

Course Name: Watershed Hydrology

Professor Name: Prof. Rajendra Singh

Department Name: Agricultural and Food Engineering

Institute Name: Indian Institute of Technology Kharagpur

Week: 10

Lecture 50: Numerical on Flood Routing

**SWAYAM NPTEL COURSE ON
WATERSHED HYDROLOGY**

By
Prof. Rajendra Singh
Department of Agricultural and Food Engineering
Indian Institute of Technology Kharagpur

Module: 10
Lecture: 05 (Numerical on Flood Routing)

Hello friends, welcome back to this online certification course on water sheet hydrology. I am Rajendra Singh, a professor in the Department of Agricultural Food Engineering at the Indian Institute of Technology Kharagpur. We are in module 10, and this is the last lecture of the module where we will be taking up numerical on flood routing.

Content- Numerical on Flood Routing

- Numerical on Reservoir Routing
- Numerical on Modified Puls or Level Pool method
- Numerical on Muskingum Method
- Numerical on Wave Celerity

So, basically, in this lecture, we will be working on numerical on reservoir routing using the modifiable pulse or level pool method, numerical on the Muskingum method of channel routing, and numerical on wave celerity.

Reservoir Routing

Example 1

The relationship between outflow Q in $\text{m}^3 \text{s}^{-1}$ and storage S in m^3 for an emergency spillway in a reservoir is $Q = S/4000$. Inflow, outflow and storage are assumed to be zero at time $t = 0$. If the inflow rate is $300 \text{ m}^3 \text{ s}^{-1}$ at the end of $t = 3$ hours, the outflow rate in $\text{m}^3 \text{ s}^{-1}$ is

- (A) 152.84
- (B) 164.84
- (C) 172.34
- (D) 184.84

GATE 2012

Let us start with example number 1: The relationship between outflow Q in cubic meters per second and storage S in cubic meters for an emergency spillway in a reservoir is given by the equation Q equals S divided by 4000. Inflow, outflow, and storage are assumed to be 0 at time t equals to 0. If the inflow rate is 300 cubic meters per second at the end of t equals to 3 hours, the outflow rate in cubic meters per second is as follows. We have a multiple-choice question with 4 choices, taken from the GATE 2012 examination.

Reservoir Routing

Solution

We know,

$$\frac{I_1 + I_2}{2} \Delta t - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1$$

Given:

$$I_1 = Q_1 = S_1 = 0, Q_2 = S_2 / 4000 \text{ m}^3 \text{ s}^{-1}, I_2 = 300 \text{ m}^3 \text{ s}^{-1}, \Delta t = 3 \text{ hour} = 10800 \text{ s}$$

Putting all given values in the above equation we get,

$$S_2 = 689361.70 \text{ m}^3$$

$$\text{Therefore, } Q = S/4000 = 172.34 \text{ m}^3 \text{ s}^{-1}$$

Option C is the correct answer



Coming to the solution, we know that for reservoir routing, the basic mass balance equation is: Inflow minus outflow equals the change in storage. Taking a delta time t and the initial and ending storage at the beginning and end of this delta t as subscripts 1 and 2, we can write the equation like this. Here, in this problem, we are given I_1 equals Q_1 equals S_1 equals 0, which means the inflow, outflow, and storage at the beginning are 0. Where Q_2 is given, S_2 , I mean, the relationship S_2 by 4000 cubic meters per second. I_2 is given as 300 cubic meters per second, and delta t is 3 hours or 10800 seconds because we have everything in cubic meters per second. So, we have to convert this into seconds. Now, putting all values in the above equation. So, we know I_1 , I_2 , we know Q_1 , we know Q_2 in terms of S , and S_1 is 0. So, putting all the values, we can find out S_2 , which comes out to be 6,89,361.7 cubic meters. We also know that Q equals S divided by 4000, which is the given relationship. So, Q comes out to be 172.34 cubic meters per second, which is one of the options, here option C, and that is why option C is the correct answer to this particular question.

Reservoir Routing

Example 2

The storage (S) and outflow (Q) of an emergency spillway are related as $S = 8000Q$, where S is in m^3 and Q is in $\text{m}^3 \text{s}^{-1}$. The inflow, outflow and storage at the beginning are assumed to be zero. The outflow rate from the reservoir at the end of 1 hour when the inflow is $400 \text{ m}^3 \text{ s}^{-1}$ will be

- (A) $16.13 \text{ m}^3 \text{ s}^{-1}$
- (B) $73.47 \text{ m}^3 \text{ s}^{-1}$
- (C) $116.13 \text{ m}^3 \text{ s}^{-1}$
- (D) $124.14 \text{ m}^3 \text{ s}^{-1}$

GATE 2005

Then we go to the next question, which has been taken from GATE 2005. It is again a multiple-choice question. It's very similar to question number 1, where the storage S and outflow Q of an emergency spillway are related as S equals $8000 Q$, where S is in cubic meters and Q is in cubic meters per second. The inflow, outflow, and storage at the beginning are assumed to be 0. The outflow rate from the reservoir at the end of 1 hour when the inflow is 400 cubic meters per second will be, which means we have to find out the outflow rate at the end of 1 hour, that is the question.

Reservoir Routing

Solution

We know,

$$I - Q = \frac{dS}{dt}$$

Given:

$$S = 8000Q \text{ m}^3, I = 400 \text{ m}^3 \text{ s}^{-1}, t = 1 \text{ hour} = 3600 \text{ s}$$

$$\frac{dS}{dt} = \text{change in the storage in 1 h} = \frac{(8000Q - 0)}{3600} = \frac{20Q}{9}$$

Putting in the basic equation,

$$400 - Q = \frac{20}{9} Q \quad \text{OR} \quad 400 = \frac{29}{9} Q$$

$$Q = 124.14 \text{ m}^3 \text{ s}^{-1}$$

Option D is the correct answer



$$\frac{I_1 + I_2}{2} \Delta t - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1$$

So, obviously, again, the same thing, I minus O equals dS by dt , which is inflow minus outflow is equal to the change in storage, that is the mass balance equation on which reservoir routing, basically hydrologic reservoir routing, is built, which we discussed while discussing the theory. And here we have been given S is $8000 Q$, Q cubic meters per second, I is 400 cubic meters

per second, T is 1 hour, that is 3600 seconds, and dS by dt is the change in storage in 1 hour, which is 8000 Q minus 0, because in the beginning it is given as 0, divided by 3600, that is the relationship because we have to convert that. So, it comes out to be 20 Q by 9. So, obviously, now we know I, Q, and dS by dt .

