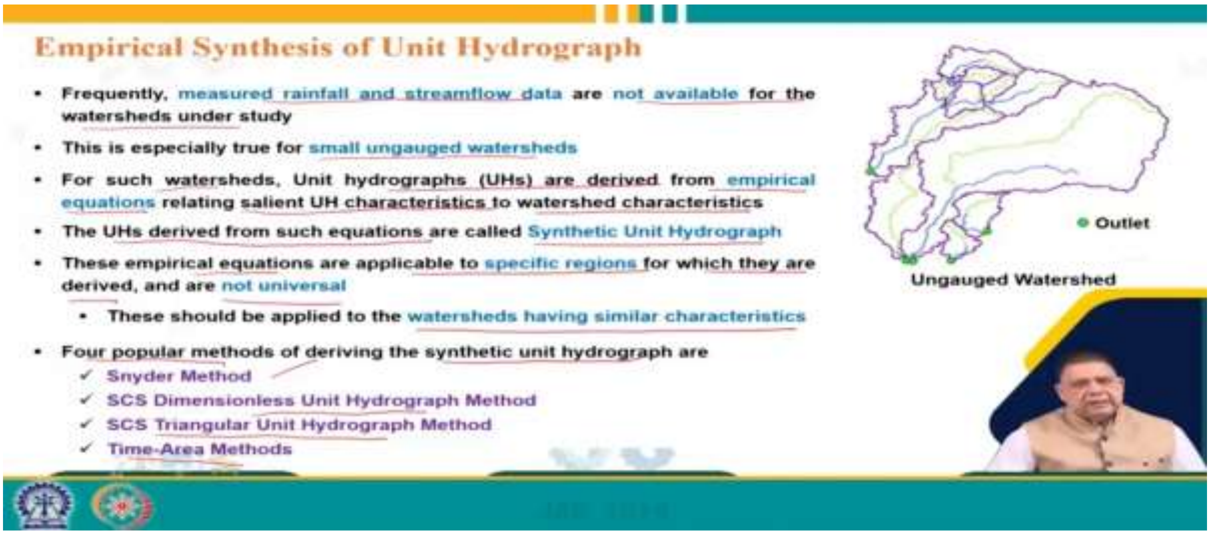
Week: 06

Lecture 26: Synthetic Unit Hydrograph I

Hello, friends! Welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the Department of Agricultural and Food Engineering at the Indian Institute of Technology Kharagpur. Today, we are beginning Module 6, and we are starting with Lecture Number 1, which is on Synthetic Unit Hydrograph, Part 1.



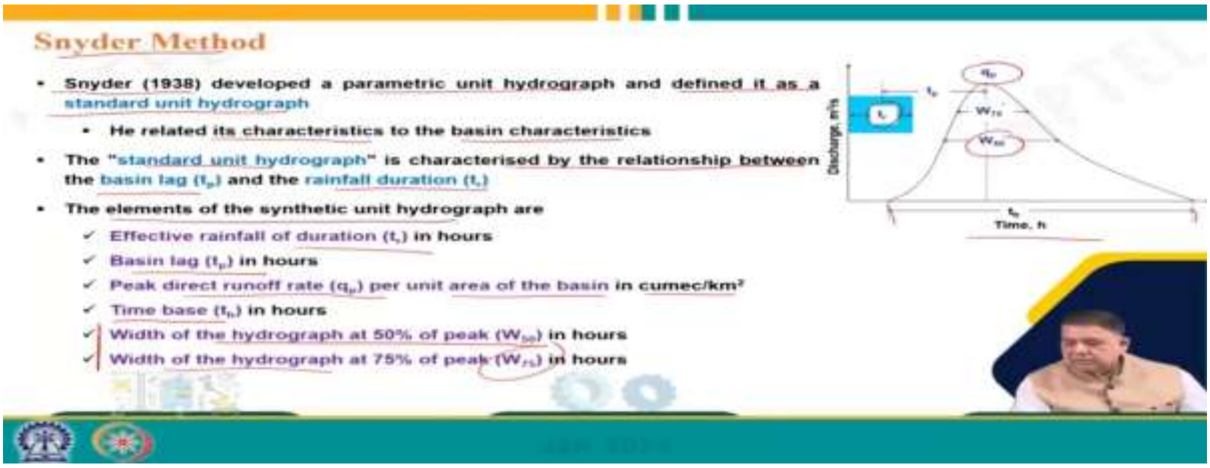
So, this lecture will consist of the empirical synthesis of unit hydrograph, meaning we will introduce synthetic unit hydrograph, discuss the Snyder method, and the SCS dimensionless unit hydrograph method of developing the synthetic unit hydrograph.



Starting with the empirical synthesis of unit hydrograph, basically, we have seen and discussed hydrograph, which is a graphical representation of discharge versus time. Of course, we have also seen how to measure discharge. Once we have measured discharge data, we plot them against time to get the hydrograph. However, many times, measured rainfall and stream flow data are not available for the watersheds under study. This is especially true for small, ungauged watersheds located in remote areas where instrumentation has not taken place.

For such watersheds, unit hydrographs are derived from empirical equations relating salient UH characteristics to watershed characteristics. Watershed characteristics are used in these empirical equations to develop the unit hydrograph characteristics. Unit hydrographs derived from such equations are called synthetic unit hydrographs. They are synthetically generated because we do not have measured discharge data. Instead, we use empirical equations that relate hydrograph characteristics to basin characteristics and synthetically generate the flow data. Hence, the name synthetic unit hydrograph. Of course, these empirical equations apply to specific regions for which they are derived and are not universal, which is true for any empirical equation. Thus, they should be applied to watersheds having similar characteristics, either in the same watershed or watersheds with similar hydrological characteristics. In hydrology, we define hydrologically homogeneous regions.

Obviously, in any watershed within a similar hydrologically homogeneous region, these equations can be applied; otherwise, their application is limited. For developing or deriving the synthetic unit hydrograph, there are four popular methods: the Snyder method, the SCS dimensionless unit hydrograph method, the SCS triangular unit hydrograph method, and the time-area method. These are the four popular methods used to derive synthetic unit hydrographs, representing relationships between hydrograph characteristics and basin characteristics. We will discuss these methods one by one in these lectures.

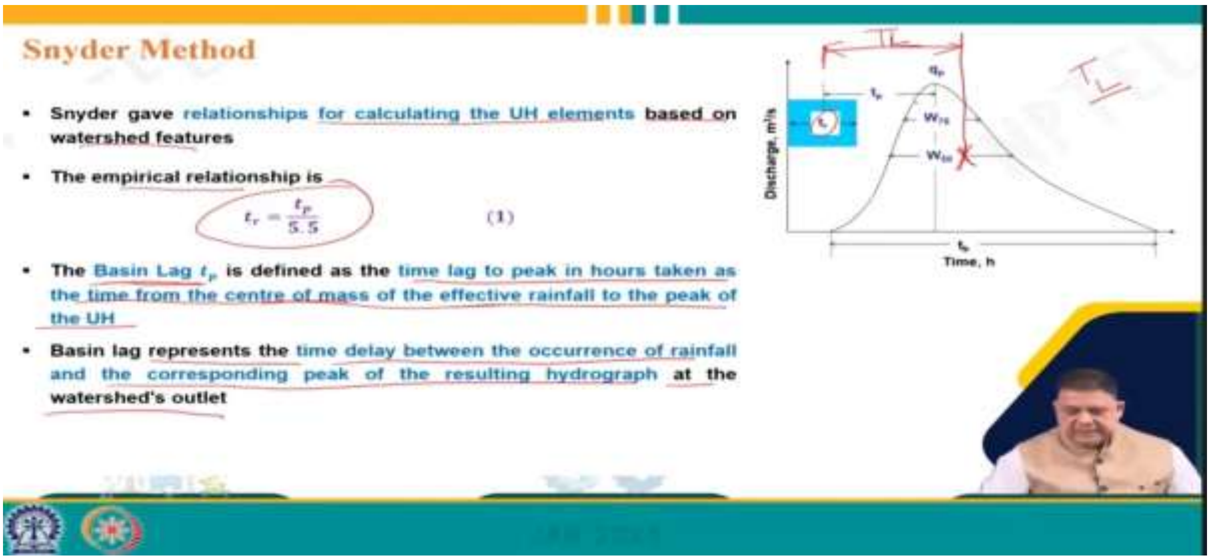


Starting with the Snyder method of generating or deriving synthetic unit hydrograph, which was developed by Snyder in 1938.

