Course Name: Watershed Hydrology

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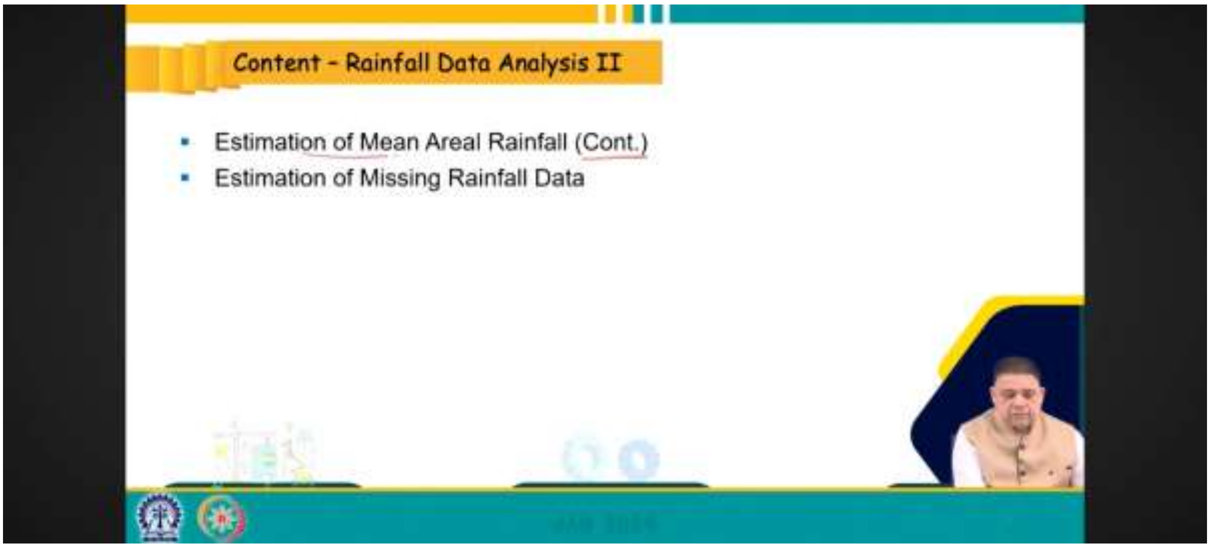
Week: 01

Lecture 04: Rainfall Data Analysis-II

Hello, friends! Welcome back to the online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are currently in Module 1, Lecture 4, focusing on rainfall data analysis part 2.

In today's lecture, we will continue our discussion on the estimation of mean aerial rainfall and then delve into the estimation of missing rainfall data.



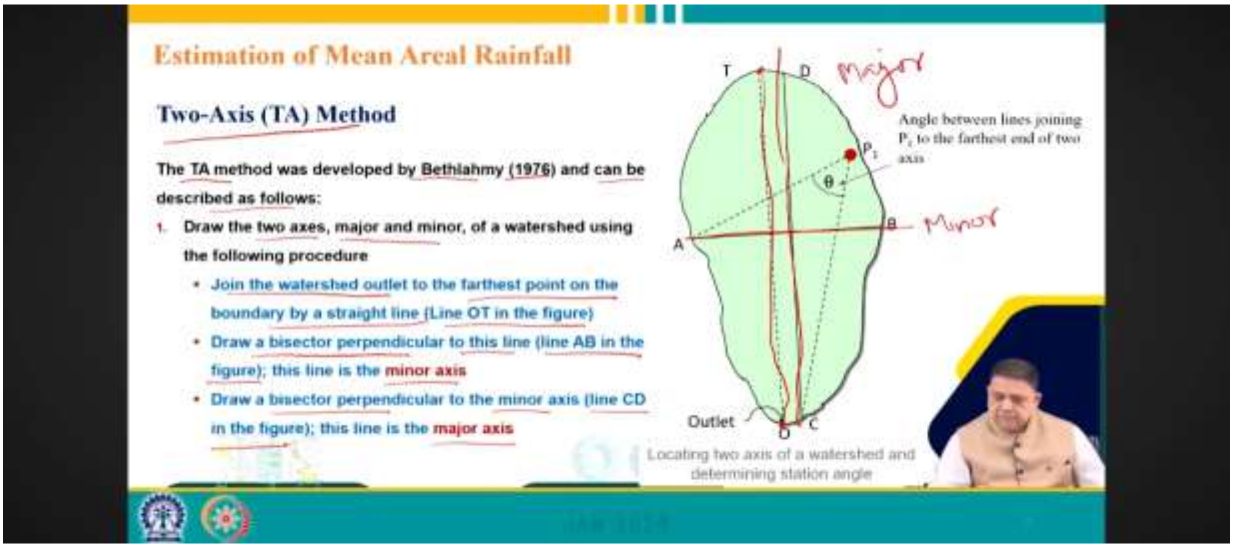
To recap from our previous lecture, we discussed the importance of estimating mean aerial rainfall, considering that within a catchment, multiple rain gauges provide point rainfall data. To derive a representative value for the entire catchment, we explored four significant methods: the arithmetic average method, the region polygon method, and the isohyetal method.

Today, we will commence our exploration of the two-axis method, another approach to determining the mean aerial rainfall of a basin. This method, developed by Bethlahmy in 1976, requires the delineation of the watershed catchment area and the identification of its outlet. These prerequisites are typically available data.

Here is how the two-axis method works:

1. First, we identify the watershed outlet and locate the farthest point on the boundary of the watershed, denoted as point "t".
2. We then draw a straight line, line "O t", joining the outlet to this farthest point.
3. Next, we draw a perpendicular bisector to the line "O t", represented as line "A B" in the figure. This line is termed the minor axis.
4. Finally, we draw a perpendicular bisector to the minor axis, forming line "CD" in the figure. This line represents the major axis.

   $P_i W_i$



To summarize, the procedure involves locating the farthest point on the basin boundary from the outlet, drawing a straight line to connect them, establishing the minor axis perpendicular to this line, and then determining the major axis perpendicular to the minor axis.

This method provides a systematic approach to estimating mean aerial rainfall, crucial for watershed management and hydrological studies.

