

Course Name: Watershed Hydrology

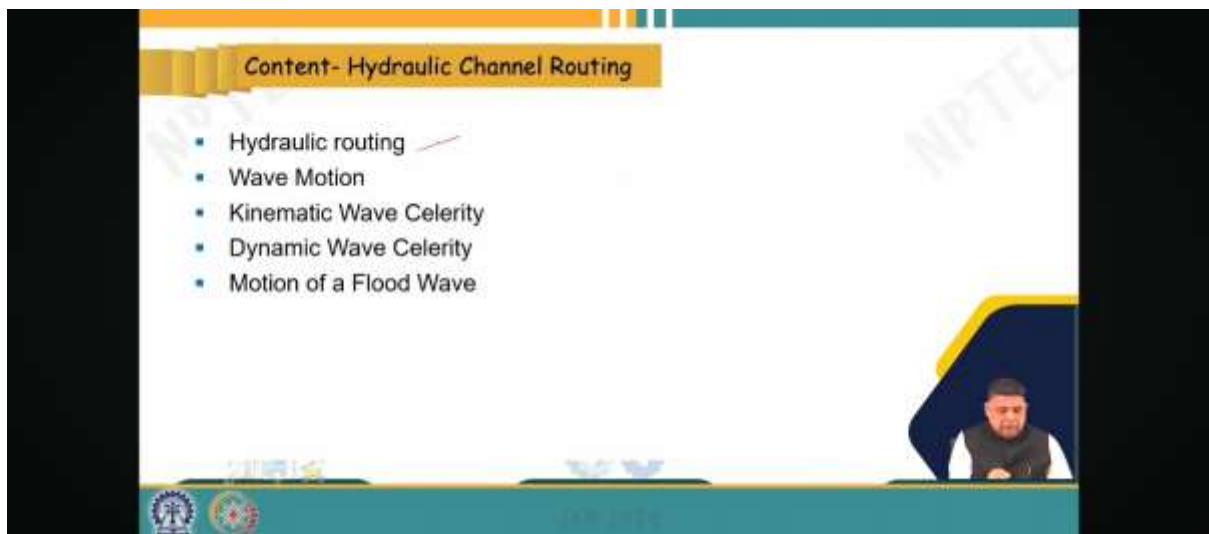
Professor Name: Prof. Rajendra Singh

Department Name: Agricultural and Food Engineering

Institute Name: Indian Institute of Technology Kharagpur

Week: 10

Lecture 49: Hydraulic Channel Routing "Hydraulic routing

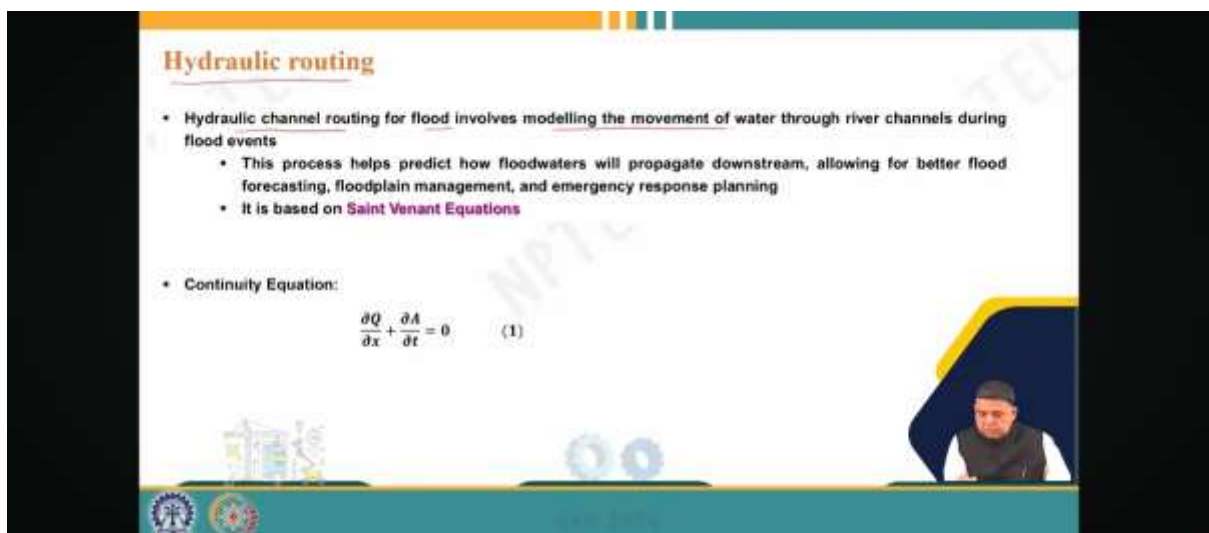


Content- Hydraulic Channel Routing

- Hydraulic routing
- Wave Motion
- Kinematic Wave Celerity
- Dynamic Wave Celerity
- Motion of a Flood Wave

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Hello friends, welcome back to this online certification course on Watershed Hydrology. I am Rajendra Singh, a professor in the Department of Agriculture and Food Engineering at the Indian Institute of Technology, Kharagpur. We are in module 10, this is lecture number 4, and the topic is hydraulic channel routing. In this lecture, we will introduce hydraulic channel routing, talk about wave motion, kinematic wave celerity, dynamic wave celerity, and the motion of a flood wave. So, basically, we already discussed that routings are of reservoir type or channel type, and then based on methods, they could be either hydrologic routing or hydraulic routing. In the previous lecture, we discussed hydrologic routing and channel routing.



Hydraulic routing

- Hydraulic channel routing for flood involves modelling the movement of water through river channels during flood events
 - This process helps predict how floodwaters will propagate downstream, allowing for better flood forecasting, floodplain management, and emergency response planning
 - It is based on Saint Venant Equations
- Continuity Equation:
$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (1)$$

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Now, we are introducing hydraulic routing here, and then we will extend that to hydraulic channel routing. Hydraulic channel routing for a flood involves modelling the movement of water through river channels during flood events. So, of course, when the flood is high, that is a flood event. If we route the movement of flow through a river channel, that is hydraulic channel routing.

This process helps predict how flood waters will propagate downstream, allowing for better flood forecasting, flood plain management, and emergency response planning. I repeat once again, as we said, in flood channel routing, when we say that means, if this is a channel or stream of our given interest and at any upstream point if we have a flood hydrograph available, that is the flow versus time graph. Then we route this particular flood and then try to see what will be the shape of this hydrograph at any downstream section of interest. So, say for example, if we know the flood hydrograph at this point and without the flow through this channel and find out the flood hydrograph at this particular point. Then basically we come to know what will be the peak flow here and at what time it is expected, how much time it will take from this for this flood to move from this point to this point we know that.

