

Course Name: Watershed Hydrology

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Week: 06

Lecture 29: Numerical on Synthetic UH, IUH and Distribution Graph I

**SWAYAM NPTEL COURSE ON
WATERSHED HYDROLOGY**

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Module: 06

Lecture: 04 (Numerical on Synthetic UH, IUH and Distribution Graph I)

Hello friends, welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology Kharagpur. We are in Module 6, and this is Lecture Number 4.

Content- Numerical on Synthetic UH, IUH and Distribution Graph I

- Numerical on Synthetic Unit Hydrograph
- Numerical on Instantaneous Unit Hydrograph (IUH)
- Numerical on Distribution Graph

Today, we will work on a numerical example involving synthetic unit hydrograph, instantaneous unit hydrograph, and distribution hydrograph. This is Part 1 of the lecture, and we will have another lecture dealing with a numerical example.

Synthetic Unit Hydrograph

Example 1

- Derive a 2-hour unit hydrograph by the Snyder method for a watershed of 350 km² area. It has a main stream that is 30 km long. The distance measured from the watershed outlet to a point on the stream nearest to the centroid of the watershed is 8 km. Take C_t as 1.5 and C_p as 0.7.

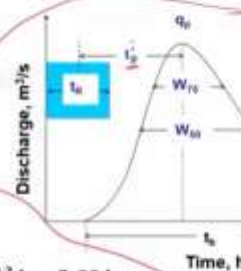
Solution:

- Watershed area, $A = 350 \text{ km}^2$
- Length of main stream, $L = 30 \text{ km}$
- Length to Centroid $L_{ca} = 8 \text{ km}$
- $C_t = 1.5$
- Therefore, basin lag can be calculated as

$$t_p = 0.75C_t(LL_{ca})^{0.3} = 0.75 \times 1.5 \times (30 \times 8)^{0.3} \text{ h} = 5.82 \text{ h}$$

- Effective rainfall duration can be calculated as

$$t_r = \frac{t_p}{5.5} = \frac{5.82}{5.5} \text{ h} = 1.06 \text{ h}$$



Now, let's start with an example on synthetic unit hydrograph. Example 1 asks us to derive a 2-hour unit hydrograph using the Snyder method for a watershed area of 350 square kilometres. The main stream is 30 kilometres long, and the distance from the watershed outlet to a point on the stream nearest to the centroid of the watershed is 8 kilometres. We are given C_t as 1.5

and c_p as 0.7. Snyder developed this synthetic unit hydrograph based on an empirical relationship between hydrograph characteristics and watershed characteristics. In this problem, we are given an area of 350 square kilometres, a main stream length (L) of 30 kilometres, and a centroid length of 8 kilometres. The coefficient C_t is 1.5. The basin lag (t_p) is related to L and L_{ca} using the relationship

$$t_p = 0.75 C_t (LL_{ca})^{0.3}$$

By substituting the given values into this equation, we can find the basin lag (T_p).

So, that gives us a value of 5.82 hours, and t_p is related to effective rainfall duration by this relationship. So, $t_r = \frac{t_p}{5.5}$,

we get t_r value as 1.06. This is where t'_p comes into the picture because we need to derive a 2-hour unit hydrograph t_r . So, t_r , is 2 times t_r , which we calculate here as 1.06. If t_r is not equal to the standard duration we want (in this case, 2 hours for the hydrograph), then we need to modify the value of t_p , hence the introduction of t'_p is used to calculate the modified t_p . The same concept is reiterated here: since the derived duration t_r , is 2 hours, we need to modify the value of t_p . This means we need to calculate t'_p , which can be calculated using the relation related to t_p , t_r , and the standard duration (t_r). We have all the values: t_p is 5.82, t_r , the duration for which we require the hydrograph is 2 hours, and the Snyder standard duration of unit hydrograph (t_r) is 1.06, which we calculated earlier. Putting these values, we get t'_p as 6.05, which is the value to use in all other equations.

Synthetic Unit Hydrograph

Solution:

- Because the desired duration t_r is 2 h, we need to modify the value of t_p :

$$t'_p = t_p + \frac{t_r - t_r}{4} = 5.82 + \frac{2 - 1.06}{4} \text{ h} = 6.05 \text{ h}$$
- $C_p = 0.7$
- Peak discharge per unit area of the drainage basin can be calculated as

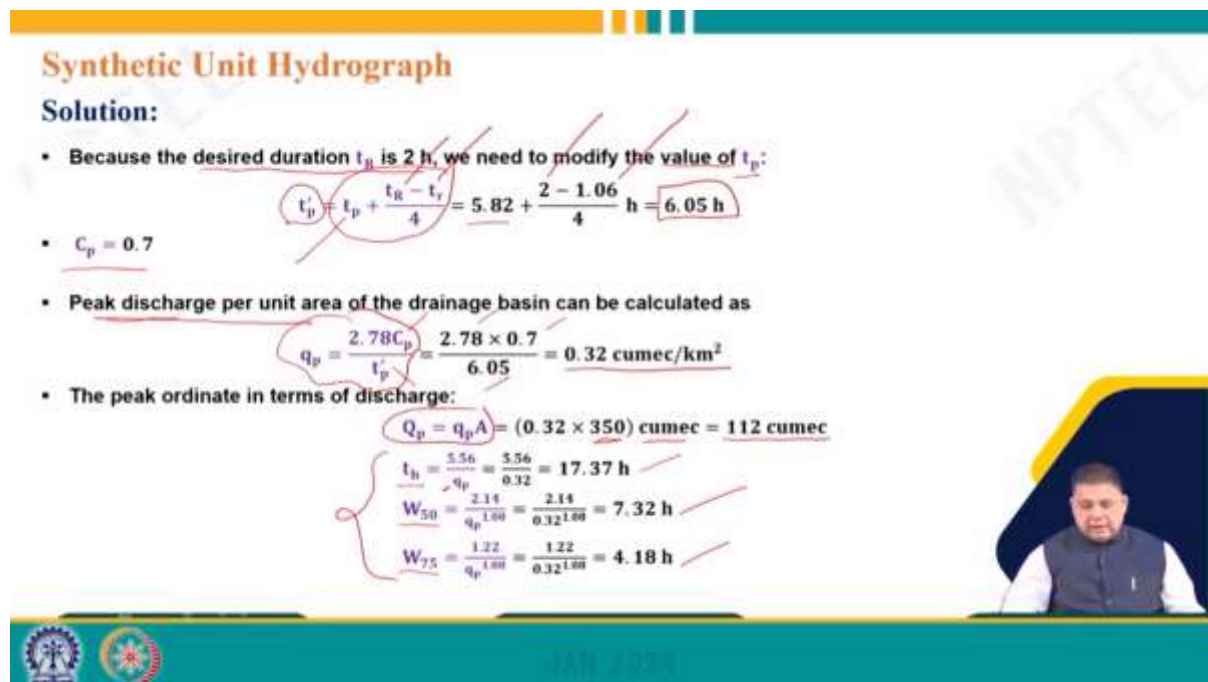
$$q_p = \frac{2.78 C_p}{t'_p} = \frac{2.78 \times 0.7}{6.05} = 0.32 \text{ cumec/km}^2$$
- The peak ordinate in terms of discharge:

$$Q_p = q_p A = (0.32 \times 350) \text{ cumec} = 112 \text{ cumec}$$

$$t_h = \frac{5.56}{q_p} = \frac{5.56}{0.32} = 17.37 \text{ h}$$

$$W_{50} = \frac{2.14}{q_p^{1.00}} = \frac{2.14}{0.32^{1.00}} = 7.32 \text{ h}$$

$$W_{75} = \frac{1.22}{q_p^{1.00}} = \frac{1.22}{0.32^{1.00}} = 4.18 \text{ h}$$



We are also given the coefficient c_p equals 0.7. Using the relationship that relates peak discharge (Q_p) to c_p and t'_p , where c_p value is known and t'_p is known, we can calculate the peak discharge per unit area of the drainage basin. The relationship is

$$q_p = \frac{2.78c_p}{t'_p}$$

where c_p dash is the modified c_p value we calculated earlier, which is 6.05. Using this, we get Q_p value as 0.32 cumec per square kilometre. Therefore, the Peak ordinate in discharge terms can also be calculated by multiplying this,

$Q_p = q_p A$, A which is given as 350 square kilometers.

So, if we multiply this 0.32 cumec per square kilometre by 350, we get the answer Q_p in cumec, which is 112 cumec. Therefore, Q_p comes out to be 112 cumec. You remember when we said that in Snyder's hydrograph, Snyder only provided a relationship up to this point, but later on, the US Army Corps of Engineers provided 3 more equations related to Q_p for finding out the time base, the width of the hydrograph at 50 percent of Q_p , and at 75 percent of Q_p . Using this relationship, we can get more points to plot a smooth unit hydrograph. Putting the values of Q_p here, we get t_b as 17.37 hours, W_{50} as 7.32 hours, and W_{75} as 4.18 hours. So, with this, we get all the elements of the unit hydrograph, which is what we need to do here.

Synthetic Unit Hydrograph

Example 2

The unit hydrograph derived from the watershed M has an 8 h duration. The peak discharge is 200 m³/s, and it occurs at 24 h from the start of the rainfall excess. The watershed N is the sub-watershed of watershed M. Derive the 4-h unit hydrograph for the watershed N using Snyder's method. The characteristics of two watersheds, M and N, are given in the table below.

Sl. No.	Parameter	Watershed M	Watershed N
1	L	100 km	50 km
2	L_{ca}	50 km	25 km
3	A	2500 km ²	1500 km ²

C_x
 C_R

Now, let's take another example on synthetic unit hydrograph. Example number 2 states that the unit hydrograph derived from watershed M has an 8-hour duration, the peak discharge is 200 cubic meters per second, and it occurs at 24 hours from the start of the rainfall excess. Watershed N is a sub-watershed of watershed M. We need to derive the 4-hour unit hydrograph

for watershed N using Snyder's method. The characteristics of watersheds M and N are given in the table below, where parameters L , L_{ca} , and NA for watersheds M and N are provided. L is 100 kilometres for M and 50 kilometres for N, L_{ca} is 50 kilometres for M and 25 kilometres for N, and the area is 2500 square kilometres for M and 1500 square kilometres for N.

For this type of problem, all the characteristics of the hydrograph are given for watershed M. Using Snyder's equations, we determine the values of the two coefficients, C_t and c_p . Once we find them, we can use watershed N to develop the desired unit hydrograph, employing Snyder's empirical relationship, as discussed earlier.

Synthetic Unit Hydrograph

Solution:

□ For the watershed M

- Duration of the hydrograph (t_H) = 8 h
- Time to peak from the start of the rainfall excess = 24 h

$$T_p = \frac{t_H}{2} + t'_p$$

$$24 = \frac{8}{2} + t'_p$$

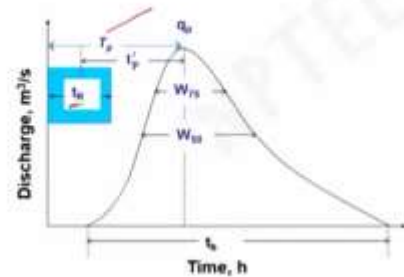
$$t'_p = 20 \text{ h}$$

- Revised time to peak from the start of the rainfall excess

$$t'_p = \frac{21}{22} t_p + \frac{t_H}{4}$$

$$20 = \frac{21}{22} t_p + \frac{8}{4}$$

$$t_p = 18.86 \text{ h}$$



This means we first need to start working on watershed M. The duration of the unit hydrograph, t_r , is given as 8 hours, and the time to peak from the start of rainfall excess is given as 24 hours.

So, the time from this t_p value is given as 24 hours, and we know that t_p from this diagram is also t_r divided by 2 plus t'_p . Because we know t_r , we can calculate the value of t'_p , which comes out to be 20 hours from the given data. Now, to revise the time to peak from the start of rainfall, we use this relationship t_r is not equal to t_r . In this problem, which is a reverse problem, we already know the t'_p value. Using this relationship between t'_p and t_r , we have to find out the t_p value, which comes out to be 18.86 hours.