So, putting all the values, we get 400 minus Q equals to 20 by 9 by Q of 400 equals to 29 by 9 by Q, or Q comes out to be 124.14 cubic meters per second, and this is one of the options, option D here. So, option D is the correct answer to this question. We can solve this variation, as I have tried to show. Otherwise, this question can also be solved by using the basic form of the equation, that is, writing in terms of inflow and outflow and ΔT , that is, using this form also we could have solved, because at the beginning we have been given I_1 , O_1 , and S_1 equal to 0. We have been given I_2 value and the S_2 relationship we know in terms of Q, and we have to find out ΔT , which is given as 1 hour, and we have to find O_2 value, which we can find out. So, using this form also, and otherwise, this direct form also, we can use. So, both forms are possible, it is possible to get the solution.

Level Pool or Modified Puls method

Example 3

A reservoir has the following elevation, discharge, and storage relationship:

Elevation, m	101.0	101.5	102.1	102.5	103.0	103.5	103.75	104.4
Storage (m ³)	45000	57000	69000	81000	93000	105000	117000	129000
Outflow (m ³ /s)	0	9	18	27	36	45	54	63

When the reservoir level was at 101.5 m, an inflow flood hydrograph, which can be approximated by a triangle as $I = 0$ at $t = 0$ h; $I = 24$ cumec at $t = 18$ h (peak flow) and $I = 0$ at $t = 36$ h (end of inflow) entered the reservoir: Route the flood and obtain the peak attenuation and time lag. Use the Level Pool method for flood routing.

Now, we move on to example number 3. The reservoir has the following elevation, and this problem is on the level pool or modified pulse method, and has the following elevation, discharge, and storage relationship. We have been given low elevation in meters, storage in cubic meters, and outflow in cubic meters per second. So, at elevation 101 meters, 204.4 meters, we have been given storage and corresponding outflow values at different elevations. When the reservoir level was at 101.5 meters, at this point, an inflow flood hydrograph, which can be approximated by a triangle, as I equals to 0 at T equals to 0, I equals to 24 hours, 24 Q mega T equals to 18 hours, that is the peak flow, and I equals to 0 at T equals to 36 hours, that is the end of inflow entering the reservoir. Route the flow and obtain the peak attenuation and time lag using the level pool method for flood routing, that is the problem. So, I think we already have discussed this method in great detail. We also took some problems here,

Level Pool or Modified Puls method

Solution

In Level Pool or modified Puls method, we need O vs S and O vs $S + O \times (\Delta t/2)$ curves.

Taking $\Delta t = 6h$ the values are tabulated below.

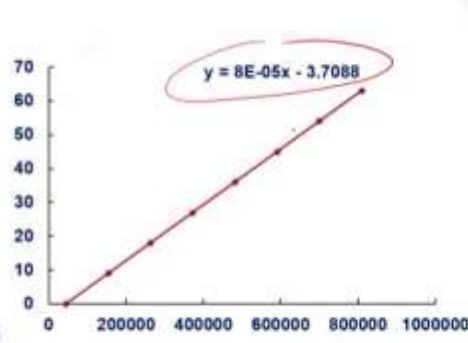
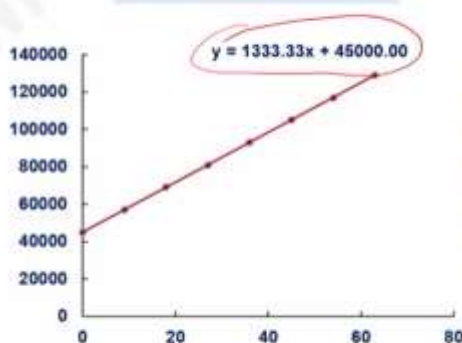
Elevation, m	Storage, S (m^3)	Outflow, O (m^3/s)	$S + O \times (\Delta t/2)$ (m^3)
101.0	45000	0	45000
101.5	57000	9	154200
102.1	69000	18	263400
102.5	81000	27	372600
103.0	93000	36	481800
103.5	105000	45	591000
103.75	117000	54	700200
104.4	129000	63	809400

And we need to know that in this method, we need to develop two curves, O versus S curve, and O versus S plus O delta T by 2 curves. So, these are two curves we need to develop, as O plus O versus S , and O versus S plus O delta T by 2. And of course, we have been given for different elevations, we have been given storage and outflow values, and delta T , because we are assuming delta T is 6 hours. So, that means, using this value, we have elevation, we have storage, we have outflow. So, of course, we can calculate S plus O delta T by 2 also. So, that is the value. So, we have all the values, requisite values; these are already given. This we have calculated in this particular column. So, now, we know O , S , and O over S plus O delta T by 2. So, we can develop the requisite curves using the data,

Level Pool or Modified Puls method

Solution

Using the data the required curves are developed



and so, this is the equation we have; this is the curve we have developed, and these are the equations we have developed. So, on the left side, we have the storage versus outflow curve,

and the relationship is in this form, where on the right side, we have outflow versus S plus O delta T by 2 curves, and this is the relationship. Both are fitting a straight-line relationship perfectly. So, I have used Excel and fitted a line; otherwise, one can draw the graph on the graph sheet and use the graph also. So, both the things are the same; probably they will give similar results, a little bit of difference maybe if you are reading the graph manually, but otherwise, more or less, the values will come out to be the same.