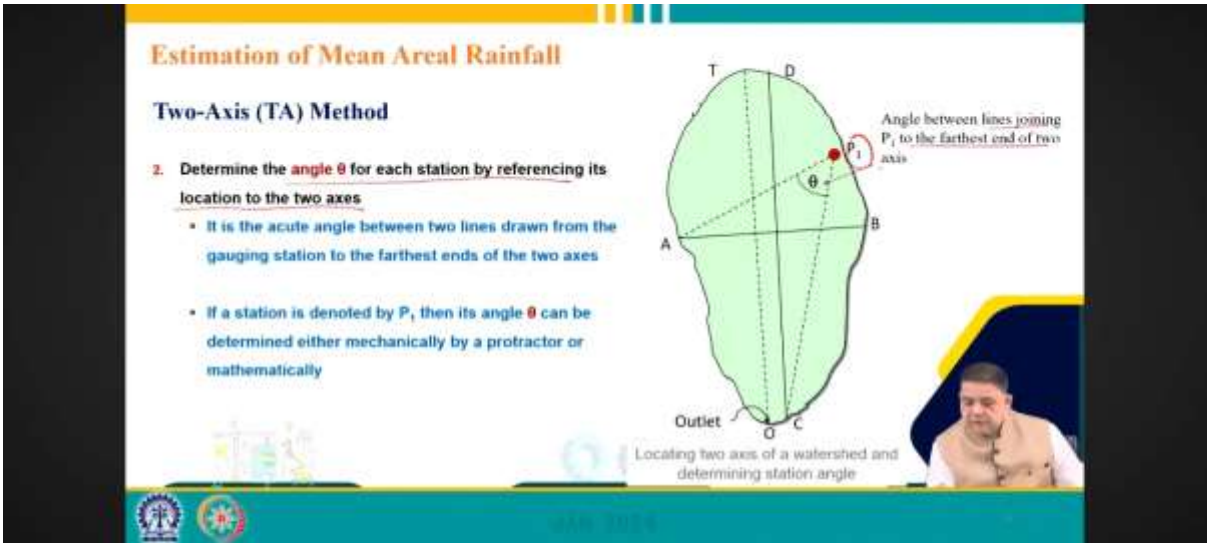
Let us proceed with our exploration into the intricacies of the two-axis method.

So, basically, in this method, our first step is to locate the major and minor axes, which serve as the main axes of the watershed. Then, the next procedure involves determining the angle ϴ for each station by referencing its position to the two axes.

For instance, let us consider station P_1, which has recorded rainfall from a point rain gauge installed in the basin. We draw straight lines joining this station to the farthest points of the two axes.

If we take P_1 and the minor axis AB and compare the distances P_1A and P_1B, we find that P_1A is greater than P_1B. Therefore, we draw a line joining P_1 to point A on the minor axis. Similarly, we determine the farthest point on the major axis concerning station P_1. In this case, point C is further away than point D.

Thus, we draw a line joining P_1 to C. The acute angle between the lines joining P_1 to the farthest ends of the two axes, namely P_1A and P_1C in this case, is the angle of interest for us.

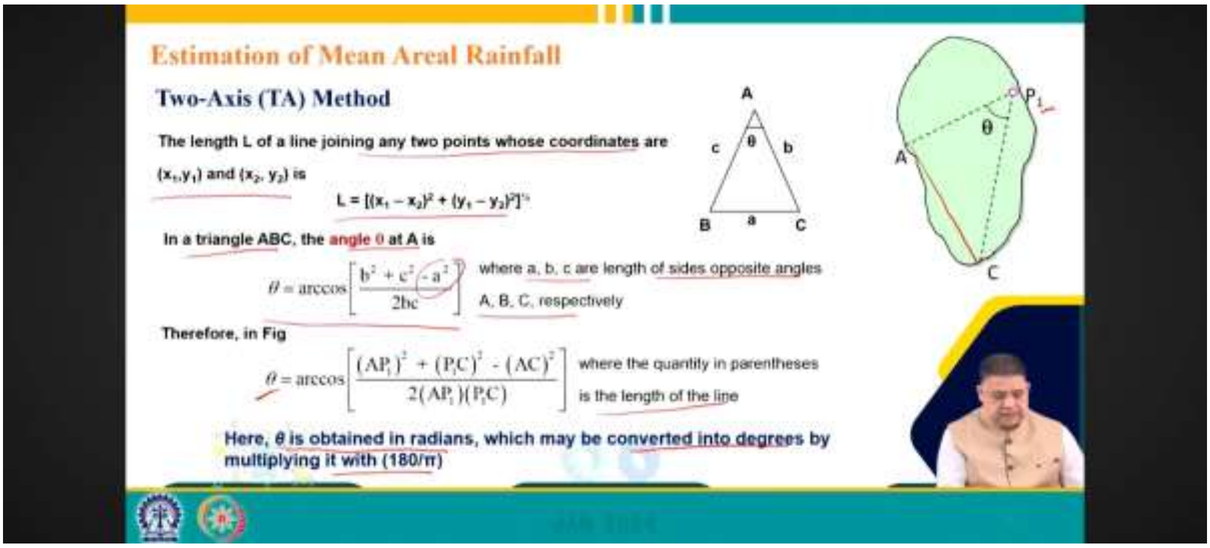


We can determine this angle ϴ either mechanically using a protractor or by using mathematical equations. Since we have located points A, B, and C, and we know their coordinates, we can find out the distances between them using this relationship: the length (L) joining any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by a certain formula.

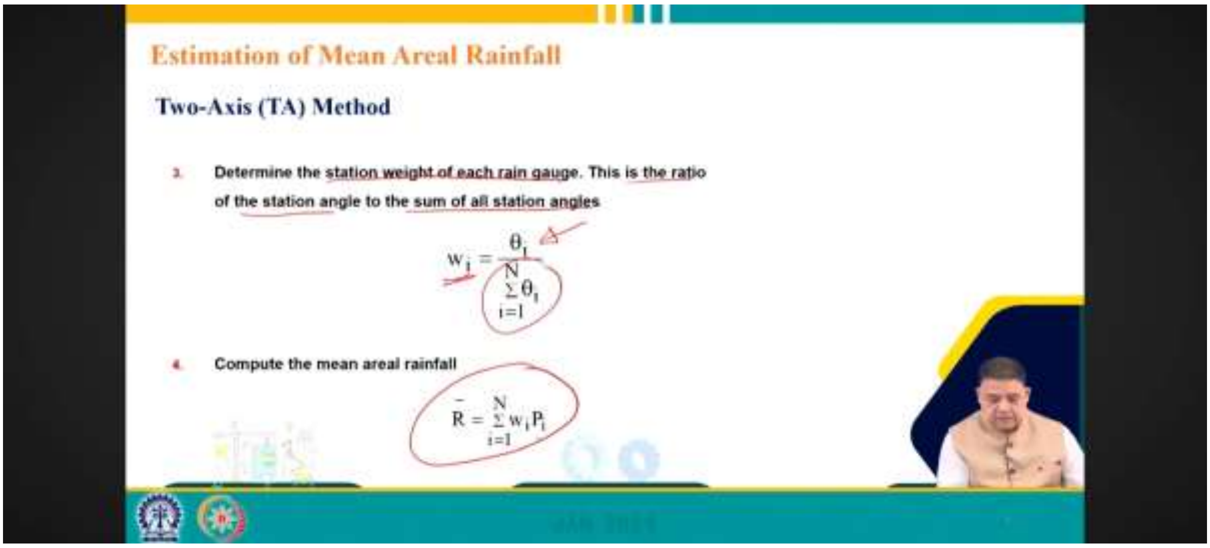
So, we can calculate these lengths, and once these lengths are known, the angle ϴ at point A can be determined using the relationship: $\Theta = \cos^{-1}((B^2 + C^2 - A^2) / (2ABC))$, where A, B, and C are the lengths of the sides opposite angles A, B, and C, respectively. Since we know the values of A, B, and C, we can find the angle ϴ at point A accordingly. Similarly, we can find out all three angles using this relationship.