Hydraulic routing

- Momentum Equation:
- Conservation form:

$$\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + g \frac{\partial y}{\partial x} - g(S_0 - S_f) = 0 \quad (2)$$

Gravity force
 Bed slope = $-\frac{dz}{dx}$
 Friction slope
 Friction Force term
 Pressure Force term
 Convective Acceleration term
 Local Acceleration term
 Accounts for unsteady flow
 Accounts for non-uniform flow

So, basically, that simply means that once we do that, we know how much time we have available in hand. So, obviously, it is nothing but flood forecasting, and then of course, we can take up the flood protection works like evacuating people from dangerous areas. We can have emergency response planning or flood plain management; anything we can do. So, that is the purpose behind carrying out the channel routing and hydraulic channel routing. And of course, we studied the basic equations which are applicable to hydrologic routing and hydraulic routing in one of the lectures.

Hydraulic routing

• **Non-conservation form:**

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g(S_0 - S_f) = 0 \quad (3)$$

◇ Avg. flow velocity \bar{y} is a dependent variable, instead of θ . This form of the equation can be derived for a unit width of flow within the channel, neglecting the lateral flow

Kinematic wave (only friction term)

$$g(S_0 - S_f) = 0 \Rightarrow S_0 = S_f \quad (4)$$

Simplest form of distributed model

Diffusion wave (Friction & Pressure terms)

$$g \frac{\partial V}{\partial x} - g(S_0 - S_f) = 0 \Rightarrow \frac{\partial V}{\partial x} = S_0 - S_f \quad (5)$$

Dynamic wave (all terms included) → **Full Hydrodynamic model**

So, basically hydraulic channel routing is based on St. Venant equations and we know that St. Venant equation has two equations one is the continuity equation that is a differential form of continuity equation that is

$$\frac{dQ}{dx} + \frac{dA}{dt} = 0$$

this is continuity equation. The momentum equation, another fundamental equation in fluid dynamics, can be expressed in conservation form as:

$$\frac{1}{A} \frac{\partial(\Delta Q)}{\partial t} + \frac{dQ}{dx} \frac{\partial}{\partial x} (Q^2) + G \frac{\partial y}{\partial x} - GS_0 - S_f = 0$$

Where:

- A represents the cross-sectional area of the flow.
- ΔQ denotes the change in discharge over time.
- Q is the discharge.
- G stands for the gravitational acceleration.
- $\frac{\partial y}{\partial x}$ represents the slope in the direction of flow.
- S_0 represents the bed slope.
- S_f represents the friction slope.

Breaking down the equation, we can identify different components. The last term, S_f , represents the friction force term.

The second term, $G \frac{\partial y}{\partial x}$ is known as the pressure force term. Then, we have two acceleration terms: local and convective. The local acceleration term, $\frac{1}{A} \frac{\partial(\Delta Q)}{\partial t}$, accounts for unsteady flow.

Whereas, the convective acceleration term, $\frac{dQ}{dx} \frac{\partial}{\partial x} (Q^2)$ accounts for non-uniform flow. So, if the flow is both unsteady and non-uniform, as we discussed earlier in a flood motion in the channel, these terms come into play.

Wave Motion

- **Kinematic Waves** govern flow when inertial & pressure forces are not important
 - ✓ Gravity and Frictional Forces are balanced (Steady, Uniform Flow)
 - ✓ Kinematic waves are often used for short-duration flood events where the time scale of the event is relatively small compared to the time it takes for the flow to accelerate or decelerate significantly
 - ✓ Flow does not accelerate appreciably
 - ✓ Froude Number, $F < 2$ (Ratio of inertial forces to gravitational forces, $v = \sqrt{gY}$)
- **Dynamic Waves** govern flow when inertial and pressure forces are important, e.g., in the movement of large flood waves
 - ✓ Dynamic wave modelling is often necessary for scenarios with longer flow durations, where the time scale for flow acceleration and deceleration becomes more significant
 - ✓ In channels with steep slopes or complex geometry, where flow acceleration and deceleration are significant, dynamic wave models provide a more accurate representation of flow dynamics compared to kinematic wave models

We can also write this equation in non-conservation form and that is basically in this case the equation is written in terms of velocity rather than Q. So, average velocity V is the dependent variable instead of Q and if that happens then we call it a non-conservation form of the equation. And this form of the equation can be derived for a unit width of flow within the channel neglecting the lateral flow. So, any flow coming from laterally entering the channel we do not consider that. So, that is how this form is there and as we earlier discussed that we can break this into different forms.

Wave Motion

Dynamic wave

Appears as:

- Gradually varied, unsteady flow
- Streamlines & water surface profiles are not parallel

Kinematic wave

Appears as:

- Uniform, unsteady flow
- Water surfaces & bed are parallel to each other and to the energy gradeline

Source: USACE-HEC

So, if we only consider the friction term of this equation and neglect the other terms, then basically we lead to the simplest form of distributed model that is $G = S_0$ or $S_0 = S_f$. We say it is a bed slope and this is a friction slope. So, bed slope and friction slope they are the same and that kind of a model is referred to as a kinematic wave model, which is the simplest form of distributed model which considers this and the Vanant equation. Incorporating equations into the lecture, we find that when we introduce both pressure and friction terms, the equation takes the form:

$$G \frac{\partial y}{\partial x} - G \frac{S_0}{S_f} = 0$$

This leads to:

$$\frac{\partial y}{\partial x} = S_0 - S_f$$

This form is termed as the diffusion wave form or diffusive wave form of the distributed model.

Kinematic Wave Celerity

- Wave is a variation in flow, i.e., a change in flow rate or water surface elevation
- Wave Celerity is the velocity with which this variation (or wave) travels along the channel
- Celerity is a core variable in a variety of hydrologic models and flow-routing algorithms that simulate changes in river discharge
 - Ranges between - 0.25 and 10 m/s [at reach to basin scale]
- Kinematic Wave Model is defined by

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \quad (\text{Continuity})$$

$$S_0 = S_f \quad (\text{Momentum})$$
- In the K-W model, the momentum equation is replaced by the uniform flow equation, e.g. Manning's equation

Now, if we include all three terms, encompassing acceleration terms as well, the equation becomes what is known as the dynamic wave model or a full hydrodynamic model. Thus, depending on the incorporation of various components within the momentum equation in a hydraulic model, the name or type of the model changes.

So, if you only consider the friction term then it is a kinematic wave model. If we consider both friction and pressure term then we say that it's a diffusive wave model, and if we consider all three terms, that is acceleration as well as friction and pressure terms, then it's fully hydrodynamic models and you will find different hydraulic models having different components or various types of hydraulic models. Some are kinematic wave, some others are diffusive wave, and some are fully hydrodynamic models. So, depending upon what component of the momentum equation has been added, based on that this categorization is done, and you will come across in literature these terms. So, that's the classification or clarification on what these terms mean, basically. Now, coming to the wave motion, if we consider the kinematic wave, then it governs flow when inertial and pressure forces are not important. So, obviously, we know that we only consider the friction term in this case, that means the gravity and frictional forces are balanced, that means the flow could be steady and non-uniform flow, the uniform flow that is the basic when if you have read fluid mechanics and hydraulics and if you remember the basics then obviously, that is how we define steady and uniform flow, that is gravitational and frictional forces balance each other.