Level Pool Method or Modified Puls

Solution $\frac{(I_1 + I_2)}{2} \Delta t \rightarrow (S_1 - O_1 \frac{\Delta t}{2}) = (S_2 + O_2 \frac{\Delta t}{2})$

From S vs O curve get S and calculate; 1st value known


Sum of col 4 and 7 (basic equation)

$(6 \times 4 \times 3600) = 86400$ 1st value is 0

Time, h	I, cumec	I_bar, cumec	I_bar*del t	O, cumec	S	S - O*(del t/2)	S + O*(del t/2)
0	0	4	86400	0	45000	45000	131400
6	8	12	259200	6.80	54070.91	-19403.65	239796.35
12	16	20	432000	15.47	65633.16	-101495.86	330504.15
18	24	20	432000	22.73	75308.63	-170191.91	261808.09
24	16	12	259200	17.24	67981.07	-116166.07	141033.93
30	8	4	86400	7.57	55098.53	-26699.74	59700.26
36	0			0			

From O versus $(S + O \frac{\Delta t}{2})$ curve corresponding to the 1st value

Correct Table Module 12



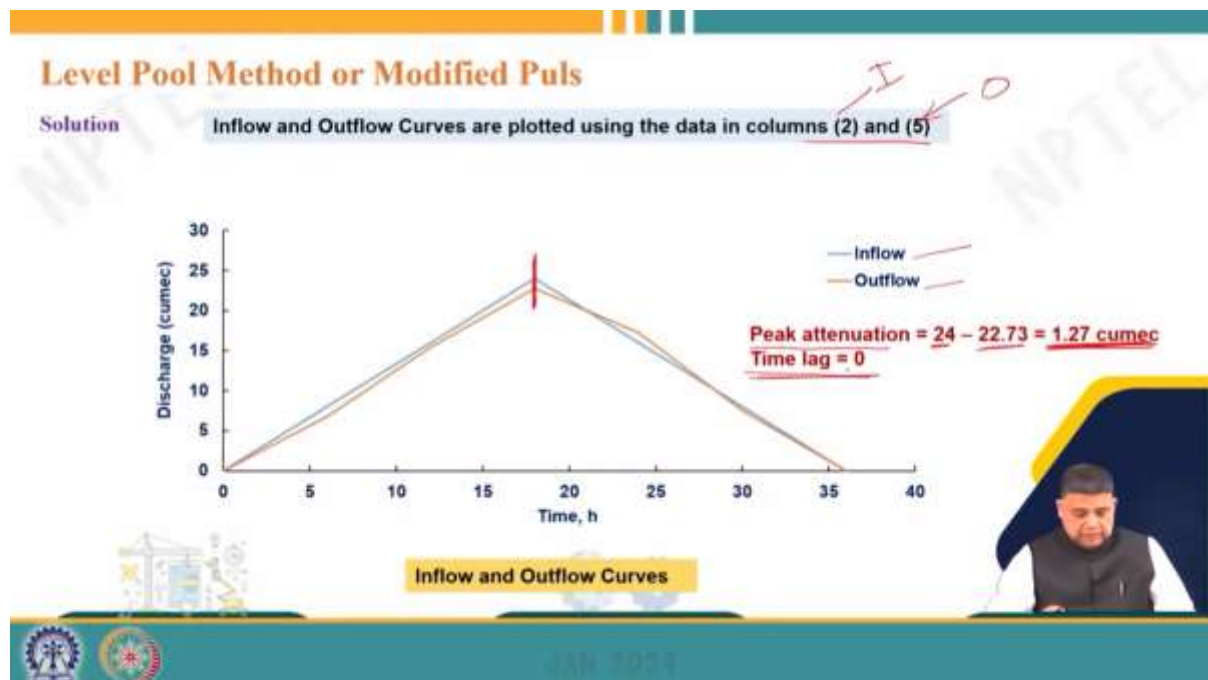
Now, we know that we have to do calculations in the tabular form. So, obviously, the first thing is that we have to have time at 6-hour intervals, that is column number 1, and then we have this inflow hydrograph; we have been given that inflow hydrograph is a triangle, 0 in the beginning, and peak at 18 hours and 36 at 0 hours.

So, these values are given to us. So, we can find out the I bar value. So, I bar value is 0 plus 8 by 2, 4; 8 plus 16, 24 by 2, 2 is 12; and then 16, 24, 40 by 2 is 20; then again, 20, 12, and 4. So, this is the I bar value, and then we need, because we have to use this form of equation where the leftmost term is I1 plus I2 by delta T. So, I bar we have already calculated; now we have to get I bar times delta T, and delta T we have taken as 6 hours already.

So, we have to, obviously, 6 hours into 3600 multiplied by 4, this value will give us 86400 is the first entry. The second entry, of course, we can go 12 into 6 into 3600; we will get this, and so on. So, I bar delta T we can calculate now. The first value of O is 0 because if you remember, this is the point where the flood was entering. So, the value is 9, and that is why the value is taken. It should be 9 here. Yeah, so this value should be 9; it should not be 0.

So, there is an error in this solution, but it does not matter if this value is 9. So, we have to make our calculations correct, and then we have to read this from S versus O curve, which will come out to be actually 45000 only because if it will come out to be 57000 here. So, this value needs to be corrected; it will be 57000 here, and that means this value will also get corrected.

So, now, the procedure is the same that we have to get this O value. I will put a corrected sheet, a corrected table, maybe in the last module, in module 12, when we discussed the miscellaneous topics and for this particular solution, but, the thing is that we have to read this O value from these tables which were provided to us, and then we have to get the value of S, S from O versus S curve, S value, and then we have to calculate S minus O delta T by 2 value. And once we have calculated, then ah then this is the sum of ah these two, that is the sum of column 4 and column 7 from the basic equation, that is the right-hand side. This is the right-hand side. So, this will give us this value, and then what we have to do is corresponding to this, we have to read the outflow from S over say S plus O delta T by 2 curve, and then this value will come, and then obviously, well, the procedure will follow the same path. So, this value onwards is correct, only the first value is not correct, but, ah, we will put a corrected solution for this problem anyway later, but the procedure remains the same. And once we have done the entire routing, then obviously, we have to plot inflow versus outflow curve, and so, column 2 versus column 5, we have to ah, ah plot, ah, against time.



So, column 2 and column 5, column 2 represents the inflow, this is I, and this is O. So, we have to represent plot this inflow and outflow hydrographs, and these are the two, not yet inflow hydrograph in blue colour and outflow hydrograph in ah, orange colour. And then, peak attenuation is the difference between inflow and outflow value. So, the peak inflow value is 24, peak outflow value is 22.73, and then ah, it comes out to be 1.27 cumec this value might change a little bit when we correct the solution, but the procedure remains the same, and of course, the time lag is 0 because we are getting the inflow and outflow peak of inflow and outflow at the same place, so the time lag is 0. So, this is the solution of this particular problem.

