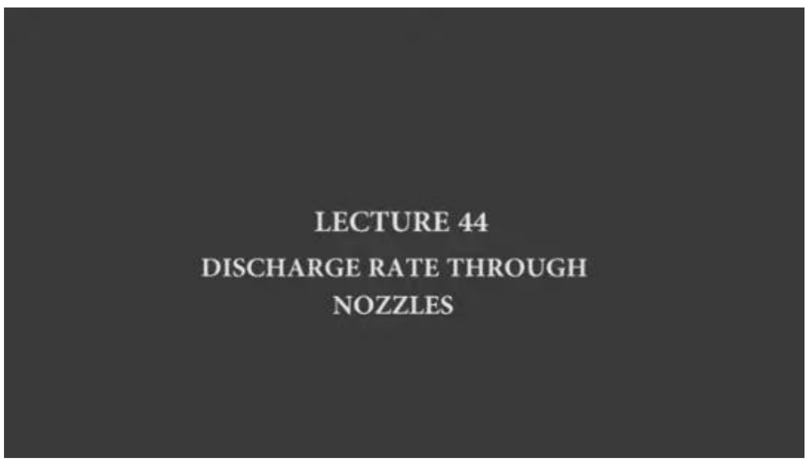
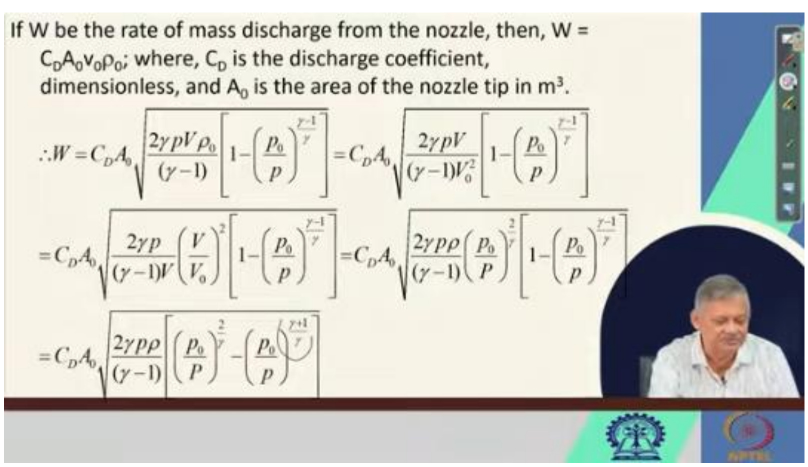
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture44

LECTURE 44 : DISCHARGE RATE THROUGH NOZZLES

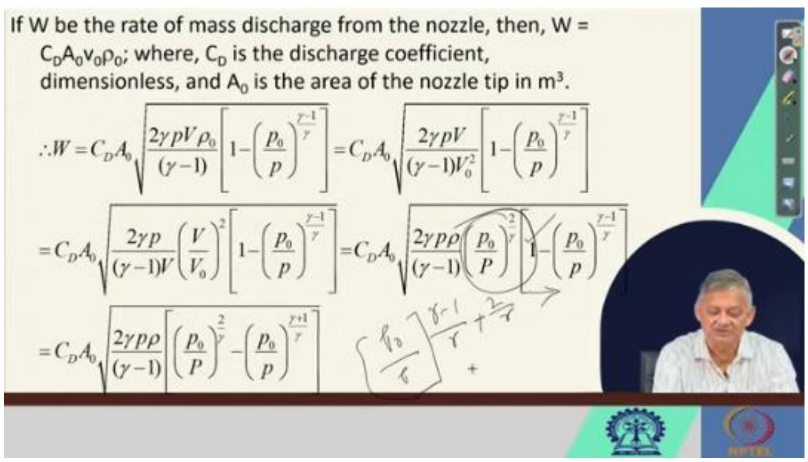
Good evening, my dear friends, students, boys, and girls. We are in the previous class, ending with the rate of discharge, that is W, when we have taken one adiabatic flow where P V gamma is constant, right? So, it is a continuation of the nozzle flow. Then we go to that where we can say that, yeah, up to this, we have done that the W is up to this, we have done that W is where is that something was here, ok.

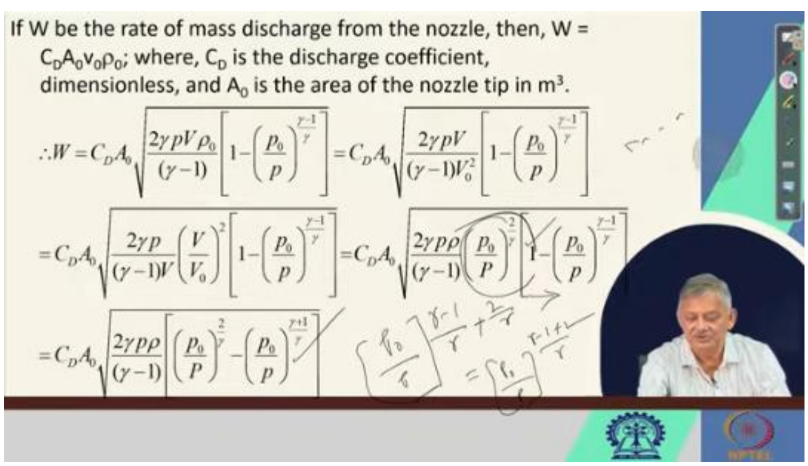




W is C_D A_o 2 gamma P rho by gamma minus 1 into P_o by P to the power 2 by gamma minus P_o by P to the power gamma plus one How gamma plus 1 came, that also we have said, right? Just for a recapitulation, that this P_o by P to the power 2 by gamma is put inside this bracket. So, it became P_o by P to the power gamma 2 by gamma into 1. Right, and then this P_o by P here it is getting multiplied.