So, Snyder developed a parametric unit hydrograph and defined it as a standard unit hydrograph. He related the characteristics of this standard unit hydrograph to basin characteristics, hence the term "synthetic unit hydrograph." The standard unit hydrograph is characterized by the relationship between the basin lag and the rainfall duration. While we know the rainfall duration, and we have also seen basin lag, in Snyder's method, basin lag is defined slightly differently, which we will explore later.

The elements of a synthetic unit hydrograph remain more or less the same, with perhaps one or two new terms introduced. We are familiar with effective rainfall duration, denoted as t_r, representing the duration in which one unit depth of effective rainfall has occurred. Then, there's the basin lag, which I will define later, and the peak direct runoff rate, q_p, representing the peak value of the hydrograph in cubic per s quare kilometer, essentially depth units. The time base remains unchanged; it's the duration from the start to the endpoint.

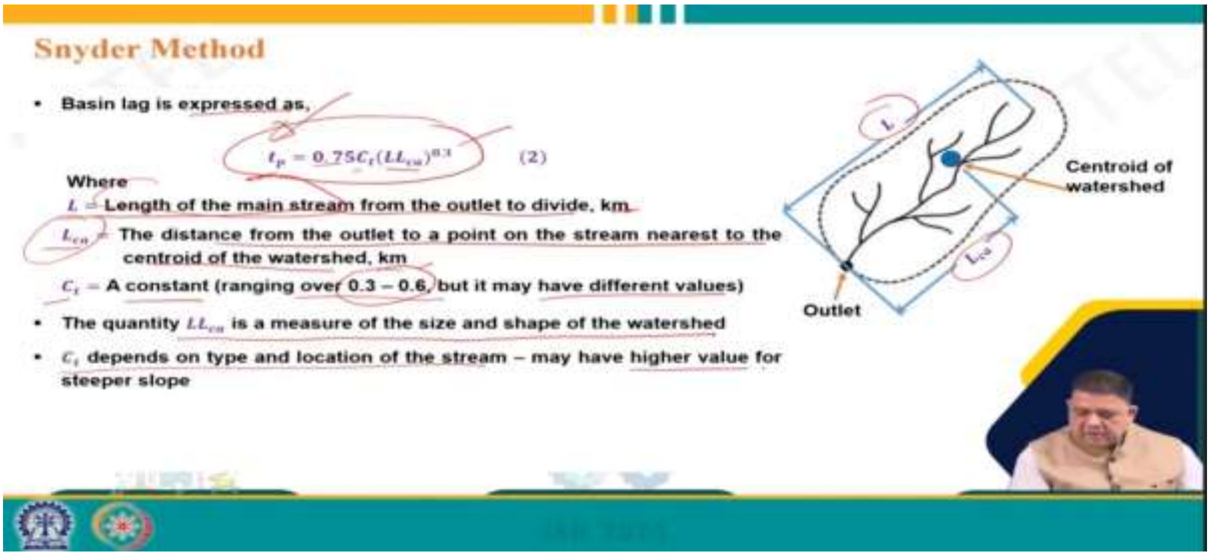
Additionally, two new dimensions are introduced: the width of the hydrograph at 50 percent of the peak (W_{50}) and the width of the hydrograph at 75 percent of the peak (W_{75}). These dimensions represent the widths of the hydrograph at these respective points.



Snyder provided relationships for calculating the UH elements based on watershed features. Now, let's see how these UH elements are linked to watershed characteristics. The first empirical relationship is T

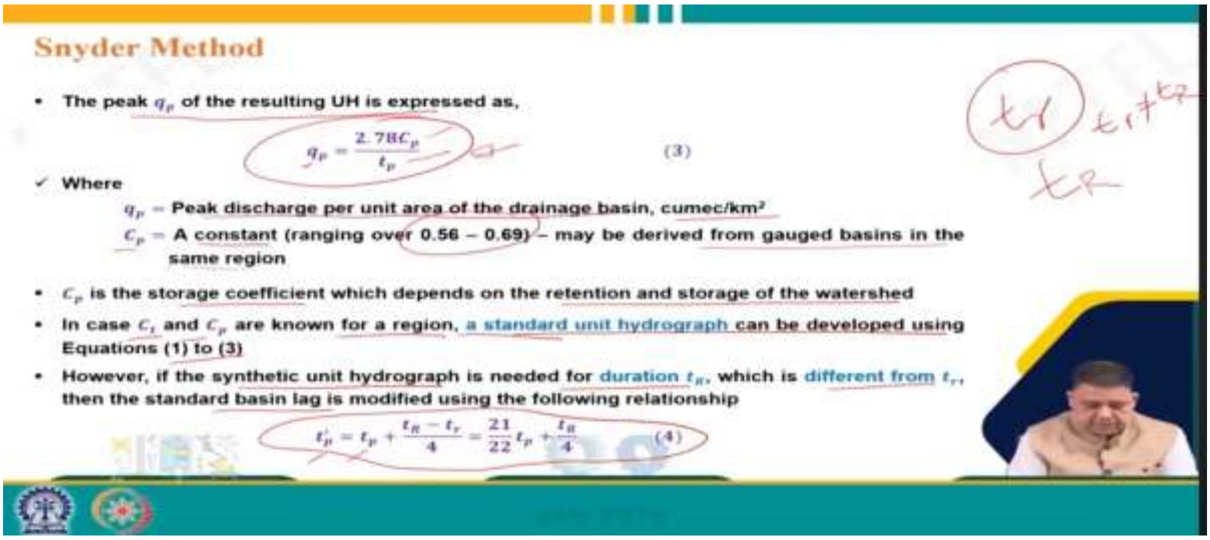
$t_r = t_p / 5.5$, where TR is the effective rainfall duration, and t_p is defined as the basin lag, the time from the center of mass of effective rainfall to the peak of the unit hydrograph.

Basin lag represents the time delay between the occurrence of rainfall and the corresponding peak of the resulting hydrograph at the watershed's outlet. It signifies the lag in the occurrence of the peak at the outlet after rainfall. This definition by Snyder highlights the basin lag in this manner.



Basin lag is expressed by the relationship given here, which is related to t_p and is defined as $0.75\ C_t\ (L\ L_{ca})^{0.3}$ is the relationship where L is the length of the mainstream from the outlet to the divide. L_{ca} is the distance from the outlet to a point on the stream nearest to the centroid of the watershed, both in kilometers, and C_t is a constant ranging from 0.3 to 0.6, but it may have different values too. So, the general range is 0.3 to 0.65, but that is not a limitation; it can take other values. Here, you can see that the basin lag, which we defined earlier, is related to t_r or storm duration, and effective rainfall duration, and is related to basin characteristics: length of the mainstream and the length to the centroid. So, that simply means that the length of the mainstream is measured from both, measured from the outlet and from the outlet to the endpoint along the stream, which measure as the length of the mainstream. And then, if we locate the centroid of the watershed and then project a point from that centroid on the stream, then we measure that length, and that is referred to as L_{ca}, the length to the centroid. So, if we have a drainage map of the basin, we can measure this L and L_{ca}, and we can obtain.