Muskingum Method

Example 4

The inflow hydrograph readings for a stream reach are given below for which the Muskingum coefficients of $K = 16$ h and $x = 0.2$ apply. Route the flood through the reach and determine the reduction in peak and the time of peak outflow. Outflow at the beginning of the flood may be taken as the same as inflow.

Time (h)	Inflow (cumec)
0	40
12	135
24	350
36	510
48	470
60	400
72	320
84	240
96	190

Time (h)	Inflow (cumec)
108	140
120	110
132	70
144	40



Then we take up example 4, and ah, this example 4 is the inflow hydrograph readings for a stream reach are given below for which the Muskingum coefficient equal to k equal to 16 and x equal to 0.2 apply. So, this is a problem on the Muskingum method route the flood through the reach and determine the reduction in peak and time of peak outflow, outflow at the beginning of the flood may be taken the same as inflow. So, this is the inflow hydrograph given to us from 0 to 144 hours, starting from an inflow of 40 to the end of inflow at 40. It is given that the first value of outflow is also the same as inflow.

Muskingum Method

Solution

Generalised Muskingum Routing equation is

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

Where,

$$C_0 = \frac{(-Kx + 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}, C_1 = \frac{(Kx + 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}, C_2 = \frac{(K - Kx - 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}$$

Substituting the values of $K = 16$ h, $x = 0.2$ and $\Delta t = 12$ h, we get

$$C_0 = 0.15, C_1 = 0.49, C_2 = 0.36$$

$$C_0 + C_1 + C_2 = 1 \text{ (Check)}$$



So, we know that the generalized Muskingum routing equation reads like this:

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

and the coefficients C_0 , C_1 , and C_2 can be found out if you know k and x , and of course, we have to assume a Δt value. So, we are taking k as 16, already given, 16 hours, x equal to 0.2, already given, and because our inflow hydrograph is given at the 12 hourly interval, so we are taking Δt equals to 12 hours. So, that we will not miss the peak value, that is one of the considerations we had. So, obviously, you can always take it as 1 by 3 or 1 by 2. So, 12 by 3, that is 4 hours, or 12 by 2, 6 hours also. So, that you will not miss. So, you can divide these 12 hours and take even smaller time such that the peak is in most mid, but just for simplicity, we are taking Δt equals to 12 hours here.


By putting k , x , and Δt values, we can calculate C_0 , C_1 , and C_2 , and the values we obtain are 0.15, 0.49, and 0.36, and the first requirement which we mentioned that we must ensure that the sum of these three coefficients C_0 , C_1 , and C_2 is equal to 1. So, obviously, by putting the values we get $C_0 + C_1 + C_2 = 1$. So, that is a check that our calculations are up to this point are correct.

Muskingum Method

Solution $C_0 = 0.15, C_1 = 0.49, C_2 = 0.36$

Time	Inflow	$C_0 I_2$	$C_1 I_1$	$C_2 O_1$	O
0	40				40
12	135	20.25	19.6	14.4	54.25
24	350	52.50	66.15	19.53	138.18
36	510	76.50	171.5	49.74	297.74
48	470	70.50	249.9	107.19	427.59
60	400	60.00	230.3	153.93	444.23
72	320	48.00	196	159.92	403.92
84	240	36.00	156.8	145.41	338.21
96	190	28.50	117.6	121.76	267.86
108	140	21.00	93.1	96.43	210.53
120	110	16.50	68.6	75.79	160.89
132	70	10.50	53.9	57.92	122.32
144	40	6.00	34.3	44.04	84.34

Reduction in peak = $510 - 444.23 = 65.77$ cumec
Time of peak outflow = 60 h



Then of course, we have to go and use the routing equation, and so, the routing equation has three components: $C_0 I_2$, $C_1 I_1$, and $C_2 O_1$, and of course, these are the three coefficients we have calculated. So, obviously, the I value are known to us, the first O value is known to us that is given in the problem already. So $C_0 I_2$, so if we consider this first-time interval 0 to 12 hours. So, I_2 is 135 here, and C_0 value is 0.15. So, obviously, $C_0 I_2$ will be 20.25. Then we have to calculate $C_1 I_1$, C_1 is 0.49, I_1 is 40 in the beginning, so that is the beginning, and so, it becomes 19.6. C_2 is 0.36, and O_1 is given as 40. So, obviously, this value comes out to be multiplied $C_2 O_1$, we get $C_2 O_1$, and then the equation sum of these three will give us O_2 value, the second at the end of the time, the value of outflow. So, this is O_2 , which will become O_1 for the next time interval, and of course, when we move to 12 to 24 hours, I_1 value 135 I_2 is 350, and O_1 is 52.5. So, obviously, this way we can calculate all the values, and then we will route the flow, and then we will get all the outflow values.

And finally, we can plot the inflow and outflow or we can just find out. So, we have to find out the reduction in peak. So, we know the peak inflow is 500 cubic meters per second and peak

outflow is coming as 444.23 cubic meters per second. So, the reduction in peak is 65.77 cubic meters per second. Also, we have to find out the time of occurrence of the peak outflow. So, obviously, this occurs at 60 hours. So, the time of occurrence of the peak is 60 hours. So, these are the two things we were asked to find out, and we have found out by routing the given inflow hydrograph over the channel reach for which the routing constants k and x were given to us. So, that is the solution of this particular problem.

Muskingum Method

Example 5

The ordinates of an inflow hydrograph are provided in the table below. If the weighing factor in Muskingum equation (x) is 0.3, and the storage-time constant (time of travel of flood wave through the channel reach) (K) is 0.8 hour, using the Muskingum method of flood routing, calculate the ordinate of the outflow (routed) hydrograph for 3rd hour in $m^3 s^{-1}$.