Synthetic Unit Hydrograph

Solution:

- Length of main stream, $L = 100$ km
- Length to Centroid $L_{ca} = 50$ km
- So, C_t can be calculated as

$$t_p = 0.75C_t(LL_{ca})^{0.3}$$
$$18.86 = 0.75C_t(100 \times 50)^{0.3}$$
$$C_t = 1.95$$

- Area of watershed M, $(A) = 2500$ km²
- Peak discharge $(Q_p) = 200$ m³/s
- Peak discharge per unit area of the drainage basin $(q_p) = \frac{Q_p}{A} = \frac{200}{2500} = 0.08$ cumec/km²
- So, C_p can be calculated as

$$q_p = \frac{2.78C_p}{t'_p}$$
$$0.08 = \frac{2.78C_p}{20}$$
$$C_p = 0.56$$

Once we know t_p , we know that t_p is related to the relationship between L , L_{ca} , and C_t which are the hydrograph variation characteristics, the length of the main stream (L) in this case being 100 kilometres, and the length to the centroid (L_{ca}) being 50 kilometres. Here, we know t_p , L , and L_{ca} . The only unknown is C_t . By putting the known values of t_p (18.86), L (100), and L_{ca} (50) into this equation, we get the value of C_t as 1.95.

Another piece of information given is that the area is 2500 square kilometres and the peak discharge is 200 cubic meters per second. This gives us Q_p value as Q_p divided by A , or 0.08 cubic meters per square kilometre. Q_p in discharge per square kilometre is related to C_p and t'_p by this relationship: $Q_p = 2.78$ times C_p divided by t'_p . Here, we already know t'_p and Q_p . By putting the value of Q_p as 0.08 (which we just calculated) and t'_p as 20 (which we calculated in the previous slide), we get the value of C_p as 0.56. So, the two unknowns, C_t and C_p , for the watershed or the catchment are known to us because N is a sub-watershed of M . Obviously, we can assume that they have similar hydrological characteristics, and the C_t and C_p values obtained from M can be used for N .

Synthetic Unit Hydrograph

Solution:

□ Watershed N

- Now $C_t = 1.95$ and $C_p = 0.56$ can be used for the watershed N, as it is the sub-watershed of M
- Length of the mainstream, $L = 50$ km
- Length to the Centroid $L_{ca} = 25$ km
- Therefore, basin lag can be calculated as

$$t_p = 0.75C_t(LL_{ca})^{0.3} = 0.75 \times 1.95 \times (50 \times 25)^{0.3} = 12.42 \text{ h}$$

- Effective rainfall of duration can be calculated as

$$t_r = \frac{t_p}{5.5} = \frac{12.42}{5.5} \text{ h} = 2.26 \text{ h}$$

- Since the desired duration t_R is 4 hours, we need to modify the value of t_p

$$t'_p = t_p + \frac{t_R - t_r}{4} = 12.42 + \frac{4 - 2.26}{4} \text{ h} = 12.86 \text{ h}$$

$(t_R = 4 \text{ h}) \# (t_r = 2.26)$



Now, for watershed N, C_t is 1.95, and c_p is 0.56, which can be used for watershed N, as it is a sub-watershed of M. For N, L is given as 50 kilometres, and L_{ca} is given as 25 kilometres.

So, basin lag can be calculated using this relationship because we know C_t , L , and L_{ca} . The value of C_t comes out to be t_p , which is 12.42 hours. Now, we know that the effective rainfall duration of Schneider's hydrograph is related to t_p by this relationship: $t_r = T_p$ divided by 5.5. So, 12.42 divided by 5.5 gives a standard duration of 2.26 hours. However, we are required to develop a unit hydrograph of 4 hours, so t_r is 4 hours, which is not equal to 2.26 hours. That's why we need to modify the value of t_p using this relationship. Using that, we get t'_p because we know t_p , T_R , and t_r . So, putting the values here, we get t'_p value as 12.86 hours.

Synthetic Unit Hydrograph

Solution:

- Peak discharge per unit area of the drainage basin can be calculated as

$$q_p = \frac{2.78C_p}{t'_p} = \frac{2.78 \times 0.56}{12.86} = 0.12 \text{ cumec/km}^2$$

- Area of watershed N, $(A) = 1500 \text{ km}^2$

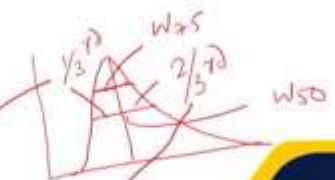
- The peak ordinate in terms of discharge:

$$Q_p = q_p A = (0.12 \times 1500) \text{ cumec} = 180 \text{ cumec}$$

$$t_b = \frac{5.56}{q_p} = \frac{5.56}{0.12} = 46.33 \text{ h}$$

$$W_{50} = \frac{2.14}{q_p} = \frac{2.14}{0.12} = 17.83 \text{ h}$$

$$W_{75} = \frac{1.22}{q_p} = \frac{1.22}{0.12} = 10.17 \text{ h}$$



And once we know t'_p , then using this relationship, the peak discharge per unit area of the drainage basin can be calculated because c_p and t'_p is known. So, Q_p value comes out to be 0.12 cumecs per square kilometre. The watershed area is 1500 square kilometres. Therefore, peak discharge in terms of discharge or cumecs can be obtained by multiplying Q_p with A, which is 0.12 times 1500, resulting in 180 cumecs.

All the desired ordinates are known, and the additional ordinates like the time base, width of the hydrograph at 50 percent, and width of the hydrograph at 75 percent can be calculated in terms of Q_p because Q_p values are known to us. Thus, we get t_b as 46.33 hours, W_{50} at 21.13 hours, and W_{75} as 12.04 hours. Remember, we said that the Schneider hydrograph.

So, when we plot, this is Q_p . We calculate W_{50} . One-third should be on this side and two-thirds should be on the right side. So, of 21.13 hours, one-third should be on the left side and two-thirds on this side. Similarly, at W_{75} , which is 12.05 hours, one-third should be on this side and two-thirds should be on the other side. This ensures that we maintain the skewness of the hydrograph, as we discussed earlier. This point also has to be remembered while plotting the hydrograph.

Dimensionless Unit Hydrograph

Example 3

Derive a 3-h unit hydrograph ordinate using the dimensionless unit hydrograph method considering lag of 4.5 h and peak discharge of 5.5 m³/s.