Kinematic Wave Celerity

- Manning's eq. with $S_0 = S_f$ and $R = \frac{A}{P}$

$$V = \frac{1}{n} \frac{A^{2/3}}{P^{2/3}} S_0^{1/2}$$

$$Q = \frac{1}{n} \frac{S_0^{1/2}}{P^{2/3}} A^{5/3}$$

$$A = \left(\frac{n P^{2/3}}{S_0^{1/2}} \right)^{3/5} Q^{3/5}$$

$$A = \alpha Q^\beta \quad \alpha = \left(\frac{n P^{2/3}}{S_0^{1/2}} \right)^{3/5} \quad \beta = 0.6$$
- Differentiating with respect to 't', we have
$$\frac{\partial A}{\partial t} = \alpha \beta Q^{\beta-1} \left(\frac{\partial Q}{\partial t} \right)$$
- Putting this in continuity equation,
$$\frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta-1} \left(\frac{\partial Q}{\partial t} \right) = q \quad [6] \quad \text{Thus, it becomes } f(Q)$$

So, if we only consider the friction term of this equation and neglect the other terms, then basically we lead to the simplest form of distributed model, which is

$$G = G_{S_0} = S_f = 0 \text{ or } S_0 = S_f$$

We say it is a bed slope and this is a friction slope. So, bed slope and friction slope, they are the same, and that kind of model is referred to as a kinematic wave model, which is the simplest form of a distributed model that considers this and the Vanant equation. Then if we add the pressure term also along with the friction term, that means we consider both pressure and friction term, that means this part of the equation,

$$G \frac{\partial y}{\partial x} - G_{S_0} S_f = 0$$

which leads to

$$\frac{\partial y}{\partial x} = S_0 - S_f$$

, then that form we call it the diffusion wave form or diffusive wave form of the distributed model. And if we take up the entire equation, that means all three terms, that is, acceleration terms are also added, that is, all terms are included, then it is referred to as a dynamic wave model or a full hydrodynamic model. So, that means, depending upon the form of the other component within the momentum equation that is considered in a hydraulic model, that is then the name or type of the model changes.

Kinematic Wave Celerity

- Manning's eq. with $S_0 = S_f$ and $R = \frac{A}{P}$

$$V = \frac{1}{n} \frac{A^{2/3}}{P^{2/3}} S_0^{1/2}$$

$$Q = \frac{1}{n} \frac{S_0^{1/2}}{P^{2/3}} A^{5/3}$$

$$A = \left(\frac{n P^{2/3}}{S_0^{1/2}} \right)^{3/5} Q^{3/5}$$

$$A = \alpha Q^\beta \quad \alpha = \left(\frac{n P^{2/3}}{S_0^{1/2}} \right)^{3/5} \quad \beta = 0.6$$
- Differentiating with respect to 't', we have
$$\frac{\partial A}{\partial t} = \alpha \beta Q^{\beta-1} \left(\frac{\partial Q}{\partial t} \right)$$
- Putting this in continuity equation,
$$\frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta-1} \left(\frac{\partial Q}{\partial t} \right) = q \quad [6] \quad \text{Thus, it becomes } f(Q)$$

Handwritten notes:
 $V = \frac{1}{n} R^{2/3} S_0^{1/2}$
 $Q = AV = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$
 $A = \frac{1}{n} \frac{S_0^{1/2}}{P^{2/3}} A^{5/3}$

So, if you only consider the friction term, then it is a kinematic wave model. If we consider both friction and pressure terms, then we say that it's a diffusive wave model. And if we consider all three terms, that is acceleration as well as friction and pressure terms, then we say fully hydrodynamic models. And you will find different hydraulic models having different components or various types of hydraulic models. Some are kinematic wave, some others are diffusive wave, and some are fully hydrodynamic models. So, depending upon what component of the momentum equation has been added, based on that, this categorization is done, and you will come across in literature these terms. So, that's the classification or clarification on what these terms mean, basically. Now, coming to the wave motion, if we consider the kinematic wave, then it governs flow when inertial and pressure forces are not important. So, obviously, we know that we only consider the friction term in this case. That means, the gravity and frictional forces are balanced. That means, the flow could be steady and non-uniform flow, the uniform flow, that is the basic when if you have read fluid mechanics and hydraulics, and if you remember the basics, then obviously, that is how we define steady and uniform flow, that is, gravitational and frictional forces balance each other.

Kinematic Wave Celerity

$$\frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta-1} \left(\frac{\partial Q}{\partial t} \right) = q \quad [6]$$

- Kinematic Waves result from changes in Q
- An increment in Q can be written as
$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt \quad [7]$$
- Dividing by dx and rearranging
$$\frac{\partial Q}{\partial x} + \frac{dt}{dx} \frac{\partial Q}{\partial t} = \frac{dQ}{dx} \quad [8]$$
- If $\frac{dQ}{dx} = q$, then equations (6) and (8) are identical, and
$$\frac{dx}{dt} = \frac{1}{\alpha \beta Q^{\beta-1}} \quad [9]$$

And kinetic waves are often used for short-duration flood events where the time scale of the event is relatively small compared to the time it takes for the flow to accelerate or decelerate significantly. That means it is applicable or this assumption is that the time scale is so short that

flow cannot accelerate or decelerate, and that means we neglect the acceleration. And that is why we assume that flow does not accelerate appreciably. That means the velocity of flow is within limits, and as a result, the Froude number is less than or equal to 1. And you remember Reynolds number; I am sure that you have read, and you remember that the Froude number is a ratio of inertial forces to gravitational forces given by this relationship v equals to $\sqrt{g y}$. So, F equals to v by $\sqrt{g y}$; this is the equation actually, not this one, F equals to v by $\sqrt{g y}$. So, basically, that is based on which the Froude number is calculated. So, if the Froude number is less than 1, then we can call it a kinematic wave. On the other hand, if you go to dynamic waves, then it governs flow when inertial and pressure forces are important, that is in the momentum movement of large flood wave. So, if there is a large flood wave and inertial and pressure force terms are getting importance, then it is a dynamic wave, and dynamic wave modelling is often necessary for scenarios with longer flow durations where the time scale for flow acceleration and deceleration becomes more significant.