Time (hour)	0	1	2	3	4	5	6	7	8	9
Inflow ($m^3 s^{-1}$)	0	34.5	85.5	178.5	147	106.5	85.5	42	16.5	0

Let us take another example, again on the Muskingum method. Example 5 says the ordinates of an inflow hydrograph are provided in the table below. If the weighing factor in Muskingum equation x is 0.3 and the storage time constant, that is, time of travel of the flood wave through the channel reach which is k , is 0.8 hours, using the Muskingum method of flood routing, calculate the ordinate of outflow routed hydrograph for the 3rd hour in cubic meters per second.

So, the inflow hydrograph is given 1 hour interval up to 0 to 9 hours, we have been given the inflow hydrograph and of course, we know the generalized Muskingum routing equation.

Muskingum Method

Solution

Generalised Muskingum Routing equation is

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

Where,

$$C_0 = \frac{(-Kx + 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}, C_1 = \frac{(Kx + 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}, C_2 = \frac{(K - Kx - 0.5 \Delta t)}{(K - Kx + 0.5 \Delta t)}$$

Substituting the value of $K = 0.8$ h, $x = 0.3$ and $\Delta t = 1$ h, we get

$$C_0 = 0.2453, C_1 = 0.6981, C_2 = 0.0566$$

$$C_0 + C_1 + C_2 = 1 \text{ (Check)}$$

So, here we have been given the value of k is 0.8 hours, x is 0.3, and our inflow hydrograph is 1 hour interval. So, we will take it 1 hour, Δt is 1 hour. Now, knowing k , x , and Δt , we can put the values here and calculate C_0 , C_1 , and C_2 . By putting and calculating, we get C_0 is 0.2453, C_1 is 0.6981, C_2 is 0.0566, and of course, as we have insisted every time when we use a Muskingum routing equation, that the sum of coefficients C_0 , C_1 , and C_2 must be equal to 1. So, we have to ensure that. And then, if you put the values in this, we find that $C_0 + C_1 + C_2$ is equal to 1, which gives us a check that the coefficient values we have calculated are perfect.

Muskingum Method

Solution

Now, for 1st hour,

$$Q_1 = 0, I_1 = 0, I_2 = 34.5$$

Putting the values in the equation,

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

$$Q_2 = 8.46 \text{ m}^3 \text{ s}^{-1}$$

Now, for 2nd hour,

$$Q_1 = 8.46, I_1 = 34.5, I_2 = 85.5$$

Putting the values in the equation,

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

$$Q_2 = 45.54 \text{ m}^3 \text{ s}^{-1}$$

$$C_0 = 0.2453, C_1 = 0.6981, C_2 = 0.0566$$

Time (hour)	Inflow ($\text{m}^3 \text{ s}^{-1}$)
0	0
1	34.5
2	85.5
3	178.5
4	147
5	106.5
6	85.5
7	42
8	16.5
9	0

Then, of course, we need to route the flow. So, our C_0 , C_1 , and C_2 are known, and the inflow hydrograph values are given to us. So, for the first hour, we do not have any information. Also, we are assuming Q_0 is 0, and I_0 , I_1 is 0, and I_2 is 34.5. So, putting the values in this equation

$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$. So, obviously, we know C_0 , C_1 , and C_2 , we know I_1 , we know I_1 and I_2 , and we also know the first value of Q .

So, obviously, at the end of 1 hour, Q_2 is 8.46 cubic meters per second, which will become the inflow value at the first value of outflow, that is, Q_1 for this second hour routing. So, Q_1 is 8.46. I_1 is 34.5 from here, and I_2 is 85.5, and C_0 , C_1 , C_2 we already know. So, putting the values in this equation, we will get Q_2 equal to 45.54 cubic meters per second, which will become the first value for the next hour.

Muskingum Method

Solution

Now, for 3rd hour,
 $Q_1 = 45.54$, $I_1 = 85.5$, $I_2 = 178.5$
 Putting the values in the equation,
 $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$
 $Q_2 = 106.05 \text{ m}^3 \text{ s}^{-1}$

Thus, the ordinate of the outflow (routed) hydrograph for 3rd hour = $106.05 \text{ m}^3 \text{ s}^{-1}$

$C_0 = 0.2453$, $C_1 = 0.6981$, $C_2 = 0.0566$

Time (hour)	Inflow ($\text{m}^3 \text{ s}^{-1}$)
0	0
1	34.5
2	85.5
3	178.5
4	147
5	106.5
6	85.5
7	42
8	16.5
9	0

So, here Q_1 will become 44.54, then 85 will become I_1 , 178 is I_2 , and knowing the values of C_0 , C_1 , C_2 , we can calculate Q_2 , that is, at the end of the time interval, at the third hour, which is the third routing time interval, the third hour, the ordinate we are getting is 106.05 cubic meters per second. So, that is the answer, that the ordinate of the outflow routed hydrograph for the third hour is 106.05 cubic meters per second.

If we know that when we are asked to come get the complete hydrograph, then we make the entire table and then do the calculations, but because only we require here for the third 3 hours. So, we did it outside. We can also do it in tabular form also. So, whatever way you like, the point is to get this answer, that the outflow hydrograph, the ordinate of the routed hydrograph for the third hour routing is 106.05 cubic meters per second.

Muskingum Method

Example 6 Estimate the routing constants K and x for the given data.

Time, days	I, cumec	O, cumec
1	3.95	3.62
2	5.82	4.34
3	8.84	5.99
4	13.59	8.72
5	18.78	12.32
6	23.19	16.14
7	26.76	19.97
8	28.80	22.89
9	29.28	25.11
10	29.40	26.63
11	29.06	27.53
12	28.50	28.04
13	27.90	28.20
14	27.11	28.04

Then we go to the next one, Muskingum method again, and now this problem says the estimate the routing cost of k and x for the given data. So, for that, for a one-day time interval for 14 days, we have been given inflow and outflow hydrograph values.

