Course Name: Watershed Hydrology

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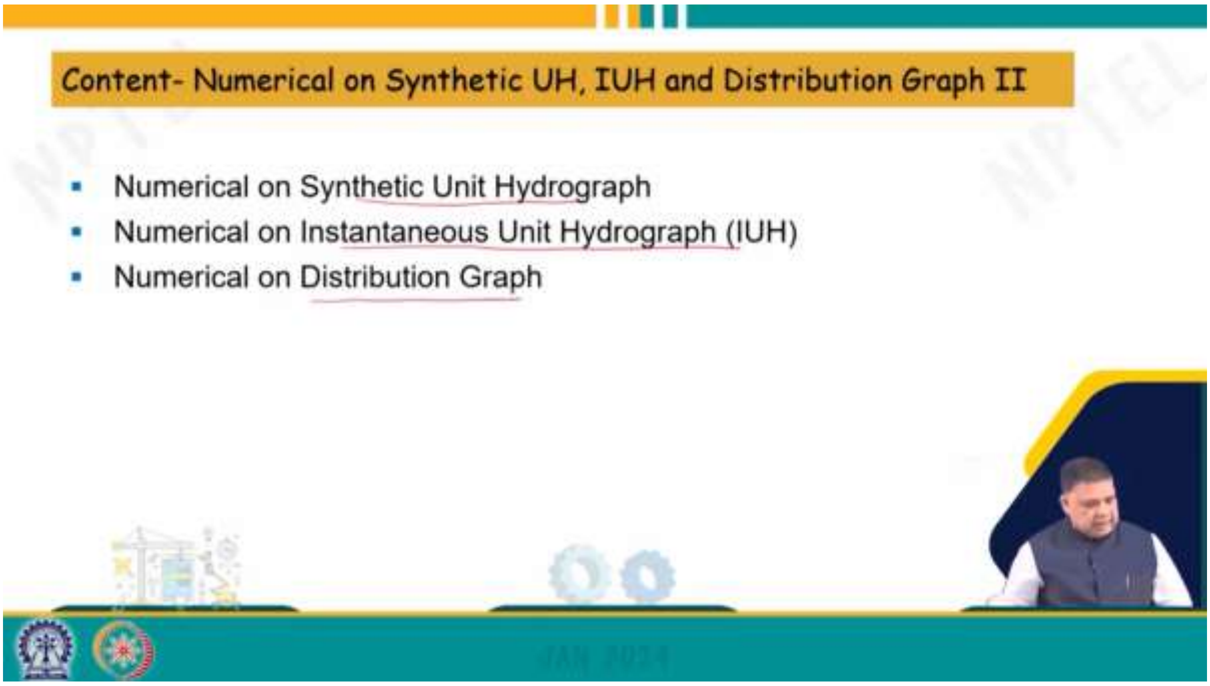
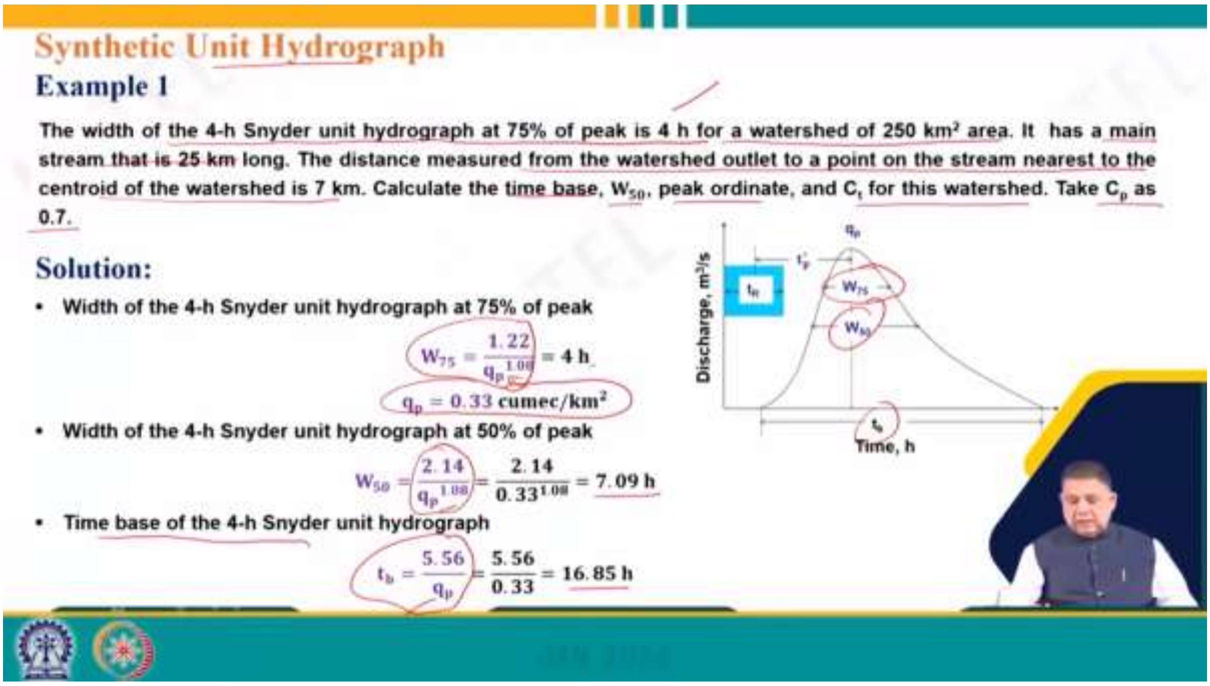
Week: 06

Lecture 30: Numerical on Synthetic UH, IUH and Distribution Graph II

Hello friends, welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a Professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are in Module 6, this is the last lecture, lecture number 5, and as we mentioned in the previous lecture.



This will be part 2 of the Numerical on Synthetic Unit Hydrograph, IOH, and Distribution Graph. In this lecture also, we will continue with the Numerical on Synthetic Unit Hydrograph, Numerical on Instantaneous Unit Hydrograph, and Numerical on Distribution Graph.



So, let us start with the Synthetic Unit Hydrograph. The width of a 4-hour unit hydrograph at 75 percent of peak is 4 hours for a watershed of 250 square kilometres area. It has a main stream that is 25 kilometres long. The distance measured from the watershed outlet to the point on the stream nearest to the centroid of the watershed is 7 kilometres. Calculate the time base W50, Peak ordinate, and CT for this watershed, take CPH 0.7.

So, this is the problem where we have been given, you remember this is the Schneider Unit Hydrograph and these are some of the elements. One of the elements is the width at 75 percent of the peak, W75, which is related to QP by this relationship

$$W_{75} = \frac{1.22}{{q_p}^{1.08}}$$

, and the value of W75 is given as 4 hours. So, from here, using W75 as 4 hours, you can get the value of QP as 0.33 cubic per square kilometre. And we know that the two additional ordinates, W50 and TB, which were provided by US Army Corps of Engineers, are all linked to QP.

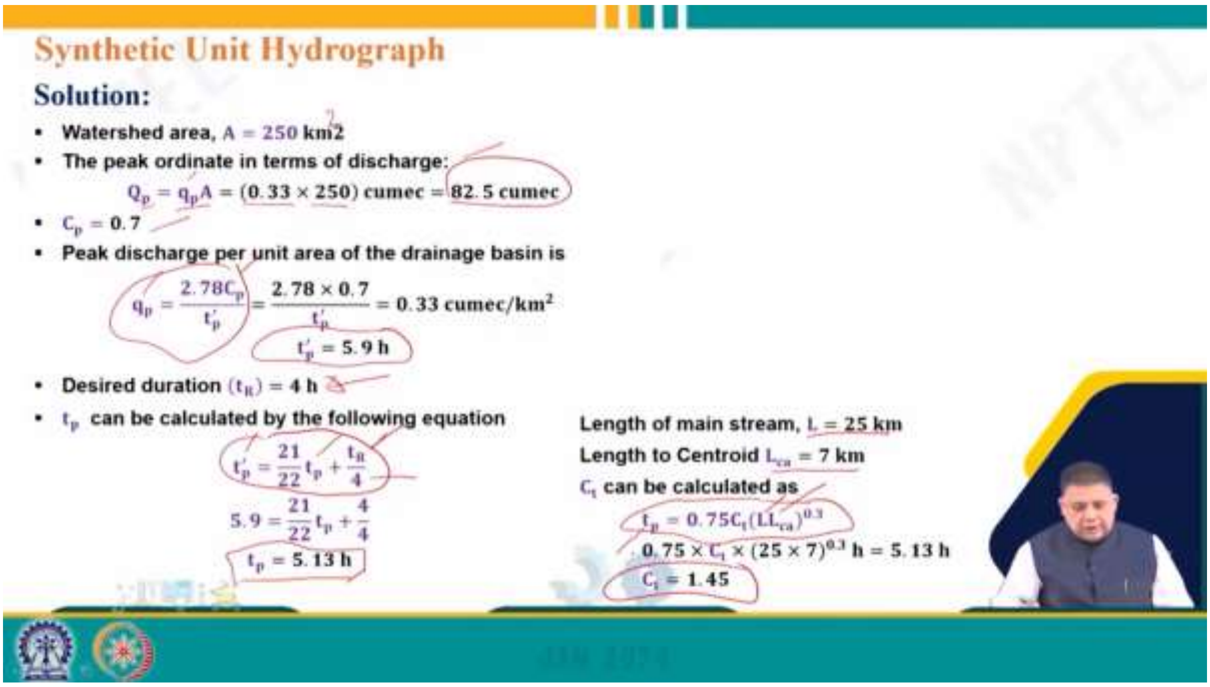
So, W50 can be calculated using this relationship,

$$W_{50} = \frac{2.14}{{q_p}^{1.08}}$$

and it comes out to be 7.09 hours. Similarly, the time base of the 4-hour synthetic hydrograph is again related to QP. So, knowing QP, we can calculate t_b,

$$t_b = \frac{5.56}{q_p}$$

which comes out to be 16.85 hours. These are the three, two that is the time base and W50 is known to us.