Kinematic Wave Celerity

- Similarly, differentiating $A = \alpha Q^\beta$ and rearranging

$$\frac{dQ}{dA} = \frac{1}{\alpha \beta Q^{\beta-1}} \quad (10)$$
- Comparing (9) and (10), we get

$$\frac{dx}{dt} = \frac{dQ}{dA} \quad (11)$$
- Or,

$$c_k = \frac{dx}{dt} = \frac{dQ}{dA} \quad (12)$$



Where c_k is the kinematic wave celerity

So, just a reverse of the kinematic wave, that means, that the time scale is so significant that the flow can appreciate, accelerate appreciably, and it channels with steep slopes or complex geometry where flow acceleration and deceleration are significant. Dynamic wave models provided more accurate represents the flow dynamics compared to kinematic wave. So, these are the conditions that if the channel's bed slope is steep or if the geometry is complex, then obviously, because of that, the flow velocities could be very high, and as a result, the flow can accelerate or decelerate significantly in a short period of time even, and that means the Froude number, the velocity of flow will be high, that means Froude number will be high, and that is why the dynamic wave is more I mean they represent such a flow condition better compared to kinematic waves. If you look at visually if you want to look at visually, as you can see here, the flow is moving, and it is just a projected view, that means there is an observer who is looking at a flow over a particular reach. So, upstream and downstream both if he is looking at, then obviously, if it is a kinematic wave, then if you look at the observer, this is what he will see that for to this observer kinematic wave will appear as uniform unsteady flow, that means, with time there will be change, but otherwise, there is no change, and water surfaces and bed are parallel to each other and to the energy grade line. So, channel bed water surface and energy grade line all will be parallel so that means, it will look horizontal to him or her, and on the other hand, if it is a dynamic wave, then it will appear as a gradually varied unsteady flow, that means, streamlines in one water surface profiles are not parallel.

Kinematic Wave Celerity

- This implies that an observer moving at a velocity c_k with the flow would see
 - ✓ A constant discharge, if $q = 0$
 - ✓ Flow rate increasing at a rate of $\frac{dQ}{dx} = q$
- The kinematic wave celerity can also be expressed in terms of the depth y as

$$c_k = \frac{dQ}{dA} = \frac{1}{B} \frac{dQ}{dy} \quad \text{where, } dA = B dy \quad (13)$$
- Both Kinematic and Dynamic waves are present in natural flood
 - ✓ In many cases channel slope dominates in the Momentum Equation, therefore, most of the flood waves move as a kinematic wave
 - ✓ However, there is no single, universal criterion for deciding when a flood wave can be approximated as a kinematic wave






So, as you can see here, at this section the depths are more. Where as it moves, this is not channel parallel to channel bed, there is a dip in this case. So, that is the major difference, that if it is an observer looking at a stream at two sections, then if it is a kinematic wave, then it will be uniform unsteady flow, or more simply, the water surface bed and water surface and the energy grade line, they will all be parallel, which will not be the case if it is a dynamic wave. That is the major difference. And this has been taken from the US Army Corps of Engineers HEC hydraulic engineering center. Now, if we talk about kinematic wave celerity, then obviously, a wave is a variation in flow, a change of flow rate or water surface elevation, that means, that is a wave, because if there is wave motion, then of course, the flow rate will change, or water surface elevation will change, and wave celerity is the velocity with which the variation or wave travels along the channel. So, of course, we have up till now, we have been talking about velocity.

Dynamic Wave Celerity

- Dynamic wave celerity is a term used in hydraulic engineering to describe the speed at which a disturbance, such as a change in water surface elevation or flow rate, travels through a river channel
- The dynamic wave celerity depends on the hydraulic properties of the channel and is influenced by factors such as channel slope, channel geometry, and flow resistance (Manning's roughness coefficient)
- In channels with gradually varying flow conditions, the dynamic wave celerity is influenced by the local slope and can vary along the channel
- For a rectangular Channel,

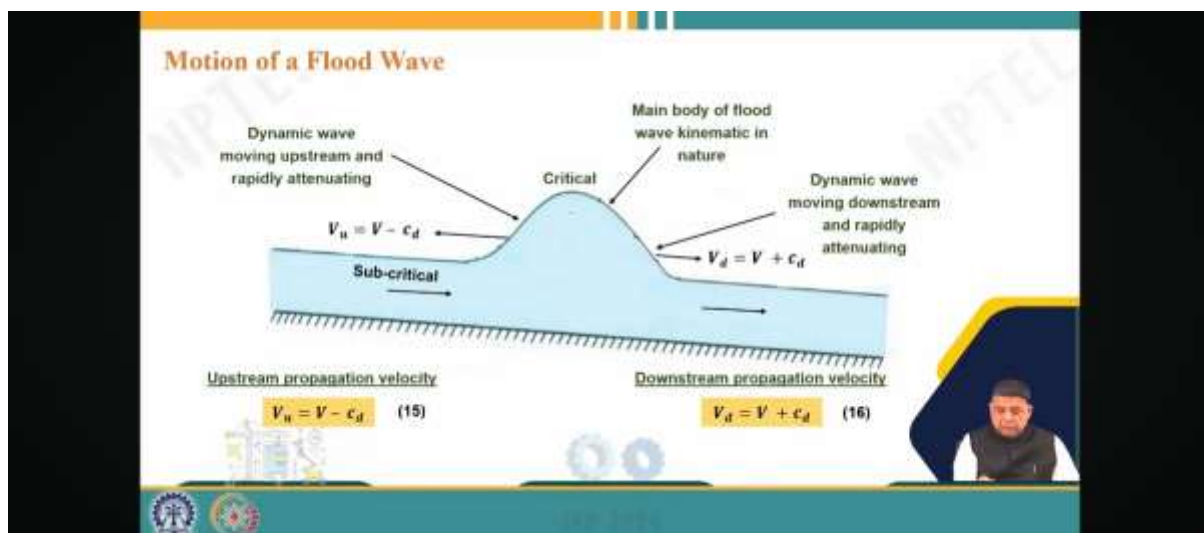
$$c_d = \sqrt{g y} \quad (14)$$

So, when velocity of flow is considered, then we are talking about the velocity of particles, but if you talk about the movement of the wave itself, then the velocity of that particular wave is referred to as wave celerity, rather than velocity. So, that is the difference between velocity and celerity. And celerity is a core variable in a variety of hydrologic models and flow routing algorithms that simulate changes in river discharge. So, in routing models, basically, we talk about celerity, rather than velocity. And the celerity basically ranges between 0.25 and 10

meters per second at reach to basin scale. So, this is typically the velocity or celerity in the value 0.25 to 10 meters per second. And if we talk about kinematic wave model, we have already seen the kinetic model, there will be continuity equation as usual, but the momentum equation only the friction term will be considered, and that means, $G H_0 - F = 0$, just now we discussed, which reduces to $H_0 = S F$, or the bed slope is same as the friction slope. And in the KW model (or the kinematic wave model), the momentum equation is replaced by the uniform flow equation.