Solution:

- Duration of rainfall excess (D) = 3 h
- Time lag (t_L) = 4.5 h
- Time to peak (t_p) = $t_L + D/2 = 4.5 + 3/2 = 6.0$ h
- Peak discharge (q_p) = 5.5 m³/s

Steps for deriving UH from DUH

- Select the time interval of 3 h and calculate T/t_p
- Determine the corresponding value of q/q_p from the dimensionless unit hydrograph table
- Multiply the values of q/q_p by Peak discharge (q_p)
- Finally, construct the unit hydrograph

Next, we move to the next problem on the dimensionless unit hydrograph, which is example 3. We need to derive a 3-hour unit hydrograph ordinate using the dimensionless unit hydrograph method, considering a lag of 4.5 hours and a peak discharge at 5.5 hours. The duration of rainfall excess is already given as 3 hours for the unit hydrograph. The time lag (T_L) is 4.5 hours. The time to peak (t_p) is calculated using the formula $t_p = T_L + 4D + D/2$, where D is the duration of the hydrograph. Graphically, this means that $T_P = D/2 + T_L$. By putting the values of T_L and D, we can get the value of time to peak as 6 hours, and the peak discharge value is already given.

The standard steps for deriving the unit hydrograph from the dimensionless hydrograph involve selecting a time interval of 3 hours and calculating T/t_p . Since we have to develop a 3-hour hydrograph, T/t_p ranges from 0 to 5. We calculate this at 3-hour intervals up to 5, and then we

can get the corresponding value of Q/Q_p from the dimensionless unit hydrograph table provided by the SCS (Soil Conservation Service). Multiplying Q/Q_p by Q_p allows us to construct the unit hydrograph.

Dimensionless Unit Hydrograph

Solution: $t_p = 6$ h, $q_p = 5.5$ m³/s

Time Ratios (t/t _p)	Discharge Ratios (q/q _p)	Mass Curve Ratios (Q/Q _p)
0	0.00	0.00
1	0.30	0.01
2	1.00	0.06
3	1.90	0.12
4	3.10	0.25
5	4.70	0.65
6	6.60	1.07
7	8.20	1.63
8	9.30	2.28
9	9.90	3.00
1.0	1.000	3.75
1.1	9.90	4.50
1.2	9.30	5.22
1.3	8.60	5.89
1.4	7.90	6.50


Time (h), t	t/t _p	q/q _p	q (m ³ /s)
0	0	0	0
3	0.5	0.47	2.59
6	1	1	5.50
9	1.5	0.68	3.74
12	2	0.28	1.54
15	2.5	0.127	0.70
18	3	0.055	0.30
21	3.5	0.025	0.14
24	4	0.011	0.06
27	4.5	0.005	0.03
30	5	0	0

$$\frac{q}{q_p} = 0.147 - \frac{(0.147 - 0.107)}{(2.6 - 2.4)} \times 0.1$$

$$\frac{q}{q_p} = 0.029 - \frac{(0.029 - 0.021)}{(3.6 - 3.4)} \times 0.1$$

Linear Interpolation

Based on the value of t/t_p, q/q_p is obtained from the standard dimensionless UH Table

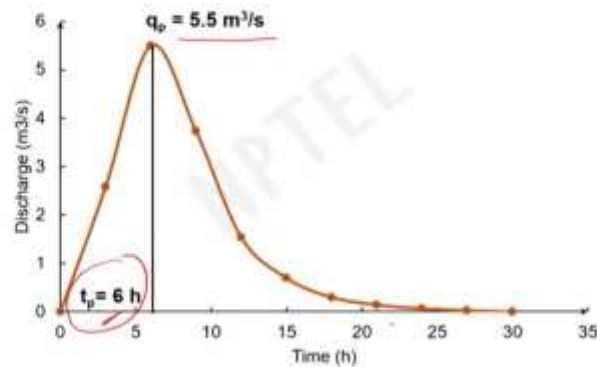


This is a simple procedure. t_p is known to us as 6 hours, and we calculated Q_p as 5.5 cubic meters per second. The standard table for this is provided by the SCS. Here, we have to develop a 6-hour unit hydrograph.

So, t_p/t_p is 6/6 times 5, which is 30. So, for 5, t_p/T is 30 hours. We need to have T/t_p so we can calculate it. For example, at 2 and 2.5, we don't have values available, but at 2.6 and 2.4, the values are 0.147 and 0.107 respectively. We need to interpolate linearly because this table allows linear interpolation for getting the desired value. So, Q/Q_p for 2.5 can be interpolated as 0.127. Similarly, for 3.5, we will use the values of 3.6 and 3.4, which are 0.029 and 0.21, and we can calculate the value at 0. So, Q/Q_p based on a value of T/t_p is obtained from the standard dimensionless unit hydrograph table, and linear interpolation is permitted. Whenever standard values are not available for direct T/t_p , we can linearly interpolate the values as shown here. Once we know that, because we already know Q_p , Q values can be obtained by multiplying 0.47 by 5.5, which is 2.59, and so on.

Dimensionless Unit Hydrograph

Solution: • 3-h UH is plotted below using the table values



We will get Q values, and then we can plot the hydrograph. So, Q_p is 5.5, t_p is 6 hours, and T/t_p in row is 5, so 6 hours. This is how we can develop the unit hydrograph of desired duration using the dimensionless unit hydrograph. It is a 3-hour unit hydrograph, but t_p value we calculated is 6 hours, which is why we are using 6 here.

SCS Triangular Unit Hydrograph Method

Example 4

The peak of a flood hydrograph due to a 1-h duration isolated storm in a catchment of area 13.5 km² is 135 m³/s. The total depth of rainfall is 54 mm. Assume a constant baseflow of 10 m³/s and ϕ -index of 4 mm/h.

1. Calculate the peak of the 1-h unit hydrograph for the catchment.
2. Assuming the above 1-h unit hydrograph to be triangular in shape with the time to peak as 1 h, calculate the peak of the 2-h unit hydrograph for the catchment.

