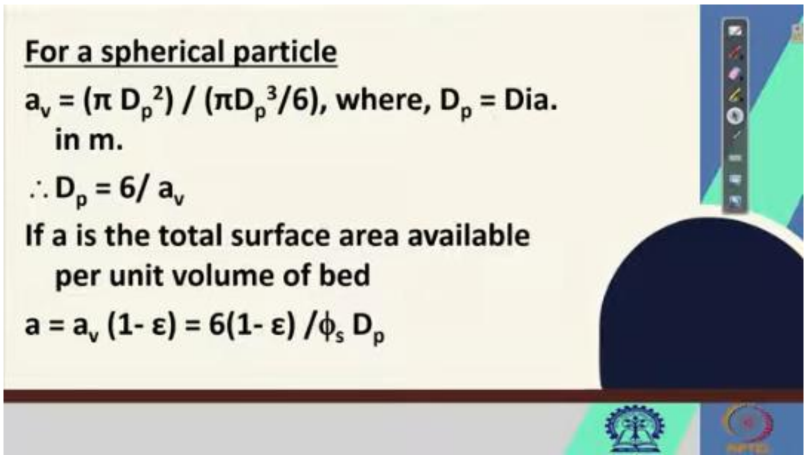
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

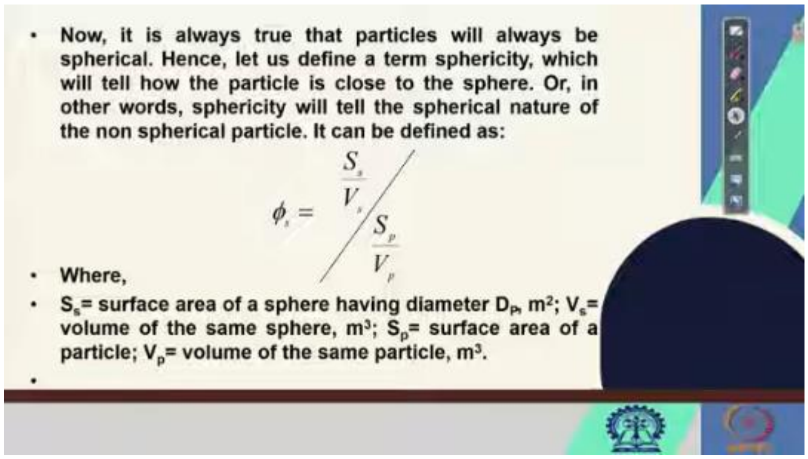
Lecture37

LECTURE 37 : HYDRAULIC RADIUS

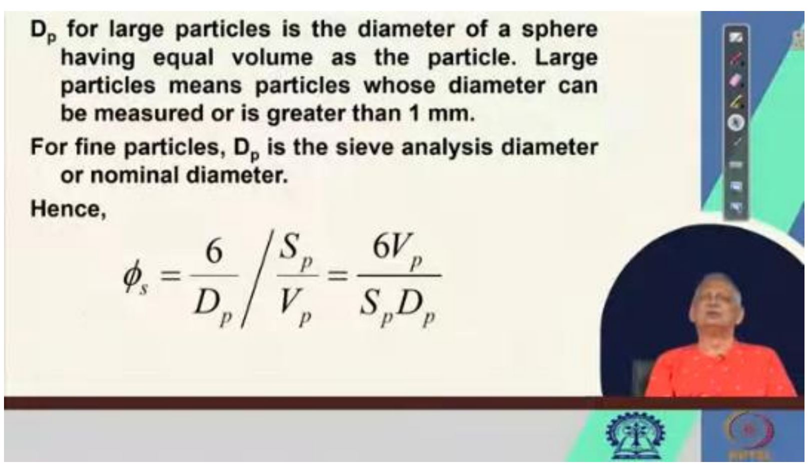
Good afternoon, my dear boys and girls and students and friends. flow through packed beds right. We have defined some of the parameters according to its definition. It may be required that some more we need to define. For example, we had said for spherical particles