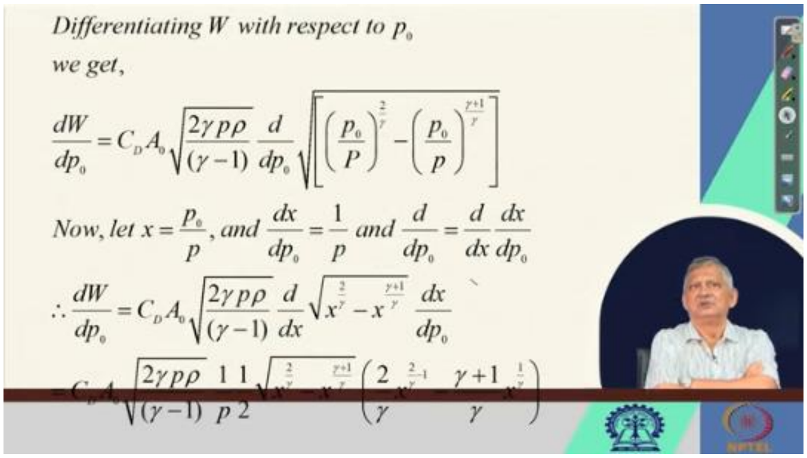
So, it is remaining P_o by P to the power here it was gamma minus 1 by gamma and here it is 2 by gamma. So, that means we have P_o by P to the power gamma remaining denominator. So, gamma minus 1 plus 2 that means it is nothing but here we are writing that P_o by P to the power gamma plus 1 by gamma, that is what we have found out, ok. So, this is nothing but the discharge rate. Now, another thing we will now do, very important, very, I do not say complicated, but very interesting, is that W we have found out, right?



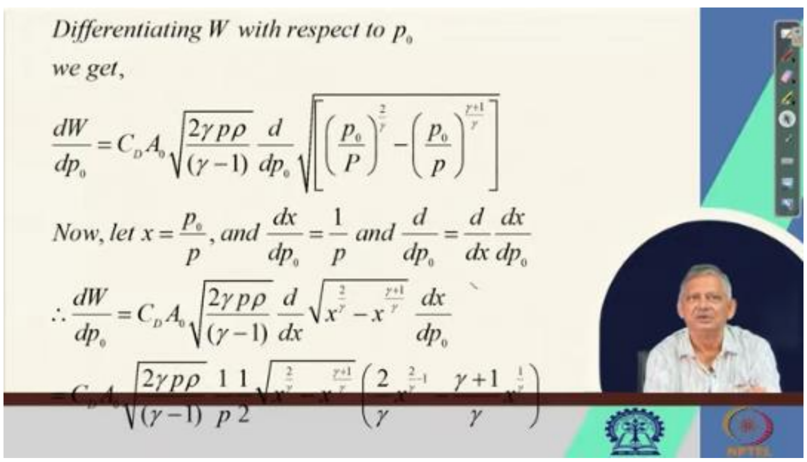


Then the question comes: what is the Differentiating W with respect to P_o, we get that dW / dP_o. Why are we differentiating? Because we want to know the meaning of dW / dP_o. dW

/ dP_o means the rate of change of P_o with the discharge. What is the effect? of the discharge pressure to the tip pressure? This tip or discharge both are the same.



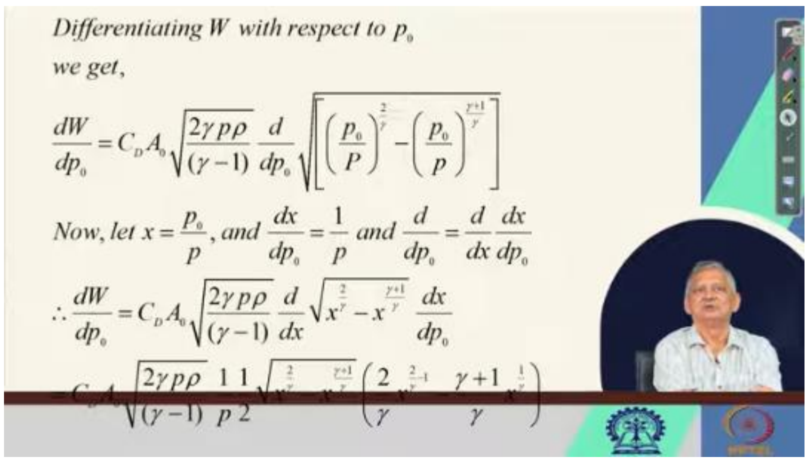
So, what is the effect of the discharge pressure on the discharge rate? That is what we are doing: we are differentiating W with Po, which means dW / dP_o. If we do the differentiation we get C_D A_o, which is beyond differentiation. Now, you see that we have shown what the discharge rate is, right? In the previous slide, we had shown it. Now, we want to show what happens if we differentiate



the rate with respect to tip pressure. Tip pressure is P_o, right? Why? Because you want to know what the effect of tip pressure on discharge is. If it is increasing, then what happens? If it is decreasing, then what happens?

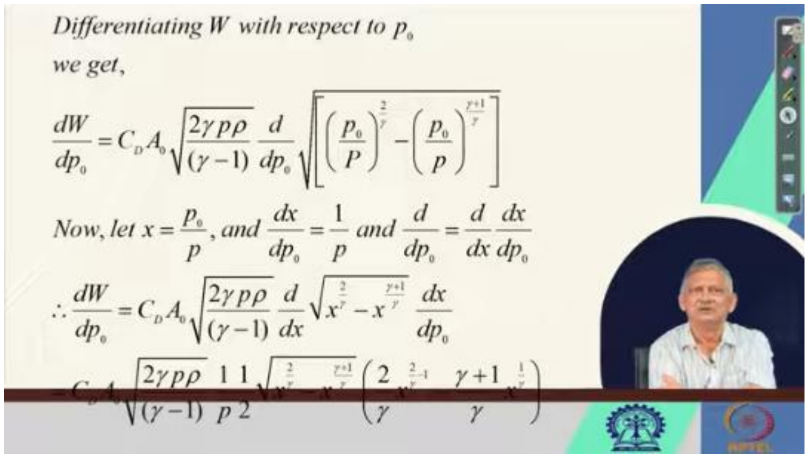
That means the effect of discharge pressure on the discharge rate is what we would like to find out. And there we are finding out that dw/dP_o, right, that is the differentiation of discharge rate with respect to discharge pressure or tip pressure. So, we can write C_D A_o is constant, right. So, it is beyond this differentiation, then under root 2 gamma P rho by gamma minus 1, this is also constant.