Therefore, in this figure, if we revisit it with point A where we joined P_1 to A and P_1 to C, we know the lengths P_1A, P_1C, and AC. These lengths can be determined because we know the coordinates. Then, the ϴ at point P1 can be found using the formula: $\Theta = \cos^{-1} ((P_1A^2 + P_1C^2 - AC^2) / (2 * P_1A * P_1C))$. The quantity in parentheses represents the length of the line, as we have already shown. P_1A is the length of the line joining these two points, corresponding to this figure, where ABC has already been defined.

An important point here is that Θ is obtained in radians, and to convert it to degrees, we must multiply it by $180/\pi$. This conversion is crucial to remember.



Next, we determine the weight for each rain gauge station, which is the ratio of the station angle to the sum of all station angles. For a particular station, Θ_i, and the sum of all thetas, we calculate the weighting factor or weight assigned based on the weight of a particular angle compared to the total weight. The mean rainfall is then computed using this relationship: r bar equals the summation from i equals 1 to "n" of $W_i * P_i$, where W_i is the weight assigned to that station, and P_i is the recorded rainfall at that station.



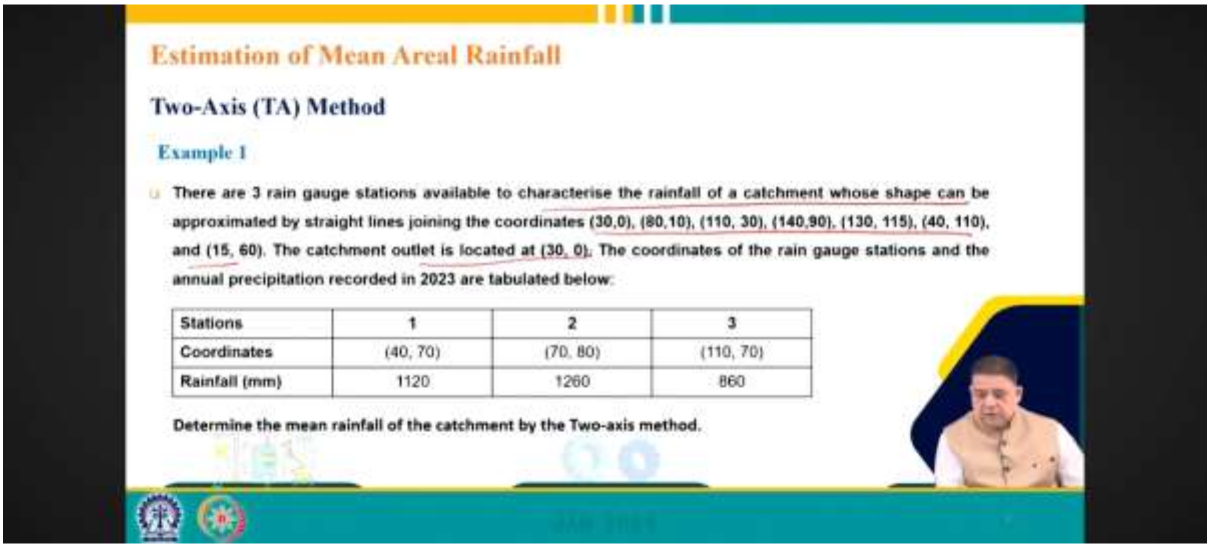
Based on this, there are certain features of this method. It assumes that all rain gauges are not equally significant regarding mean rainfall. A rain gauge located near the center of the watershed should be weighted more than one located farther out, and the weight of the rain gauge depends on its location with respect to the two axes of the watershed.

This simply means that if we have a watershed and there are two, suppose we have located the major and minor axes, and suppose we have a rain gauge station at point A at the center, and a station at point X. Obviously, if we must join the farthest points, this is the farthest point and this is the farthest point. So, the angle Θ for station A, where in this case the farthest point is this one, and the farthest point on D1. This is Θ of X.

You can see that the weight assigned to Θ A will be much larger compared to Θ X. That is how a rain gauge located in the center is more representative of the mean aerial rainfall, and that is why it is given more weight. This is a feature of this method, and this method is fast, efficient, and easily amenable to computer programming. You can easily write a computer program and use it to measure rainfall using this method.

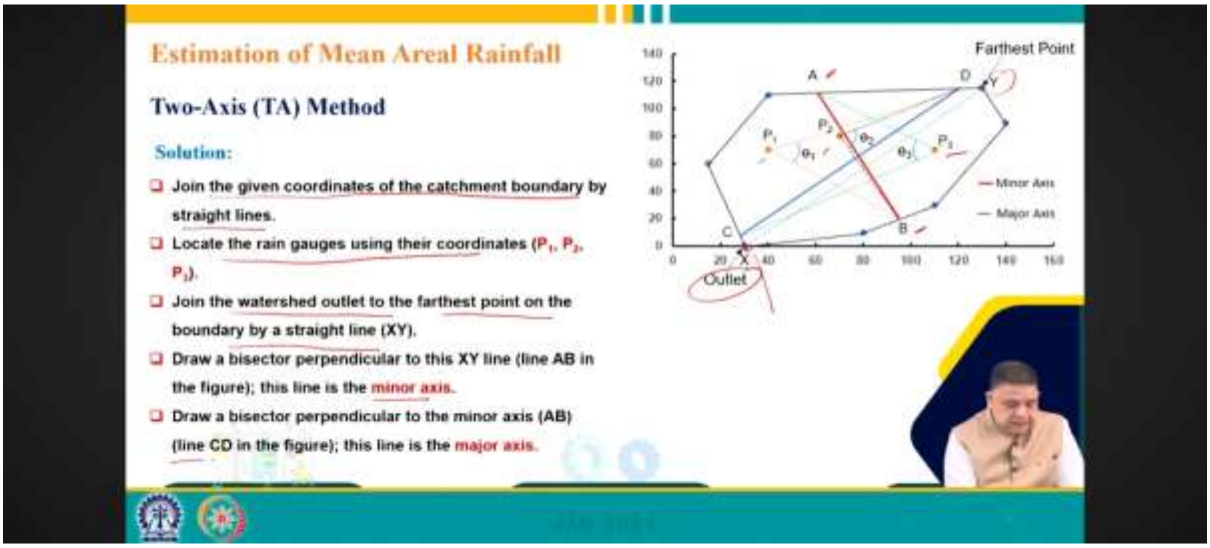
Now, let us consider an example. There are three rain gauge stations available to characterize the rainfall of a catchment, whose shapes can be approximated by straight lines joining the coordinates.

These are the coordinates provided for the boundary of the catchment, and the location of the catchment outlet is given as one of these points. Additionally, the coordinates of the rain gauge stations and the annual precipitation recorded in 2023 are tabulated below. Our task is to determine the mean rainfall of the catchment using the two-axis method.