A, these we have defined, but we also defined the sphericity and the sphericity is defined as like this. This is true that the particles will always be spherical, hence let us define a term sphericity which will tell. how the particle is close to the sphere or in other words sphericity will tell the spherical nature of the non spherical particle. it can be defined as phi$_s$ is S_s over V_s over or divided by S_p over V_p S_s over V_s divided by S_p over V_p where obviously, S_s is the surface area. of a sphere having a diameter of D_p in meter square, V_s is the volume of this same sphere in meter cube, S_p is the surface area of a particle and V_p is the volume of the particle same particle in meter cube right.

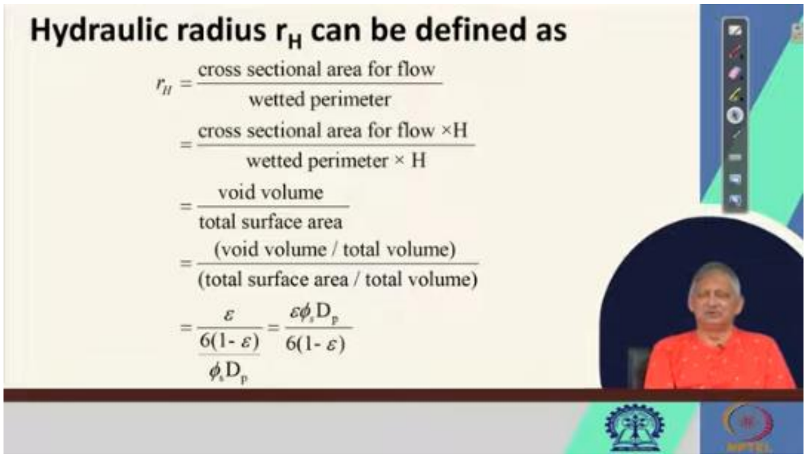


So, if this is the definition then we can say D_p for large particles is the diameter of a sphere having equal volume as the particle. Large particles means particles whose diameter can be measured or is greater than 1 millimeter you see 1 millimeter is very low right. and more than a millimeter that could be as large as small you see as much. I gave you example of grinding.



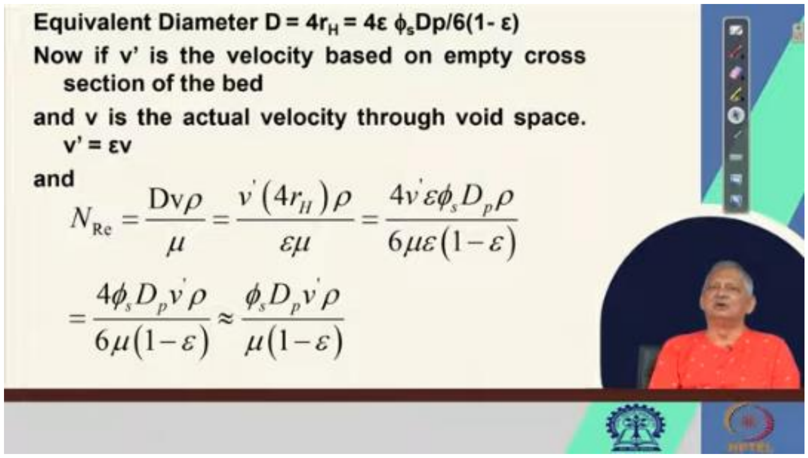
So, ground particles are not in millimeter they are in micrometer right. For the fine particles D_p is the sieve analysis diameter. or nominal diameter. Hence, we can write phi$_s$ is equal to 6 over D_p over S_p over V_p or that can be equated as 6 V_p over S_p over D_p right. Now, we also

Define another term called hydraulic radius. Though you have done it in the previous class, but since these are very new, so a little repeat may not be bad for the new people who are not accustomed with this. That R_h is the hydraulic radius. is defined as cross sectional area for flow over weighted perimeter that is cross sectional times if we multiply both numerator and denominator with H. So, cross sectional area of our flow times H over weighted perimeter times H, this becomes void volume over total surface