(GATE 2010)

Solution:

- ϕ -index of the catchment = 4 mm/h
- The total depth of rainfall (P) = 54 mm
- Excess rainfall duration (t_e) = 1 h

$$\phi = \frac{P - ER}{t_e}$$

$$4 = \frac{54 - ER}{1}$$

$$ER = 50 \text{ mm} = 5 \text{ cm}$$

- Thus, $ER = 5 \text{ cm}$
- Peak of the flood hydrograph due to a 1-h duration = 135 m³/s
- Peak of DRH = Peak of the flood hydrograph - Baseflow = 135 - 10 = 125 m³/s
- Thus, the peak of 1-h unit hydrograph for the catchment = $\frac{\text{Peak of DRH}}{ER} = \frac{125}{5} = 25 \text{ m}^3/\text{s}$

Next, we come to problem number 4, which is on the SCS triangular unit hydrograph method. A peak of a flood hydrograph due to a 1-hour duration isolated storm in a catchment of area 13.5 square kilometres is 135 cubic meters per second. The total depth of rainfall is 54 millimetres, assuming a constant base flow of 10 cubic meters per second and a 5 index of 4 millimetres per hour. We need to calculate the peak of the 1-hour unit hydrograph for the catchment, assuming the above unit hydrograph to be triangular in shape with the time of peak

at 1 hour. We also need to calculate the peak of the 2-hour unit hydrograph for the catchment. This question is from the GATE 2010 examination.

We are given the Φ –index value for the catchment as 4 millimetres per hour. The total depth of rainfall is given as 54 millimetres, and the excess rainfall duration is 1 hour. Using the relationship $\Phi = \frac{P-ER}{t_e}$, we can calculate the effective rainfall. Given that we know Φ , P, and t_e , putting the values, the ER value comes out to be 50 millimetres or 5 centimetres. The effective rainfall is 5 centimetres. The peak of the flood hydrograph due to 1-hour duration is already given as 135 cubic meters per second. Therefore, the peak of DRH will be the peak of the flood hydrograph minus the base flow value, which is also given as 10 cubic meters per second. So, 135 minus 10 equals 125 cubic meters per second. Therefore, the peak of the 1-hour unit hydrograph for the catchment is the peak of DRH divided by the effective rainfall, giving us the peak of the unit hydrograph. So, we know the peak of DRH. Dividing by ER, we get the peak of the 1-hour unit hydrograph as 25 cubic meters per second. That is the first bit.

SCS Triangular Unit Hydrograph Method

Solution:

- ❑ Time to peak of 1-h UH (t_{p1}) = 1 h
- ❑ Duration of excess rainfall (D_1) = 1 h
- ❑ Thus, the time lag for 1-h UH can be calculated as

$$t_{p1} = \frac{D_1}{2} + t_L$$

$$1 = \frac{1}{2} + t_L \implies t_L = 0.5$$
- ❑ Thus, the time to peak for 2-h UH can be calculated as

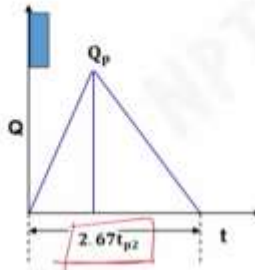
$$t_{p2} = \frac{D_2}{2} + t_L$$


$$t_{p2} = \frac{2}{2} + 0.5 = 1.5 \text{ h}$$
- ❑ Peak of the 2-h unit hydrograph can be calculated as

$$\frac{1}{2} \times t_b \times Q_p = ER \times A$$

$$\frac{1}{2} \times 2.67 \times 1.5 \times 3600 \times Q_p = \frac{1}{100} \times 13.5 \times 10^6$$

$$Q_p = 18.73 \text{ m}^3/\text{s}$$





Then, the time to peak of the 1-hour unit hydrograph is given as 1 hour, and it is triangular in shape, as mentioned earlier, with a duration of rainfall of 1 hour. The time lag for the 1-hour unit hydrograph (TP1) can be calculated as $D_1/2$ by T_L . This is how we can find out the TL value, which comes out to be 0.05 because is 1, as already given, and D_1 is 1 hour. Therefore, the TL value comes out to 5.

The time to peak of the 2-hour unit hydrograph can be calculated as $TP_2 = D_2/2$ plus TL. Since TL value remains the same and D_2 is 2, TP_2 becomes 1.5 hours. We also know that the peak of the 2-hour unit hydrograph can be calculated because a triangle has half the time base into Q_p , which means the area under the curve will be the same as the area of effective rainfall times the area of the catchment, representing the total volume of runoff generated.

Here, T_V is 2.67 because T_P_2 is 1.77 hours. So, it is 2.67. We remember that the time base is 2.67 times T_P . So, 2.67 times 1.5 is the time base. Half into time base into Q_p , where 3600 is coming because it is in hours, so we are converting that into seconds. E_R is 1 unit depth,

representing 1 centimetre, which is converted into meters. Everything is known except Q_p . Putting the values, we can calculate that Q_p is 18.73 cubic meters per second. Therefore, the peak of the 2-hour unit hydrograph is 18.73 cubic meters per second, which is the answer to the second part.

SCS Triangular Unit Hydrograph Method

Example 5

The 1-h unit hydrograph of a catchment area is triangular in shape. The details are as follows:

- (i) Catchment area = 25 km²
- (ii) Time base = 10 h
- (iii) Peak discharge = 21 m³/s
- (iv) Time to peak = 3 h

Calculate and plot the flood hydrograph ordinates for a storm event of 40 mm in the first hour, 60 mm in the second hour and 20 mm in the third hour. Assume a uniform baseflow of 5 m³/s and a ϕ -index of 2.5 mm/h.

Solution:

Time interval (h)	1	2	3
Rainfall in the interval (mm)	40	60	20
ϕ -index (mm/h)	2.5	2.5	2.5
Losses (mm)	2.5	2.5	2.5
Effective rainfall (mm)	37.5	57.5	17.5

$$\phi - \text{index} = \frac{P - ER}{t_e}$$



Now, we move on to example number 5, which is again on the SCS triangular unit hydrograph method. The 1-hour unit hydrograph of a catchment area is triangular in shape, with details given here. The catchment area is 25 square kilometres, the time base is 10 hours, the peak discharge is 21 cubic meters per second, and the time to peak is 3 hours. To plot the flood hydrograph ordinates for a storm event of 40 millimetres in the first hour, 60 millimetres in the second hour, and 20 millimetres in the third hour.