So, that means, if you just need a stream network in a basin, which is easy to obtain, and of course, nowadays with remote sensing data, we can easily get these L and L_{ca} using GIS software. So, L and L_{ca}, if obtained, then we can calculate t_p. The quantity LL_{ca} is a measure of the size and shape of the watershed. So, the length is considered, and the centroid considers the shape also. C_t depends on the type and location of the stream; it may have higher values for steeper. So, if the watershed has a steeper slope, then the value of CT will be a little bit higher.

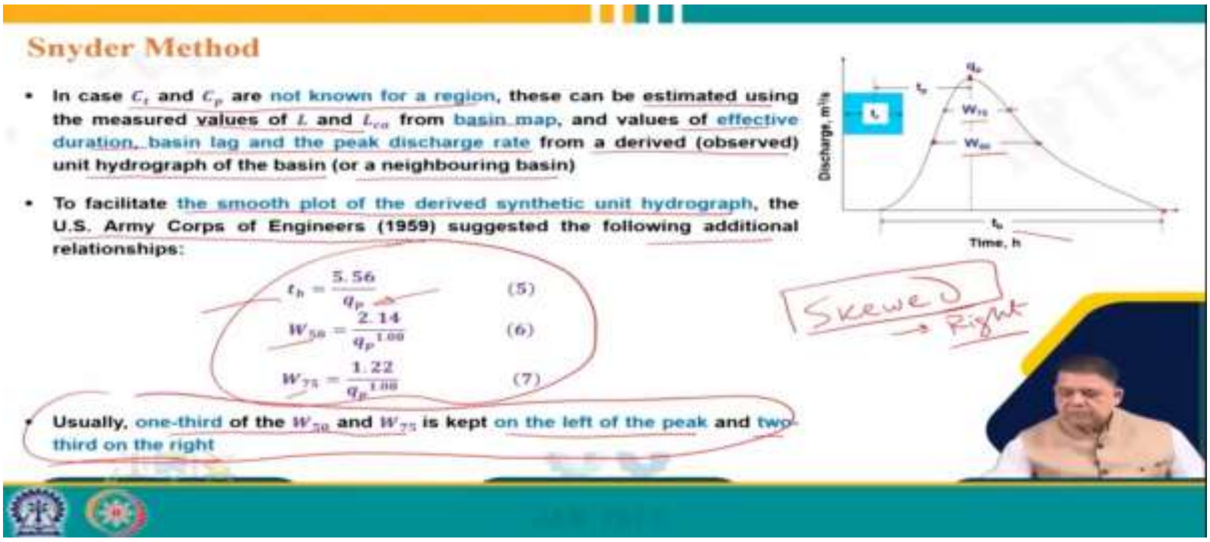


Now, the q_P of the resulting UH can be expressed using this relationship. So, $q_P = 2.78C_p/t_P$, where the value of t_p is already obtained. q_P is the peak discharge per unit area of the drainage basin, cubic per square kilometer, and C_p is a constant ranging from 5.56 to 0.669, and it may be derived from gauge basins in the same region. So, if you have data from nearby watersheds in the same hydrological conditions, then we can use the C_p value from there. This is a storage coefficient that depends on the retention storage characteristics of the watershed. So, in case C_t and C_p are known for a region, a standard unit hydrograph can be developed using the relationships we have seen, that is t_r equals t_p by 5, and $t_p = 0.75\ C_t\ (LL_{ca})^{0.3}$, and then $q_p = 2.78\ C_p/t_p$. So, with these relationships, we can develop the standard unit hydrograph. However, if the synthetic unit hydrograph is needed for a division TR, which is different from t_r, so Snyder's formula always develops a standard unit hydrograph for the storm duration of t_r.

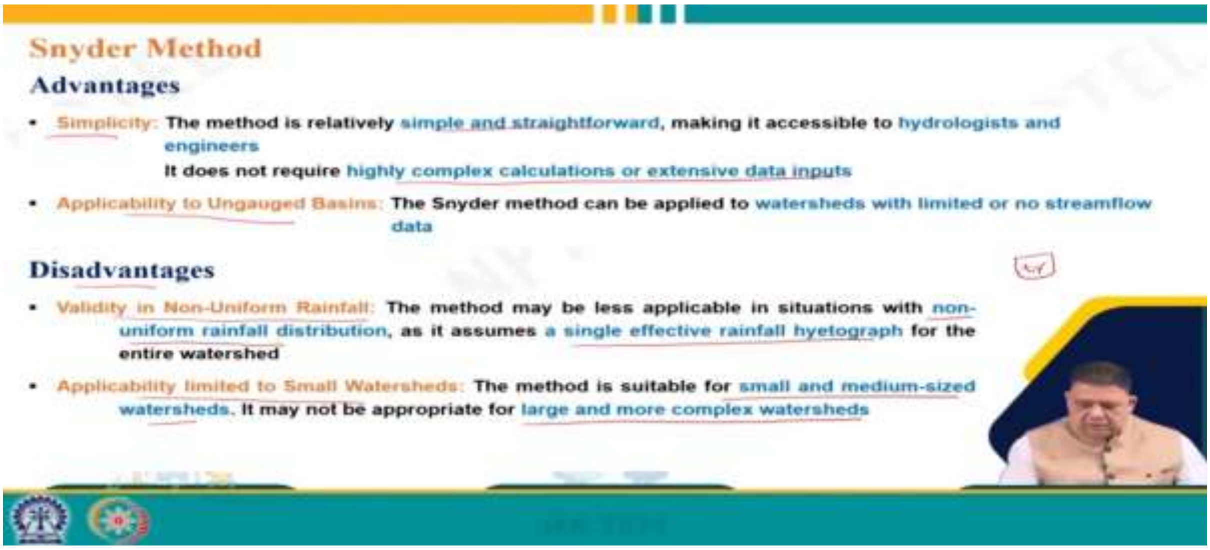
But suppose we are interested in some other duration, that is when t_R is not equal to t_r, then we need to modify the standard basin lag, and for that, we use this relationship. And quite often, we have to use this relationship to modify basin lag. So, this TP' is TP, which is the calculated value we have for t_r, and TR, TR is the required storm duration. TR is a standard storm duration divided by 4. So, this relationship can be used for getting the modified value of basin lag. And, once we have this modified value, this value will be used in equation number 3 to get the q_p value.

So, this is one we have to remember. Now, in case C_t and C_p are not known for a region, these can be estimated using the major value of L and L_{ca} from the basin map and values of effective duration, basin lag, and peak discharge rate from the derived unit hydrograph of the neighbouring basin if you have all the data, you do not need to bother for a synthetic unit hydrograph. So, you will have data from the neighbouring basins. Now, if you just saw the equation. So, from there, we got this value and we got this t_r and t_p only these 3 we got.

So, knowing only this much, plotting the hydrograph was challenging. To facilitate the smooth plot of the derived synthetic unit hydrograph, the US Army Corps of Engineers in 1959 suggested the following additional relationship. And here, as you can see, t_b, that is the time base, W_{50}, that is the width at 50 percent of q_P, and W_{75}, which is width 75 at 75 percent of q_P, can be obtained in terms of q_p, and q_p value we have already calculated using equation number 3, that is $2.78\ C_P/t_p$ or t_p or t_p' depending upon that, but if you know the q_P, we can find out these.