Muskingum Method

Solution

Time, days	I, cumec	O, cumec	Ave I	Ave O	ΔS	S	$xI+(1-x)O$				
							$x=0.1$	$x=0.2$	$x=0.3$	$x=0.4$	
1	3.95	3.62									
2	5.82	5.34	4.88	4.48	0.41	0.41	4.52	4.56	4.60	4.64	
3	8.84	7.99	7.33	6.66	0.67	1.07	6.73	6.80	6.86	6.93	
4	13.59	12.72	11.21	10.36	0.86	1.93	10.44	10.53	10.61	10.70	
5	18.78	16.79	16.19	14.76	1.43	3.36	14.90	15.04	15.18	15.33	
6	23.19	22.11	20.99	19.45	1.54	4.90	19.60	19.76	19.91	20.06	
7	26.76	25.34	24.98	23.73	1.25	6.15	23.85	23.98	24.10	24.23	
8	28.80	27.88	27.78	26.61	1.17	7.32	26.73	26.84	26.96	27.08	
9	23.45	24.44	26.13	26.16	-0.04	7.28	26.16	26.15	26.15	26.15	
10	18.23	18.97	20.84	21.71	-0.86	6.42	21.62	21.53	21.45	21.36	
11	14.44	15.33	16.34	17.15	-0.81	5.60	17.07	16.99	16.91	16.82	
12	9.23	10.22	11.84	12.78	-0.94	4.66	12.68	12.59	12.49	12.40	
13	5.22	6.21	7.23	8.22	-0.99	3.67	8.12	8.02	7.92	7.82	
14	1.15	2.34	3.19	4.28	-1.09	2.58	4.17	4.06	3.95	3.84	

Given

Col. (4-5) Cumulative

$xI+(1-x)O$

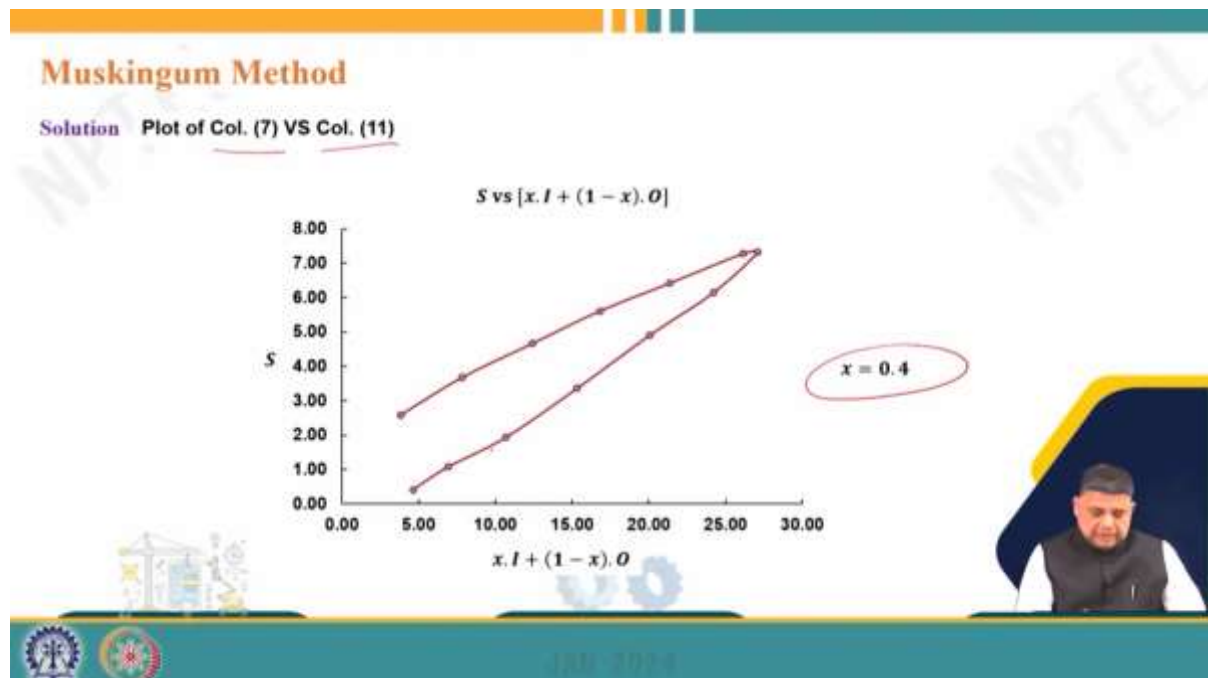
So, that means, for a given reach, we have been given inflow and outflow, and for that particular reach, we need to estimate k and x values. We know that basically for estimating k and x values, what we do is that we assume different values of x and then we calculate $xI + (1 - x)O$, as shown here, and of course, we also calculate the corresponding storage values. Then, you plot this S versus $xI + (1 - x)O$ because this equation is of linear form k times $xI + (1 - x)O$, and that is the property we try to use. Because we said that basically this relationship S versus $xI + (1 - x)O$ will give us a loop, will not give a straight line, but kind of a loop. So, we choose the narrowest loop and fit a straight line, and the slope of that line gives us a value

of k and the value of x for which we get the narrowest line, that is the value of x . That is the procedure we discussed earlier, that is the procedure we use.

Basically, here we have been given I and O values for different time intervals. It is a daily time step we are using here. So, we have to calculate the average I and average O . So, obviously, here, average I means average of the first 2 values, and average O is that, this is the average of this, and the difference between these 2 will be the change in storage during that particular time interval. This is also the beginning of S , and then for the second routing period, we will again get this average I , average S , and we get this, and then we will have a cumulative value.

So, in this column, column 7, we put the cumulative storage value, and that is the storage value we need to plot. As you can see, the change in storage in different time intervals, it reduces, it increases, it is increasing, and then it is reducing also. So, that is why we get a loop, it is not a straight line, but we get a loop because storage goes up and down. Then, what we do is that once we have calculated storage cumulative storage values for all the time steps, then we take different values. We said that x varies between 0.1 and 0.4. So, we are taking 4 values here, x equal to 0.1, x equal to 0.2, x equal to 0.3, and x equal to 0.4. One may also take 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, and 0.4, I mean, so many values you can take, or any value of x between 0 and 0.4 can be assumed here. So, we are assuming the value of x , and then knowing I and O , we are calculating these values for different x values. These columns are for different x values, and I and O we already know, corresponding I and O value we already know.