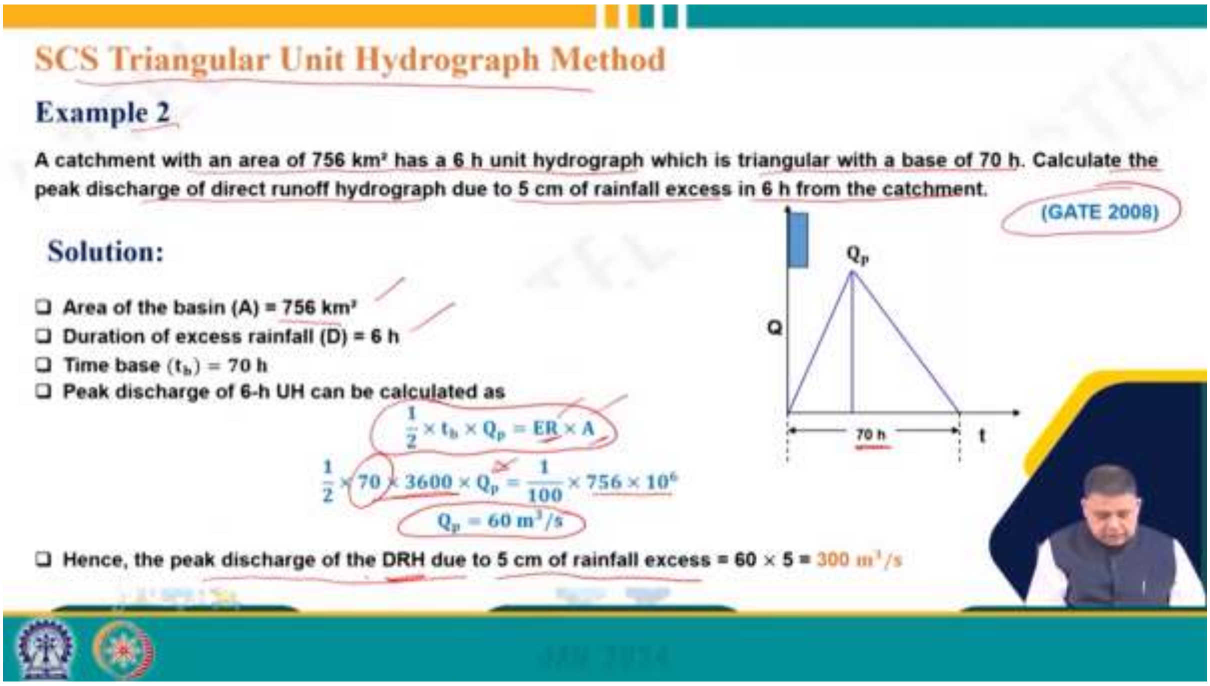


Now, the watershed area is given as 250 square kilometres. So, the peak ordinate in terms of discharge is QP times A, where QP is cubic per square kilometre already calculated as 0.33, and the area is 250 square kilometres. So, the peak discharge comes out to be 82.5 cumecs. We have been given CP value of 0.7. So, using the relationship that is QP is 2.78 CP by Tp dash because we know QP, we know CP.

Using this relationship, we can calculate Tp dash, which comes out to be 5.9 hours. And we have been given the desired duration of 4 hours. So, Tp dash, we using this relationship that Tp dash is 21 by 22 Tp by Tr by 4, we can calculate the value of Tp. As we know Tp dash, we know Tr. So, knowing these two, Tp is the only unknown comes out to be 5.13 hours. We are given the length of the main stream is 25 kilometres, length to centroid is 7 kilometres. We know that this is the main relationship given by Snyder that is

$$t_p = 0.75c_t(LL_{ca})^{0.3}$$

We have been given all, we know that Tp value, we know L value, we know L Ca value. So, the only unknown is Ct here. So, by putting the values here, we can calculate Ct value, which comes out to be 1.45. With this way, we have answered all the bits of this particular problem.

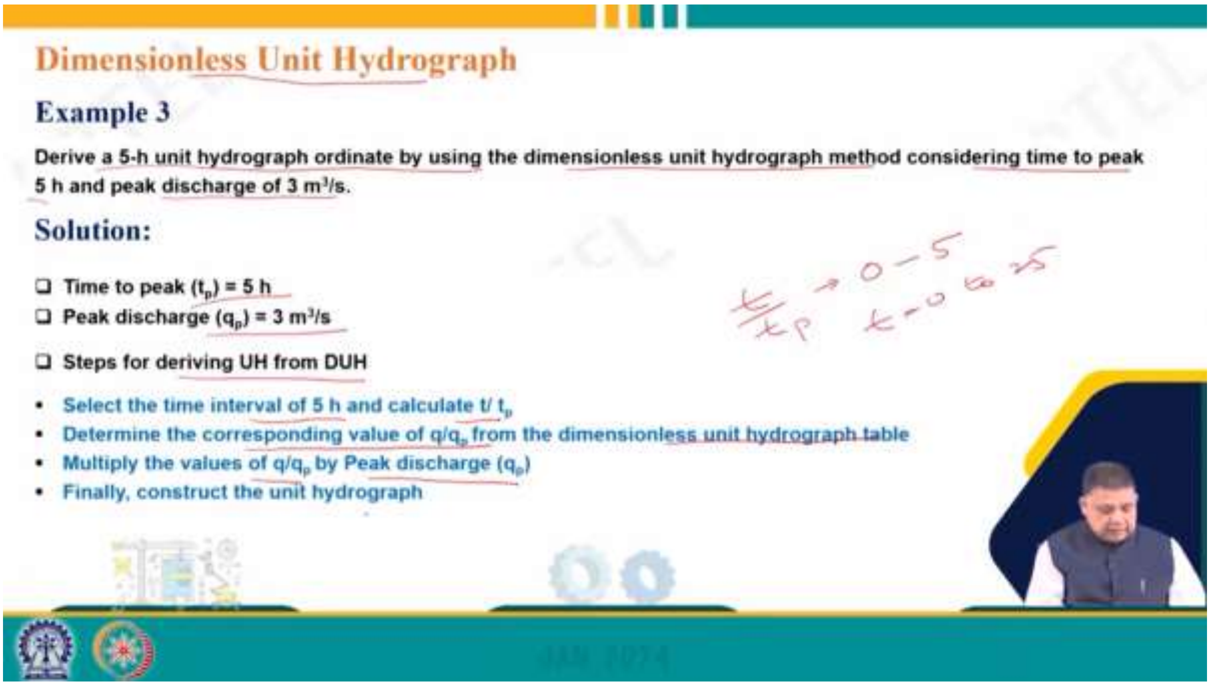


Then we go to the second problem, SC, which is on SCS triangular unit hydrograph method, which is example 2 of this lecture. A catchment with a catchment area of 750 square kilometres has a 6-hour unit hydrograph, which is triangular with a base of 70 hours. Calculate the peak discharge of direct runoff hydrograph due to 5 centimetres of rainfall excess 6 hours from the catchment. This question is taken from GATE 2008 examination.

So, here we have been given the time base of 75 70 hours, the area of the basin is given, and the duration of excess rainfall is given. We know this relationship that area under the curve represents the volume of runoff. So, the area is half Tb times Qp, and we also know that a unit hydrograph means 1 unit depth of effective rainfall over the entire basin area.

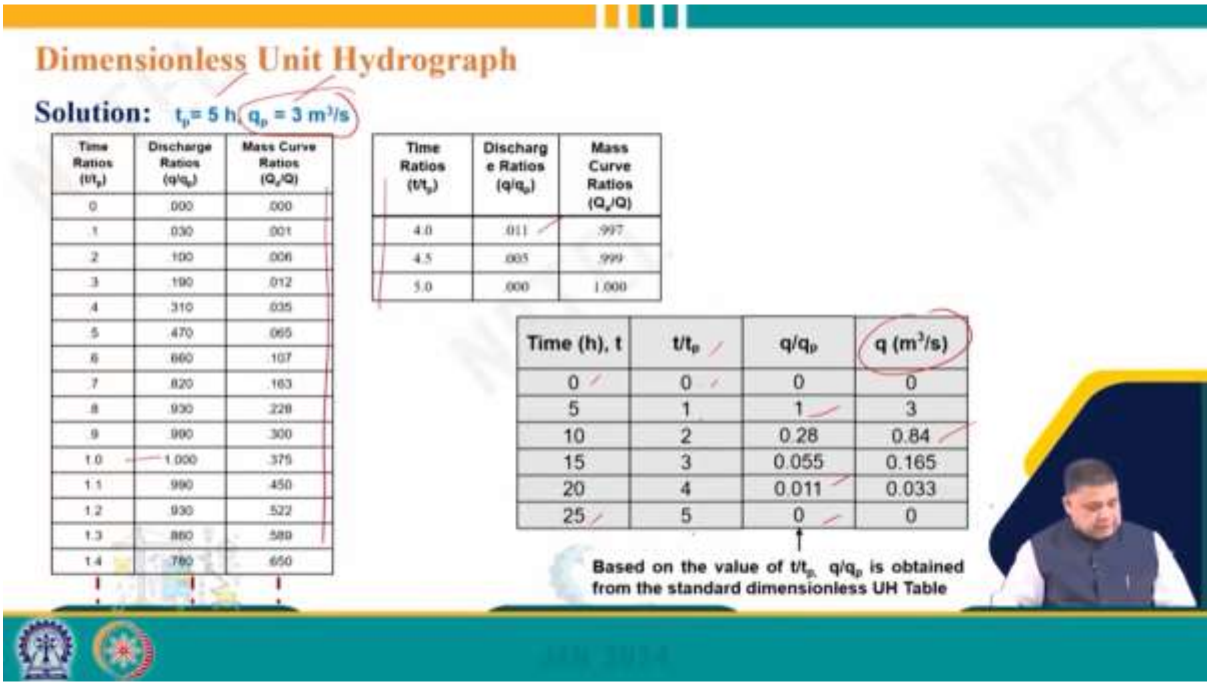
So, Er times a. So, here Tb is given as 70. So, we are putting 70, but it is in hours. So, we are converting it into seconds. QP is the only unknown here. Er is 1 centimetre. So, that means, in meters we are putting 1 by 100, and area is given as 750 square kilometres.

So, we are putting 750 into 10 to the power 6 square meters and then solving for we get QP equals to 60 cubic meters per second. So, the peak discharge of the DRH due to 5 centimetres of this is the peak discharge of this unit hydrograph. So, for DRH, we need to multiply with effective rainfall magnitude. So, peak of discharge of DRH due to 5 centimetres of rainfall will be 60 times effective rainfall that is 5 centimetres. So, the answer comes out to be 300 cubic meters per second. So, a pretty simple problem which was asked in the GATE 2008 examination.



We go to example number 3, which is on the dimensionless unit hydrograph. Derive a 5-hour unit hydrograph ordinate by using the dimensionless unit hydrograph method considering time to peak 5 hours and peak discharge of 3 cubic meters per second. So, time to peak Tp is given here, peak discharge is given here. The steps we have already seen in previous problems also select a time interval of 5 hours and calculate T by Tp. Remember T by Tp has a range of 0 to 5.