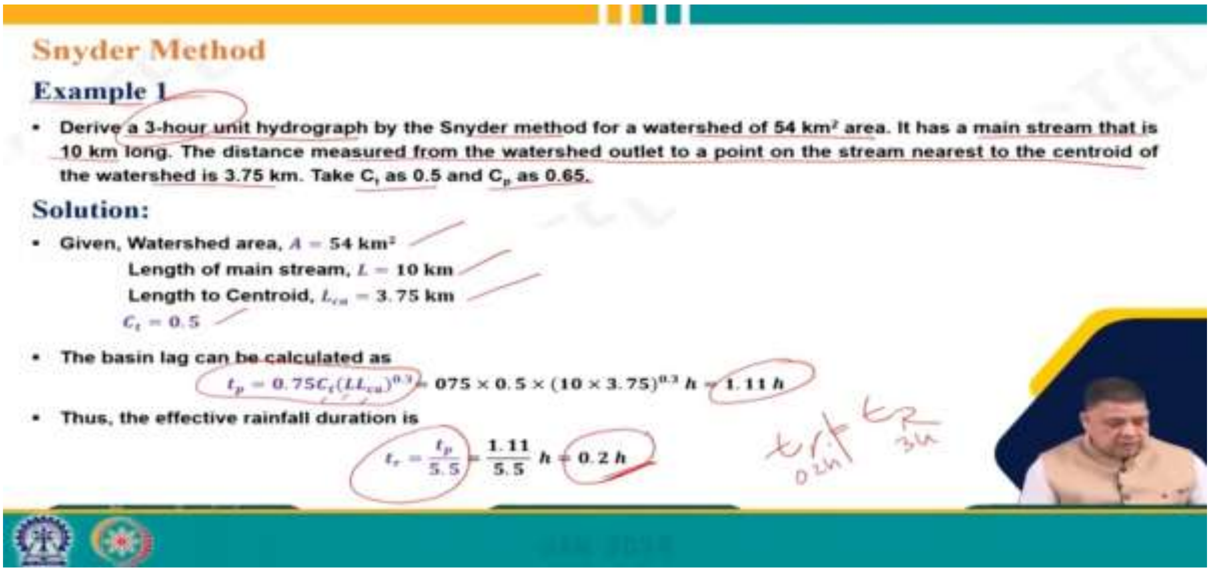


So, now, what happens is that you have an additional point. So, it is much easier to plot a smooth curve, and there is a recommendation that while plotting, one-third of W_{50} and W_{75} are kept on the left of the peak and two-thirds on the right. So, one-third here, two-thirds here, and remember it is a skewed distribution, the hydrograph is skewed to the right, and to ensure that skewness towards the right, this particular recommendation is being done, that is one-third should be on W_{50} on the left-hand side and two-thirds should be on the right-hand side so that we automatically get a curve which has a lot more area on the right-hand side, that is the recession side, which we discussed earlier while discussing the hydrograph.



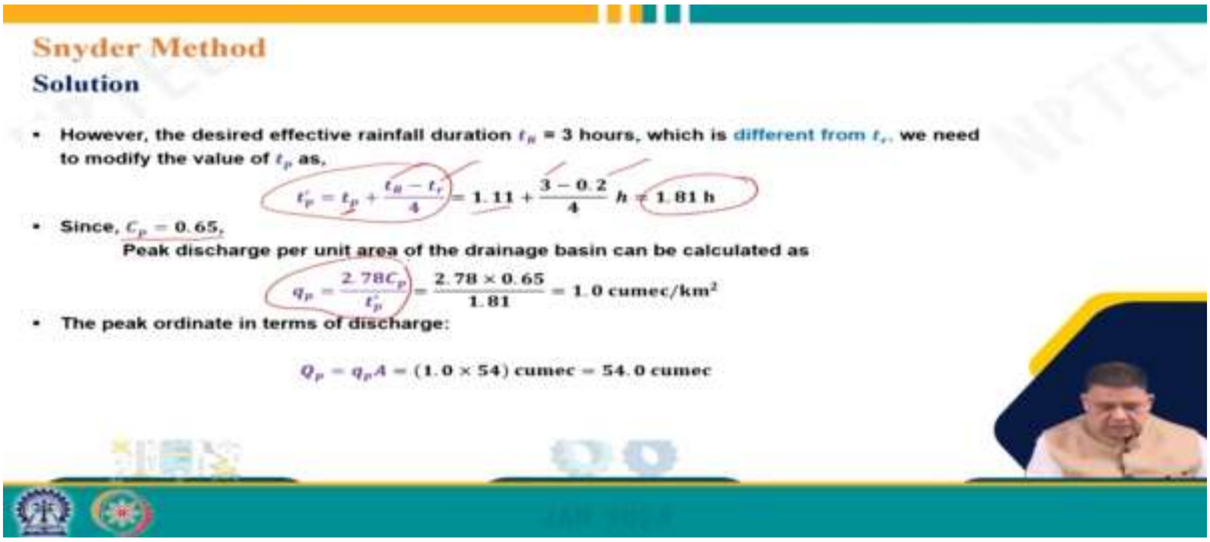
Now, coming to advantages and disadvantages, there are certain advantages of the slider method, that is, it's very simple. The method is relatively straightforward, as you can see that we need L and L_{ca} and then C_t values, and then you can develop a unit hydrograph, it does not require highly complex calculations or extensive data inputs, and it applies to ungauged basins, which is obvious, the prime purpose that we need to develop a synthetic unit hydrograph for a watershed which is ungauged or where we do not have data.

But there are certain disadvantages also like validity and non-uniform rainfall. Now we have only a single storm here with t_r, and so if we deal with any non-uniform rainfall distribution or complex storm event, then this method will fail. Because it assumes a single effective rainfall hydrograph, its applicability is limited to small watersheds. The method is suitable for small and medium-sized watersheds but may not be appropriate for large and more complex watersheds. Obviously, because it is synthetic, you would not like to take that kind of risk if you use it for large watersheds where a huge magnitude of discharge is coming. So, it is recommended to use it for small or medium-sized watersheds.