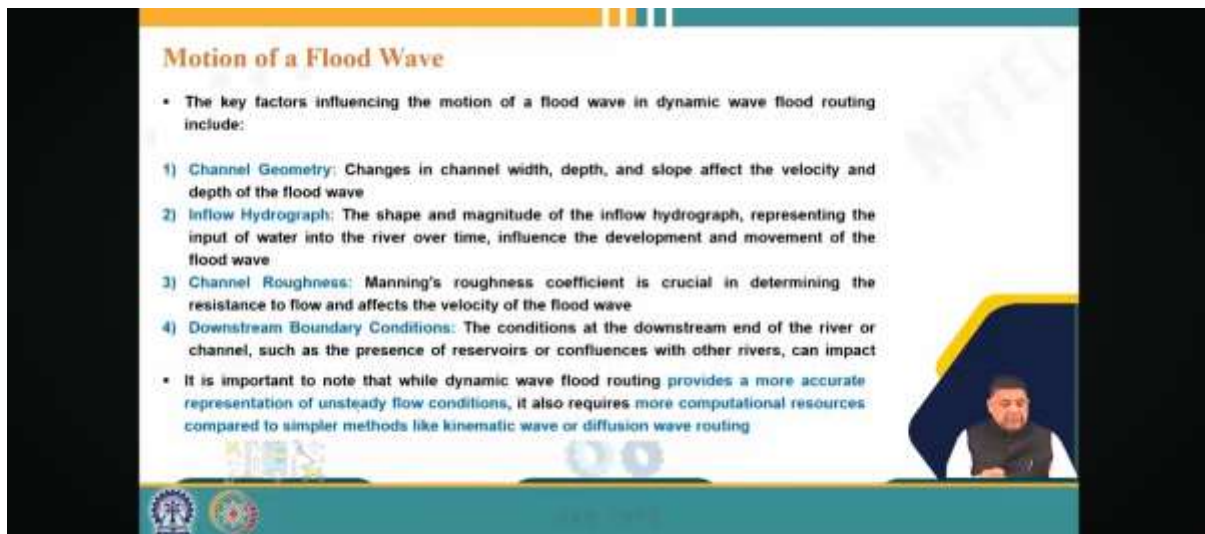
So, $S_0 = F/S$ so, we can replace this momentum equation by any uniform flow equation and say for example, we can use the Manning's equation in place of this equation. Now, if you talk about the Manning's equation with $S_0 = F/S$ and $R = A/P$ that is how we define. So, we know that ah Manning's equation is $V = \frac{1.49}{n} R^{2/3} S_0^{1/2}$ that is what we know or so, we are calling it as $V = C R^{2/3} S_0^{1/2}$ here. So, basically $1/n$ remains as it is ah R we are writing A/P so that means, it becomes $A^{2/3} / P^{2/3} S_0^{1/2}$ remains $S_0^{1/2}$. So, we term of Q if you write it will be $Q = AV$ so that means, we can write ah it is $1/n$ and A already exists here.



So, there will be basically multiplication of V here. So, basically ah Q equals to A times V so that means, you see it is $1/n$. So, it is $1/n A^{5/3} S_0^{1/2}$ so, we can write $1/n S_0^{1/2} P^{2/3} S_0^{1/2}$. And A basically $1 + 2/3 R^{1/n} S_0^{1/2} P^{2/3} 2/3 A$ to the power $5/3$. So, that is why we are writing here Q equals to $1/n S_0^{1/2} A^{5/3}$.

And then we can also write the same equation like A equals to in terms of Q if we write A equals to $n^{3/5} P^{2/5} Q^{3/5} S_0^{-1/5}$ that means, this portion divided by this this portion we can write. So, this just gets ah inverted so $n^{3/5} P^{2/5} S_0^{-1/5}$ and Q power of Q becomes ah the $3/5 A^{5/3} S_0^{1/2}$. So, it is $3/5$ and here also power of $3/5$ comes. So, we can write this is A equals to αQ^β where α is this and $P^{2/5} S_0^{-1/5}$ to the power $2/5$ to the power 0.6 and β is 0.6 . So, αQ^β we can write from Manning's equation ah a form of ah kinematic wave celerity we can express. And differentiating with respect to t we get $\frac{\partial A}{\partial t}$ is if we write this $\alpha \beta Q^{\beta-1} \frac{\partial Q}{\partial t}$ and putting this in the continuity equation the continuity equation, we know that $\frac{dQ}{dx} + \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t}$ is Q is a lateral flow. So, that means, in the continuity equation this entire thing we are introducing in place of $\frac{\partial A}{\partial t}$. And of course, here as you can see that this term was in

terms of Q this also is comes in terms of over the entire equation becomes a function of Q or discharge basically in this case. And if you continue that is this is a form of equation then ah kinetic wave changes



result due to change in Q.

So, obviously, if discharge changes, everything changes and increment and Q can be written as $\frac{dQ}{dx} = \frac{dQ}{dt} \frac{dt}{dx}$ that means, $\frac{dQ}{dx}$ terms you are considering and if we divide that by $\frac{dQ}{dt}$ and rearrange then this is the form we get $\frac{dQ}{dx} \frac{dt}{dx}$ that we are dividing everything by $\frac{dQ}{dt}$. So, $\frac{dQ}{dx} \frac{dt}{dx}$ this becomes $\frac{dQ}{dt}$ is get cancelled then we have $\frac{dQ}{dx} \frac{dt}{dx}$ will be here we are multiplying with $\frac{dQ}{dt}$ in numerator and denominator both and we are just manipulating or rearranging and this part remains $\frac{dQ}{dx}$. So, here if $\frac{dQ}{dx}$ is called Q then this equation 6 and 8 are identical that is this equation this equation radical and we get $\frac{dQ}{dx}$ by $\frac{dQ}{dt}$ from here we get $\frac{dQ}{dx} \frac{dt}{dx}$ that means, here $\frac{dQ}{dx} \frac{dt}{dx}$ is 1 by this this is here. So, this is $\frac{dQ}{dx} \frac{dt}{dx}$ is this or $\frac{dQ}{dx} \frac{dt}{dx}$ is 1 by $\alpha \beta Q \beta - 1$.

So, this is what it is coming. So, if you compare equation 6 and 8 you will get $\frac{dQ}{dx} \frac{dt}{dx}$ is 1 by $\alpha \beta Q \beta - 1$ that is equation number 9. And similarly, differentiating $\alpha \beta Q \beta - 1$ and rearranging we get $\frac{dQ}{dx} \frac{dt}{dx} = \frac{dQ}{dA} \frac{dA}{dx}$ and that is nothing, but kinematic wave celerity that is $C_k = \frac{dQ}{dA}$. So, in terms of space change with time or discharge with area it can be expressed this is the kinematic wave celerity. This implies that in an observer moving at velocity C_k with the flow would see a constant discharge if Q is 0 that means, he will not find any change in discharge because $C_k Q$ is nothing, but change in flow and flow rate will increase $\frac{dQ}{dx}$ if Q if there is a lateral flow in flow coming. So, kinematic wave celerity can also be expressed in terms of depth.