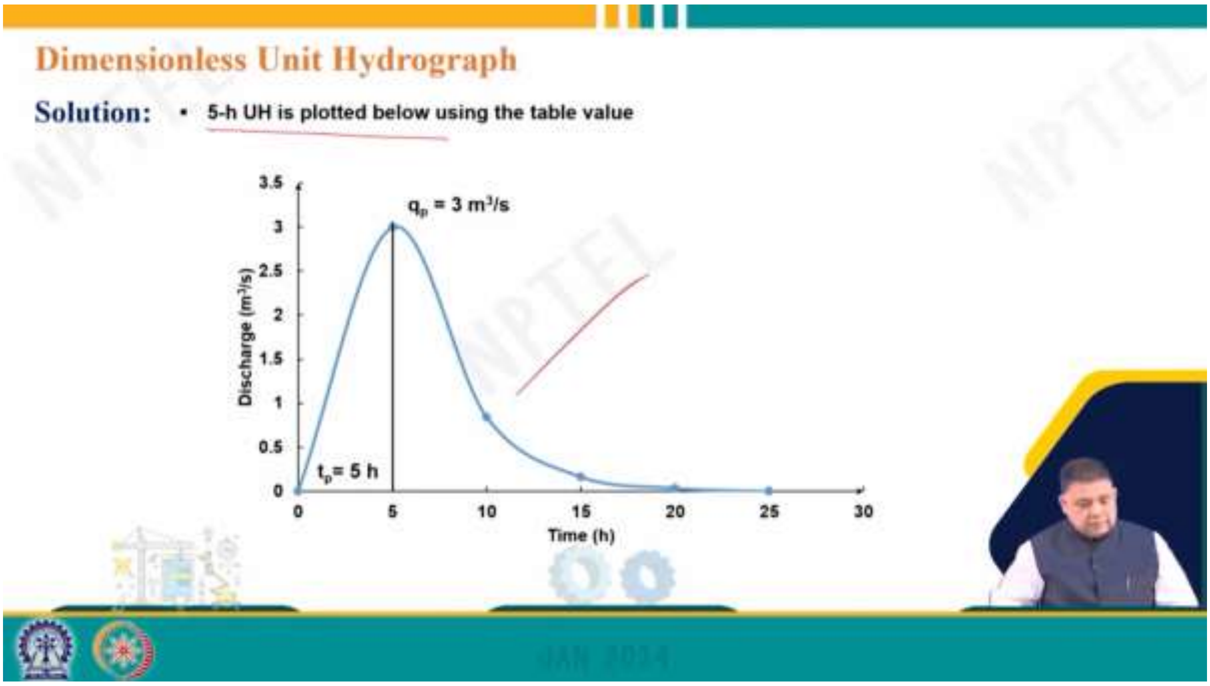
So, that means, T value will go from T will be from 0 to 25. So, every 5 hours, we have to determine the corresponding values of Q by Qp from the dimensionless unit hydrograph table, which is provided by SCS or NRCS multiply the values of Q by Qp by peak discharge Qp, which is given to us to get the ordinate ordinates of the unit hydrograph and then concentrate the unit hydrograph.



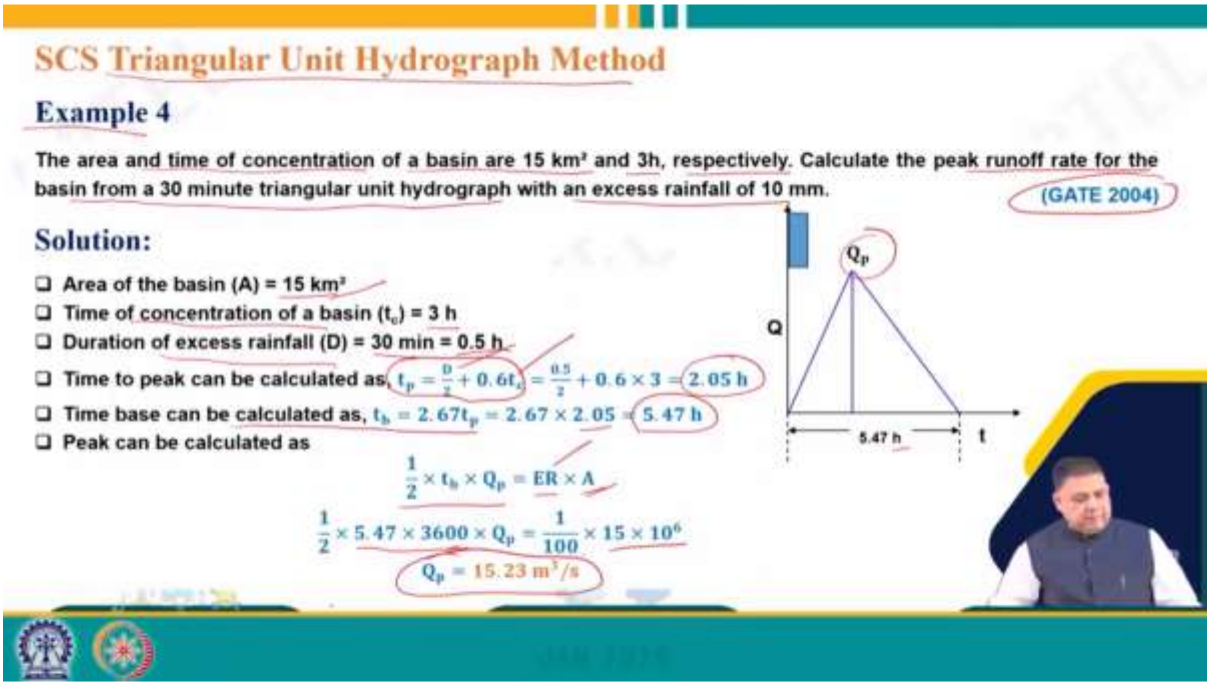
SO, Tp is given as 5 hours Qp is given 3 cubic this is the truncated SCS standard dimensionless unit hydrograph table. So, where we said that time will vary range from 0 to 35 because T by Tp will be max value is 5 which ranges between 0 and 5. So, 0 1 2 3 4 5 and thus we can because it these values are all integers straight away, we can read the values from the table like for 1 it is 1 in this table it is 4 is coming here 4 value is 0.

With intermediate values, I have not shown here, but you can straight away get all these values from the table itself. So, that means, knowing T by Tp, we are reading these values from Q by Qp values from this standard table and putting here, and then Qp will be nothing but these multiplied by Qp, that is the Q value. So, obviously, we are multiplying by Qp, which comes out to be 3Q, which is given as 3 cubic meters per second.

So, 3.84 and so on. So, the other ordinates of this unit hydrograph.



This is a 5-hour unit hydrograph plotted using the developed data, even the given data developed using the dimensionless standard table as well as the given data to us.

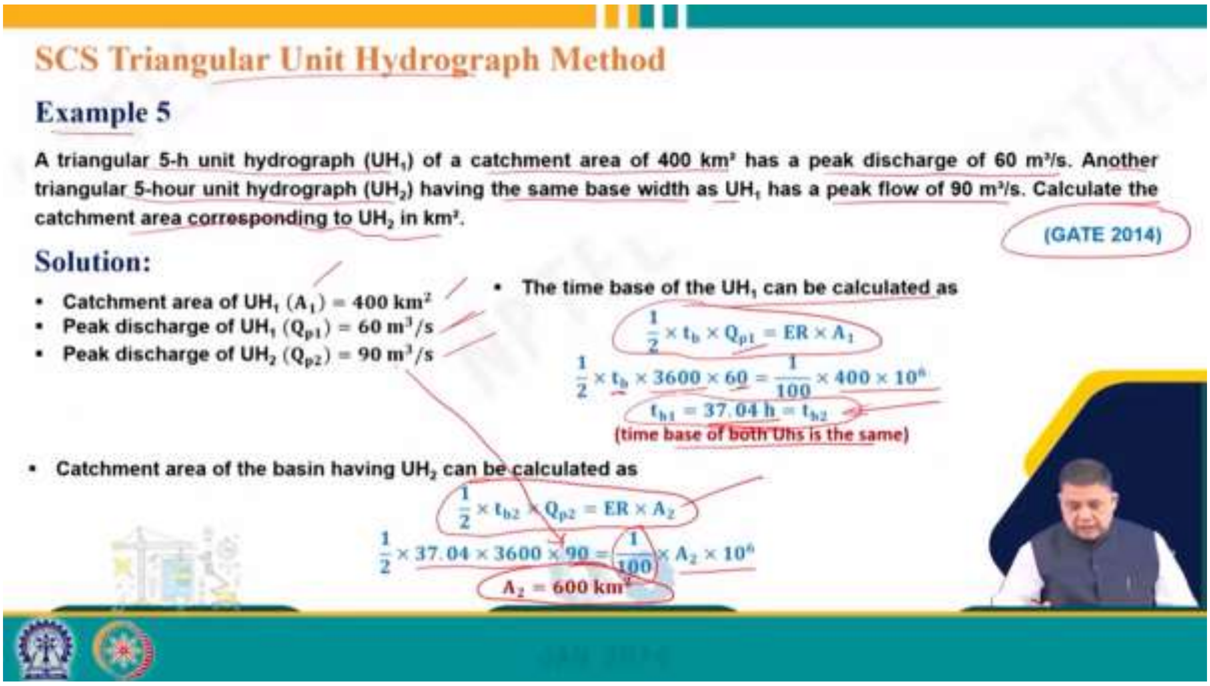


Then we go to the next problem, which is example 4, SCS triangle unit hydrograph method. The area and time of concentration of a basin are 15 square kilometres and 3 hours, respectively. Calculate the peak runoff rate for the basin from a 30-minute triangular unit hydrograph with an excess rainfall of 10 centimetres.

So, we have to calculate the peak runoff rate, and this question was asked in GATE 2004 examination. The area of the basin is given as 15 square kilometres, time of concentration of basin is given as 3 hours, and the duration of excess rainfall D is 30 minutes or 0.5 hours is given here. So, time to peak we know from this here we know that this is Tc Tp is related to D by 2 and Tc by this relationship that is D would D by 2 plus 0.6 Tc D is given known to us Tc is known to us.