So, it is a problem where we have a complex storm, assuming a uniform base flow of 5 cubic meters per second and a ϕ index of 2.5 millimetres per hour. These values we can assume. The first thing obviously, we have to find out is the effective rainfall, that is the ERH. So, the time intervals 1, 2, and 3, representing the first hour, second hour, and third hour, have rainfall magnitudes given as 60, 40, 60, and 20. The Φ -index value is given as 2.5 millimetres per hour. The total loss, because it is a 1-hour period, will also be 2.5 millimetres per hour. Therefore, the effective rainfall in millimetres comes out to be 40 minus 2.5, which is 37.5 for the first hour, 60 minus 2.5, which is 57.5 for the second hour, and 20 minus 2.5, which is 17.5 for the third hour. These are the effective rainfall amounts for the three consecutive hours, which we need to operate on the unit hydrograph to get the DRH in the flood hydrograph.

SCS Triangular Unit Hydrograph Method

Solution:

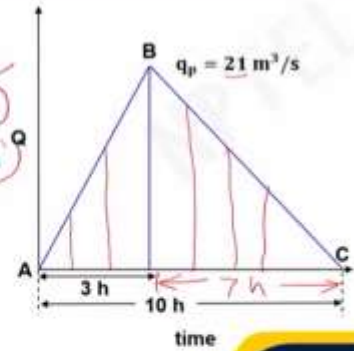
- ✓ Time to peak (t_p) = 3 h
- ✓ Time base of the hydrograph (t_b) = 10 h
- ✓ Peak of the UH (q_p) = 21 m³/s

- ✓ The slope of line AB = $\frac{21-0}{3-0} = 7 \frac{\text{m}^3}{\text{s}/\text{h}}$
- ✓ The slope of line BC = $\frac{21-0}{10-3} = 3 \frac{\text{m}^3}{\text{s}/\text{h}}$

Time (h)	Ordinates of 1-h UH
0	0
1	7
2	14
3	21
4	18
5	15
6	12
7	9
8	6
9	3
10	0

At t h the discharge = $q_p + \text{slope of AB} \cdot t$
 At 2 h the discharge = $7 + 7 \cdot 1 = 14 \text{ m}^3/\text{s}$

At t h the discharge = $q_p - \text{slope of BC} \cdot t$
 At 4 h the discharge = $21 - 3 \cdot 1 = 18 \text{ m}^3/\text{s}$



We have been given the peak and time base, but we are not given the ordinate. So, in order to find that, we have to use a technique. The time to peak is 3 hours, the time base of the hydrograph is 10 hours, and the peak is given at 21 hours. Because we need the intermediate ordinates, even though it is a Eq, because it is triangular, we still need to calculate. So, we calculate the slope of the line A to B, which is 21 minus 0 over 3 hours, giving us 21 divided by 3, which equals 7 cubic meters per second per hour. The slope of line B to C is 21 over 7, which equals 3 cubic meters per second per hour, as this part is 7 hours, which is 10 hours. This simply means that at different hours, we can calculate the ordinate.

So, basically, with a slope of 7, it will be 1 hour at 7, 2 hours at 14, 3 hours at 21, and here it starts reducing, becoming 18, 15, 12, 9, 6, 3, and so on. This is how we can calculate the ordinates of the 1-hour unit hydrograph at different hours.

SCS Triangular Unit Hydrograph Method

Solution:

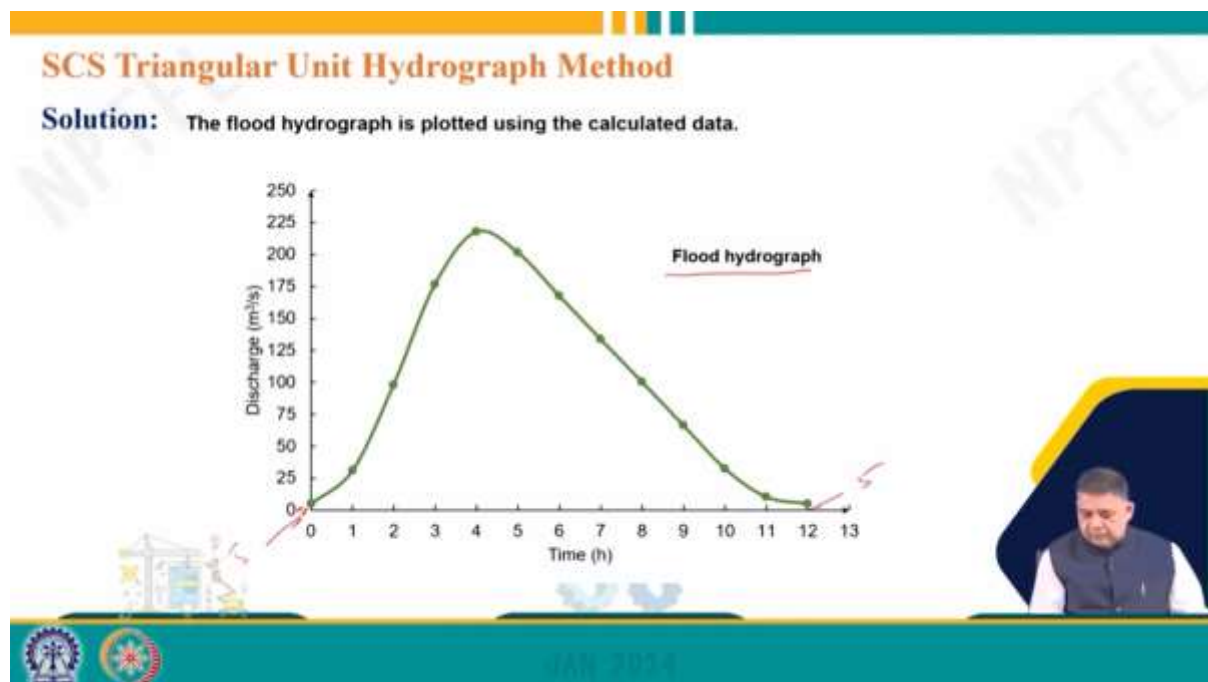
Time (h)	Ordinates of 1-h UH	DRH of ER 3.75 cm	DRH of ER 5.75 cm	DRH of ER 1.75 cm	DRH (m ³ /s)	Baseflow (m ³ /s)	Flood hydrograph (m ³ /s)
0	0	0			0	5	5
1	7	26.25	0		26.25	5	31.25
2	14	52.5	40.25	0	92.75	5	97.75
3	21	78.75	80.5	12.25	171.5	5	176.5
4	18	67.5	120.75	24.5	212.75	5	217.75
5	15	56.25	103.5	36.75	196.5	5	201.5
6	12	45	86.25	31.5	162.75	5	167.75
7	9	33.75	69	26.25	129	5	134
8	6	22.5	51.75	21	95.25	5	100.25
9	3	11.25	34.5	15.75	61.5	5	66.5
10	0	0	17.25	10.5	27.75	5	32.75
11			0	5.25	5.25	5	10.25
12				0	0	5	5



Now, we need to operate for calculating the DRH. So, these are the ordinates of the 1-hour unit hydrograph. We have three ERH events: 3.75 centimetres in the first hour, 5.75 in the second hour, and 1.75 in the third hour. Therefore, we need three ordinates superimposed, each lagging the previous one by D hours, which is 1 hour in this case.