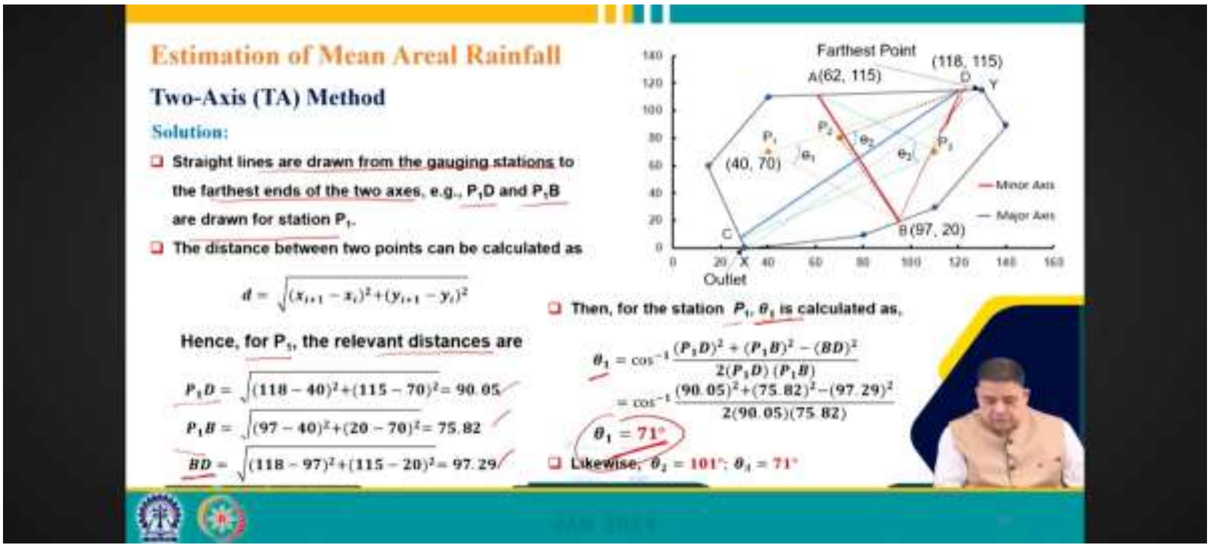
Now, applying the two-axis method to this problem, we start by joining the given coordinates of the catchment boundary with straight lines, as we approximate their shape. These various coordinates are connected by straight lines, and the rain gauge stations, P_1, P_2, and P_3, are located using their given coordinates.

The procedure begins by determining or locating the two axes: the minor and major axes. First, we join the watershed outlet to the farthest point on the boundary with a straight line. This outlet serves as our reference point. Then, we find the farthest point on the boundary, denoted as point Y in this case. By drawing a straight-line connecting points X and Y, we establish the minor axis, represented by line AB. Next, we draw a perpendicular bisector of this minor axis AB, resulting in line CD, which represents the major axis.



With the major and minor axes located, the next step is to draw straight lines from the gauge stations to the farthest ends of the two axes. For example, for station P_1, considering the minor axis, point B is the farthest, and for the major axis, point D is the farthest. Hence, we join lines P_1D and P_1B. Similarly, for station P_2, we join lines P_2B and P_2D, and for station P_3, we join lines P_3C and P_3A. The coordinates of these points can be determined once the minor and major axes are drawn.

For station P_1, the relevant distances are P_1D, P_1B, and the distance between PD. Knowing these coordinates, we can calculate the value ofΘ_1 using the relationship we previously discussed: $\Theta 1 = \cos^{-1}((P_1D^2 + P_1B^2 - BD^2) / (2 * P_1D*P_1B))$. The obtained value for$\Theta_1$ is 71 degrees. Similar calculations are performed for stations P_2 and P_3, yielding Θ_2 as 101 degrees and Θ_3 as 71 degrees.

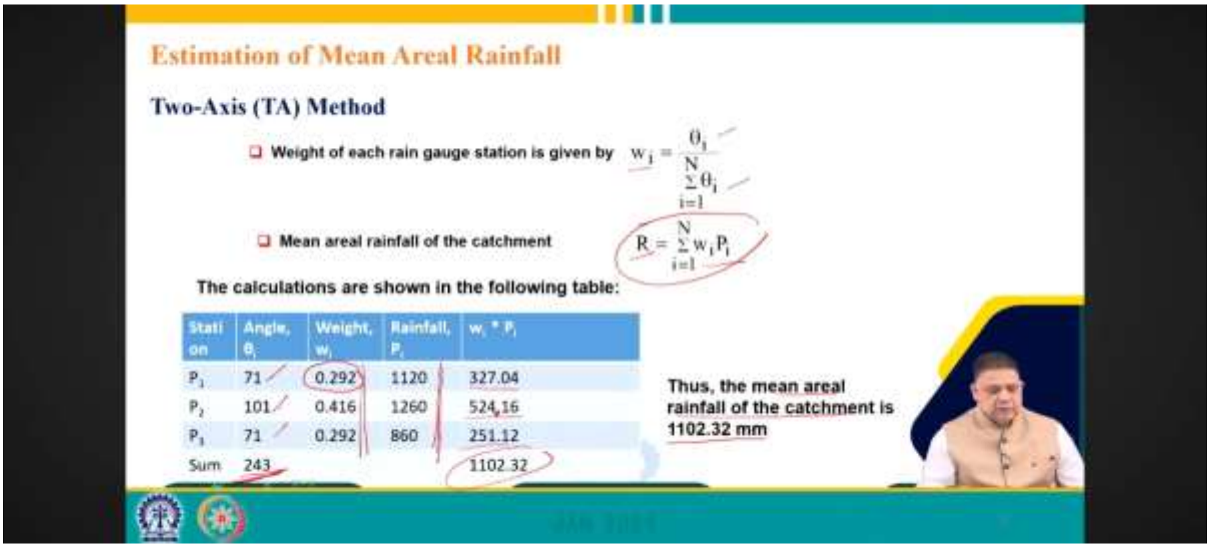


Next, we calculate the weights for the different stations. The weight (Wi) for each station (i) is determined by dividing $\Theta(i)$ by the sum of all thetas. The mean rainfall (r bar) is then calculated using the formula: r bar = Σ (Wi * Pi) / ΣWi, where Pi represents the recorded rainfall at station i.

These calculations are shown below:

So, we have determined that Θ_1, Θ_2, and Θ_3 for stations P_1, P_2, and P_3 are 71, 101, and 71, respectively. The total angle, Θ, is 243 degrees. The weights for each station will be $\Theta_1/243$, $\Theta_2/243$, and $\Theta_3/243$. Therefore, for station P_1, the weight is 71/243, for station P_2 it is 101/243, and for station P_3 it is 71/243. Multiplying each weight by the corresponding rainfall value from the table, we get the products: $71/243 * P_1$, $101/243 * P_2$, and $71/243 * P_3$. The sum of these products yields 1102.32.