Wave Celerity


Example 1

- A rectangular channel is 45 m wide and has a Manning roughness coefficient of 0.025. Calculate the bed slope considering flow velocity of 3 m/s, kinematic wave celerity c_k of 5 m/s and the flow rate of 150 cumec.

Solution:

- From Manning's Eq. $Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$
- Assuming $R = y$ and $A = (By)$ (for wide channel)
- Hence, $Q = \frac{1}{n} (By)y^{2/3} S_0^{1/2}$
- Kinematic Wave Celerity, $c_k = \frac{1}{B} \frac{dQ}{dy}$

$$c_k = \frac{1}{B} \frac{d}{dy} \left(\frac{S_0^{1/2} B}{n} y^{5/3} \right)$$

$$c_k = \left(\frac{S_0^{1/2}}{n} \right) \left(\frac{5}{3} \right) y^{2/3}$$


So, obviously, same equation area we know area is B times y or d A is B times d y. So, obviously, we can also express in terms of depth that is equation number 3. So, both kinematic and dynamic waves are present in natural flood in many cases channel slope dominates the momentum equation therefore, most of the flood waves move it is kinematic wave and however, there is no single universal criteria for deciding when a flood wave can be approximated is a kinematic wave. So, obviously, it depends on how you look at and what assumptions you make for deciding or whether you are going to take into dynamic or kinematic form of the modelling. So, that all depends, but this is how kinematic wave celerity can be defined.

Now, if we look at the dynamic wave celerity, then dynamic wave celerity is the term used in hydraulic engineering to describe the speed at which a disturbance such as a change in water surface elevation or flow rate travels through a river channel. So, basically, it is a movement of the dynamic wave where of course, you know that acceleration will be more so that means, water surface elevation or flow rate will be changing rapidly. The dynamic wave celerity depends on the hydraulic properties of the channel and is influenced by factors such as channel slope, channel geometry, and of course, flow resistance, same and x of next coefficient. In channels with gradually varying flow conditions, the dynamic wave celerity is influenced by the local slope and can vary along the channel and for a rectangular channel like we have we have derived for kinematic wave celerity, we can also derive the equation for a rectangular channel, the dynamic wave celerity C D is given by root G y, and that is equation number 14. So, if you look at the motion of a flood wave then obviously, the flow here is subcritical, that means, velocity low, but depth is more, but obviously, because of the conditions if the flow becomes critical here that means, from subcritical to critical then obviously, later on it has to become supercritical.

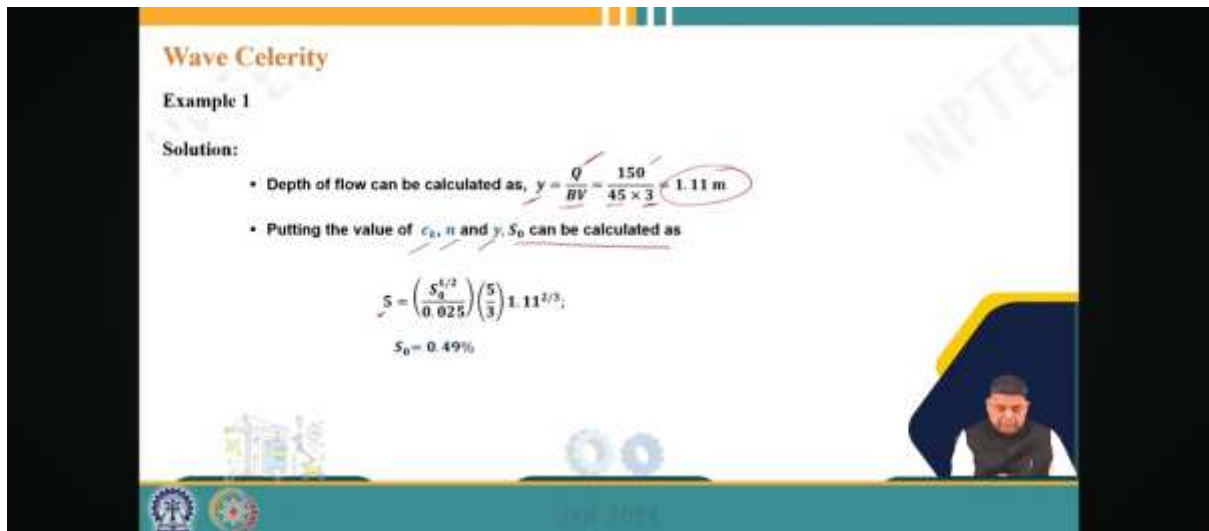
Wave Celerity

Example 1

Solution:

- Depth of flow can be calculated as, $y = \frac{Q}{BV} = \frac{150}{45 \times 3} = 1.11 \text{ m}$
- Putting the value of c_2 , π and y , S_0 can be calculated as

$$S = \left(\frac{S_0^{4/3}}{0.025} \right) \left(\frac{5}{3} \right) 1.11^{2/3};$$

$$S_0 = 0.49\%$$


So, down dynamic wave moving upstream and rapidly attenuating, and this is the main body of flood wave kinematic in nature, and this is the dynamic wave moving downstream and rapidly attenuating. So, it is moving upstream and moving downstream. So, upstream propagation basically that is in this case it is here in this direction if you consider the velocity upstream propagation velocity is v minus C . D already, we know is the kinematic wave celerity and in the downstream propagation velocity that is on downstream section is v plus C . So, it is basically C D will act in negative negatively in upstream wave and positively in downstream wave at the move wave move. So, this how it is upstream and downstream propagation velocity can be calculated.

Now, if we look at the dynamic wave celerity, then dynamic wave celerity is the term used in hydraulic engineering to describe the speed at which a disturbance such as a change in water surface elevation or flow rate travels through a river channel. So, basically, it is a movement of the dynamic wave where, of course, you know that acceleration will be more so that means, water surface elevation or flow rate will be changing rapidly. The dynamic wave celerity depends on the hydraulic properties of the channel and is influenced by factors such as channel slope, channel geometry, and of course, flow resistance, and the coefficient of next. In channels with gradually varying flow conditions, the dynamic wave celerity is influenced by the local slope and can vary along the channel, and for a rectangular channel like we have, we have derived for kinematic wave celerity, we can also derive the equation for a rectangular channel, the dynamic wave celerity C D is given by root G y , and that is equation number 14. So, if you look at the motion of a flood wave, then obviously, as you can see here, the flow here is subcritical, that means, velocity low, but depth is more, but obviously, because of the conditions if the flow becomes critical here, that means, from subcritical to critical, then obviously, later on, it has to become supercritical.


Wave Celerity

Example 2

- A rectangular channel is 60 m wide, has bed slope 1 per cent and Manning roughness 0.035. Calculate the water velocity V , the kinematic and dynamic wave celerity c_k and c_d , and the velocity of propagation of the dynamic waves $V \pm c_d$ at a point in the channel where the flow rate is 140 cumec.