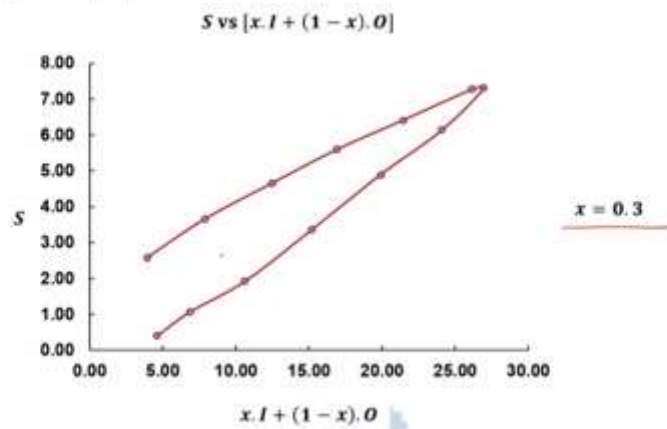
So, now we have S and we have, say for example, x , x equal to 0.1, we have $xI + (1 - x)O$ value. So, we will plot that. Similarly, we will plot S versus $xI + (1 - x)O$ for x equal to 0.2, x equal to 0.3, and x equal to 0.4, and we will get a loop for each value.



So, like here, we are plotting column 7 versus column 11, that means, for x equal to 0.4, we can start with x equal to 0.1, it does not matter, because you have to plot each. So, this is what we get. You see we are getting a loop, but it is quite a gap, a broad gap, between two tails.

Muskingum Method

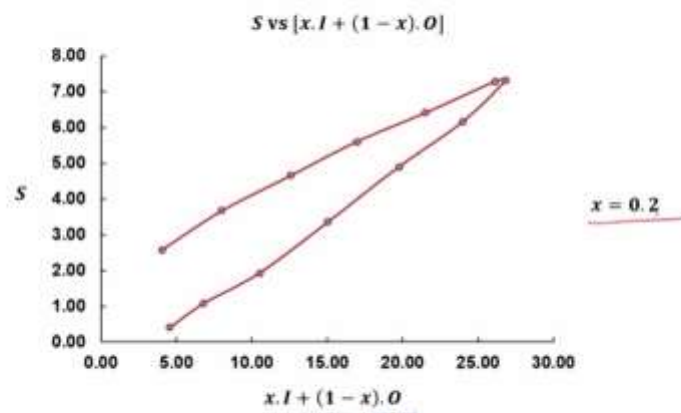
Solution Plot of Col. (7) VS Col. (10)



Then this one is a plot we get for x equal to 0.3, a little bit narrower than 0.4,

Muskingum Method

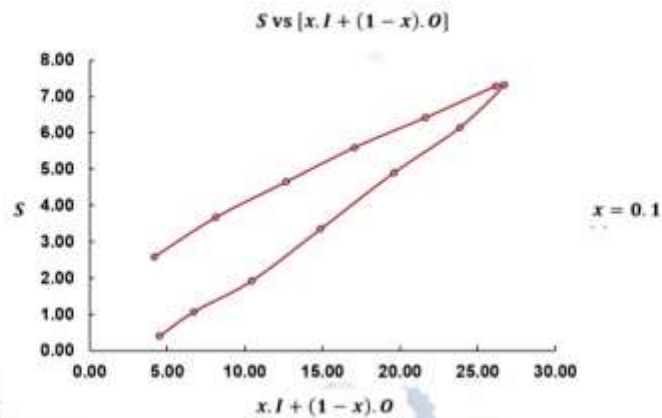
Solution Plot of Col. (7) VS Col. (9)



then we get 0.2,

Muskingum Method

Solution Plot of Col. (7) VS Col. (8)



And then finally, we get 0.1.

So, these are the four different plots we get, and of course, we have to make adjustments, which one is the narrowest. Because here, there is not a significant difference. Remember when in the classroom, we plotted a problem, we got a very significant difference for different x values, but here, we are not getting a very significant difference. So, of course, you have to use judgment,

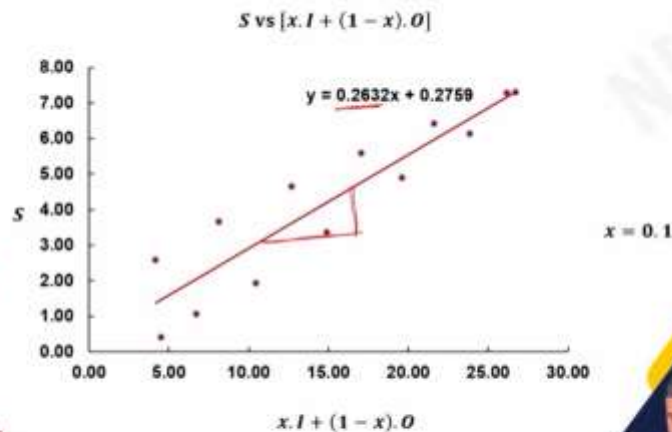
Muskingum Method

Solution

- Since Narrowest loop is obtained for $x = 0.1$,
 $x = 0.1$
 $K = 0.26$ (slope of the line)

The routing constants K and x for the given reach are $K = 0.26$ days and $x = 0.1$

K & x should have the same time units



And then our judgment says that the narrowest lift is obtained for x equal to 0.1. So, 0.1 and 0.2, 0.1 and 0.2, of course, very different, a little difference, but still we are choosing x equal to 0.1 graph. So, x we are choosing x value is 0.1, and then what we do with the values we have got, we fit a straight line, and then we get the slope of this line. The idea is to get the slope of this line, and by putting this curve, we get 0.2632. So, the value of k becomes 0.26, the slope

of the line. So, the routing constants k and x for the given reach are 0.26 days and x equal to 0.26 days, because you remember we said that k and Δt should have the same time units. So, in this problem, because Δt was in days, that is why we are saying k is equal to 0.26 days. Had it been in hours, then k would have been hours. So, that is, you have to remember, and that what will be the unit of k that will correspond to the unit of Δt given in the problem.

Wave Celerity

Example 7

A 160 m wide rectangular channel has a bed slope of 1 per cent and a Manning roughness coefficient of 0.035. Calculate the water velocity V , the kinematic (c_k) and dynamic wave celerities (c_d) and the velocity of propagation of dynamic waves at a point in the channel where the flow rate is 100 m³/s.