Thus, the mean aerial rainfall of the catchment is 1102.32 millimetres using the two-axis method. Although this process may seem lengthy, it can be easily programmed and is a popular method for determining mean aerial rainfall.

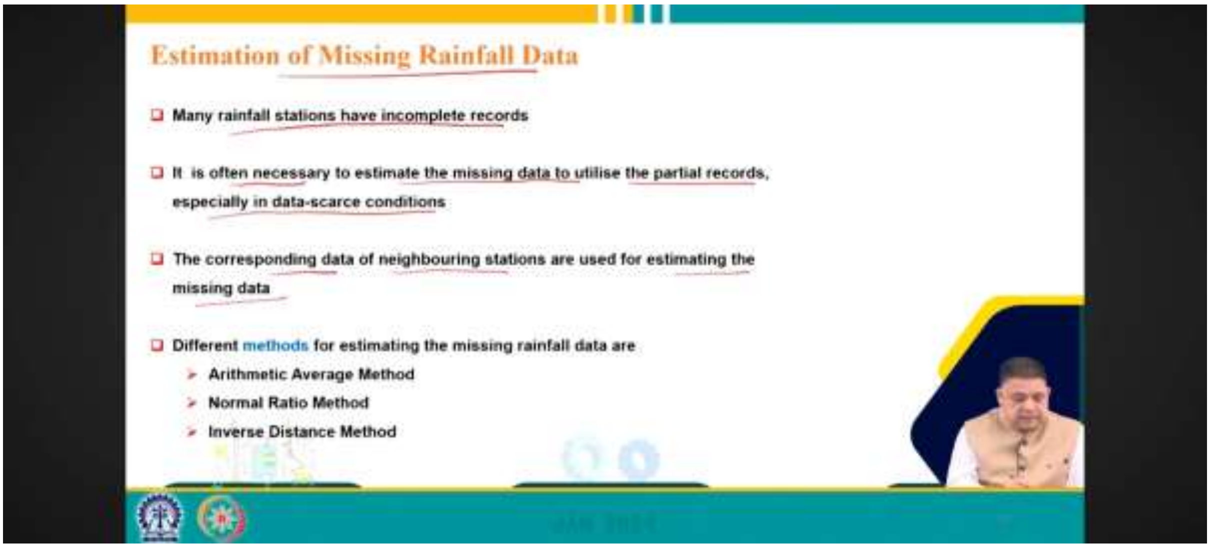


Now, let us discuss the estimation of missing data.

In the previous lecture, I mentioned the sources of rainfall data, with one major source in India being the India Meteorological Department (IMD). Data can be obtained from their website or through CDs they typically provide. However, it's common to encounter incomplete records at rainfall stations. For instance, in a one-year dataset, there might be missing data for 10 days in May or 5 days in December.

Missing data is a prevalent issue, often resulting in incomplete records. In such cases, it becomes necessary to estimate the missing data to utilize these partial records, especially in areas with limited data availability. While discarding incomplete records might be feasible with abundant data, it's essential to estimate missing data when dealing with limited datasets.

To estimate missing rainfall data, we typically rely on data from neighboring stations. Various methods can be employed for this purpose, including the arithmetic average method, normal ratio method, and inverse distance method. We will explore these methods one by one.

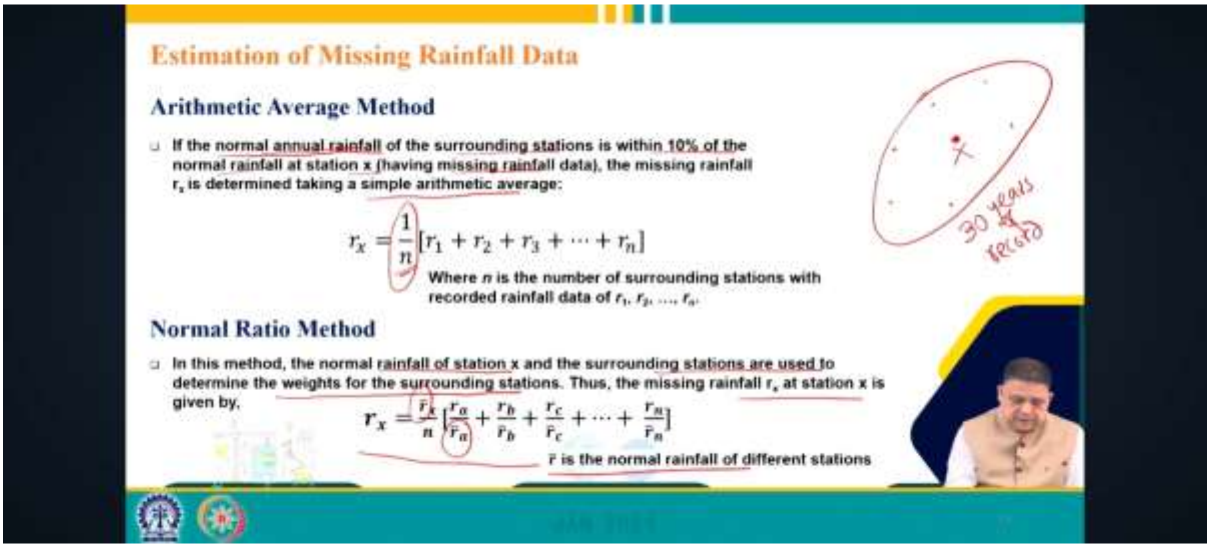


The first method is the arithmetic average method. If the normal annual rainfall of the surrounding stations is within 10 percent of the normal rainfall at station x, which has missing rainfall data, we can proceed with this method. Essentially, in a catchment where we're analyzing rainfall data and find a missing record, we gather corresponding data from neighboring stations. The initial analysis involves examining the normal annual rainfall of these stations.

Normal rainfall represents the average rainfall over a long period, typically using 30 years of record. By comparing the normal rainfall of station x with that of neighboring stations, we assess whether the variation falls within 10 percent. If the normal rainfall of station x is within this threshold of the normal rainfall of surrounding stations, we can simply compute a simple arithmetic average of the recorded data from the neighboring stations to fill the missing record at station x. Mathematically, this involves summing the rainfall values from neighboring stations (R_1, R_2, ..., R_n) and dividing by the number of neighboring stations.