So, it is also beyond differentiation, but this is definitely under differentiation. Differentiation that dPo with respect to P_o by P to the power 2 by gamma minus P_o by P to the power gamma plus 1 by gamma, right. So, this differentiation we have to do. Now, definitely, it is not that easy because there are two terms, P_o by P to the power 2 by gamma and P_o by P to the power gamma plus 1 by gamma. Now you will say why C_D Ao under root 2 gamma P rho by gamma minus 1, that went beyond the differentiation.



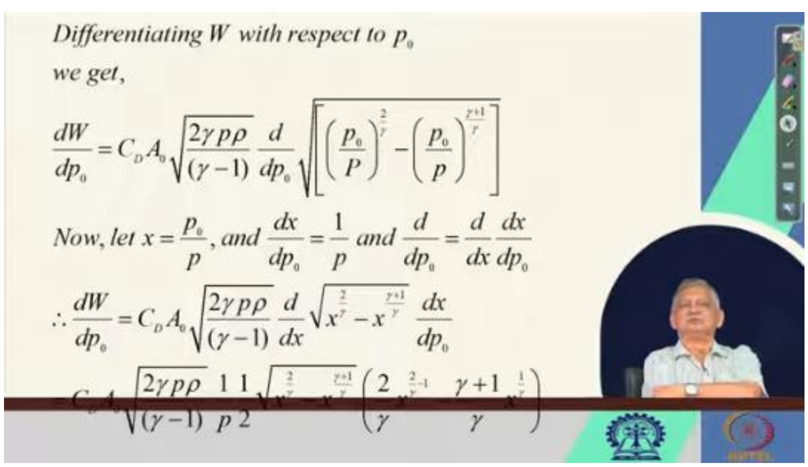
What are you differentiating? You are differentiating with respect to P_o. So, where P_o is there, then only it will be, because this P_2 gamma P rho by gamma minus 1, this P is the internal or inside pressure, right, where it is the reservoir or where from it is coming that pressure. But P_o is the tip pressure. So, we are differentiating with respect to tip pressure, Po.

So, wherever Po is, there will be under differentiation. So, that is why we wrote C_D A_o under root 2 gamma P rho by gamma minus 1, dP_o under root Po by P to the power 2 by gamma, minus P_o by P to the power gamma plus 1 by gamma, right. Now, for this differentiation, let us do a trick to make it easy, that is, let x be P_o by P. Therefore, dx / dP_o is nothing but 1 by P, right. and d dP_o is again nothing, but d dx of d x dP_o, right.

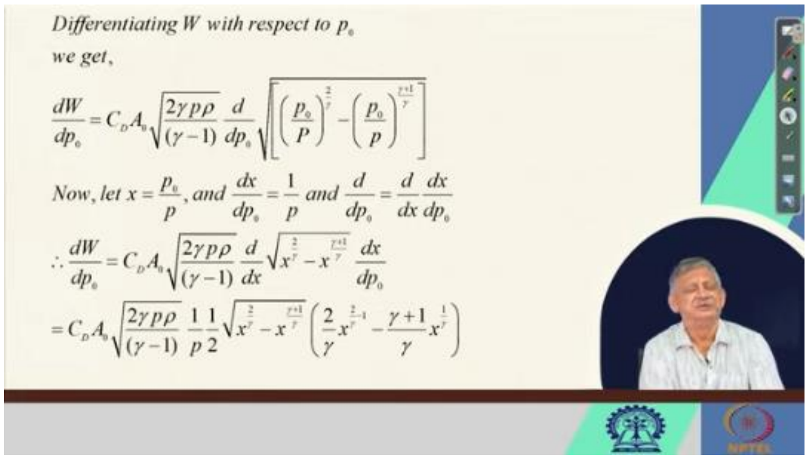


So, if we substitute them properly, then we get d w / dP_o is C_D A_o, constant which was 2 gamma p rho by gamma minus 1, that is also a clear constant, d now here you see d d P_o. So, d d P_o, how much we have seen? d d x of d x / dP_o. So, it is d d x of what x to the power 2 by gamma minus, x to the power gamma plus 1 by gamma d x / dP_o, right which we can write C_D A_o rho under root 2 gamma P rho by gamma minus 1 into that d x / dP_o, we have already seen this is nothing, but 1 by p.

So, 1 by p has come and d dx of x to the power 2 by 7 is how much? 1 by 2 under root x to the power 2 by 7, 2 by gamma rather not, 7, 2 by gamma and the other one is x to the power gamma plus 1 by gamma into 2 by gamma right, x to the power 2 by gamma minus 1 and x to the power gamma plus 1 by gamma, right. Then, C_D A_o 2 gamma P rho by gamma minus 1 into 1 by P into 1 by 2 because the differentiation gives, into under root x to the power 2 by gamma minus x to the power gamma plus 1 by gamma.