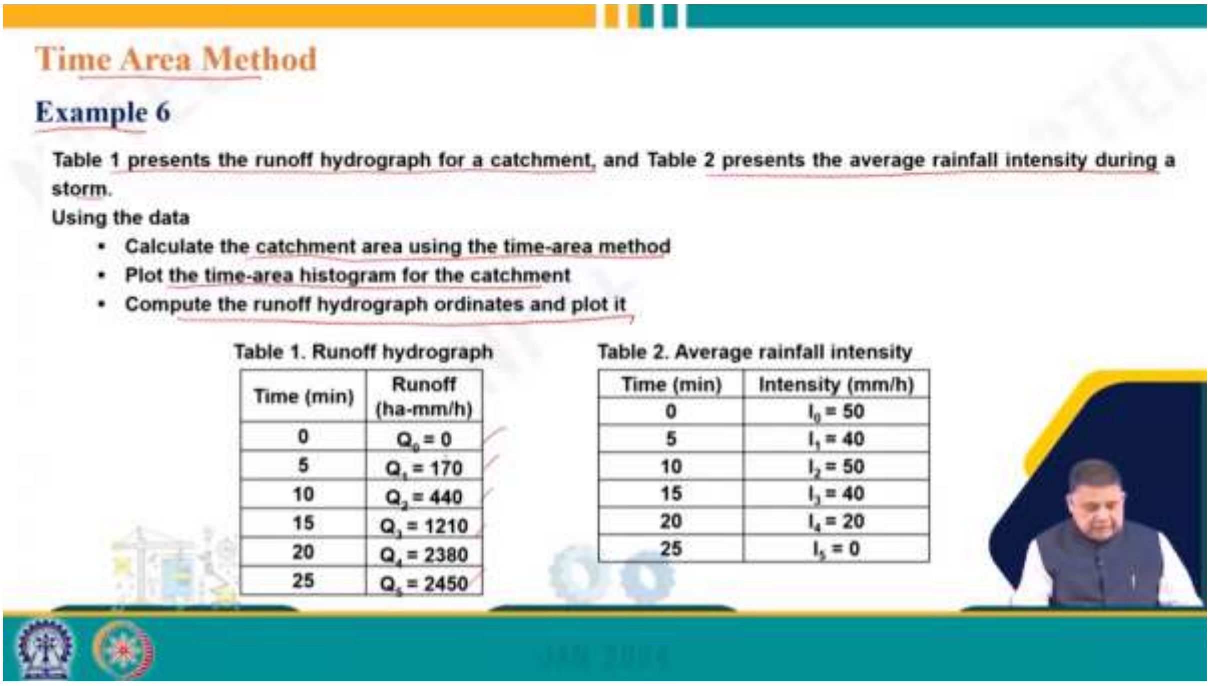
So, by putting these values here, we can calculate the value of Tp which comes out to be 2.05 hours, and time base is 2.67 Tp that is a standard triangular unit hydrograph relationship we know. Because Tp is calculated so we know the time base is 5.47 hours. Now, once we know that for obtaining the value of Qp we have to use the concept that area under the curve is equal to the volume of runoff generated by 1 centimetre of effective rainfall. So, half into Tv into Qp, that is the area under the curve and is equal to Er, which is 1 centimetre. So, 1 by 100 meters and area is 15 square kilometres.

So, 15 into 10 to the power 6 square meters. So, that gives us a value of Qp equals to 15.23 cubic metre per second, that is the answer to the particular question. So, it is a very straightforward question.

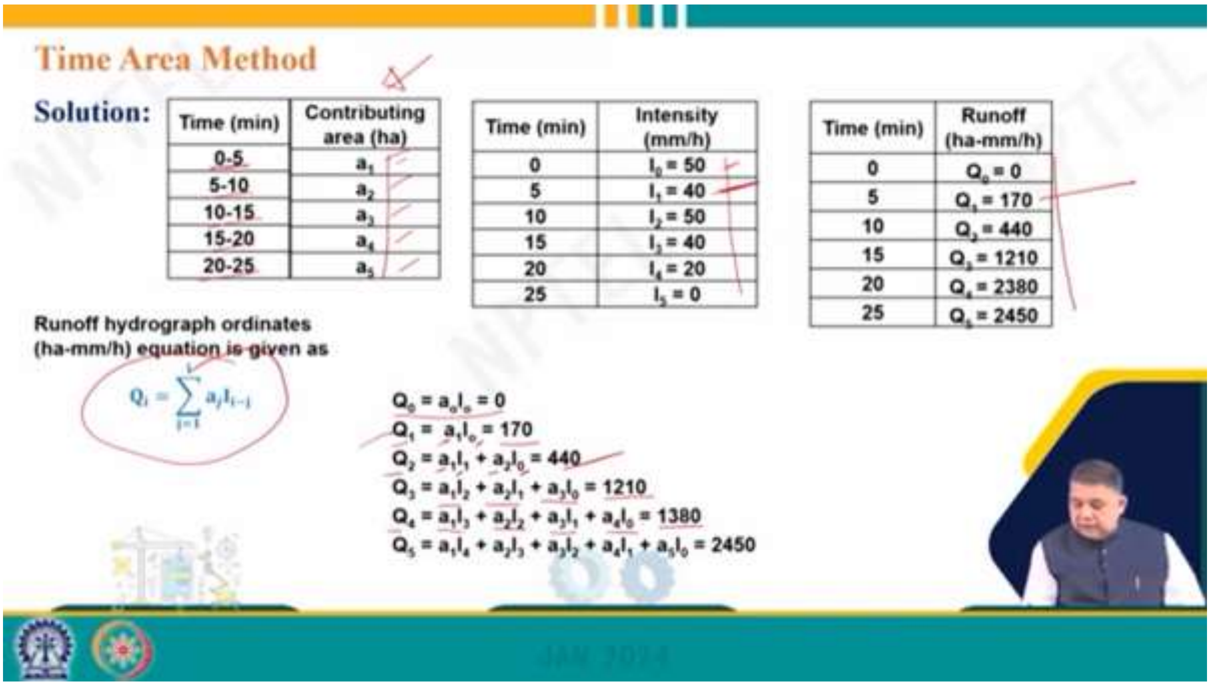


Now, we go to example number 5, which is again on the triangular unit hydrograph method. A triangular 5-hour unit hydrograph UH1 of a catchment area of 400 square kilometres has a peak discharge of 60 cubic meters per second. Another triangular 5-hour unit hydrograph UH2 having the same base as UH1 has a peak flow of 90 cubic meters per second. Calculate the catchment area corresponding to UH2 in square kilometres, and this question has been taken from GATE 2014 examination. So, the catchment area value information provided is a catchment area of UH1 is 400 square kilometres, which we are calling A1. Peak discharge of UH1, which we are calling Qp1, is 60 cubic meters per second, and peak discharge of UH2, Qp2, is 90 cubic meters per second. So, the time base of the unit hydrograph can be calculated by the same formulation, the same concept that the area under the curve represents the volume of runoff generated because of 1 centimetre of effective rainfall over the catchment area.

So, here Qp1 value is given to us as 60 cubic meters per second. So, here, and we want the answer in hours. So, we are calculating the hours itself, and here 1 by 100 times the area of the basin in square meters. So, putting this, we get the time base of the hydrograph as 37.04 hours, which is the same as the time base of the second unit hydrograph also, because it says that the time base of the two hydrographs is the same, both are 5 hours, and the time base is the same, only the peak values are 60 and 90. The first area is 400 square kilometres, and we have to find the second area. So, again using the same concept, that is half times Tb2 times Qp2 is Er times A2, A2 is the unknown here. So, half into Tb, time base is 37.05 04 hours, so that is in seconds. Qp2 is 90, so we are putting it here, and ER is 1 by 100, and A2 again in square meters or square kilometres. So, we are putting it here into 10 to the power 6. So, from here, we get the value of A2 as 600 square kilometres, which is the answer to this particular question, that the area of the second catchment is 600 square kilometres.

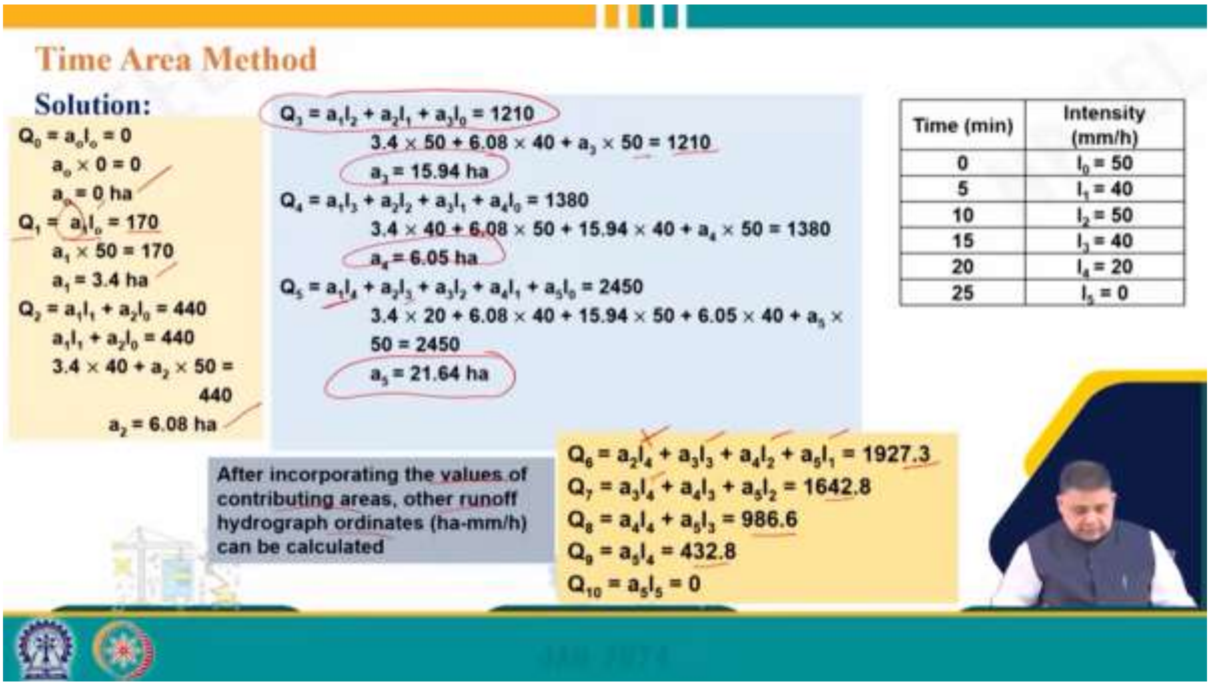


We go to the next problem, which is on the time-area method. This is the first problem we are going to discuss on this particular method. Table 1 presents the runoff hydrograph for a catchment, and table 2 presents the average rainfall intensity during a storm. So, these are the two tables here. Calculate the catchment area using the time-area method. Plot the time-area histogram of the catchment. Compute the runoff hydrograph ordinates and plot it. So, earlier, in the previous problems, we were given the area under the histogram, but in this case, we are given the discharge represented by different hydrograph histograms, that is isochrones, and the average rainfall intensity's time duration is given. Delta T value should be the same, which is 5 minutes in both cases. So, that is what it is. So, basically, we now have to find out these contributing areas.



So, let us say that for different isochrones the areas are A1, A2, A3, A4, A5, that is for isochrones represented by 5 minutes 5 to 10, 10 to 15, 15 to 28, 20 to 25. Intensity values are already given to us and runoff value is also given, and you know that in earlier problems we were calculating runoff using this relationship, and we also remember that we said that there will be a time lag because the first intensity will occur in the first area and reach the ordinate at the first hour, at the second hour the second will contribute, third hour three will contribute, and so on and so forth three different areas will contribute. So, this is how the relationship will flow. So, at 0 there is no problem. Q1 will be the first area due to the first rainfall intensity, that is I0.