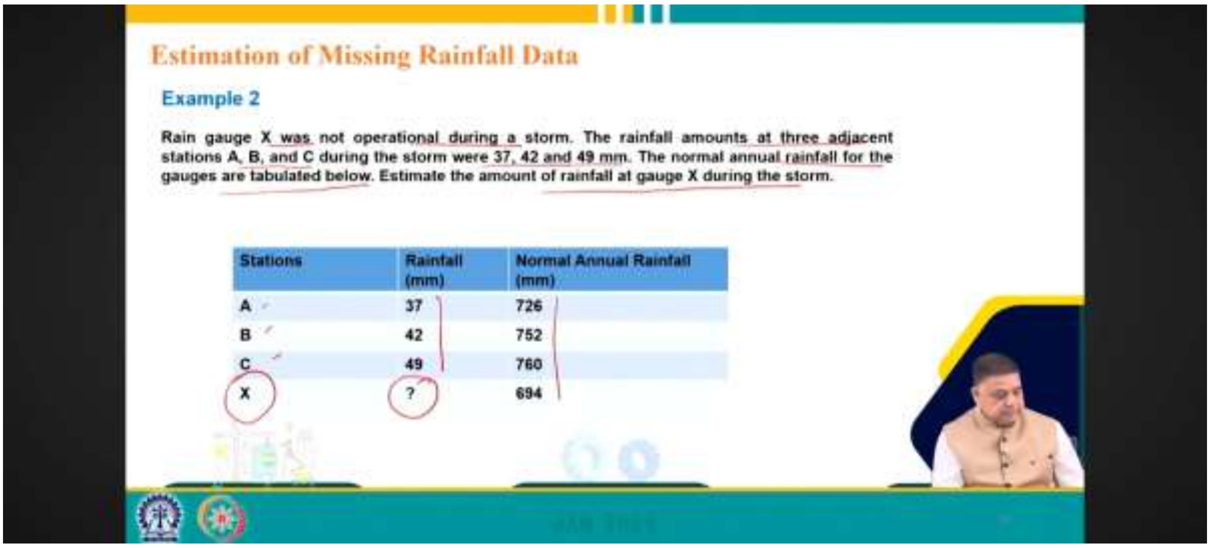
The next method is the normal ratio method. In this approach, the normal rainfall of station x and the surrounding stations are utilized to determine the weights for the surrounding stations. Unlike the arithmetic average method where equal weight is assigned (1/n), in the normal ratio method, the weight assigned is the ratio of the normal rainfall of station x to that of the neighboring station.

So, basically, in this case, $\bar{R}$ represents the normal rainfall. $\bar{R}_x$ and $\bar{R}_a$ or $\bar{R}_x$, $\bar{R}_b$, $\bar{R}_c$ assign the weights. In this method, the normal rainfall of station x and surrounding stations are used to determine the weights for surrounding stations, and then the missing rainfall record is calculated using this relationship. Essentially, it involves multiplying the weight assigned by the rainfall at that particular station and dividing by the total number of stations.



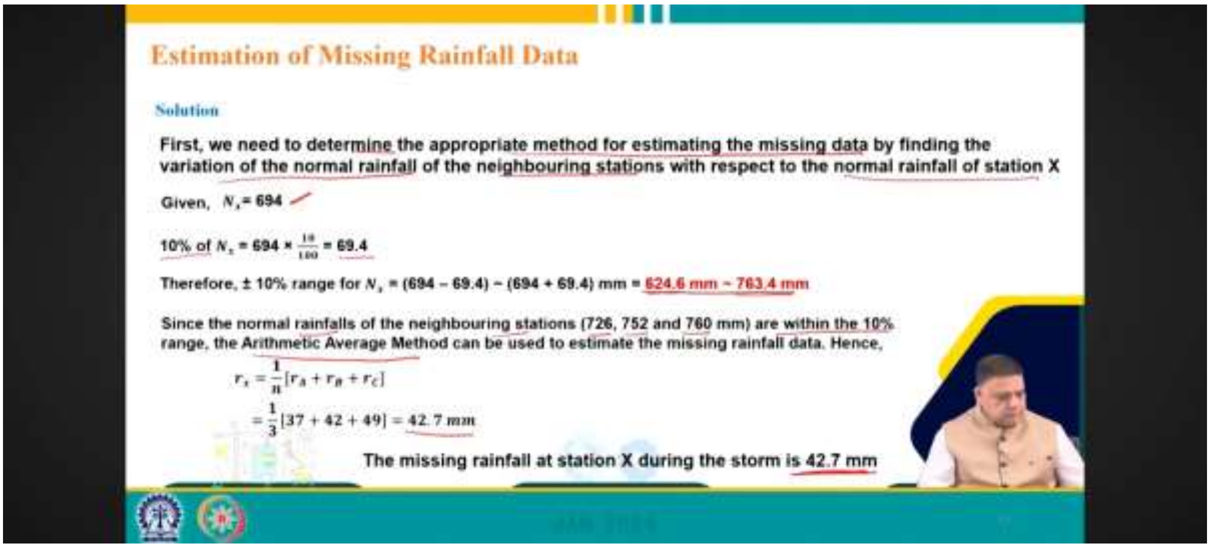
Let's consider an example of the estimation of missing rainfall:

Rain gauge x was not operational during a storm. The rainfall amounts at three adjusted stations, A, B, and C, during the storm were 32, 42, and 49 millimetres, respectively. The normal rainfall for the gauges is tabulated below. Estimate the amount of rainfall at station x during the storm.



To determine the missing rainfall data, we first need to identify the appropriate method for estimating the missing data by assessing the variation of normal rainfall of neighboring stations with respect to station x. The normal rainfall of station x is given as 694, and we consider a 10 percent variation. So, 10 percent of this value is 69.4, yielding a range of 624.6 to 763.4. Therefore, the normal rainfall of neighboring stations should fall within this range to use the arithmetic average method; otherwise, we must use the normal ratio method.

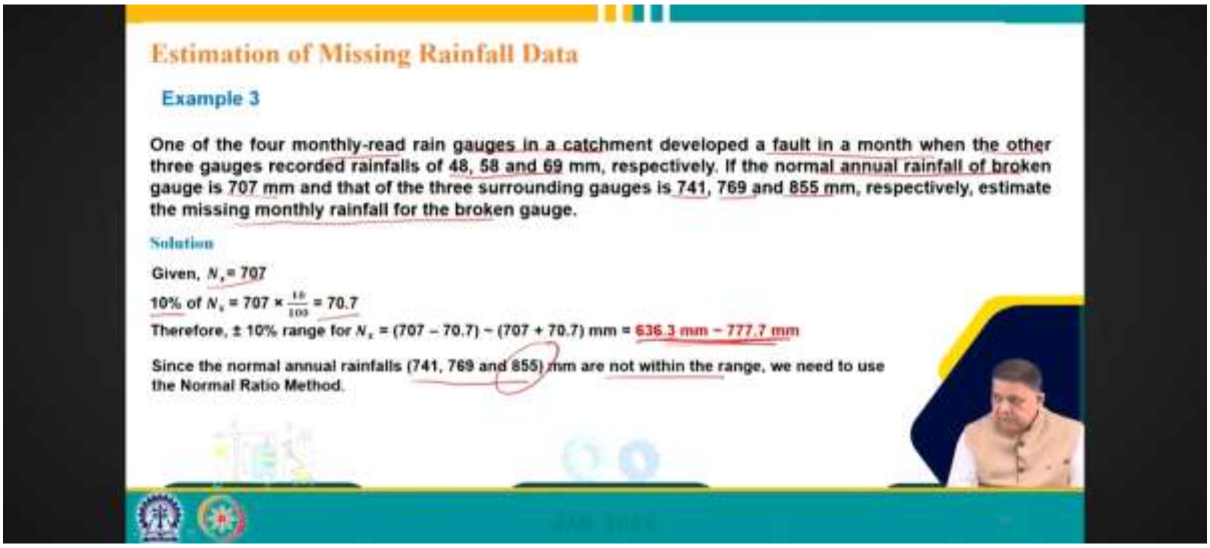
Since the normal rainfalls of the neighboring stations (726, 754, and 760) are within this range, we can use the arithmetic average method. In this case, as we saw, equal weights are assigned. Therefore, the average rainfall will be the sum of the rainfalls at neighboring stations divided by the number of stations, resulting in (32 + 42 + 49) / 3 = 41 millimeters. Thus, the missing rainfall at station x during the storm is 41 millimeters.