The DRH due to ER of 3.75 centimetre, which is right at the beginning. So, basically, here we have 1, 2, 3 hours. 3.75 in the first hour, 5.75 in the second hour, and 1.75 in the third hour. So, 1, 2, 3. This is 3.75, this is 5.75, and this is 1.75. This is our ERH. So, the first one will start immediately and it will be simply 1-hour ordinates multiplied by 3.75 here. The second one will be 1-hour ordinate multiplied by 5.75, but there is a lag of 1 hour because there is a lag of 1 hour in this second storm. The DRH due to the third storm will be in the third hour, 1.75 times the UH ordinates, but there will be a lag of 2D hours, which means 2 hours. So, 0-hour, 1 hour, and 2 hours. It's simply multiplication, and then we need to sum up to get the DRH ordinate. So, 0: 26.5, 52.5 plus 40.25 is 92.75 and then sum. So, these are DRH ordinates and the VS base flow value is 5, a constant value is given. So, flood hydrograph ordinates are simply DRH ordinates plus base flow.

So, simply a sum of that, 26.5 plus 5 is 31.25, 92.5 plus 5 is 97.5 and so on. This is how we will get the flood hydrograph for this particular problem.



And this is the flood hydrograph plotted here. Here, this value is close to 5, but this value is 5. It does not start from 0, and the same is true here. The value is 5 here, this ordinate.

Instantaneous Unit Hydrograph (IUH)

Example 6

Derive a 3-h unit hydrograph using the following IUH ordinates of a catchment:

Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
IUH (m ³ /s)	0	12	35	50	44	36	29	25	18	8	3	0

Use the S-hydrograph method.



Then we go to problem number 6 where we need to derive an IOH, derive a 3-hour unit hydrograph using the following IOH ordinate of a catchment. IOH ordinates are given, and we have been asked to use the S hydrograph method.

Instantaneous Unit Hydrograph (IUH)

Solution:

Time (h)	IUH	IUH (m ³ /s) lagged by 1 h	Sum of Col. 2 and Col. 3	1-h UH	S-Hydrograph addition	S-Hydrograph	S-Hydrograph lagged by 3-h	3-h DRH for 3-cm (S _A -S _B)	3-h UH
0	0		0	0		0		0	0
1	12	0	12	6	0	6	-	6	2
2	35	12	47	23.5	6	29.5	-	29.5	9.8
3	50	35	85	42.5	29.5	72	0	72	24.00
4	44	50	94	47	72	119	6	113	37.67
5	36	44	80	40	119	159	29.5	129.5	43.17
6	29	36	65	32.5	159	191.5	72	119.5	39.83
7	25	29	54	27	191.5	218.5	119	99.5	33.17
8	18	25	43	21.5	218.5	240	159	81	27
9	8	18	26	13	240	253	191.5	61.5	21
10	3	8	11	5.5	253	258.5	218.5	40	13.33
11	0	3	3	1.5	258.5	260	240	20	6.67
12	0	0	0	0	260	260	253	7	2
13	0	0	0	0	0	260	258.5	1.5	0.50
14	0	0	0	0	0	260	260	0	0
15	0	0	0	0	0				
16	0	0	0	0	0				

Sum of Col.2 and Col.3 is divided by 2



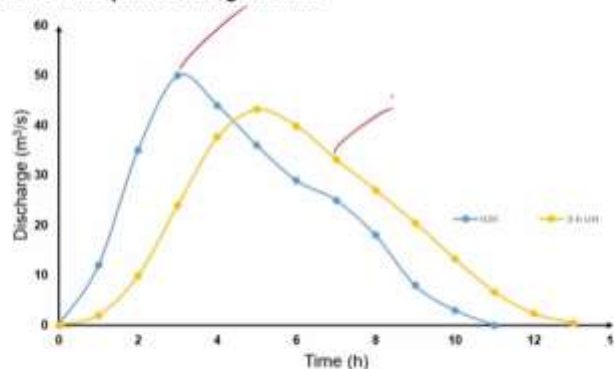
So, this is the IOH ordinate, and we know that for developing a 1-hour unit hydrograph, we can develop IOH 1-hour unit hydrograph. We need to develop in this case because the problem says. So, for the first 1-hour unit hydrograph, we need to develop and lag by 2-unit hydrograph hours and take the average of those ordinates. So, here we are lagging by 1 hour, copying the same ordinates, summing up, and then dividing by 2. So, the sum of columns 2 and 3 divided by 2 gives us 1-hour unit hydrograph ordinates. Because we have to develop a 2-hour unit hydrograph, we will, of course, have to develop the S hydrograph. Once we know UH is given, either we can superimpose an n number of unit hydrographs or T by D number of unit hydrographs, or you can use the summation hydrograph addition. So, here we lag by starting lagging by 0, 1 hour, and in this case, 0. Then we start summing up.

So, 0 plus 6 is 6, 6 plus 23. So, on the front side, 29, this sum, and so on, that is how. So, we put the addition, and then we get this error by adding up summing up these 2 columns. So, 0, 6, this is 29.5, this is 72, and so on, and it gets a constant value of 260 at 11 hours. Then we have to develop a 3-hour hydrograph. So, we will lag this S hydrograph by 3 hours. Lagging by that, we get S a minus S b, which is the 3-hour DRH due to 3 centimetres of effective rainfall, and 3-hour UH will be this divided by 3. So, because 3 centimetres effective rainfall is there, HBR lagging by this by 3 hours, that means, 6 by 2, 29.5, and so on, we get the 3-hour UH ordinate.

Instantaneous Unit Hydrograph (IUH)

Solution:

- The IUH and the 3-h UH are plotted using the data



This is our IUH, and this is the 3-hour unit hydrograph developed using the data.