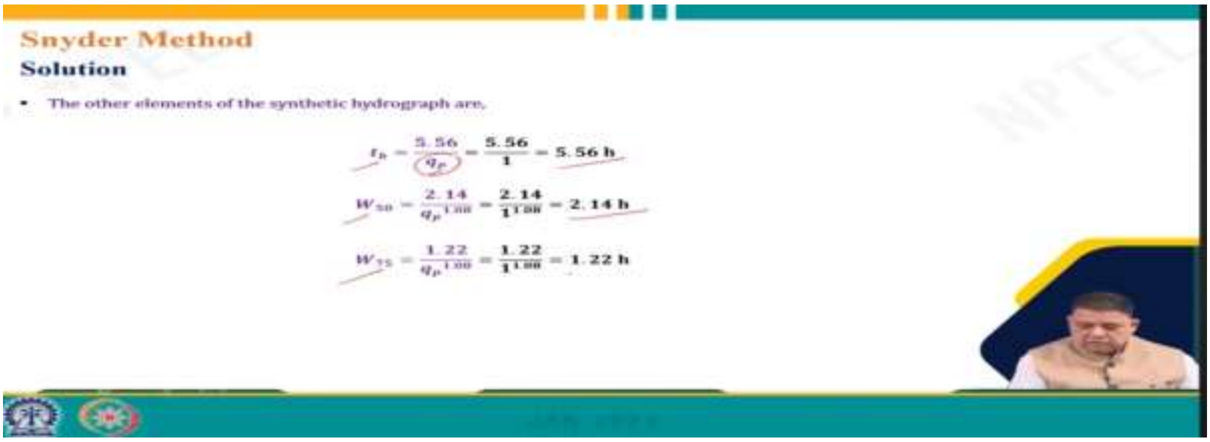


Now, let us take an example of this and try to see how to handle that, that is derive a 3-hour unit hydrograph by the Snyder method for a watershed of 54 square kilometers area. It has a mainstream that is 10 kilometers long, and the distance measured from the watershed outlet to the point on the stream nearest to the centroid of the watershed is 3.7 kilometers. Take C_t as 0.5 and C_p as 0.65. So, the information that is given is a 54 square kilometer length to the main length of mainstream L is 10 kilometers length to centroid L_{ca} is 3.7. C_t is 0.5. So, we know this relationship $t_p=0.75\ C_t\ (LL_{ca})^{0.3}$. So, we know C_t, we know L, we know L_{ca}. Putting these values, we can calculate the value of t_p, which comes out to be 1.11 hours.

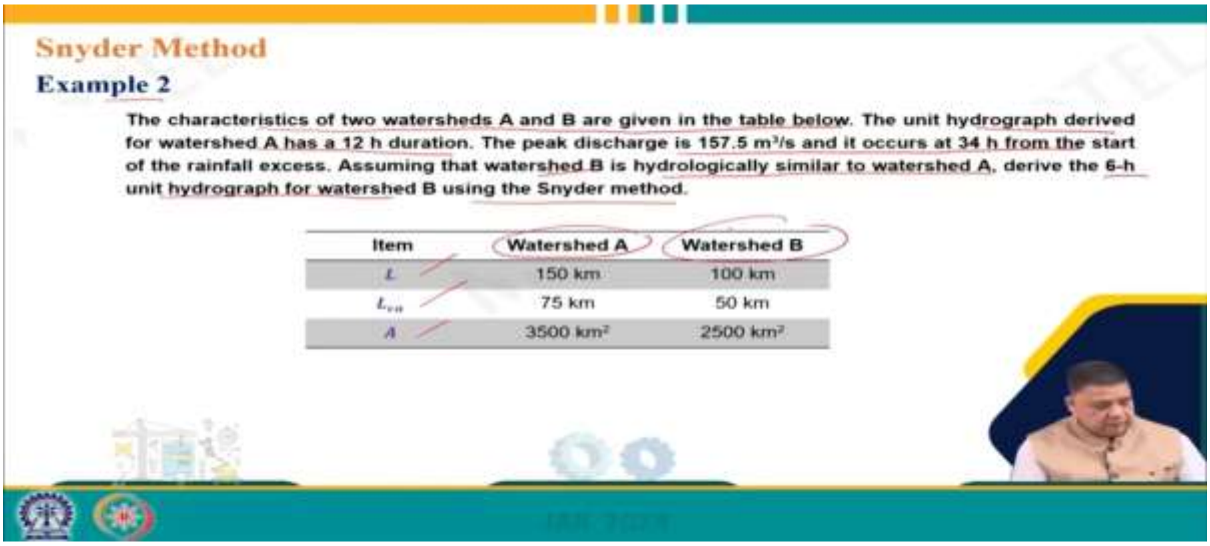
Now, we know that effective rainfall duration is related to t_p by this relationship $t_r = t_p /5.5$. So, by calculating t_p, we get this value as 2 hours, but we have been asked to derive a 3-hour unit hydrograph, wherein we got the duration as 2 hours. So, in this case, t_r is not equal to t_R. So, we are interested in 3 hours; this is 0.2 hours; we are interested in 3 hours. So, we need to modify the value of t_p.



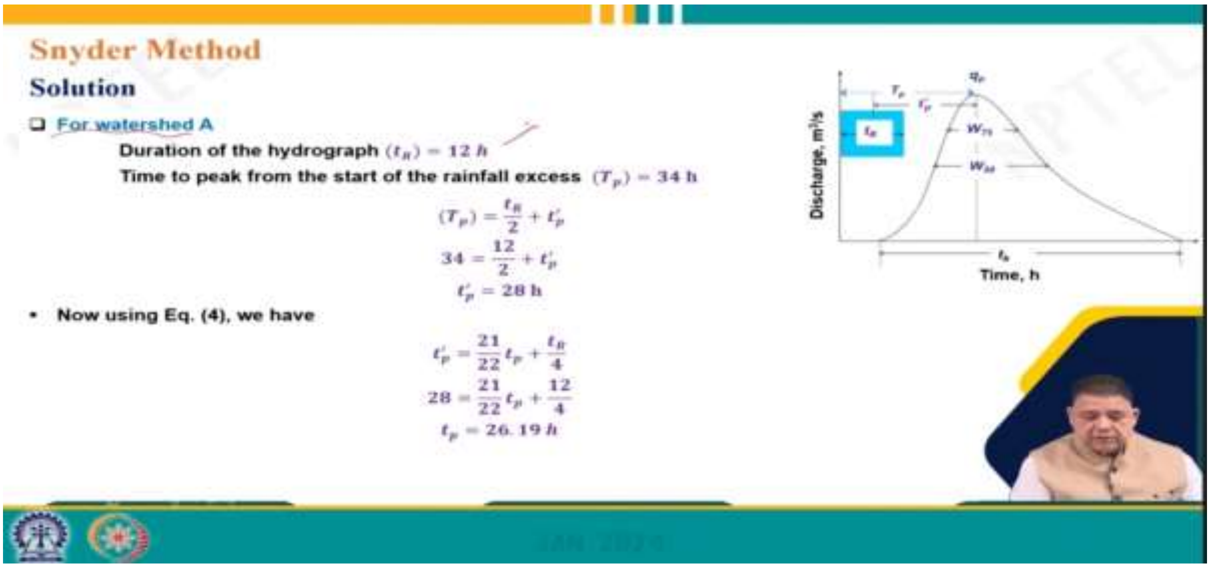
So, t_p we can modify t_p using this relationship. So, t_p value is 1.11, t_R is 3, t_r is 0.2, 4 is, of course, constant. So, using this relationship, we get t_p' dash is 1.81 hours. Now, the value of C_p is known to us. So, we can calculate the peak discharge q_p using this relationship of 2.78 C_p/t_p'. Now, we know C_p, we know t_p', putting the value, remember we are using t_p' here. So, that is why 1.81 hours. So, this comes out to be 1 cubic per square kilometer in terms of discharge; we multiply by area, and we get 54 cumecs. So, that is the peak discharge.