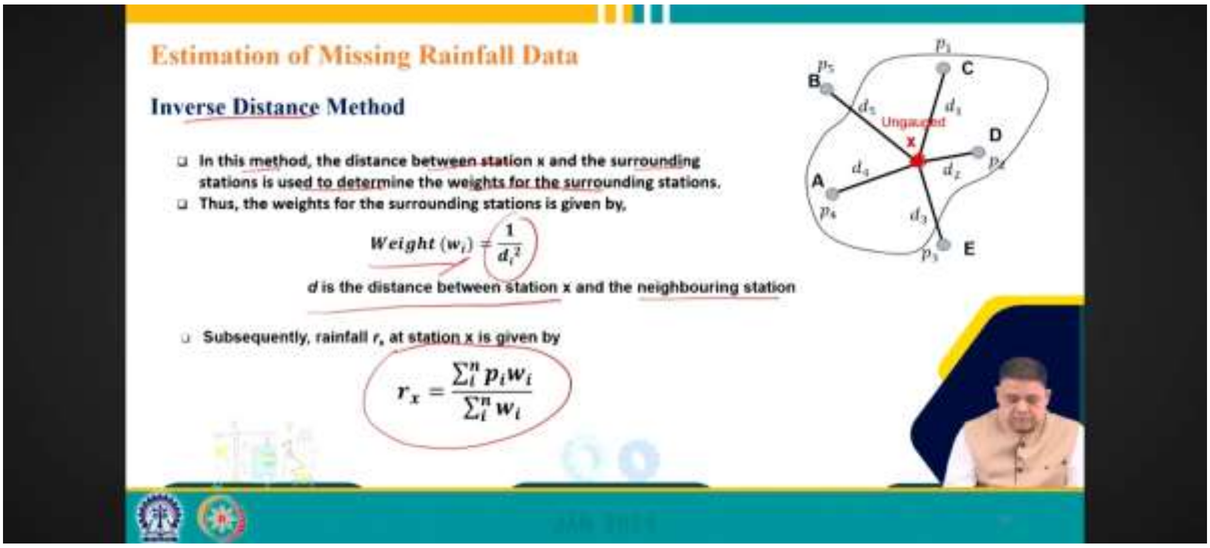
For another example, let's consider that one of the four monthly rain gauges in a catchment developed a fault in a month when the other three gauges recorded rainfall of 48, 458, and 69 millimeters, respectively. If the normal rainfall of the broken gauge is 707 millimeters, and that of the three surrounding gauges is 741, 769, and 855 millimeters, respectively, estimate the missing monthly rainfall for the broken gauge.

In this case, the normal rainfall for station x, for which we need to find the missing data, is given as 707, with a 10 percent range of 70.

So, basically, in this case, the 10 percent range for *RxRx* is 636.3 mm and 777.7 mm. That means, this is the cutoff to decide the method as far as the normal rainfall at neighboring stations is concerned. Now, if we look at the values of normal interval rainfall at the neighboring stations, we find that a value of 855 is beyond this range. So, that means, the normal interval rainfalls are not within the range. Therefore, we cannot use the arithmetic average method and need to resort to the normal ratio method, which is what we will use here.

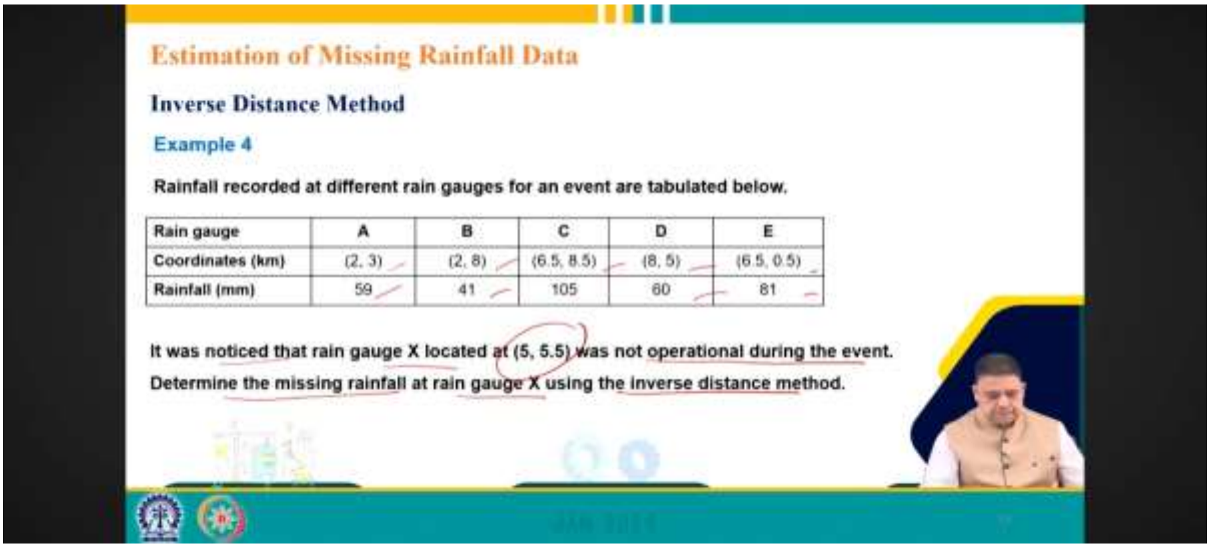


In this case, the formula is given, but we can also calculate the weights individually for each station. That is, by the normal range rainfalls of station x and ab, ca, bc, combined, you can use the formula like this, where the values of $r_a, r_b, r_c,$ are the recorded rainfalls, $\bar{r}\bar{r}$ represents the normal rainfalls, and the number of stations is 3. By inputting the values here, we get an r_x value of 52 mm. That means, the missing monthly rainfall at the broken gauge is 52 mm. So, this data, as I already mentioned, will be filled in the record, and then subsequent analysis using the other data will be done.