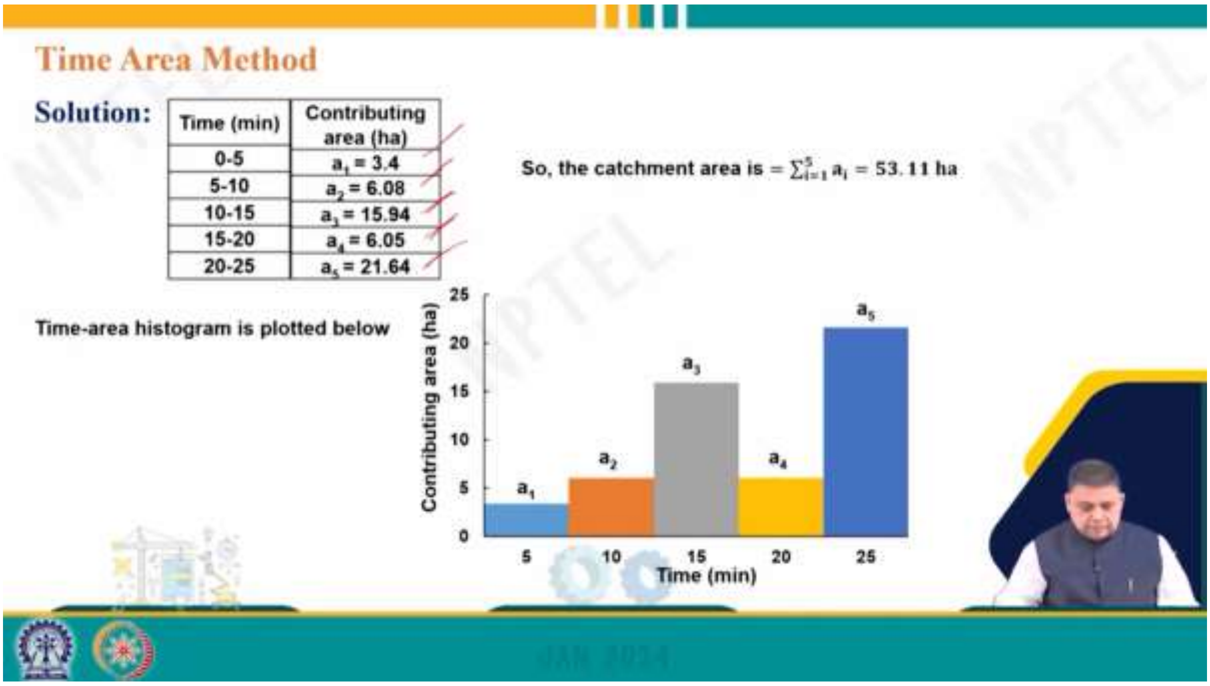
So, that value is given as 170. Q2, where A1 will have the second rainfall intensity, that is I1, and second area will have I0, and that value is given as 440. Similarly, Q3 is A1 I2 A2 I1, and now the third will also come into picture with I1, so that value is given at 1210. Q4 is I A1 I3 A2 I2 A3 I1 A4, fourth area will also come, which is given as 2380. So, this is 2380, and Q5 value is 2450. So, A1 I4 A2 I3 A3 I2 A4 I1 A5 I0.



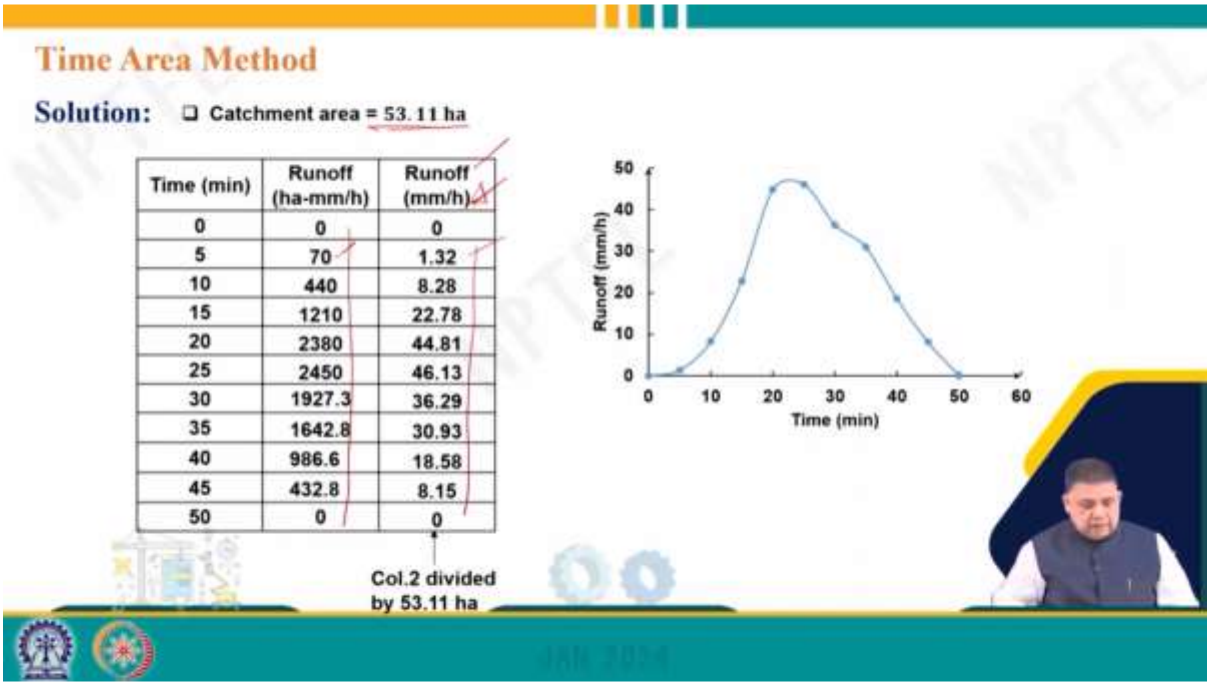
So, putting these values, because in all the 0s this is Q1 A10 is 170, only unknown is the area because I0 value is known. So, we can calculate the value of A1 as 3.4 hectares from this equation. A2 will come out to be 6.08 hectares from this relationship putting the values of A1 I2 A2 I1, and A3 value is unknown, I0 value is known, and right-hand side is known.

So, A3 comes out to be 15.94, A4 comes out to be 6.05, and A5 comes out to be 21.64. So, different areas we have calculated, and then for the remainder of the unit hydrograph, we will continue this relationship because after 5 hours, the first rainfall, all 5 rainfalls have gone from the first area. So, that will no longer exist. For A2, it was A3 I3. So, it is I4, this I3 I2 I1 here, the seventh one next time step I4 for this will go out here, and then it will be A3 I4 A4 I3 A5 I2. From here, 3 will also go, only 4 and 5 will contribute.

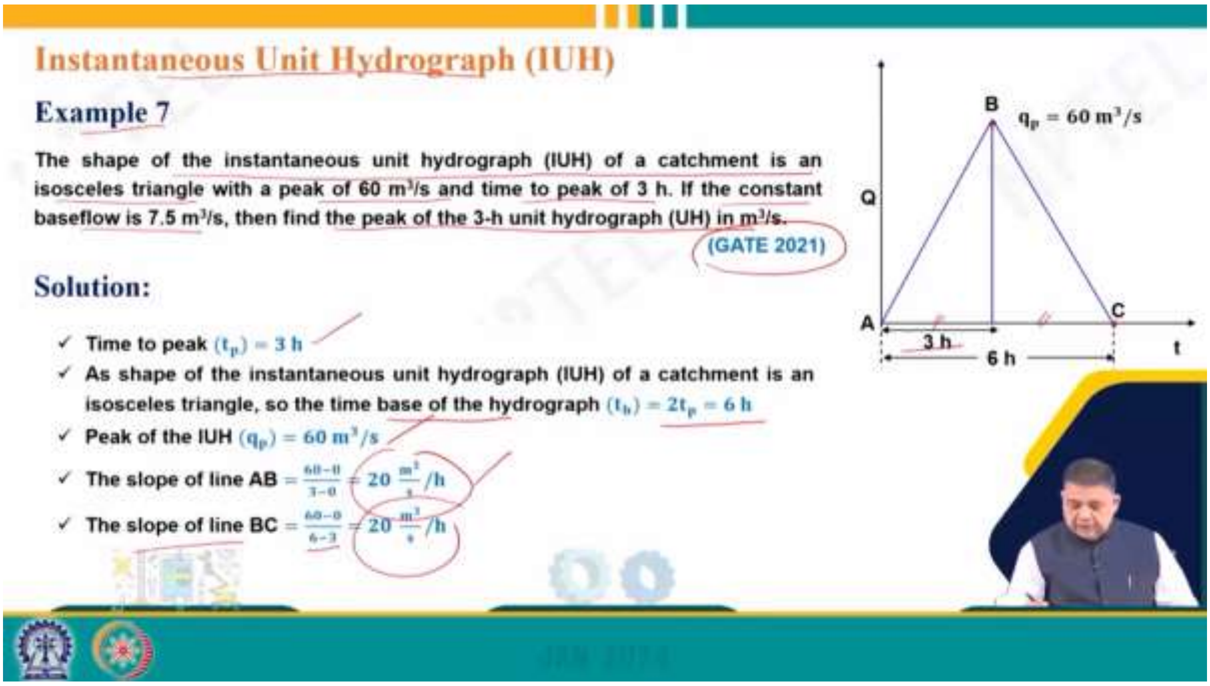
So, A4 I4 A5 I3, and then only the fifth will contribute, A5 I4, and lastly, it will be 0. So, these values we calculate because now we know A2 A3 A4 A5, all values are known to us. So, we calculate these values of different discharges, which were these are the calculations we have made after incorporating the values of contributing areas. Runoff hydrograph ordinates we are calculating here actually.



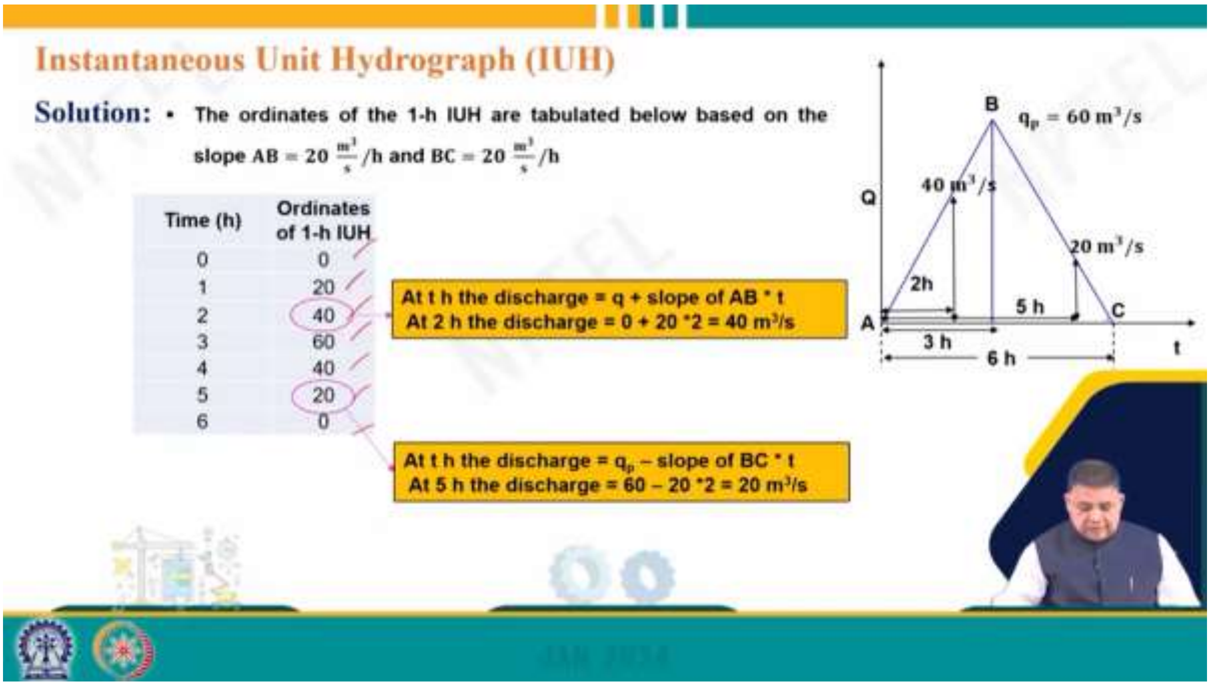
So, now, we know the contributing area, and we also know all the other answers. So, this is the first answer we have to give, that is what is the area, that is the area is 3.4, 6.08, 15.94 6.05 and 21.64 that is plotted here for different isochrones, that is the area under isochrones is calculated here.