Now, void, if we now introduce another thing that if we divide both numerator and denominator with total volume, then it becomes void volume by total volume divided by total surface area by total volume. Now void volume by total volume by definition we have already given that is nothing but epsilon and this total surface area by total volume just now we have shown it to be for spherical particles it is 6 by ϕ_s D_p into 1 minus epsilon. On rearrangement it can be said that epsilon ϕ_s D_p over 6 into 1 minus epsilon right. So, hydraulic radius is expressed in terms

void volume, particle size and sphericity as epsilon ϕ_s D_p by 6 into 1 minus epsilon. This may be required subsequently. Now, another also important that is called we have said hydraulic radius. So, equivalent of that is equivalent diameter. So, equivalent diameter is capital D that is nothing, but 4 times hydraulic radius that is 4 R_h.

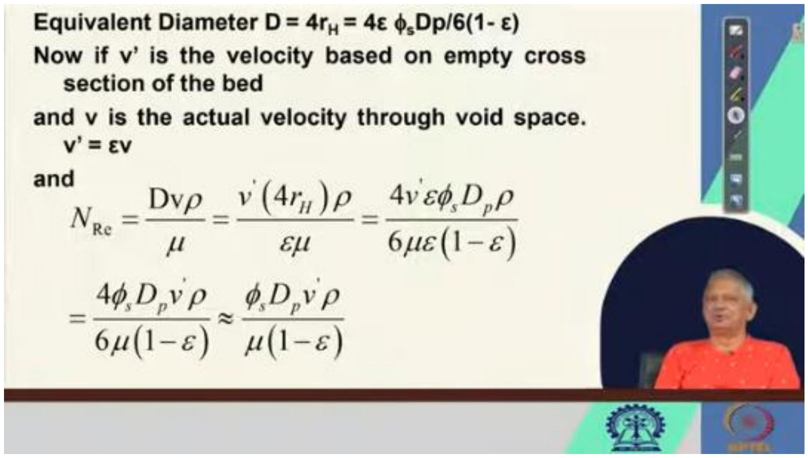


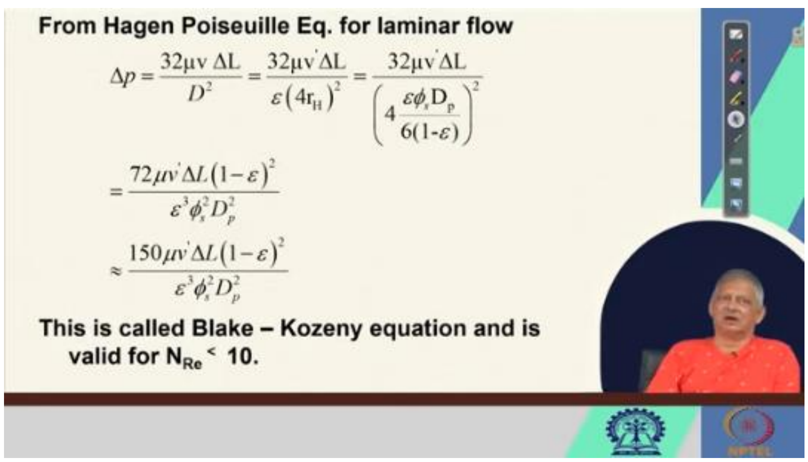
So, if we substitute R_h with whatever just in the previous slide we have said that 4 epsilon (ε) phi$_s$ (Φ_s) D_p over 6 into 1 minus epsilon. Now, if v prime (v') is the velocity based on empty cross section which I have shown in the previous class with a water bottle that if it is empty, then the velocity if there will be any flow of air or any fluid. that is v prime. So, v prime is the velocity based on empty cross section of the bed and v is the actual velocity through void space right. So, therefore, we can write v prime is equal to epsilon into v right.