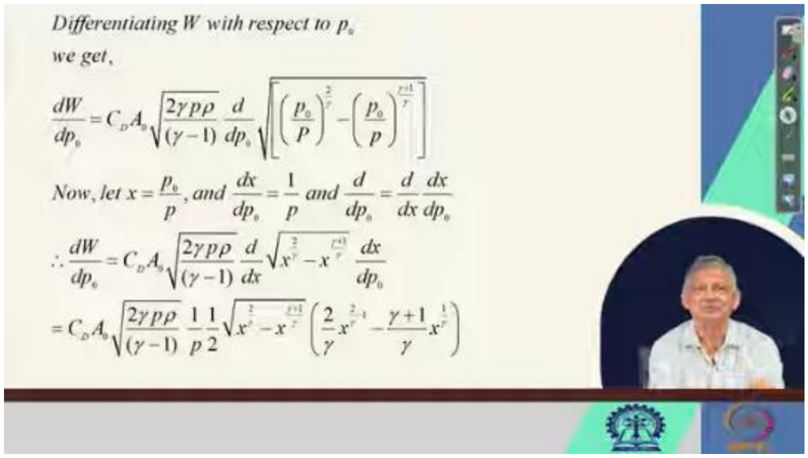


Now, differentiation of this part is 2 by gamma into x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma, right. This is the differentiation of dW / dP_o, when we replaced x as P_o by p and then we changed x to, we made x to P_o by P. P_o by P, rather, and we made d x / d P_o as 1 by P and d d P_o as d x / d P into d d x of d x d P_o, right. So, this we substituted into the original equation. There we got d w / d P_o is C_D A_o

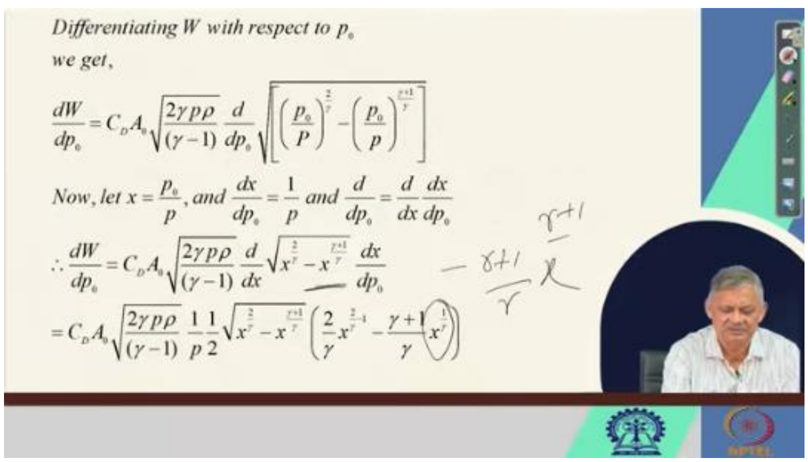


under root 2 gamma P rho by gamma minus 1, what it was, then d dx of by substituting under root x to the power 2 by gamma minus x to the power gamma plus 1 by gamma and this multiplied by dx / dP_o, right. Now, dx / dP_o, already we have seen it is 1 by P that it has come out. So, that is C_D Ao under root 2 gamma P rho by gamma minus 1 into 1 by P into 1 by 2 under root x to the power 2 by gamma minus x to the power gamma plus 1 by gamma it remains. Now, again differentiation of this is 2 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma into x to the power 1 by gamma because it is gamma plus 1 by gamma.

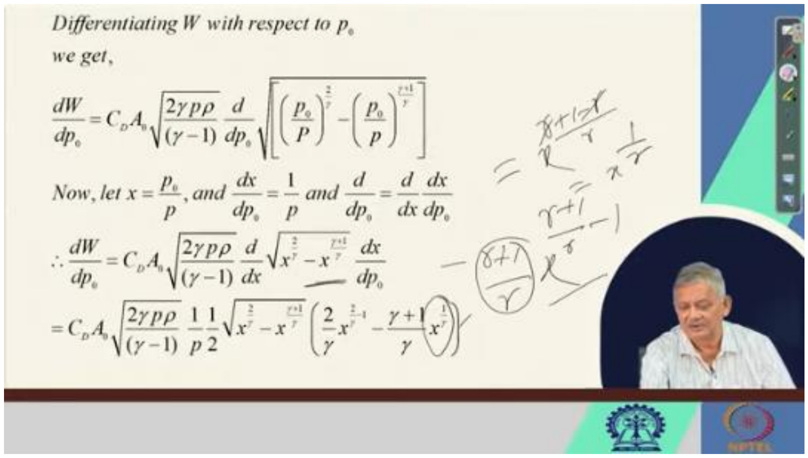
Right, with a negative it was there and now it has become gamma plus 1 by gamma minus 1 that means, 2 gamma goes out. So, 1 by gamma remains. You understood, hopefully, right? Hopefully, you understood that. How did it come like this? So, we are differentiating this, right? x to the power gamma plus 1 by gamma, right. So, that was minus 1 gamma plus 1 by gamma that has come out and x to the power what we have gamma plus 1



By gamma minus 1, right. This can be written as x, leaving this only for this x part, x to the power of this gamma and this gamma. So, x to the power of gamma, common. So, gamma plus 1 minus gamma. So, gamma gamma goes out.

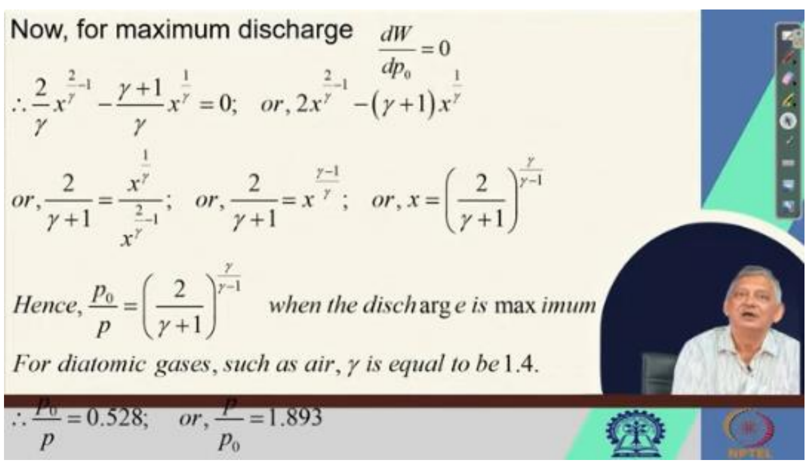


So, it becomes x to the power of 1 by gamma, that is what has happened. Right. Hope you could have followed it properly, right. Then, if that be true, then we got dW / dP_o is C_D A_o under root 2 gamma P rho over gamma minus 1 into 1 by P into 1 by 2 into root over x to the power of 2 by gamma minus x to the power of 2 gamma plus one by gamma