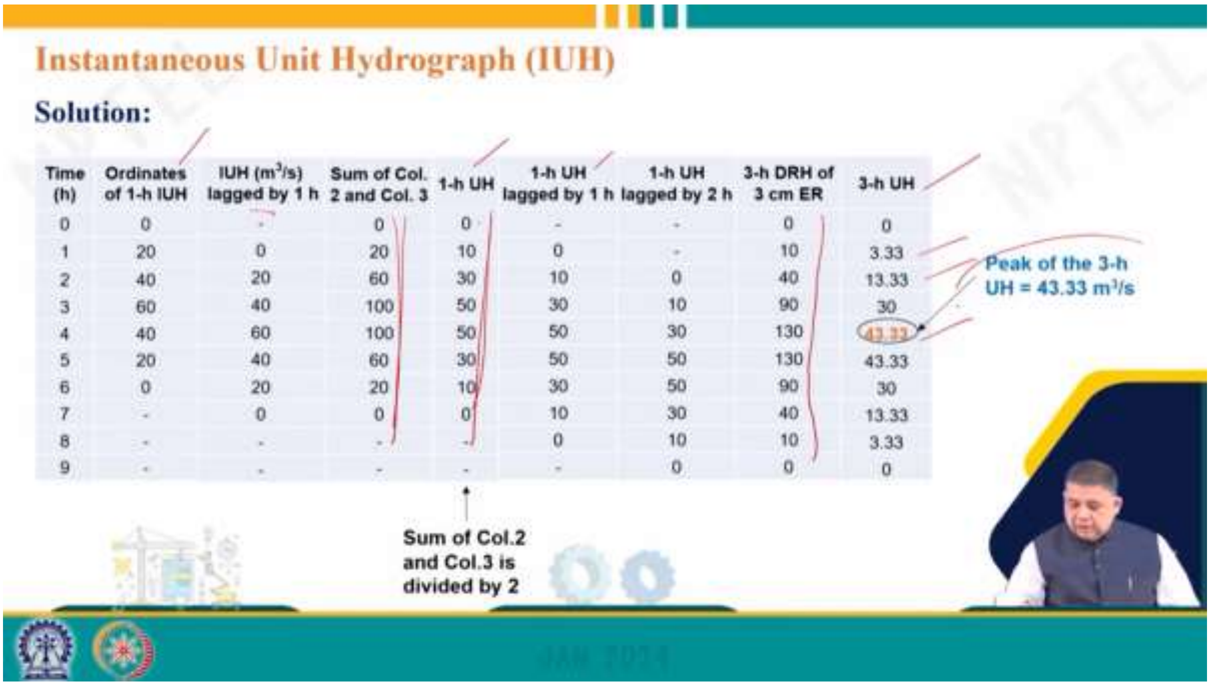
And then we can also get the runoff in millimetres per hour because it is in hectare millimetres per hour. We know the total catchment area because we know this area. So, the sum of the areas comes out to be 53.11 hectares. So, putting these values, multiplying or dividing these runoff hectare millimetres per hour, we can get the values in one runoff in millimetres per hour. These are the ordinates and these are plotted here. We can plot in other units also, like you can calculate in cubic meters per second and also plot the hydrograph. So, the hydrograph can be plotted with different units. This is where you meet millimetres per hour. It could be in cubic meters per second also. So, you can choose different units, but once you have calculated these runoff values here, you can convert the units the way you want and then you can plot it here. So, that is how the runoff can be obtained.



The next problem is on instantaneous unit hydrograph. The shape of an instantaneous unit hydrograph of a catchment is an isosceles triangle with a peak of 60 cubic meters and time to peak 3 hours. If the constant base flow is 7.5 cubic meters per second, then find the peak of 3-hour hydrograph in cubic meters per second. This question is taken from GATE 2021 examination. So, we have been given time to peak is 3 hours and it is an isosceles triangle. So, the time base, of course, this will be equal to this. So, that is why time base will be 2Tp or 6 hours. Peak is given 60 cubic meters per second. So, we can find the slope of these lines. The slope of line AB is 60 by 0 divided by 3, which is 20.

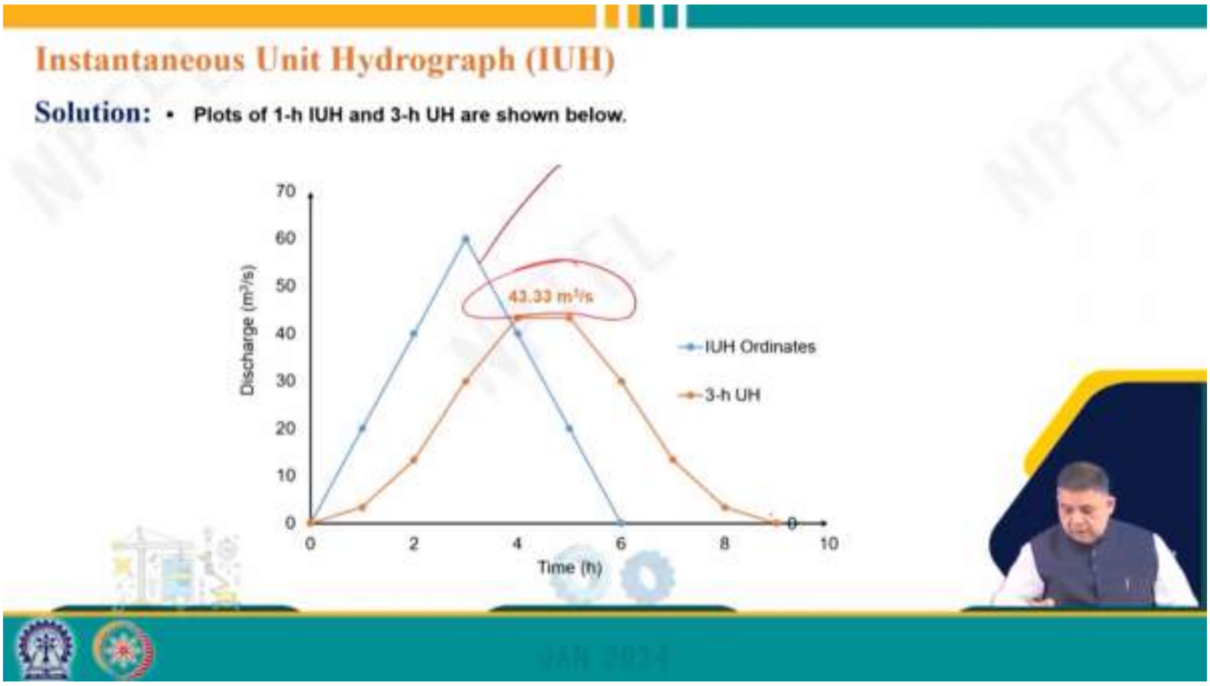


Similarly, the slope of line BC will also be 20 because this is 60 here and this is 0 here. So, 60 minus 0 by 3 is 20. So, same slope. So, that means, we can calculate the ordinates. It will be 0 20 40 and so on and so forth. So, 0 20 40 60 40 20 0. These will be ordinates of an instantaneous unit hydrograph. So, basically, this is just how we are calculating, but because this is a triangle. So, it is very easy to find the values here. Similarly, we can also show how to calculate the slope, but we do not need to do any fancy calculations because the slope value we know. So, we can straight away get the values here.

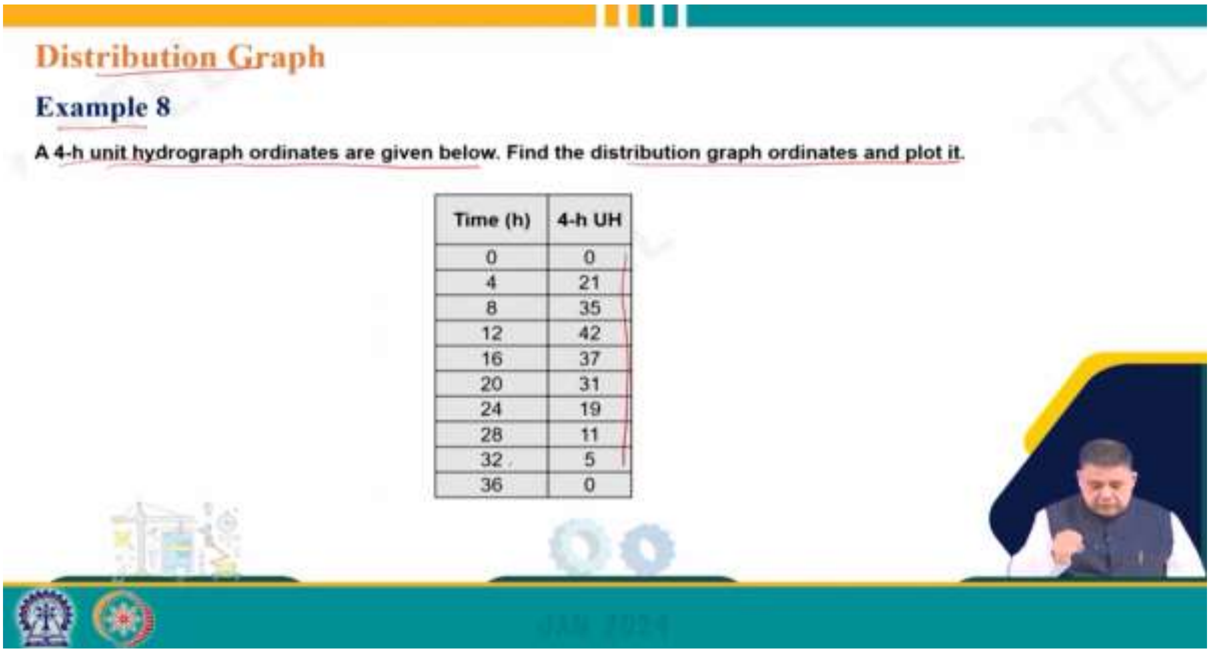


Time (h)	Ordinates of 1-h IUH	IUH (m^3/s) lagged by 1 h	Sum of Col. 2 and Col. 3	1-h UH	1-h UH lagged by 1 h	1-h UH lagged by 2 h	3-h DRH of 3 cm ER	3-h UH
0	0	-	0	0	-	-	0	0
1	20	0	20	10	0	-	10	3.33
2	40	20	60	30	10	0	40	13.33
3	60	40	100	50	30	10	90	30
4	40	60	100	50	50	30	130	43.33
5	20	40	60	30	50	50	130	43.33
6	0	20	20	10	30	50	90	30
7	-	0	0	0	10	30	40	13.33
8	-	-	-	-	0	10	10	3.33
9	-	-	-	-	-	0	0	0

