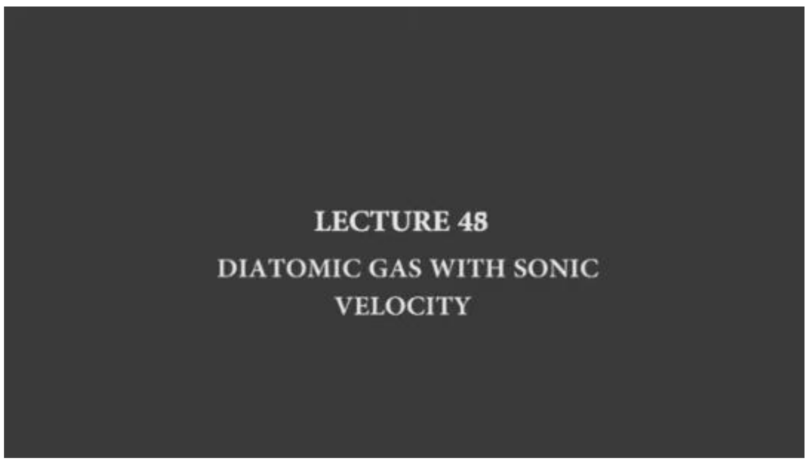
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture48

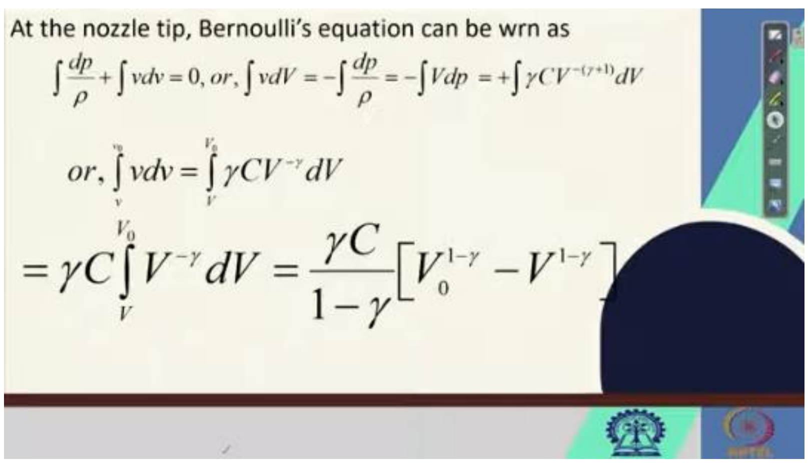
LECTURE 48 : DIATOMIC GAS WITH SONIC VELOCITY

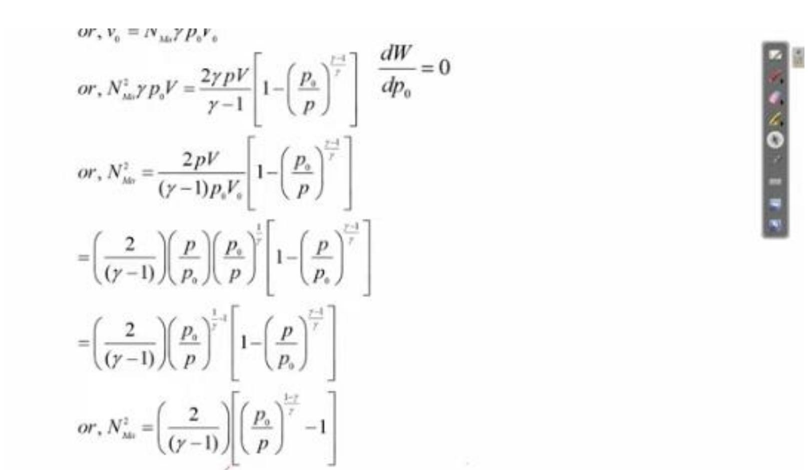
Good morning, my dear boys and girls, students, and friends. Right? We are discussing the different characteristics of the velocity of sound. Right? We have shown you earlier how sounds, such as sirens or other things, work.