So, another term that is the non-dimensional, N_{Re}. So, on non-dimensional N_{Re} or Reynolds number we can define as D V rho by mu that is by definition. So, if we substitute appropriately then we can write v as v prime d as 4 hydraulic radius 4 R_h rho remains same and v is v prime by epsilon right. So, mu is there that means, dv rho by mu is v prime by epsilon.

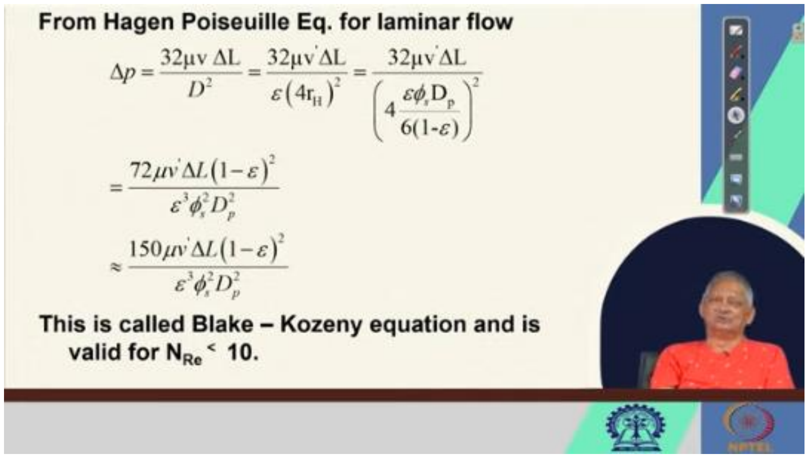
4 R_h into rho by mu. So, this on rearrangement we can write 4 v prime epsilon phi$_s$ D_p rho by 6 mu into epsilon (ε) into 1 minus epsilon. On rearrangement, this you can write to be 4 phi$_s$ into D_p into v prime rho over 6 mu into 1 minus epsilon, that is equal to phi$_s$ D_p into v prime rho over mu into 1 minus epsilon right. So, this is roughly equals to because 4 by 6 is what is almost 0.7 right. So, that means, it is close to 1, that is why,

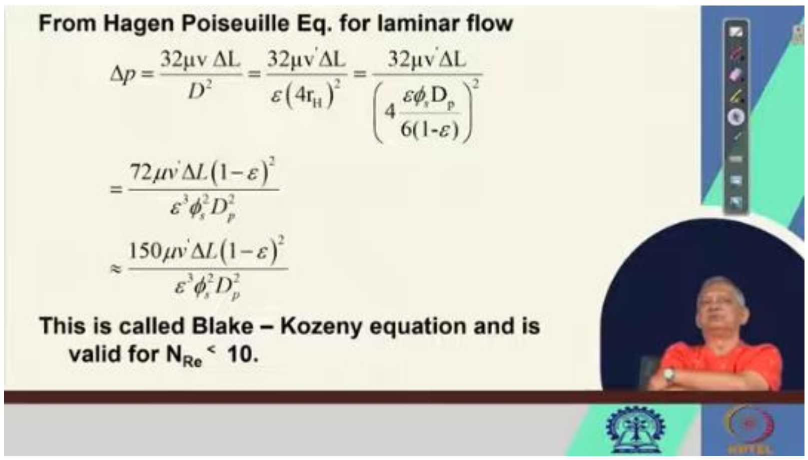
we can say it to be roughly equal to, by neglecting 4 by 6, ok. That is phi$_s$ D_p v prime rho over mu into 1 minus epsilon. This you keep in mind that this is the Reynolds number, that is N_{Re}. for packed bed right. So, there phi$_s$ is the sphericity, D_p is the diameter of the particle, v prime is the velocity through empty cross section, rho is the density of the fluid flowing, mu is the viscosity of the fluid flowing and epsilon is the void fraction, right. So, this N_{RE} definition we have taken. Now, if we start from Hagen-Poiseuille equation for laminar flow, what it was? It was, delta P is equal to 32 mu v L by D square, right, delta P is equal to 32 mu v prime delta L by epsilon into 4 Rh whole square.