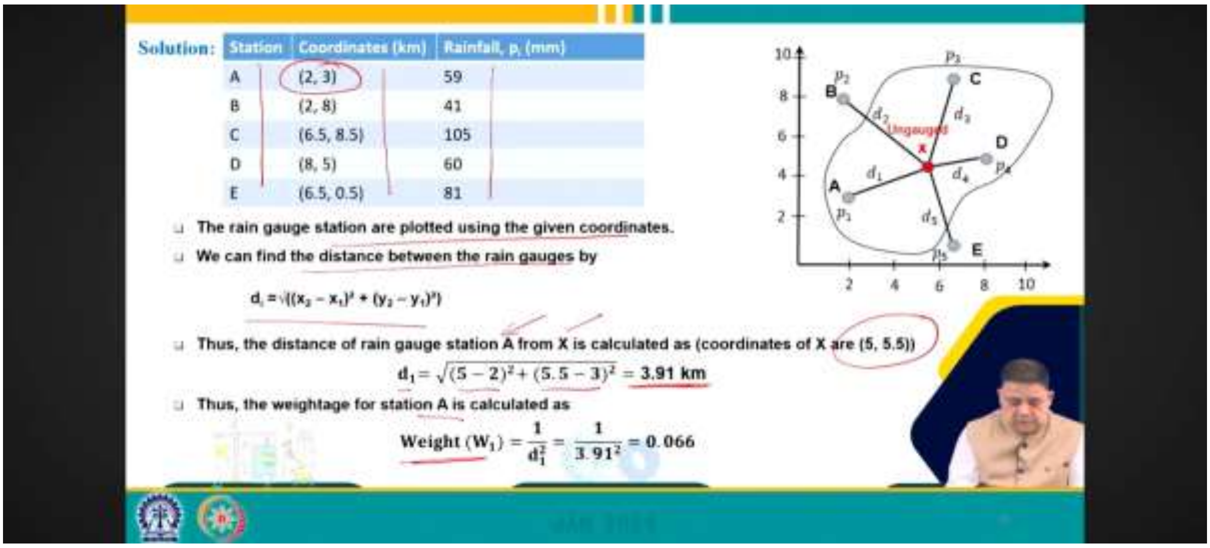


Now, we come to the next method, which is the inverse distance method. In this method, the distance between station x and the surrounding stations is used to determine the weights for the surrounding stations. So, that simply means we must rely on the weights, and the weight assigned to a particular station is given by $1/d_i^2$, where these are the distances between station x and the neighboring stations.

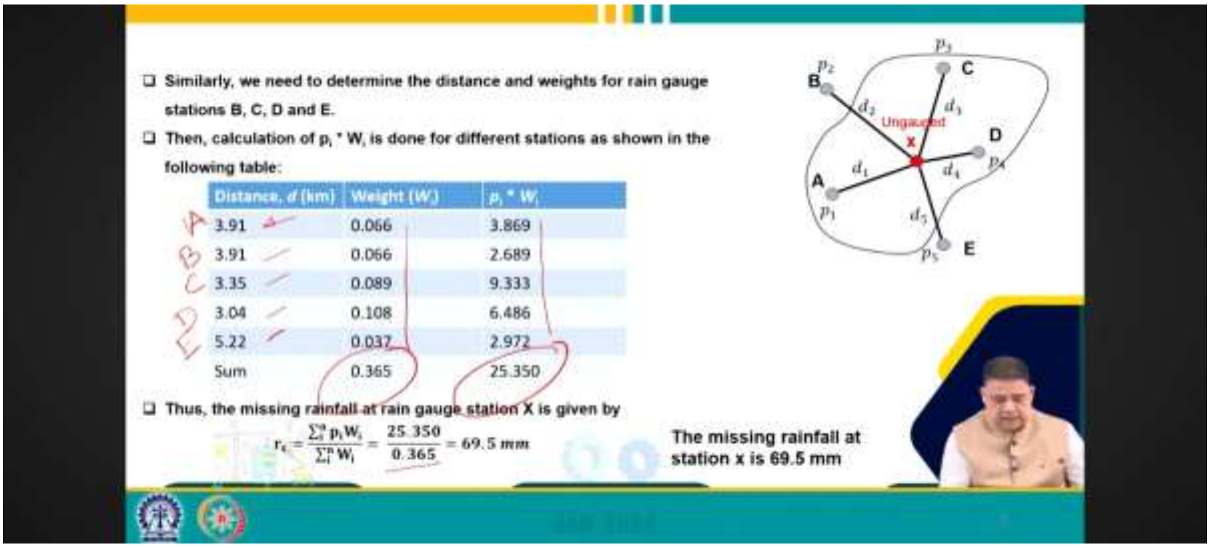
In this case, what happens is that suppose this is the problem station xx and these are our neighboring stations. So, the first thing we have to do is measure the distances by knowing the coordinates. We already know we can easily calculate the distances between two points. These distances d_1 to d_5 are estimated, and once the distances are known, then we can find out the weights assigned to different stations. And then using this equation where the numerator is the sum of P_iW_i and the denominator is the sum of the weights assigned to different stations, and using this, we can find out the missing rainfall at station x.



Let us take an example based on this method. Rainfall recorded at different rain gauges during an event is tabulated below for rain gauges A, B, C, D, and E. The coordinates are given as 2, 3, 2, 8, and so on, and the rainfalls recorded at these stations are also provided. It was noticed that rain gauge x, located at 5, 5.5, was not operational during the event. Determine the missing rainfall at station x using the inverse distance method. So, that is the method we must use in this case.



Now, we know these stations, we know their coordinates, and we have been given the rainfall values. So, obviously, we know most of the things. The rain gauge stations are plotted using the given coordinates, and we can find the distance between the rain gauges by this formula. We already have used this. For example, the distance of rain gauge station A, which has a coordinate of 2, 3, and station x, which has a coordinate of 5, 5.5, can be calculated by this relationship using this D_1 as $\sqrt{((5-2)^2+(5.5-3)^2}$, which comes out to be 3.91 kilometers. And we know that the weight is $1/D_i^2$. So, the weight for station A will be $1/D_1^2$, which comes out to be 0.066. That is how we can assign the obtained weight to this particular station.



Using a similar procedure, we can determine the distance and the weights for different rain gauge stations B, C, D, and E as well. And then, obviously, we need to calculate P_iW_i. So, the distances shown here, station A we already calculated, and these are the distances for other stations B, C, D, and E. So, this is station A which we already calculated, and this is for B, C, D, and E. So, these are the distances, these are the weights we calculated, and this is the P_iW_i, and the sum of W_i is this, the sum of P_iW_i is this. We know that the ratio of these two, that is, missing rainfall at station x, can be obtained by finding out the ratio of these two. So, (25.350 / 0.365) gives us a value of 69.5. So, missing rainfall at station x is 69.5 mm using the inverse distance method.

With this, we come to the end of this chapter, where we saw the two-axis method for determining the mean annual rainfall. Then, we saw the three different methods of estimating the missing rainfall record. I hope you got the points. In case of any doubt, please raise questions on the forum, and, please give your feedback so that we can improve things. Thank you very much.