Solution

Manning's Eq. $Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$

Here, $B = 160$ m, $n = 0.035$, $S_0 = 1\%$, and $Q = 100$ m³/s

Assuming $R = y$ (for wide channel)

$$Q = \frac{1}{n} (B y) y^{2/3} S_0^{1/2} = \frac{1}{n} B y^{5/3} S_0^{1/2}$$

With the known value of Q , n , B and S_0 ,

$$y = 0.4 \text{ m}$$



Then we come to the next problem, example 7, which is on wave celerity. A 60-meter-wide rectangular channel has a bed slope of 1 percent, a Manning roughness coefficient of 0.035. Calculate the water velocity, kinematic and dynamic characteristics C_k and C_d , and velocity of propagation of dynamic waves at point in the channel where the flow rate is 100 cubic meters per second.

So, the value of q is given to us, the channel width is given to us as 160 meters, Manning's roughness coefficient is given as 0.035, and the slope of the channel is given as 1 percent, with q given as 100 cubic meters per second. Now, typically for wide channels, we assume hydraulic radius equal to the depth of flow, and thus putting this equation here, we get

$$Q = \frac{1}{n} (By) y^{2/3} S_0^{1/2} = \frac{1}{n} B y^{5/3} S_0^{1/2}$$

that is putting in the Manning's equation. And with the known values of q , b , and h_0 , we can calculate the value of y . So, y comes out to be 0.4 meters here and.

Wave Celerity

Example 8

A 50 m wide rectangular channel has a Manning roughness coefficient of 0.035. Calculate the bed slope considering wave velocity of 3.5 m/s, kinematic wave celerity c_k of 4.8 m/s and the flow rate of 125 cumec.

Solution

- Kinematic Wave Celerity is given by

$$c_k = \frac{1}{B} \frac{dQ}{dy}$$

$$c_k = \frac{1}{B} \frac{d}{dy} \left(\frac{S_0^{1/2} B}{n} y^{5/3} \right)$$

$$c_k = \left(\frac{S_0^{1/2}}{n} \right) \left(\frac{5}{3} \right) y^{2/3}$$

- Depth of flow can be calculated as,

$$y = \frac{Q}{BV} = \frac{125}{50 \times 3.5} = 0.71 \text{ m}$$

- Putting the value of c_k , n and y , S_0 can be calculated as

$$4.8 = \left(\frac{S_0^{1/2}}{0.035} \right) \left(\frac{5}{3} \right) 0.71^{2/3} \quad S_0 = 0.02 = 2\%$$



So, wave velocity v will be $\frac{q}{b \cdot y}$. So, q is known to us, b is known to us, y we have calculated. So, that comes out to be 1.56 meters per second and the kinematic wave celerity C_k we know is that $\frac{1}{b} \cdot \frac{dq}{dy}$ and that means, $\frac{1}{b} \cdot \frac{dq}{dy}$ of this value q value already we know, and which ah on ah ah which comes out to be $\frac{a \cdot h}{n \cdot \left(\frac{5}{3} \cdot y \right)^{5/3}}$.

Ah. So, putting the known values, we get C_k equals to 2.58 meters per second. Dynamics ah wave celerity $C_d = \sqrt{g \cdot y}$, g we take is 9.81, y we have already calculated. So, that value gets is 1.98 a meter per second, and the velocity of propagation of the upstream down wave is $v - C_d$. We know v , we know C_d . So, that is minus 0.42 meter per second, and that of a downstream dynamic wave, which is $v + C_d$, 1.5 plus 1.98. So, it comes out with 3.454 meters per second. So, those are the different answers to different bits in this problem. We take example 8, a 50-meter-wide rectangular channel has a Manning's roughness coefficient of 0.035. Calculate the bed slope considering wave velocity of 3 meters per second, kinetic wave celerity of 4.8 meters per second, and flow rate of 125 cubic. So, we know C_k is given by this relationship, and we have also ah ah done that by differentiating we get this the form depth of flow can be calculated as $\frac{q}{b \cdot v}$. v q is given as 125 cubic meters, b is given as 50 meters, and 3.5 is velocity we have already it is also given wave velocity. So, value of depth ah y we get is 0.71 meters, and then putting the value of C_k and $n \cdot y$ in this particular equation here and only unknown is ah S_0 which we need to find out supporting that value we get S_0 is 0.02 or 2 percent. So, the bed slope of the channel is 2 percent for the given problem.

Wave Celerity

Example 9

A 55 m wide rectangular channel has a bed slope of 1.5%. Calculate the Manning's roughness coefficient considering wave velocity of 2.5 m/s, kinematic wave celerity c_k of 4.1 m/s and the flow rate of 115 cumec.

Solution • Kinematic Wave Celerity is given by

$$c_k = \frac{1}{B} \frac{dQ}{dy}$$

$$c_k = \frac{1}{B} \frac{d}{dy} \left(\frac{S_0^{1/2} B}{n} y^{5/3} \right)$$

$$c_k = \left(\frac{S_0^{1/2}}{n} \right) \left(\frac{5}{3} \right) y^{2/3}$$

• Depth of flow can be calculated as,

$$y = \frac{Q}{BV} = \frac{115}{55 \times 2.5} = 0.84 \text{ m}$$

• Putting the value of c_k, S_0 and y, n can be calculated as

$$4.1 = \left(\frac{0.015^{1/2}}{n} \right) \left(\frac{5}{3} \right) 0.84^{2/3}; \quad n = 0.044$$

We take the last problem, a 55-minute-wide rectangular channel has a bed slope of 1.5 percent. Calculate the Manning's roughness coefficient considering wave velocity of 2.5 meters per second, kinetic wave celerity of 4.1 meters per second, and flow rate of 115 cubic. So, it is similar problem only thing is that in this case Manning's roughness coefficient we have to find out.

So, basically first we find out the y value and then C_k 0 y ah known values in this equation if we put ah then we get n equals to 0.044. So, the Manning's roughness coefficient for the given data comes out to be n equal to 0.044. So, with this, we come to the end of this lecture. So, we have taken ah numerical on various aspects of flood routing and of course, you need to practice more problems, in fact, so that you are tuned to various conditions, various variations which you can find in different problems. ah Please give your feedback and also raise your questions or doubt we will be happy to answer. Thank you very much.

THANK YOU