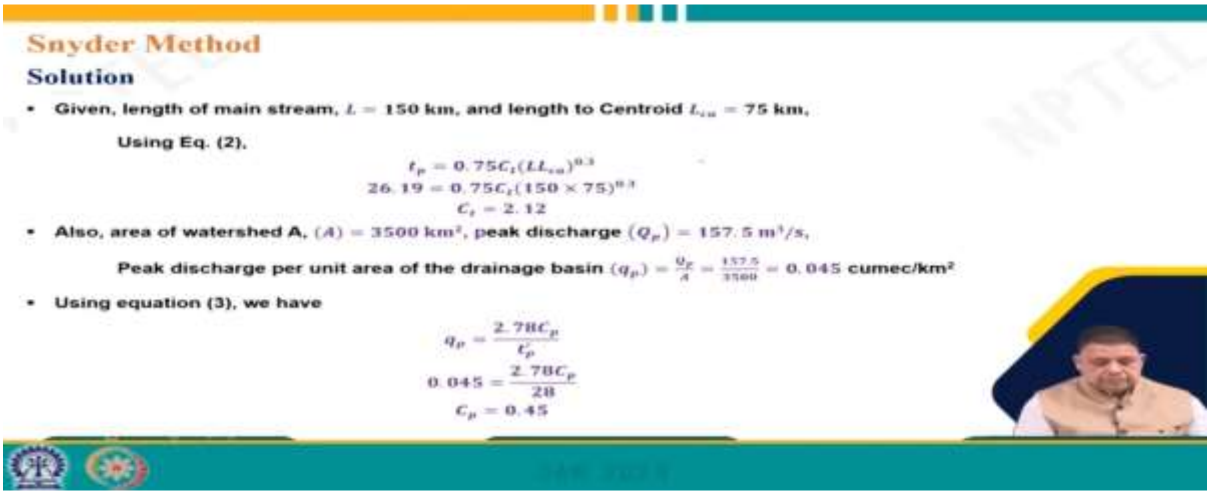
Now, other elements are that like t_b, W_{50}, W_{75}, knowing q_p, we can calculate this, and then we can q_p that is, we have to use the in cumecs per square kilometer as per this relationship. So, putting the values, we get t_b equal to 5.56 hours, W_{50} is 2.14 hours, and W_{75} is 1.22 hours. So, now, that means, we know everything, and knowing that, we can plot the hydrograph quite easily.



We take another example: the catchment characteristics of two watersheds A and B are given in the table below. Unit hydrograph derived for watershed A has a 12-hour duration, peak discharge is 157.5 cubic meters per second, and it occurs at 34 hours from the start of rainfall access. Assuming that watershed B is hydrologically like watershed A, derive the 6-hour unit hydrograph for watershed B using the Snyder method. So, here, the length of the mainstream, length to the centroid, and area of watersheds A and B are given to us.

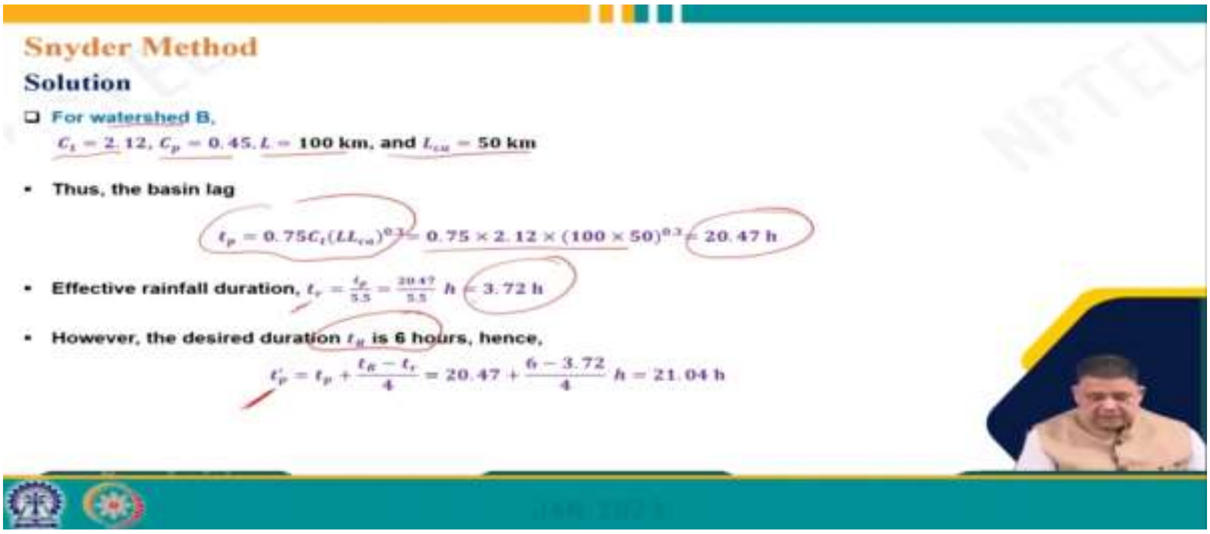


First, we will use the data for watershed A. So, the duration of the hydrograph we are given is 12 hours. Remember, it is t_R, not a standard duration, and the time to peak is 34 hours, t_p. So, we know that t_p is t_R divided by 2, t_p'. So, 34 TR we know, putting these values, we get t_p' is 28 hours. And then, we know this relationship: t_p' is (21 /22) *t_p. So, putting the known values here, we get the value of t_p, the basin lag, and the standard basin lag, which can be defined in terms of Snyder's equations, is 26.19 hours, which was different than when we started.

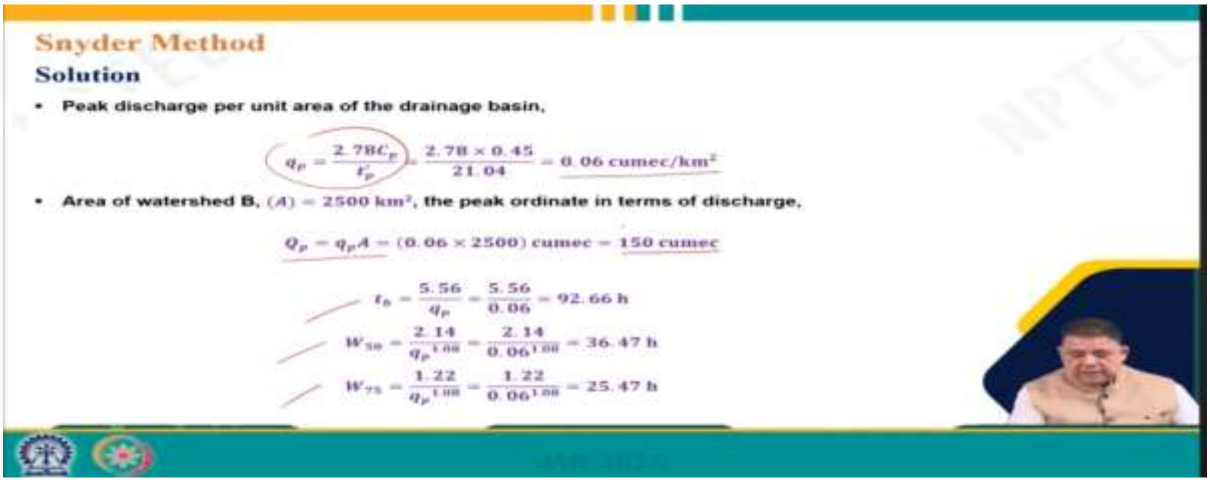


Now, the length of the mainstream is 150, length of the centroid is 0.75. So, using this relationship and knowing t_p, we can calculate t_p LL_{ca} is known. So, we can find out the value of C_t, which comes out to be 2.12. Also, we know the watershed area and peak discharge; these values are also given.

So, we can find out q_p in terms of cubic meters per square kilometer, which comes out to be 0.05. So, now, putting in this equation q_p=2.78 $C_p/t_p{'}$, we can find out the only unknown, which is C_p. So, using the information given for watershed A, what we have done is we have found out the 2 constants, C_t and C_p, which we can utilize for watershed B.