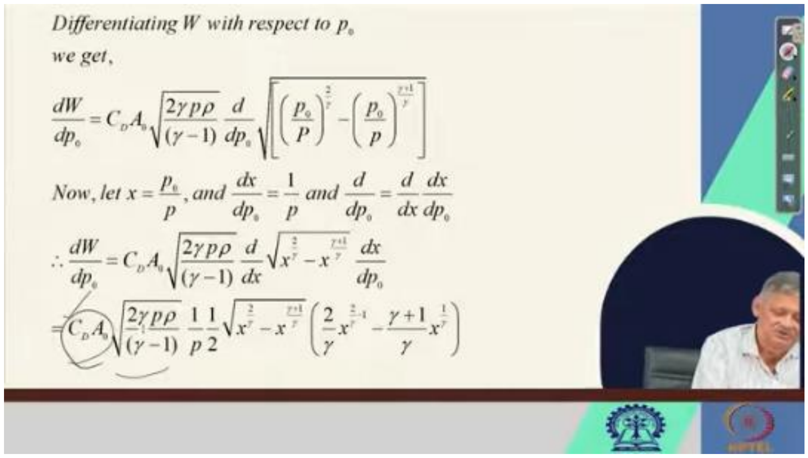


into 2 by gamma x to the power of 2 by, sorry, x to the power of 2 by gamma minus 1 minus gamma plus 1 by gamma to the power of x to the power of 1 by gamma, right. This is our dW / dP_o. Now, the rate of change of W, discharge, right. If that becomes 0, that is dW / dP_o, if it becomes 0, right. So, when will it happen?

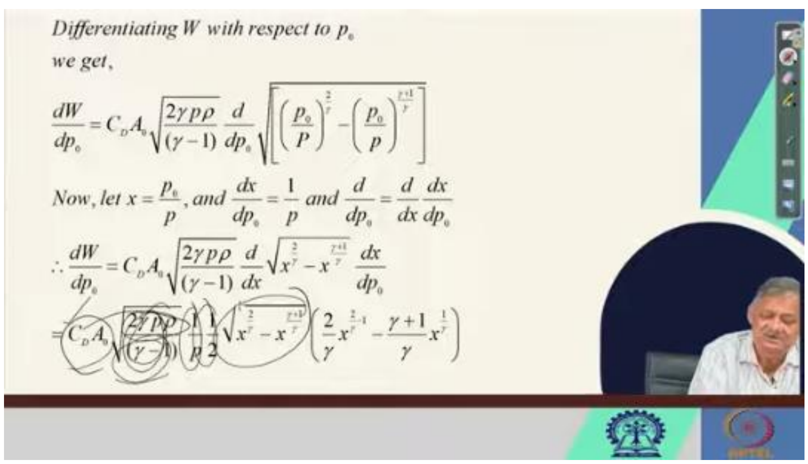
This will happen when the discharge is maximum, right. For maximum discharge, what do we need? That dW / dP_o must be equal to 0, right. So, if it is to be 0, then 2 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma should be equal to 0. Because you see,



What could be 0? dW / P_o is 0, then what could be 0, right? This cannot be 0, where is that, ok. You see, this cannot be C_D A_o, it cannot be 0 because A_o is a definite number, C_D is also a definite number, this cannot be 0. 2 gamma P rho are all some positive quantities.

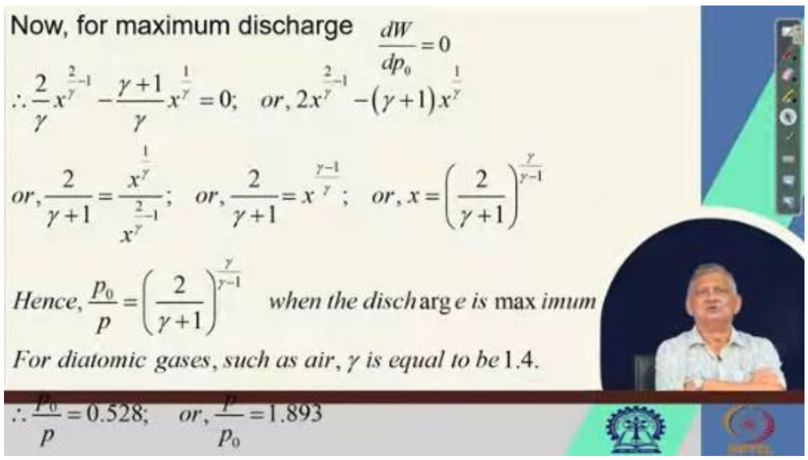


So, it cannot be 0, gamma also has some value, maybe more than 1. So, it is this cannot be also 0, 1 by P, since definite pressure is there, that cannot be 0, 1 by 2, this also cannot be 0. It is very unlikely that under root of x to the power 2 by 7 minus x to the power gamma plus 1 by gamma becomes 0, there, because it becomes more or less 1. One number which we, which we, which we called what as imaginary, right. So, that cannot be 0.