Instantaneous Unit Hydrograph (IUH)

Example 7

- The ordinates of a 6-h S-hydrograph for a rainfall intensity of 10 mm/h of a catchment are given in the table below. Derive the IUH for this catchment.

Time (h)	0	6	12	18	24	30	36	42	48	52
Ordinates (m ³ /s)	0	30	90	180	252	306	342	360	360	360

We take this example 7, which is again on IUH. The ordinates of 6-hour S hydrograph for a rainfall intensity of 10 millimetres per hour of catchment are given in the table below. Derive the IUH of this coordinate. So, this is the S hydrograph ordinate given to us,

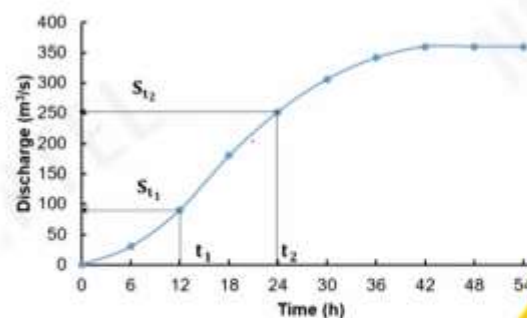
Instantaneous Unit Hydrograph (IUH)

Solution:

- S-hydrograph is plotted using the data
- The slope of the S-hydrograph will give the ordinates of the IUH
- The slope can be calculated as

$$h(t) = \frac{1}{i} \times \frac{dS}{dt}$$

$$h(t) = \frac{1}{i} \times \frac{S_{t_2} - S_{t_1}}{t_2 - t_1}$$

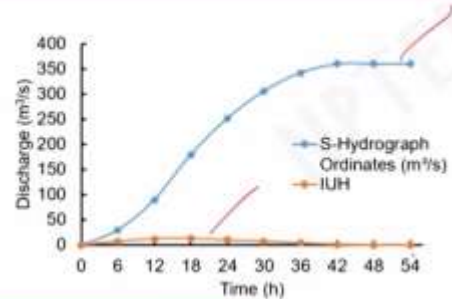


we know that the slope of S hydrograph will give us the ordinates of IUH, provided we take the small-time intervals using this relationship H_t is 1 by I_t , is slope, I mean at 2-time intervals we select and then ordinate differences, that will, that is how we calculate the slope. So, slope will give you the H_t ordinates.

Instantaneous Unit Hydrograph (IUH)

Solution:

Time (h)	S-Hydrograph Ordinates (m ³ /s)	IUH (m ³ /s)
0	0	0
6	30	7.5
12	90	17.5
18	180	28.5
24	252	40.5
30	306	49.5
36	342	55.5
42	360	58.5
48	360	30



$$h(t) = \frac{1}{i} \times \frac{S_{t_2} - S_{t_1}}{t_2 - t_1}$$

$$h(6) = \frac{1}{1} \times \frac{90 - 0}{12 - 0} = 7.5 \text{ m}^3/\text{s}$$

$$h(30) = \frac{1}{1} \times \frac{342 - 252}{36 - 24} = 49.5 \text{ m}^3/\text{s}$$



Here the S hydrograph ordinates are given to us. So, IUH ordinate we can calculate like, for example, at 6 hours, we have given 0 and 90. So, between 0 and 90, that is 12 hours. So, 90 by 12, it is 7.5. So, it is here 7.5. Similarly, H30 we can calculate by taking the 2.2 for 52 and 342 and 36 and 24 hours, that is the average we have to take or the slope we have to calculate basically. So, that slope comes out to be 49.5 cubic meter per second here. So, these are the IUH ordinates. So, this is the S hydrograph, this is IUH ordinate because the scale, I could have put a secondary scale for IUH, but of course, we know that once we have IUH, we can plot that IUH and we will get the desired shape.

Distribution Graph

Example 8

A distribution graph ordinates are given below. The sum of the ordinates of the 2-h unit hydrograph is $500 \text{ m}^3/\text{s}$. Find the peak of the 2-h UH and plot it.

Time (h)	Distribution hydrograph ordinates (%)
0	0
2	8
4	10
6	9.5
8	20
10	12
12	15
14	12.5
16	13
18	0

Now, we come to the last problem on the distribution graph, example 8. A distribution graph ordinates are given below. The sum of the ordinates of 2-hour Intergraph is 500 cubic meter per second. Find the peak of 2-hour Intergraph and plot it.

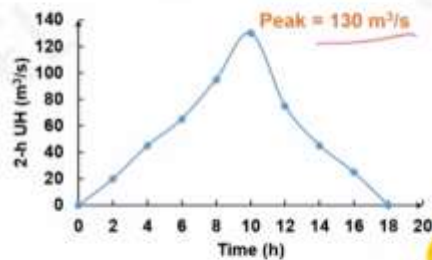
Distribution Graph

Solution: Sum of the ordinates of the 2-h unit hydrograph is $500 \text{ m}^3/\text{s}$

Time (h)	Distribution hydrograph ordinates (%)	2-h UH
0	0	0
2	4	20
4	9	45
6	13	65
8	19	95
10	26	130
12	15	75
14	9	45
16	5	25
18	0	0

↑
(Distribution hydrograph ordinates * $500 \text{ m}^3/\text{s}$) / 100

□ The 2-h UH is plotted below



So, this is the distribution hydrograph. Values are given here. So, distribution hydrograph values are here. We know that the sum is 500, that means, by multiplying this, we can get the 2-hour unit hydrographs. So, 4 into 500, that is 500 between percent. So, that is why 5 multiplied by 5 4 500 divided by 100. So, that is why it is multiplied by 5, that means, so effectively 45, 65, 95, 130, and all, and that is a 2-hour unit hydrograph we have plotted. So, very simple problem where we have to just get the ordinates if not given, then we have to find out the ordinates sum and then multiply to get the ordinates.

So, with this, we have seen some problems, and I said that we will have another part, another lecture devoted to numerical. These are important because after understanding the concept, you must know how to solve and you must know how to attack a particular problem. Hydrograph is a very important topic, which is asked in almost all competitive examinations. So, it should be thought of the concept and how to handle the problems.

So, that is why we are devoting some time to the problems. Please feel free to give your feedback and raise your doubts, which we will be happy to answer in the forum. Thank you very much.