Solution:

- From Manning's Eq. $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$
- Assuming $R = y$ and $A = (By)$ (for wide channel)
- Hence, $Q = \frac{1}{n} (By)^{2/3} S_0^{1/2}$
- With known values of Q , n , B and S_0 , $y = 0.88$ m
- Hence, the Wave Velocity,

$$V = \frac{Q}{By} = 2.65 \text{ m/s}$$


So, down dynamic wave moving upstream and rapidly attenuating, and this is the main body of the flood wave kinematic in nature, and this is the dynamic wave moving downstream and rapidly attenuating. So, it is moving upstream and moving downstream. So, upstream propagation basically, that is in this case it is here in this direction, if you consider the velocity upstream propagation velocity is v minus CD , CD already we know is the kinematic dynamic wave celerity, and in the downstream propagation velocity that is on downstream section is v plus CD . So, it is basically CD will act in negatively in upstream wave and positively in downstream wave as the wave moves. So, this is how upstream and downstream propagation velocity can be calculated.

Now, coming to the key factors influencing the motion of flood wave in a dynamic wave flood routing, we already discussed that it is channel geometry, inflow hydrograph, channel roughness, and downstream boundary condition, that is, the condition at downstream end of the river or channels such as presence of reservoir or confluence with other rivers, and that can impact the flow. And it is important to note that while dynamic wave flood routing provides a more accurate representation of unsteady flow condition, it also requires more computational resources compared to simpler methods like kinematic wave or diffusive wave model. So, depending upon again the computational effort or the accuracy you require, then that also governs whether you are going to consider the dynamic wave or the kinematic wave or diffusive wave. So, that depends on how different kinds of models are developed based on different assumptions. Now, we take an example on wave celerity, say example 1.

Wave Celerity

Solution:

- Kinematic Wave Celerity is $c_k = \frac{1}{B} \frac{dQ}{dy}$

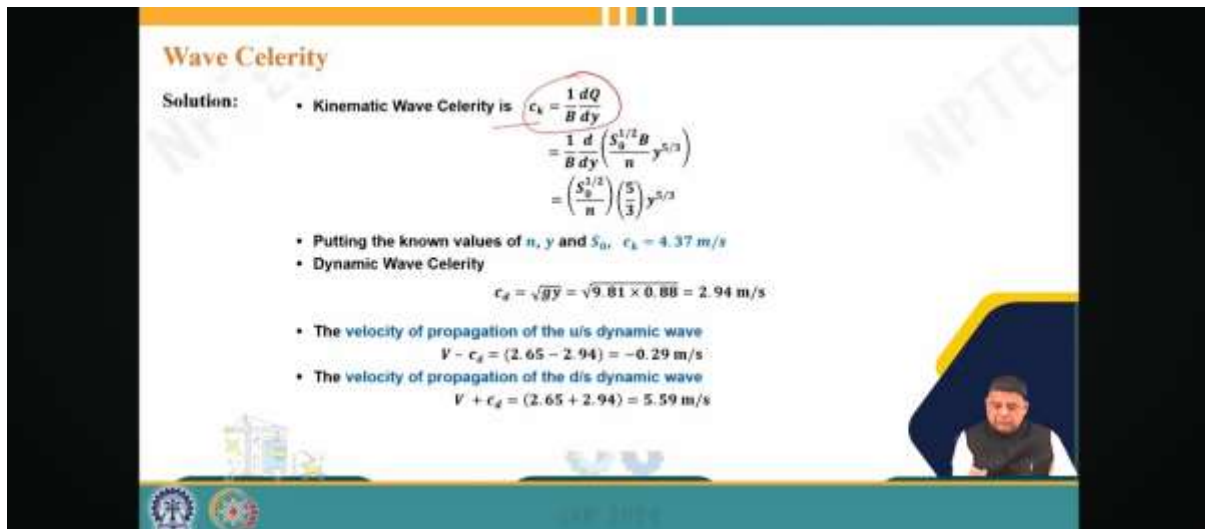
$$= \frac{1}{B} \frac{d}{dy} \left(\frac{S_0^{1/2} B}{n} y^{5/3} \right)$$

$$= \left(\frac{S_0^{1/2}}{n} \right) \left(\frac{5}{3} \right) y^{2/3}$$
- Putting the known values of n , y and S_0 , $c_k = 4.37 \text{ m/s}$
- Dynamic Wave Celerity

$$c_d = \sqrt{gy} = \sqrt{9.81 \times 0.88} = 2.94 \text{ m/s}$$
- The velocity of propagation of the *u/s* dynamic wave

$$V - c_d = (2.65 - 2.94) = -0.29 \text{ m/s}$$
- The velocity of propagation of the *d/s* dynamic wave

$$V + c_d = (2.65 + 2.94) = 5.59 \text{ m/s}$$



So, a rectangular channel is 45 meters wide and has a Manning roughness coefficient of 0.025. Calculate the bed slope considering a flow velocity of 3 meters per second, kinematic wave celerity c_k of 5 meters per second, and flow rate of 150 cumec. So, from Manning's equation, we already know this is the form of Manning's equation, and if it is a wide channel then typically in hydraulics, we assume r equals to y or n equals to b , that is, the hydraulic radius assumed equal to the bed width. And thus, by putting these assumed values the equation reduces to this form and we know that kinematic wave celerity is c_k equals to $\frac{1}{B} \frac{dQ}{dy}$ and so from here we can get this form.

So, it is $\frac{1}{B} \frac{d}{dy} \left(\frac{S_0^{1/2} B}{n} y^{5/3} \right)$. So, from here S_0 is already taken here $y^{5/3}$. So, $S_0^{1/2}$ is a constant and B gets cancelled here. So, it is $\frac{5}{3} \left(\frac{S_0^{1/2}}{n} \right) y^{2/3}$.

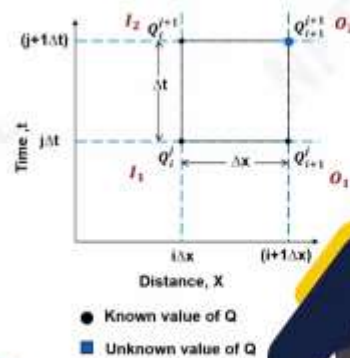
So, this is how c_k reduced to this form. Now, depth of flow can be calculated because we know Q , we know B and velocity. So, we have 150 (given), channel width (given), and velocity (given). So, we get 1.11 meters as the depth of flow and putting c_k , n , and y we can calculate S_0 , that is, S_0 equals to $\frac{3}{5} \left(\frac{c_k n}{S_0^{1/2}} \right)^2 y^{-2/3}$. So, S_0 is the unknown value, n is known as 0.025 and this $\frac{3}{5}$ is a constant anyway, and y to the power $\frac{2}{3}$.