Once we have a 1-hour unit IUH, then obviously, we will first develop the 1-hour unit hydrograph, that is by lagging IUH by another hour and then taking the average. So, it is summing up the ordinates and dividing by 2, that is taking the average, that gives us a 1-hour unit hydrograph. Then we have to lag, basically we will use the matter of superposition. So, lagging by 1 hour, lagging by 2 hours, and we can then sum up the ordinates. So, here 0, 10, 40, 80, and so on. These are the 3-hour ordinates and 3-hour UH ordinates will be this divided by 3. So, 10 divided by 3 is 3.33, 40 divided by 13.33, 130 divided by 3 is 43.33, and so on. So, this is the peak of the 3-hour ordinate, which is 43.33, and there are 2 values which are coming up at the same.

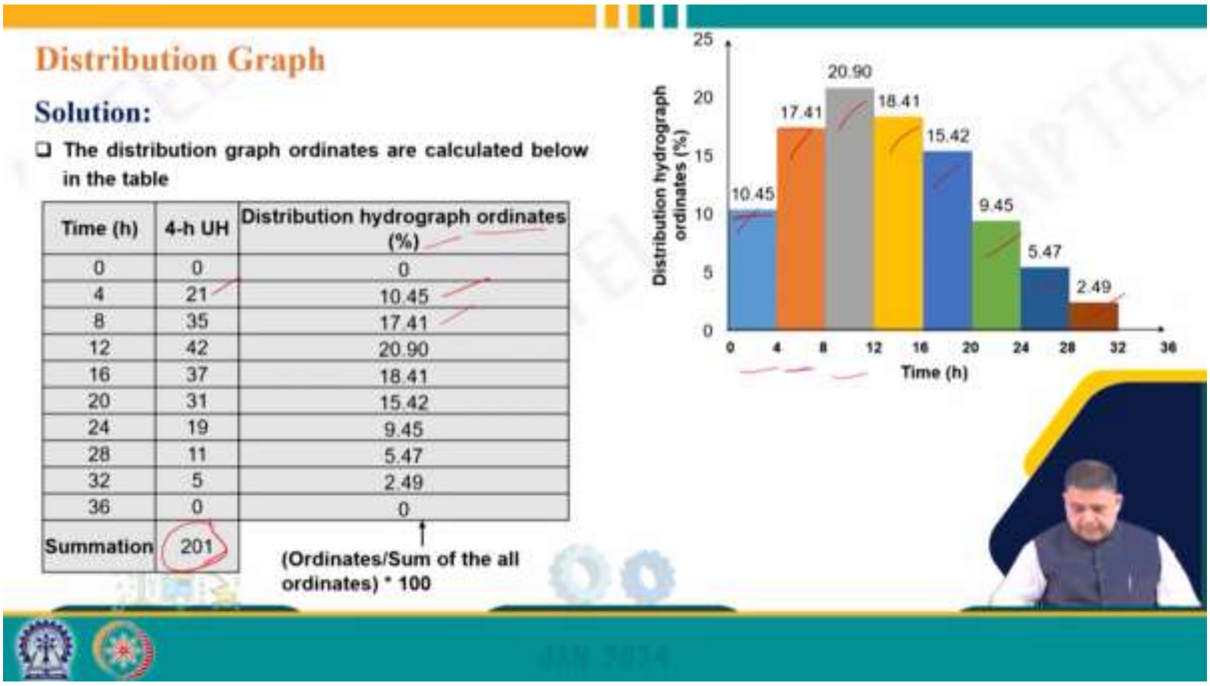


So, this is typically the IUH ordinate, and this is the derived 3-hour unit hydrograph from this IUH. These are the ordinates here. That is how we are getting the values here.



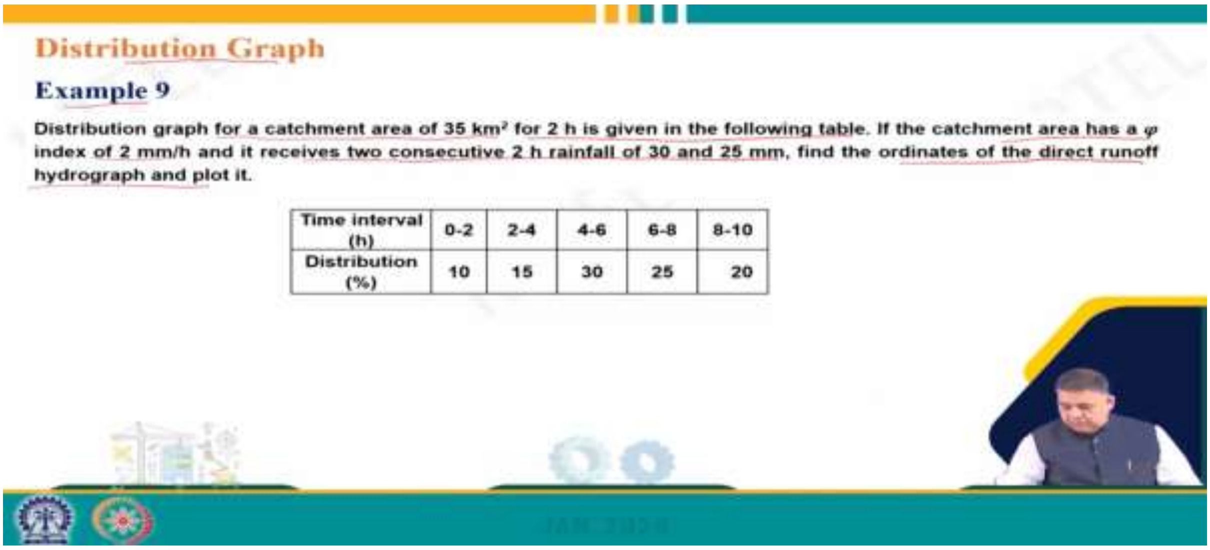
Time (h)	4-h UH
0	0
4	21
8	35
12	42
16	37
20	31
24	19
28	11
32	5
36	0

We move to the next example, which is example number 8, a 4-hour distribution graph. A 4-hour unit hydrograph ordinates are given below. Find the distribution graph ordinates and plot it. So, 4-hour unit hydrograph ordinates are given over 0 to 36 hours.



Time (h)	4-h UH	Distribution hydrograph ordinates (%)
0	0	0
4	21	10.45
8	35	17.41
12	42	20.90
16	37	18.41
20	31	15.42
24	19	9.45
28	11	5.47
32	5	2.49
36	0	0
Summation	201	

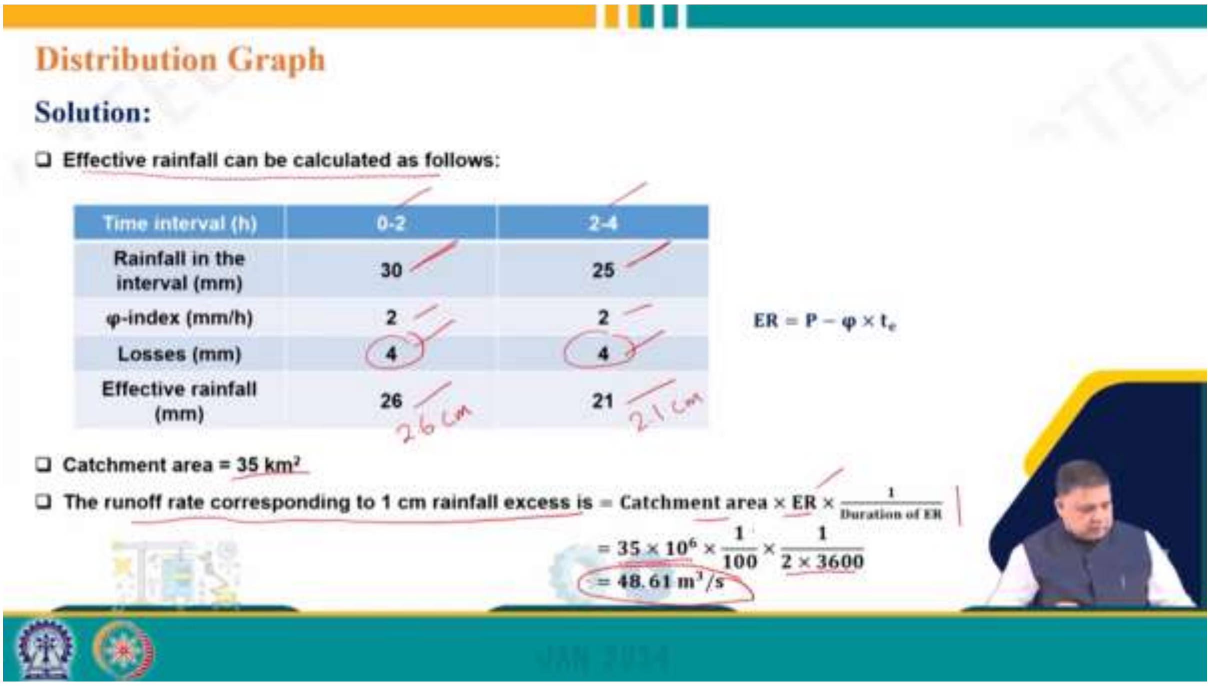
So, we have to obtain the distribution graph ordinates. The first thing we have to obtain is the sum of the ordinates of the given hydrograph, which we can simply sum up, which comes out to be 201. Then, the distribution hydrograph ordinates are nothing but this divided by the sum and of course, percent, so on, multiplied by 100. 21 by 201 multiplied by 100 comes out to be 10.45. So, it comes to be 17.4 and so on and so forth. So, for different distributions, we will get the distribution graph, and this is how the distribution graph is plotted here. So, it is a very simple problem. Typically, distribution graph problems are pretty straightforward. Here, different distribution graphs represent the areas differently. As we have already seen, it is just a unit hydrograph represented differently, which presents the area of the curve represented in different time intervals. 0 to 4 represents 10.45 percent, 4 to 8 represents 27.41 percent, and so on and so forth. That is how we get the distribution graph in this case.