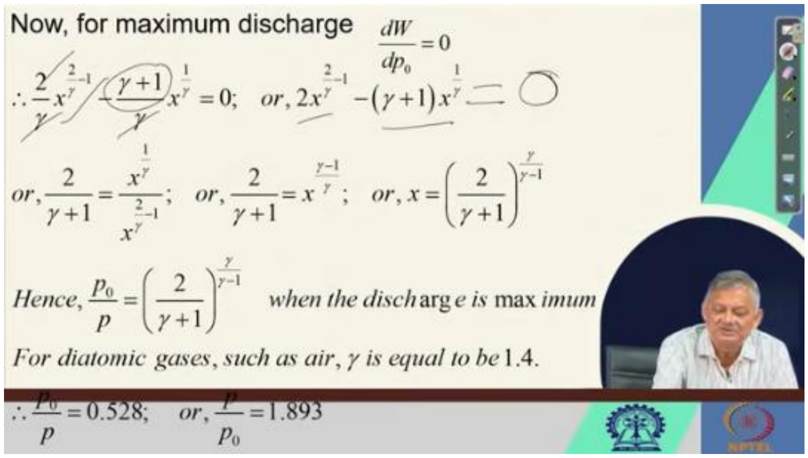
So, only this can be equal to 0, because this is this minus this. There is a possibility that this can be equal to 0. So, if this is becoming 0. That is what we have taken into the next slide: if the rate of discharge with respect to, if the differentiation of the rate of discharge with respect to Po, that is tip pressure, is 0, then we can write 2 by gamma. Into x to the power 2, x to the power 2 by gamma minus 1, minus gamma plus 1 by gamma into x to the power 1 by gamma, that is equal to 0, or 2 into x to the power 2 by gamma minus 1, minus gamma plus 1 into x to the power 1 by gamma, is equal to 0.

Now we can write that 2 by gamma plus 1 is equal to x to the power 1 by gamma by x to the power 2 by gamma minus 1. So, earlier you see, we started with 2 by gamma x to the power 2 by gamma minus 1, minus gamma plus 1 by gamma into x to the power 1 by gamma, is equal to 0, because that is the only thing which can be 0 when dW / dP_o is 0. dW / dP_o means it has to be maximum discharge. Maximum discharge is only possible when dW. The rate of change of P_o has no effect on W. So, dW / dP_o is 0. So, that is possible only when 2 by gamma x to the power 2 by gamma minus 1, minus gamma plus 1 by gamma x to the power 1 by gamma, is equal to 0, right. So, we can write.

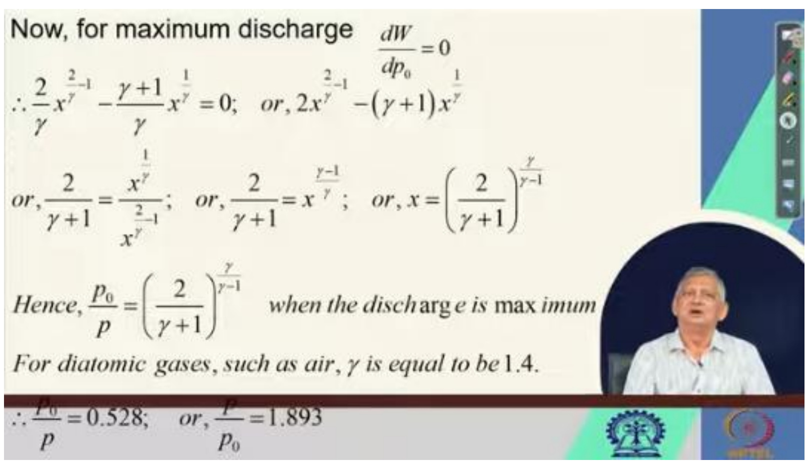


That 2, by rearranging 2 x to the power 2 by gamma minus 1, this side is remaining, right, and this side we are writing this gamma, and that gamma goes out. Right. This gamma and that gamma, both 2 by gamma and gamma plus 1 by gamma, this 2 gamma goes off, right. Then what remains? So, this minus gamma plus 1 into x to the power 1 by gamma is equal to 0, right.

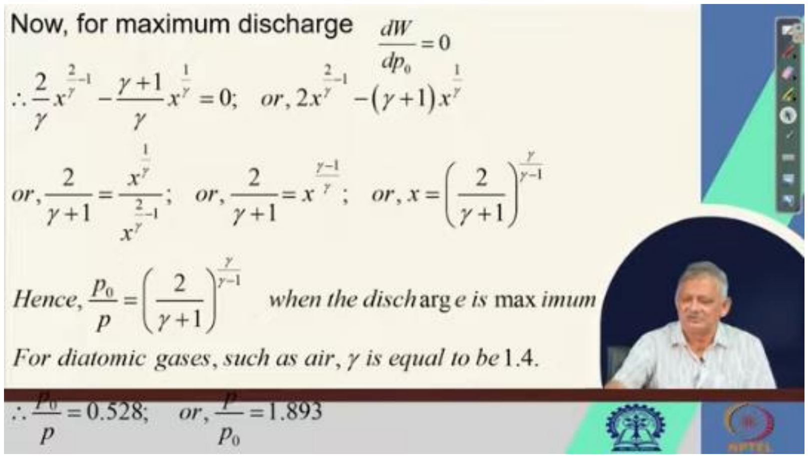
That is what it is not written here. It should have been somehow. This is equal to 0, right? Because this gamma and this gamma, they go out. So, 2 into x to the power 2 by gamma minus 1, that is what is here, and this is minus gamma plus 1 into x to the power 1 by gamma. So, that is equal to 0, right?