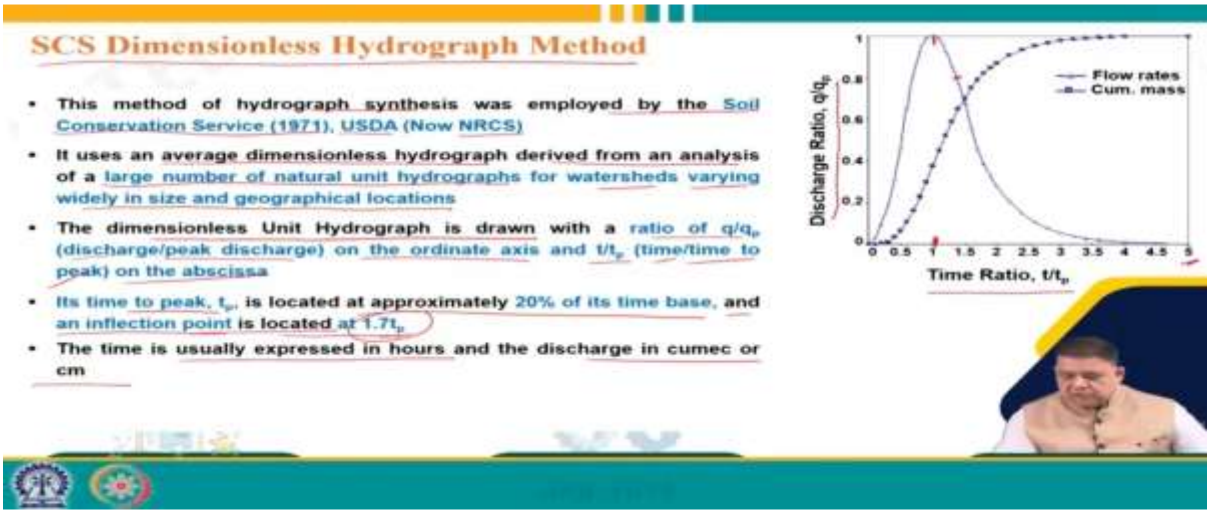


Now, we will get into watershed B. So, C_t, C_p, LL_{ca}, these pieces of information we know now. So, using this relationship t_p= 0.75 C_t $LL_{ca}^{0.3}$, and putting the values, we get t_p = 20.47, and standard duration t_r is t_p/5.5, which comes out to 3.72 hours. But we need a unit hydrograph for a 6-hour duration; that is what we have been asked for. So, that means, we have to modify the basin lag, and $t_p{'}$, and use this relationship because we know t_p, we know t_R, we know t_r, and putting the values, we get $t_p{'}$ as 21.04 hours.



And then q_p, you can find out, which comes out to 0.06 cubic meters per square kilometer. Multiplying by area, we can get the q_p value, which comes out to be 150 cubic meters, and then we can calculate t_b, W_{50}, W_{75}.

So, all the elements are known, and we can draw the unit hydrograph quite easily. So, that is how we can use Snyder's method to develop a unit hydrograph.



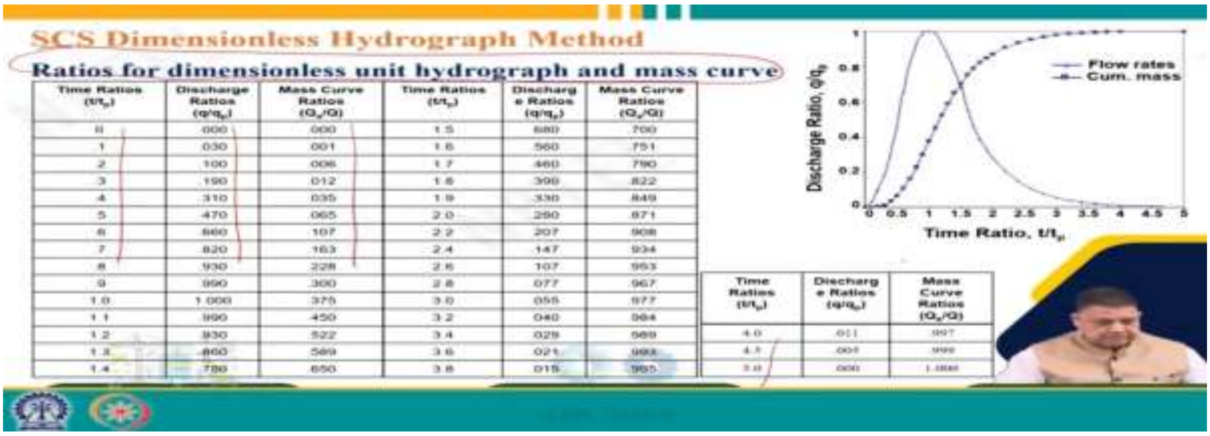
Now, we get into the SCS dimensionless hydrograph method, which is the other method, and this method of hydrograph synthesis was employed by or developed by the Soil Conservation Service. They developed this in 1971, and SCS, we know that now is known as NRCS, which is the Natural Resource Conservation Service, and of course, it belongs to USDA, US Department of Agriculture.

So, what this method uses is an average dimensionless hydrograph derived from the analysis of many natural unit hydrographs for watersheds varying widely in size and geographical location. So, what SCS did was they collected the unit hydrograph data from many watersheds from various regions and various sizes, and then they developed this dimensionless unit hydrograph. This dimensionless unit hydrograph is drawn with the ratio of q by qP, that is discharge divided by peak discharge on the ordinate axis, that is, discharge ratio here, and T by TP, that is time to time to peak on the abscissa.

So, the time ratio is here. So, it is discharge is q by q_p versus time ratio t by t_p and. So, as you can see here, this is a curve. So, when t by t_p is 1, q_p by q_p is 1, and you can see that the t by t_p value ranges from 0 to 5. So, that is the range for which this hydrograph is available, and of course, the discharge ratio goes up to 1 because the max could be only 1, and its time to peak t_p is located approximately 20 percent of the time base, and an explicit point is located to 1.7 t_p.

So, becauseit is up to 5 and t_b by t_p will be a to 1. So, that is why it says that 20 percent, and it is one of the inflection points, that inflection if you remember we were discussing hydrograph, and we decided on the peak segment, and we said that it lies between 2 inflection points, one on the rising side, one on the restriction limb. So, it talks about 1.7 t_p.

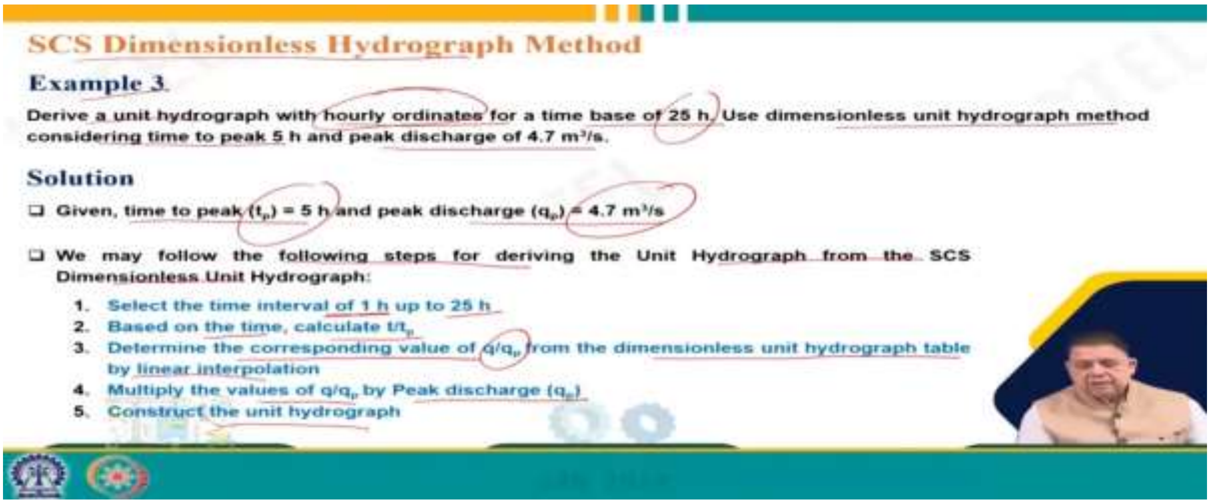
So, time to peak is obviously, 1 in this case. So, 1.7, that means, it talks about there will be an inflection point which lies at 1.7 t_p. The time is usually expressed in hours, and discharge in cubic meters or centimeters, depth units also, but time usually is hours. So, this is what the dimensionless hydrograph looks like, and of course, there is a cumulative mass curve also plotted here.



Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)	Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000	1.5	.680	.700
.1	.030	.001	1.6	.560	.751
.2	.100	.006	1.7	.460	.790
.3	.190	.012	1.8	.390	.822
.4	.310	.035	1.9	.330	.849
.5	.470	.065	2.0	.280	.871
.6	.660	.107	2.2	.207	.908
.7	.820	.163	2.4	.147	.934
.8	.930	.228	2.6	.107	.953
.9	.990	.300	2.8	.077	.967
1.0	1.000	.375	3.0	.055	.977
1.1	.990	.450	3.2	.040	.984
1.2	.930	.522	3.4	.029	.989
1.3	.860	.589	3.6	.021	.993
1.4	.780	.650	3.8	.015	.995

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

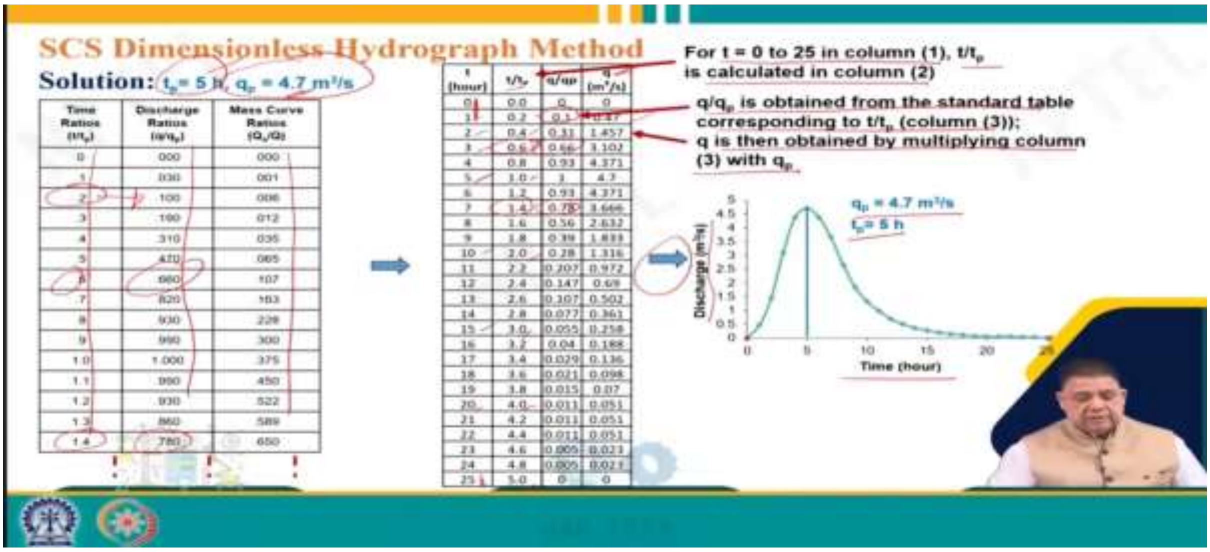
Now, this graph to represent this graph ratio for dimensionless unit hydrograph in mass curve SCS provided is the standard table where you can see here the time ratio 0-time ratio is here t by t_p 0 to 5 you can see here and the corresponding q by t_p ratios and mass curve values are given here, and this is the plot. So, this is a standard plot we get.



Now, we will take an example here, example 3. So, to see the application of the SCS dimensionless hydrograph method, derive a unit hydrograph with hourly ordinates for a time base of 25 hours, and use the dimensionless unit hydrograph method considering time to peak at 5 hours and peak discharge of 4.

7 cubic meters per second. So, we have been given the time to peak is 5 hours and peak discharge is 4.7 cubic meters per second. So, we can follow the following steps for deriving a unit hydrograph from the SCS dimensionless unit hydrograph method. So, we have been asked to derive hourly ordinates.

So, the time interval is 1 hour, and up to a time base of 25 hours. So, up to 25 hours. So, 1, 2, 3, 4, till 25, and based on the time, we calculate t/t_p because tp is already given. So, we can calculate t by t_p and determine the corresponding values of q/q_p from the dimensionless unit hydrograph table by linear interpolation. So, if the values do not match exactly, we can interpolate the given values and multiply the value of q by qp by the peak discharge, which is known to us, and construct the unit hydrograph. So, straightforward method as far as if you have the dimensionless unit hydrograph table available with you.



So, as you can see here, we have written here that t_p is 5 hours, q_p is 4.7 cubic meters per second, and this is the standard dimensionless unit hydrograph table. As you can see, this is a time ratio, of course, only a truncated part. I put the larger one that was given in the previous slide, and these are the discharge ratios, and these are the mass ratios. Of course, we do not use the mass of ratios. We only are concerned with the time ratio and discharge ratio. Now, what we have done is time 1-hour hourly time up to 25 we have 0, 1, 2, 3. So, these are the ordinates we must find out. So, t/t_p for t equal to 0 to 25 in column 1 t/t_p is calculated in column 2, and t_p value already we know, 5.

So, that means, 1 by 5 is 0.2, 2 by 5 is 0.4, 3 is 0.5, 5 at 5 hours it is 1, at 10 hours it is 2, at 15 it is 3, 24 and 25 it is 5. So, that is what we said that t by tp ranges from 0 to 5, which is the upper limit of 5. Now, q by q_p is obtained from the standard table corresponding to t by t_p in column 3.

So, this is t by tp is 0.2. So, this is 0.2 here, corresponding value is 0.1 q by q_p. So, that is why we are saying q qp is 0.1. Similarly, we have only put a truncated table up to 1, 1.4. So, as we can see when 1.54, the value is 0.78. So, 1.4 we are using a value of 0.78. It, we have 0.6 here. So, at 0.6 here, the value is 0.660. So, we are using 0.660 here. So, obviously, for the different t values we were interested in, we first calculate t by t_p, and then from the standard dimensionless unit hydrograph table provided by SCS, we obtain the q by q_p values.

Now, we know the qp value. So, we can find out the q value by multiplying the q by q_p values with qp values. So, we get q values as 0.47, 1.47. So, q is then obtained by multiplying column 3 with q_p, which we know. So, these are the values here, and then using all of those we can plot this dimensionless unit hydrograph. So, its q_p is 4.7 cubic meters per second, its t_p is 5 hours, and it starts at 0 and ends at 25, that is what we have been asked for, and this is the discharge in cubic meters per second and time in hours. So, this is how we can use the SCS dimensionless hydrograph method and develop the unit hydrograph synthetic unit hydrograph. So, we discussed in this lecture synthetic unit hydrograph, and we saw there are 4 different methods.

Today, we discussed two of the methods: the Slider method and the SCS dimensionless unit hydrograph method. Other methods will be covered in the next class. Thank you very much. If you have any questions or doubts, please raise them in the forum, and we will be happy to answer. Thank you very much.