So, this is equal to 32 mu v prime delta L over 4 epsilon ϕ_s D_p over 6 into 1 minus epsilon whole square right. This can be written as 72 mu v prime delta L into 1 minus epsilon whole square over epsilon cube ϕ_s square D_p square. That can roughly be said, instead of 72, it has been found by the scientist that the prediction of delta P is more closer to the actual if instead of 72 it is taken 150. That is why it was changed to 150 mu v prime into delta



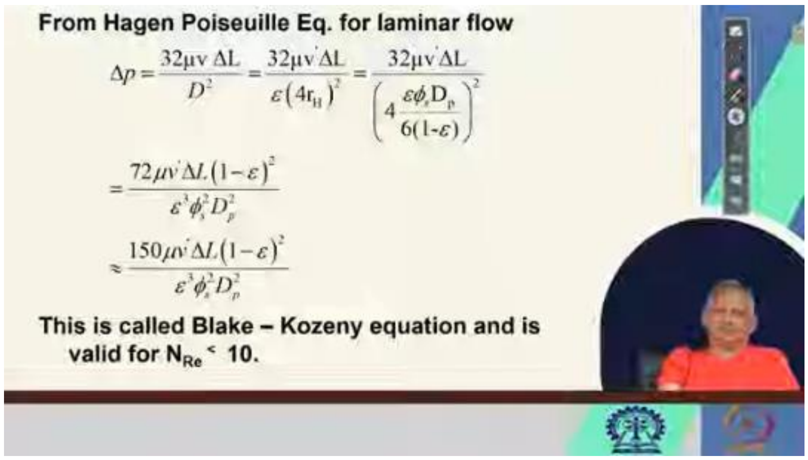


L into 1 minus epsilon whole square by epsilon cube ϕ_s square D_p square right. So, we have done very quickly, but we can repeat because a new thing that is the application of the Hagen-Poiseuille equation. So, we start with Hagen-Poiseuille equation. It says that delta P is equal to 32 mu v delta L by D square.

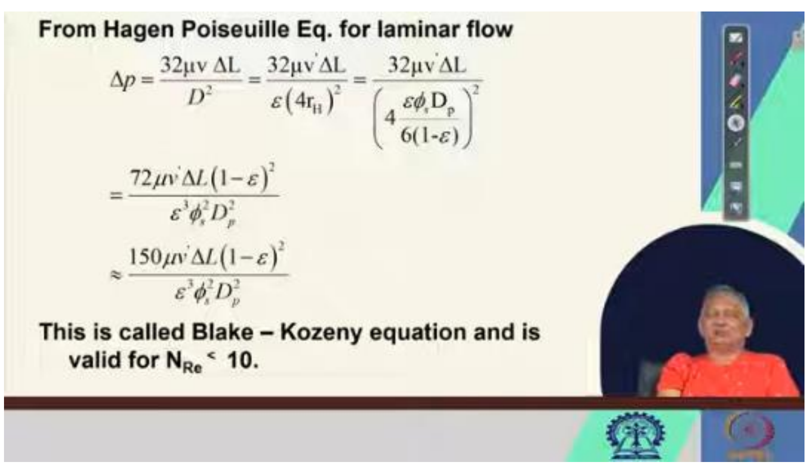
That was the application of Hagen-Poiseuille's equation. We come for laminar flow as delta P is equal to 32 mu v delta L over D square. This was for Hagen Poiseuille for laminar flow right in pipe flow. So, this is equal to 32 mu instead of v we write v prime delta L, the moment we are writing it is v prime, obviously, epsilon is also coming.