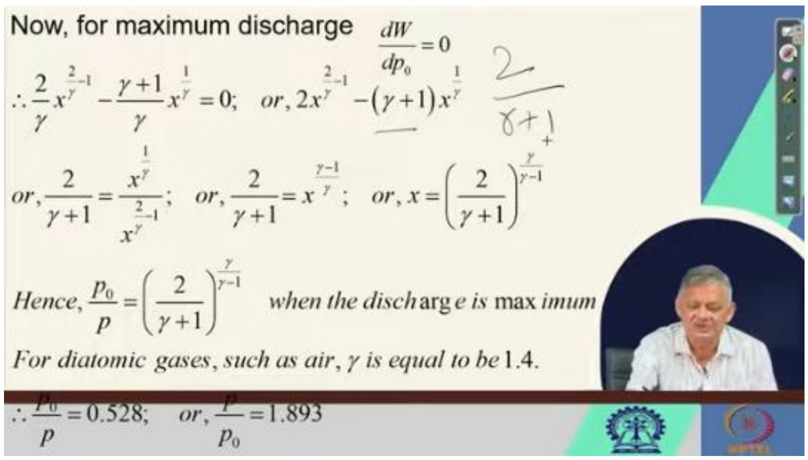


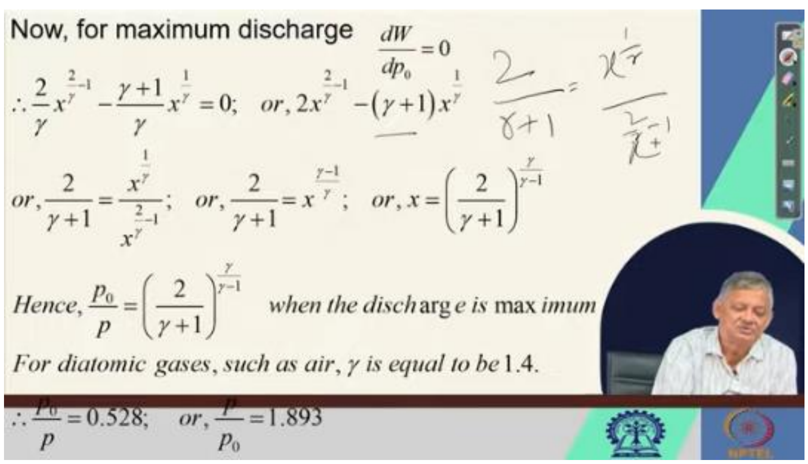
So, that correction, please do. Now, what is it that we have? 2 x to the power 2 by gamma minus 1 minus gamma plus 1 into x to the power 1 by gamma, right? Therefore, we can write 2 by gamma plus 1 that is equal to x to the power 1 by gamma. Over x to the power 2 by gamma minus 1, right? Gamma plus 1.



So, that means, what did we do? We divided the 2 by this gamma plus 1, right? This we have taken to one side, and this is equal to, because that gamma has gone there, x to the power 1 by gamma. Divided by x to the power 2 by gamma minus 1, that is what explicitly we have done, right? So, 2 by gamma plus 1 remains, and this x to the power 1 by gamma divided by x to the power 2 by gamma minus 1, that means, that x to the power 1 by gamma minus 2 by gamma.



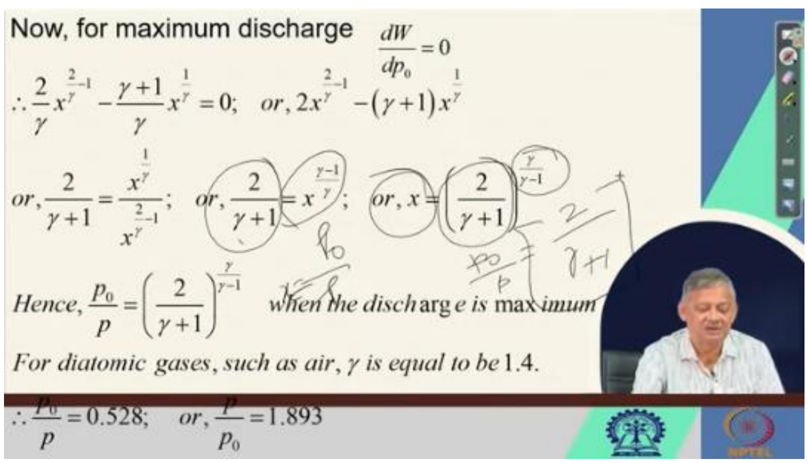




Plus 1, that means x to the power this gamma is common. So, it is 1, this is minus 2, and here it is plus 1 gamma, right? That means equals to x to the power gamma plus 1 by gamma. Is it coming? 2 by gamma plus 1 x to the power gamma, oh sorry, yes, gamma minus 1, it is minus 2 plus 1. So, gamma minus 1, sorry, gamma minus 1, because it is coming, sorry, it is coming that this, yeah. This is 1 minus 2 plus gamma by gamma, that means gamma x to the power gamma, and minus 2 plus 1 is minus 1, gamma minus 1 by gamma, yeah, that is what it has come, x to the power gamma minus 1 by gamma, right.

So, if that be true, then we can say that. 2 by gamma plus 1 that has become 2 by gamma plus 1 that has become x to the power gamma minus 1 by gamma, right. Or, in other words, we can say x has become 2 by gamma plus 1. To the power gamma by gamma minus 1, right. That is what explicitly has happened, 2 by gamma plus 1, is x to the power gamma minus 1 by gamma, or x is equals to 2 by gamma plus 1, that 2 by gamma plus 1, this as inverse.

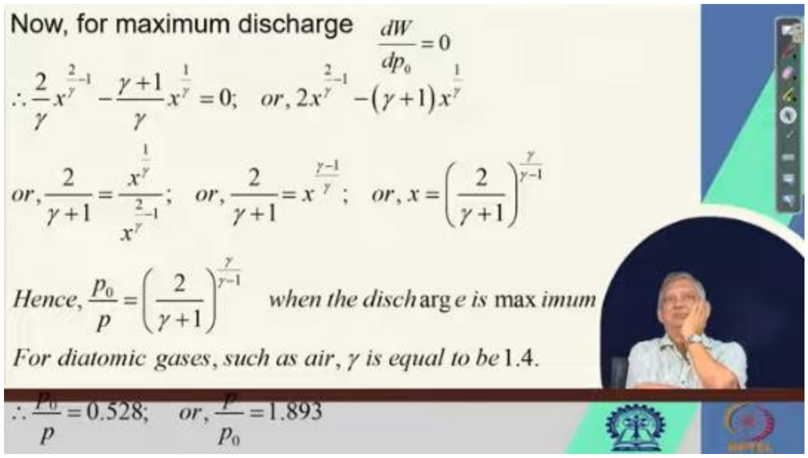
So, gamma by gamma minus 1, right. So, if that be true, now we replace x, now we replace x, x with what originally we have taken, p is equals to x P_o by p is equals to x, which originally we took. So, P_o by p, right, that is becoming 2 by gamma plus 1 whole to the power. Gamma by gamma minus 1, right. So, if that be true, then we can write now that P_o by P is equal to 2 by gamma plus 1 by gamma plus 1 by.