Time interval (h)	0-2	2-4	4-6	6-8	8-10
Distribution (%)	10	15	30	25	20

We now come to the last problem, which is a distribution graph problem again, which is example number 9. The distribution graph for a catchment area of 35 square kilometres for 2 hours is given in the following table.

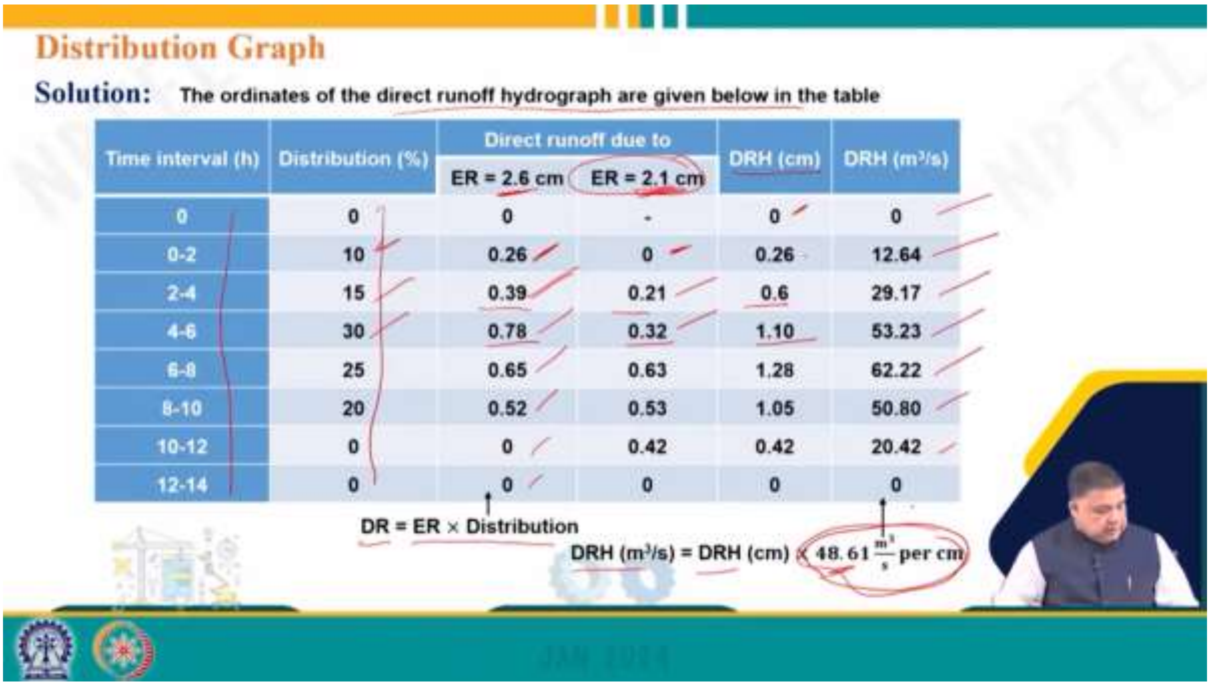
So, we have been given the distribution hydrograph here. In the earlier problem, we developed the distribution graph. If the catchment area has a phi index of 2 millimetres per hour and receives 2 consecutive 2-hour rainfall events of 30 and 25 millimetres, find the ordinates of the direct runoff hydrograph and plot it. This simply means that this is the unit hydrograph ordinate. Distribution hydrographs mean the unit hydrograph ordinate is given here, and we have a complex system. So, we have to operate that complex system on the unit hydrograph to get the direct runoff ordinates, and we have to plot that. That is the problem here.



Coming to the solution, the first thing we have to calculate is the effective rainfall, that is ERS. The time intervals are given as 0 to 2 and 2 to 4, and rainfall values in the 2 events are given as 30 and 25 mm. So, these 2 events, 30 and 25 mm, are given. Also, we have been given the phi index value of 2 millimetres per hour. So, 2 times 2 millimetres per hour is given here, the phi index value. But because this is a 2-hour storm, basically the total loss will be over 2 hours, so 2 mm per hour times 2, that means, losses will be 4 and 4 millimetres in 2-hour intervals.

And so, effective rainfall is 30 minus 4, 26; 25 minus 4 is 21. So, these are the effective rainfall values in millimetres. But as you will remember, we use basically centimetres. So, this will be basically 2.6 centimetres, which we will use later on, and this will be 2.1 centimetres, which we will use later on for deriving the DRH ordinates.

The catchment area is given as 35 square kilometres. So, the runoff rate corresponding to 1 centimetre of rainfall excess is catchment area multiplied by ER multiplied by the duration of ER, which is what we already used every time. So, 35 multiplied by 10 to the power of 6, which is the area in square meters; ER is 1 centimetre per hour, so 1 by 100 meters; and the duration is 2 hours, so 2 multiplied by 3600. For 1 centimetre of rainfall, the runoff rate is 48.61 cubic meters per second. So, that is what we have obtained.



Distribution Graph

Solution: The ordinates of the direct runoff hydrograph are given below in the table

Time interval (h)	Distribution (%)	Direct runoff due to ER = 2.6 cm	Direct runoff due to ER = 2.1 cm	DRH (cm)	DRH (m³/s)
0	0	0	-	0	0
0-2	10	0.26	0	0.26	12.64
2-4	15	0.39	0.21	0.6	29.17
4-6	30	0.78	0.32	1.10	53.23
6-8	25	0.65	0.63	1.28	62.22
8-10	20	0.52	0.53	1.05	50.80
10-12	0	0	0.42	0.42	20.42
12-14	0	0	0	0	0

DR = ER × Distribution

DRH (m^3/s) = DRH (cm) × $48.61 \frac{m^3}{s}$ per cm

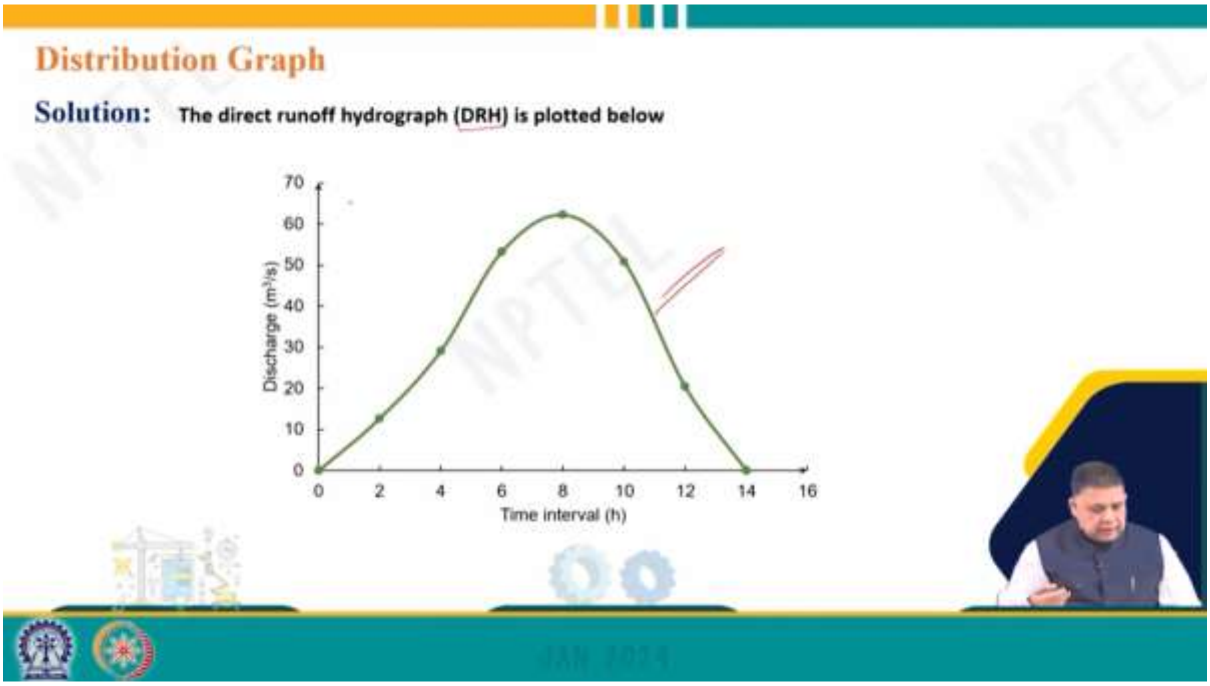
The ordinates of the direct runoff hydrograph can be obtained because here we have different hours and the distribution percentage is given. So, basically from here, we can calculate, I mean, either you can do it one way, that you first calculate the UH unit and then get the DRH ordinate, or you can simply operate on this. The effective rainfalls are 2.6 and 2.1. We know that they are from a complex storm, so there will be a lag of D hours, in this case, 2 hours, that is the thing.

So, 0 to 10, that means 2.6 centimetres, it is 0.26; 15 multiplied by 2.6, 30 multiplied by 2.6, divided by 100, actually, because of the percent. So, 26 divided by 100 is 0.26, 15 multiplied by 2.6, divided by 100 is 0.39, and so on: 0.78, 0.65, 0.72, 0.55, and so on. So, basically, we are directly using the distribution percentages and multiplying here. We could have always developed the UH ordinate first and then multiplied here, but we are doing it directly.

In this case, there will be a lag of D hours, that is 2 hours in this case. So, that is why the ordinate starts here.

So, 10 multiplied by 2.1, that is 21, divided by 100 is 0.21; 15 multiplied by 2.1, divided by 100 is 0.32. So, these are the DRH due to 2.1 centimetres of effective rainfall in the second 2-hour period. So, this is the first 2-hour period, and this is the second 2-hour period, and the sum of these two. This is the math principle of superposition we have studied several times and solved many problems. So, we know it is a method of superposition we are using here.

So, DRH ordinate will be the sum of these two columns. So, 0.26 here, 0.39 plus 0.21 is 0.6, 0.78 plus 0.32 is 1.10, and so on. So, already it is given here DR is ER into distribution and distribution that is percent and here it is already given DRH ordinate is in centimetres, it is given, and we know that for each centimetre, the value we have calculated is 48.61. So, we can multiply this DRH to get the DRH ordinate. We multiplied DRH in centimetres by this discharge value, which we calculated as 48.61 cubic meters per second per centimetre. So, that is why 0.26 multiplied by 48.61 is 12.64, 29.17, 59.23, and so on.



These are the DRH ordinates, and then we can plot these DR or DRH ordinates to get this final hydrograph. So, this is the plotted DRH, and that is how we have used a distribution graph to operate a complex system to develop this DRH.

So, I mean we have tried to handle various kinds of problems, various variations of different kinds of problems on hydrograph. And remember, in the previous module also, one lecture we entirely dedicated to numerical, and in this case too. So, enough of problems you have handled, please practice more and more problems to be efficient, to be clear on the concepts, and because while solving the problem, time-saving is very important. So, you have to be efficient. So, with this, if you have any doubt, please raise questions, and we will be happy to answer on the forum. Thank you very much.