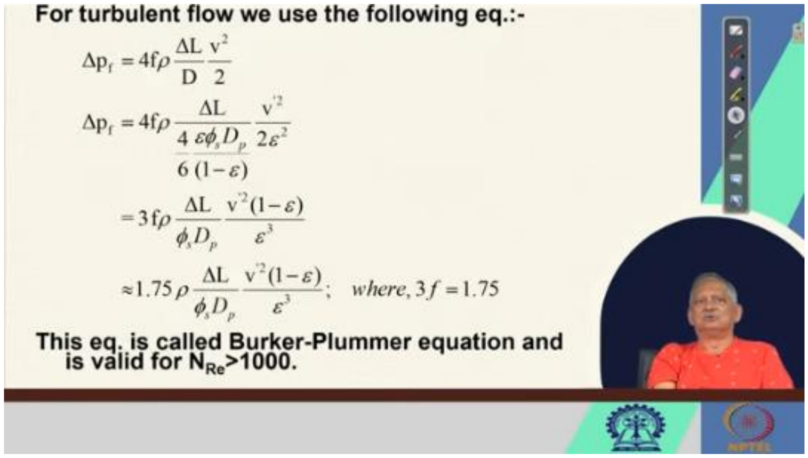
So, v prime by epsilon where we are replacing D with R_h hydraulic radius and that hydraulic radius is 4 R_h whole square right. So, 32 mu v prime delta L that hydraulic radius again we are substituting with the value of hydraulic radius, that is, 4 times epsilon, it was already there and ϕ_s D_p by 6 into 1 minus epsilon whole square. So, rearranging it we can write it to be 72 mu v prime delta L into 1 minus epsilon whole square divided by epsilon cube into ϕ_s square Dp square right, ϕ_s square D_p square yes. some scientists found that instead of this 72, if 150 is taken, then the prediction of delta P is more closer to the actual.

So, that is why they substituted it with 150 So, delta p is 150 mu v prime delta L into 1 minus epsilon whole square by epsilon cube ϕ_s square D_p square. This was as I said by some scientist did it. So, this was done by Blake Kozini. and that in their name the equation is named as Blake Kozeny equation right, Blake Kozeny, K O Z E N Y and this is applicable for or valid for N_{Re} less than 10, right mind it that Blake Kozeny equation



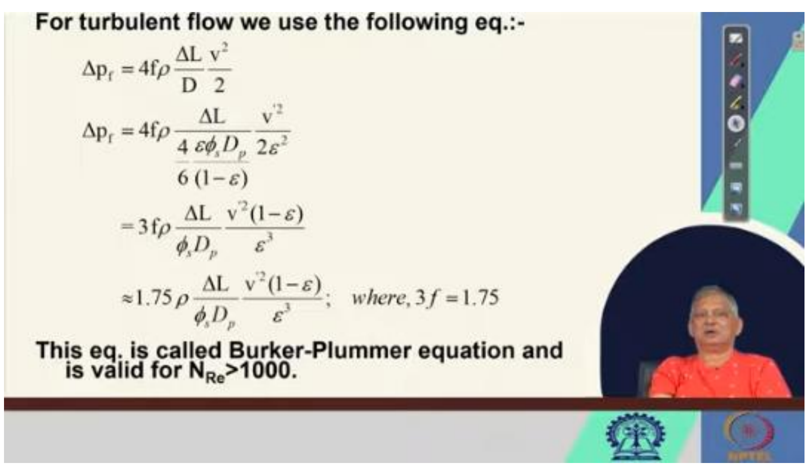
derived from the Hagen Poiseuille's equation for laminar pipe flow is valid for N_{Re} less than 10 right. Now, the other one for turbulent flow we can use the equation delta P_f is 4 f rho delta L by D into v square by 2. Now, if you substitute, delta P_f is 4 f rho into delta L. So, D, we can substitute with 4 R_h, and in turn it is 4 by 6 epsilon ϕ_s D_P by 1 minus epsilon into, that v as v prime square by 2 epsilon square,