So, $y^{2/3}$ and or putting all the values and making calculations we get S_0 is 0.49 percent. So, the channel bed slope is 0.49 percent, that is what we get in this equation from this problem. Then another example if we take a rectangular channel is 60 meters wide, has a bed slope 1 percent, and Manning's coefficient 0.35 Calculate the water velocity v that kinematic and dynamic waves it is c can see d and the velocity of propagation of the dynamic waves b plus minus c d at a point of the channel where flow rate is 140 cumec. So, Manning's equation we already know, assume the same assumptions we will make that r equal to y for a wide channel and then thus Manning's equation becomes this and with the known value of Q and v and S_0 we can calculate y which comes out to be 0.88 meters and wave celerity if you consider wave velocity if you consider v is equal to Q by $b y$ and we know Q we know b we know y .

Muskingum-Cunge Routing Method

- It is proposed by Cunge (1969) based on Muskingum method.
- The Muskingum-Cunge method incorporates the diffusion wave approximation, making it more suitable for simulating the routing of flood waves through river channels.
- This method is commonly used in hydrological and hydraulic modeling for river flow forecasting, floodplain management, and engineering studies.
- Referring to the time-space computational grid, the Muskingum routing equation

$$O_2 = C_2 I_2 + C_1 I_1 + C_3 O_1 \quad (17)$$



So, velocity comes out to be 2.65 meters per second. So, kinematic wave celerity already we calculated in the previous equation that is $1 \text{ by } b \text{ d } q \text{ by } d \text{ y}$. So, that means, the same thing we differentiate ah this with respect to y and this is the form we get finally. Now, we know n we know y we know S_0 . So, from here the c_k or kinematic wave celerity comes out to be 4.3 meters per second. Dynamic wave celerity we already know is equal to $\sqrt{g y}$. So, \sqrt{g} we are containing x direction due to gravity 9.81 y value we will have already calculated 0.88. So, c_d that is dynamic wave celerity comes out to be 2.94 meters per second and velocity of propagation of the upstream down wave we had known we consider minus c_d and in velocity of propagation of downstream wave we consider plus c_d . So, $v \text{ minus } c_d \text{ v plus } c_d$ we know velocity value and we also know c_d . So, this comes out to be minus 0.29 meters per second where downstream wave it is 5.59 meters per second. So, obviously, here there will be deceleration where there will be acceleration ah because of the higher velocity in the dynamic wave. Now, we come to Muskingum-Kunge routing method, it was proposed by Kunge based on Muskingum method. So, it incorporates the diffusion wave approximation making it more suitable for simulating the routing of flood waves through river channels and this method is commonly used in hydrologic and hydraulic modelling for river flow forecasting front plane, that means, in channel routing basically and we already know that this is the equation ah this is the Muskingum routing equation. So, basically because say dynamic routing.

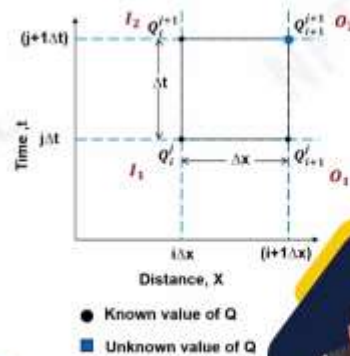
So, basically, we consider the space and time horizons. So, obviously, we consider the upstream reach, downstream reach, and initial time in the and the t_1 and t_2 , that means, beginning. So, $J \Delta t \ J + 1 \ \Delta t$ here we are saying $I \ \Delta x \ I + 1 \ \Delta x$ we are calling it. We know that inflow hydrographs are known to 1, that means, I_1 and I_2 are known to us. So, Q_{ij+1} is known to us. We also know that downstream section O_1 , the first outflow value is known to us, that means, Q_{ji+1} is known to us. So, only unknown is O_2 or $Q_{i+1, j+1}$, that means, at this particular point that is what is known.

Muskingum-Cunge Routing Method

- Equation (17) can be written for the discharge at $x = (i+1)\Delta x$ and $t = (j+1)\Delta t$ as

$$Q_{i+1}^{j+1} = C_0 I_2^{j+1} + C_1 Q_i^j + C_2 Q_{i+1}^j \quad (18)$$

- With definition of C_0 , C_1 and C_2 same as earlier with K , x and Δt as constant



So, these are known value of Q already we have shown here and this is the unknown value of Q . Now, we can also write in terms of these grid values. So, O_2 that is $O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$. So, we all already know C_2 , C_1 , we already know the equation and with definition of C_0 , C_1 and C_2 remaining the same and $k n x \Delta t$ remaining the constant.

So, if we know basically so, we can carry out this routing. So, also Kunge showed that k and Δt are taken as constant equation 18 is approximate solution of kinematic wave equation. Kunge further demonstrated that equation 8 can be considered as appropriate solution for the modified diffusive wave also. If k is $\Delta x C_k$ or C_k we already know at dQ/dx or we can consider x is $1/2 \left(1 - \frac{Q}{B C_k S_0 \Delta x}\right)$ so that means, 19 and 20. So, Muskingum-Kunge routing is carried out by Solm equation 18 that is the previous equation and the coefficient in 18 are computed using 19 and 20 that is k and x are computed from here with equation for C_0 , C_1 and C_2 for each time and space point of computation since, k and x both change with respect to time and space. Thus, using the appropriate conditions, the Muskingum-Kunge routing method can be employed to solve kinematic wave or diffusive equations.

Muskingum-Cunge Routing Method

- Cunge showed that when K and Δt are taken as constant, Eq. (18) is an approximate solution of the kinematic wave equation
- Cunge further demonstrated that Eq. (18) can be considered an approximate solution of a modified diffusion equation, if

$$K = \frac{\Delta x}{c_k} = \frac{\Delta x}{dQ/dA} \quad (19)$$

$$x = \frac{1}{2} \left(1 - \frac{Q}{B C_k S_0 \Delta x}\right) \quad (20)$$

- Muskingum-Cunge routing is carried out by solving equation (18)
 - The coefficients in (18) are computed using (19) and (20) along with equations for C_0 , C_1 and C_2 for each time and space point of computation, since K and x both change with respect to time and space
- Thus, using the appropriate conditions, the Muskingum-Cunge routing method can be employed to solve kinematic wave or diffusion wave equations

So, can Muskingum-Kunge routing method by changing the value of k and x we can also go for diffusive wave equations. So, with this we come to end of this lecture where we have talked about the hydraulic channel routing, we saw wave motion the kinematic and dynamic waves

and celerity how to derive at least the kinematic wave celerity and we also showed that Runge–Kutta routing method which is basically hydraulic routing method how that is related to hydrologic channel routing or Muskingum equation basically. So, with this please give your feedback and also raise your questions or doubts we will be happy to answer on the forum. Thank you very much.