2 by gamma plus 1 to the power gamma by gamma minus 1, right? That is the pressure ratio. Now, mind it here, our pressure ratio is outlet, that is tip to the inlet, that is more, right. So, that is why it is coming from high to low. That is why we gave the example of refrigerant.

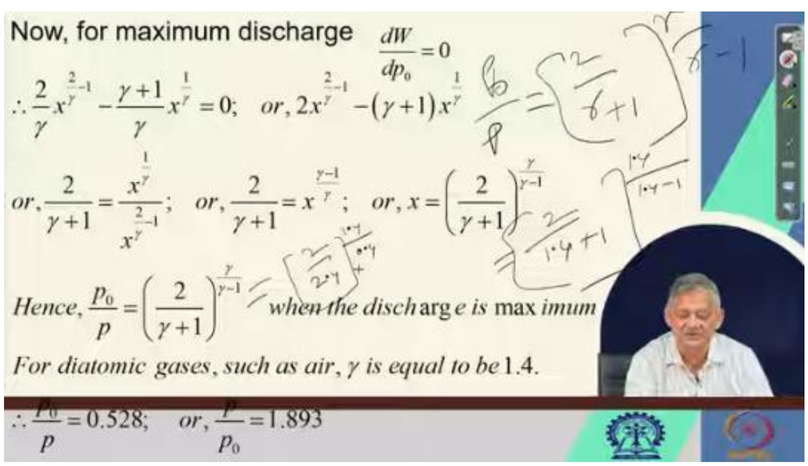
That is why we gave the example of whistling, all these. That means, P_o is the tip having low pressure, and P is the high pressure. So, P_o by P is a ratio. And it should be less than 1 because P_o is less than P, right. So, if we now say that P_o by P is equal to 2 by gamma plus 1 to the power gamma by gamma minus 1, right.

So, when the discharge is maximum, then it becomes P_o by P equals to 2 by gamma plus 1 to the power gamma by gamma minus 1, right? And if that be true, then if we now take that one gas which is diatomic in nature. If diatomic gases are there, then its gamma value is 1.4, it is known. It is not that it is coming from somewhere else, it is known.

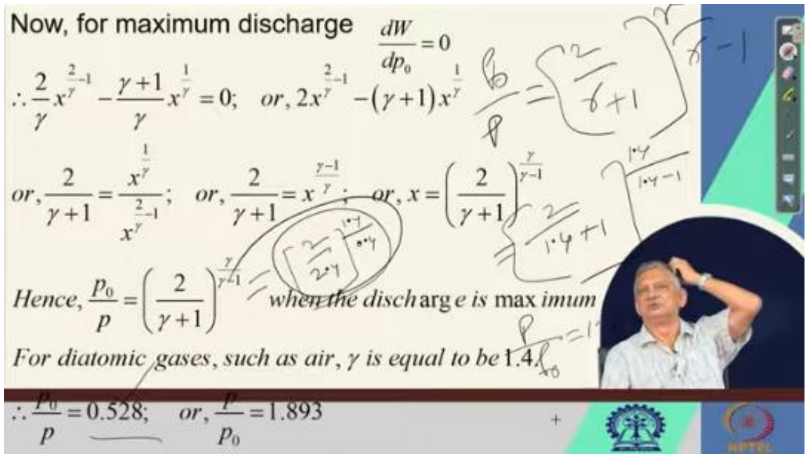


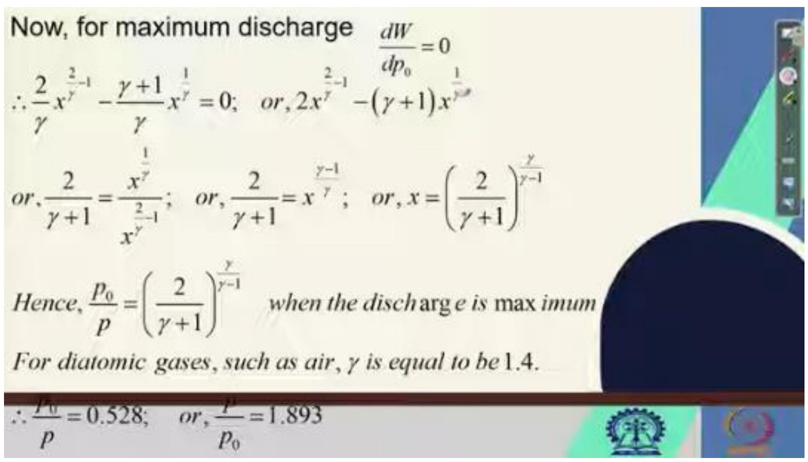
A diatomic gas has a gamma, which is the heat capacity ratio C_P over C_V, of 1.4, right. So, therefore, if it is a diatomic gas, then we write P_o over P to be equal to 0.528, P_o by P to be 0.528, because P_o by P is equal to 2 by gamma plus 1. to the power gamma by gamma minus 1. Now, P_o by P is equal to 2 by 1.4 plus 1 to the power 1.4 divided by 1.4 minus 1. So, this is equal to 2 by 1.4 plus 1, which means 2 by 2.4

to the power gamma, that is 1.4 by 0.4, 1.4 minus 1, which is 0.4. So, if you solve it, then it comes to 0.528, right, it comes to 0.528. So, this is the critical value of the pressure that pressure ratio, outlet tip to the inlet, when it becomes 0.528, then the discharge becomes maximum, or inversely, this is tip to inside, or if it is inside to tip, P by P_o, then it is reversed, it is 1.893, right.



So, this we have to keep in mind for maximum discharge, the pressure ratio that is tip to inlet is 0.528, or the inlet to the tip is 1.893. With this, we have come to the end of this class. So, we will meet again to carry forward the remaining part. I am so thankful to you that you are carefully listening to the class.





Thank you all.