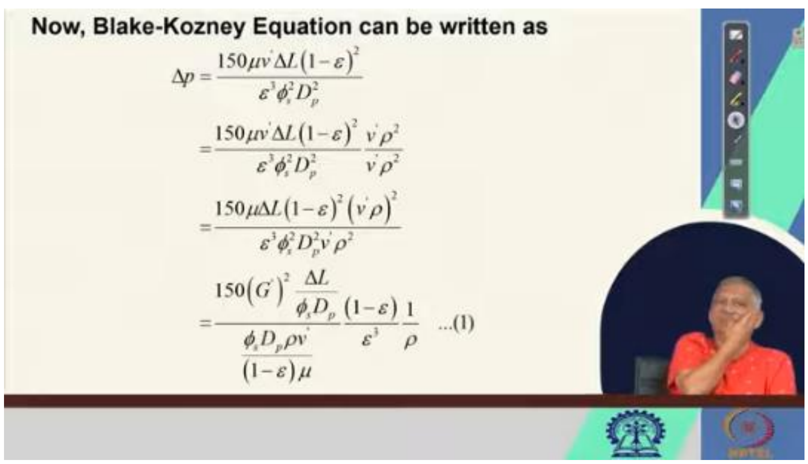
right, because this v prime square by epsilon square and there was a 2. So, these on rearrangement we can write 3 f rho delta L by phi$_s$ D_p into v prime square into 1 minus epsilon by epsilon cube right. That again, some of the scientist have found that, instead of 3 f rho, if we take, instead of 3 f, 1.75, then the prediction is more closer to the actual. So, if this be then they have said that it is 1.75 delta P_f, is 1.75 into rho into delta L by phi$_s$ Dp into v prime square into 1 minus epsilon by epsilon cube. Obviously, in this case 3f is nothing but equal to 1.75.

This was done by Barker Plummer, B-U-R-K-E-R Plummer, P-L-U-M-M-E-R, these two scientists and according to their name this equation is named as Barker Plummer, right? This equation and this is valid, which is very, very important that this is also valid for NRe greater than 1000. that means, from the Hagen Poiseuille and the from the turbulent, for both laminar and turbulent flow. We have studied from the very beginning that is Hagen Poiseuille's equation and the friction factor equation and then they came to two equations, two relations of delta P.

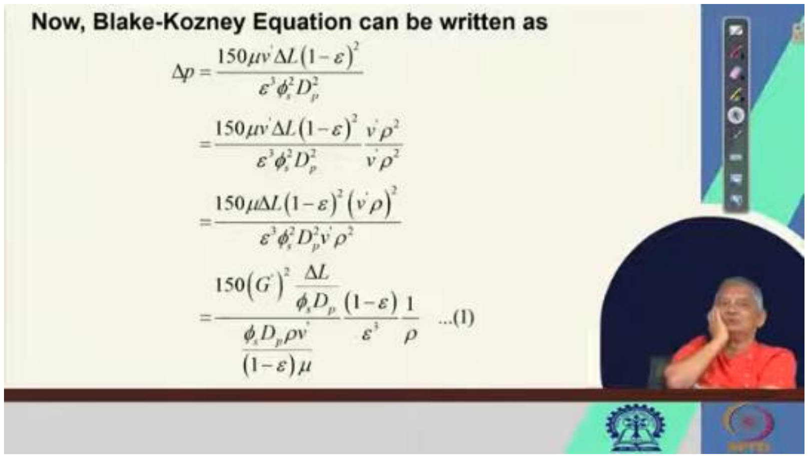


that is finding out the pressure drop theoretically, but both has the limitation that for the Hagen Poiseuille equation that is Blake Kozeny equation. It is valid up to N_{Re} less than 10 and for Barker Plummer for the turbulent flow. it has been shown that the relation delta P is valid for N_{Re} greater than 1000. Then the question comes obviously that what about, between 10 to 1000.

Is there anything? Yes. Subsequently, some scientists came forward and they started by taking both the equations, right. So, from the black cousin equation, they had taken, and I do not say, it is renovated obviously, they did something with that and the delta P that became equal to 150 mu v prime delta L into 1 minus epsilon whole square by epsilon cube phi$_s$ square D_p square right. So, this is equals to 150 mu v prime delta L into 1 minus epsilon square by epsilon cube phi$_s$ square D_p square into v prime into rho square by v prime into rho square. That means, they have introduced both in the numerator and denominator a new term or not new term some parameters like v prime rho square both in numerator and denominator, right and then reshuffled the equation.



as 150 mu delta L into 1 minus epsilon whole square into v prime rho whole square because one v prime was here inside. So, that they have taken inside and it has become v prime rho whole square. and the denominator has become epsilon cube phi$_s$ square D_p square then v prime into rho square right epsilon cube phi$_s$ square D_p square v prime into rho square. Now, they rearranged this equation as 150 instead of mu and v instead of rho v, right they have taken as rho v prime equal to G prime, right, 150 G prime whole square into delta l by phi$_s$ D_p into 1 minus epsilon right divided by phi$_s$ D_p rho v prime into 1 minus epsilon into mu by epsilon cube and divided by rho. So, this equation some people have given a numbering of the equation as equation 1. I repeat from the Blake-Kozeny equation delta p is equal to 150 mu by mu v prime into delta l into 1 minus epsilon rho square by epsilon cube phi$_s$ square D_p square by introducing both numerator and denominator with v prime rho rho square. They have introduced and rearranged to the level that delta p becomes equal to



150 G prime which is nothing by it, but rho rho v prime right into delta L by phi$_s$ D_p into 1 minus epsilon divided by phi$_s$ D_p rho v prime over 1 minus epsilon into mu by epsilon cube and also divided by rho right. I should say the other way around that 150 G prime delta L by phi$_s$ D_p divided by phi$_s$ D_p into rho v prime over 1 minus epsilon into mu times 1 minus epsilon by phi$_s$ epsilon cube. times 1 by rho is the right way of reading it, right. Then from the Barker Plummer equation, they have done that delta P is equal to 1.75

rho into delta L by phi$_s$ D_p into v prime (v') square into 1 minus epsilon by epsilon cube is equal to 1.75 rho delta L by phi$_s$ D_p times v prime square into 1 minus epsilon by epsilon cube into rho by rho. they multiplied rho both in the numerator and denominator. Now, they rearranged in the form like 1.75 V rho rather V prime rho whole square into delta L by phi$_s$ into D_p into 1 minus epsilon by epsilon cube into 1 by rho. And this is equals to 1.75 G prime square delta L by phi$_s$ D_p into 1 minus epsilon by epsilon cube into 1 by epsilon right this on rearrangement.

That means, from the Berker Plummer equation they have multiplied both numerator and denominator with rho and then rearranged to get the final of delta p as delta p is equal to 1.75 G prime whole square delta l by phi$_s$ D_p into 1 minus epsilon by epsilon cube into 1 by rho right. Then they have added these two equation both the outcome of the Blake-Kozeny as well as the Berker-Plummer equation.

They have added these two and found out that delta P is equal to 150 G prime (G') whole square into delta L by phi$_s$ D_P into 1 minus epsilon by epsilon cube into 1 by rho divided by phi_s Dp rho v prime 1 minus epsilon into mu plus 1.75 into G prime square delta l by phi_s D_p into 1 minus epsilon by epsilon cube into 1 by rho. These on rearrangement they have done delta p rho by G prime square into phi_s D_P by delta L into epsilon cube by 1 minus epsilon that is equal to 150 by N_{Re} plus 1.75. So, by adding some people have both by nurturing equation by nurturing Blake Kozini and Barker

plumber equation some people have made adding them and rearranging making a final expression like delta p is equal to delta p rho by G prime square into phi_s D_p by delta l into epsilon cube by 1 minus epsilon that is equals to 150 by N_{Re}. plus 1.75 and this equation is known as very famous equation called Ergun's equation. It is valid very much in pack bed right. Now, the time is up we will come and discuss in the next class.

Thank you very much.

Thank you.