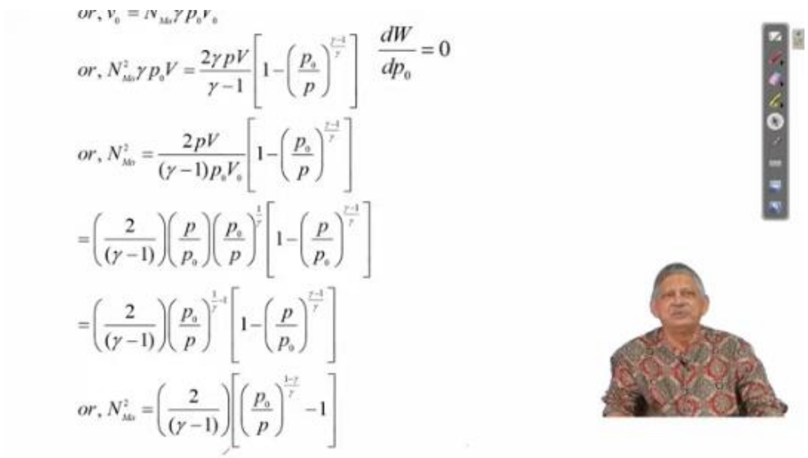
We have also discussed that. Now, we are dealing with the velocity of sound in a medium, and in this case, we are talking about the air medium, right? And we have defined a number called the Mach number. So, the Mach number N_{Ma}, we have equated it with the velocity at the tip, that is, v_0 equals N_{Ma} under

capital V_o, right? We will go to that slide specifically. So, here we have seen that V_o is N_{Ma} under root gamma P_o V_o, where N_{Ma} is the Mach number, right? Now, V_o, as we have defined, is N_{Ma} under root gamma P_o V_o, then V_o squared—we are making a square.

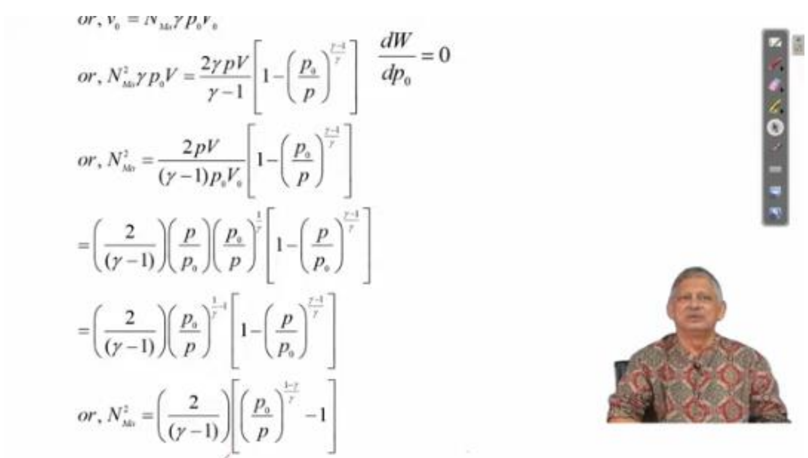




The v_o square is equal to N_{Ma} gamma P_o V_o, right. Of course, N_{Ma} is also under square, right, because v_o we have made square, then N_{Ma} is also square, but that square root under gamma P_o rhoo or V_o rather gamma P_o and capital V_o that is now going without root, right. So, if we rearrange, then we can write N_{Ma} square is gamma P_o V, capital V is equal to 2 gamma PV by gamma minus 1 into 1 minus P_o by P to the power gamma minus 1 by gamma. Right. And we have also shown earlier that dW/dP_o, that is discharge with respect to tip velocity, is 0 when the flow is maximum.

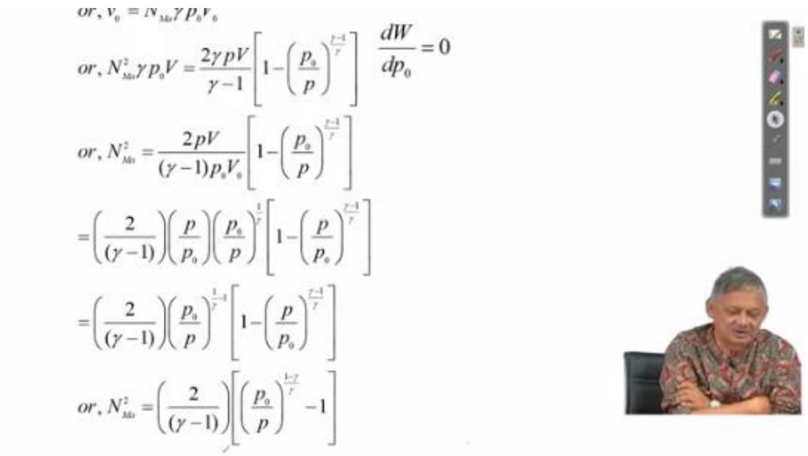


Therefore, we can write N_{Ma} square is equal to gamma 2 P capital V by gamma minus 1 into P_o V_o into 1 minus P_o by P to the power gamma minus 1 by gamma, right. So, this is equal to 2 by gamma minus 1 into P by P_o into P_o by P to the power 1 by gamma because here it is P by P_o. So, here we have inverted it that as P_o by P to the power 1 by gamma into 1 minus P by P_o to the power gamma minus 1 by gamma, right.



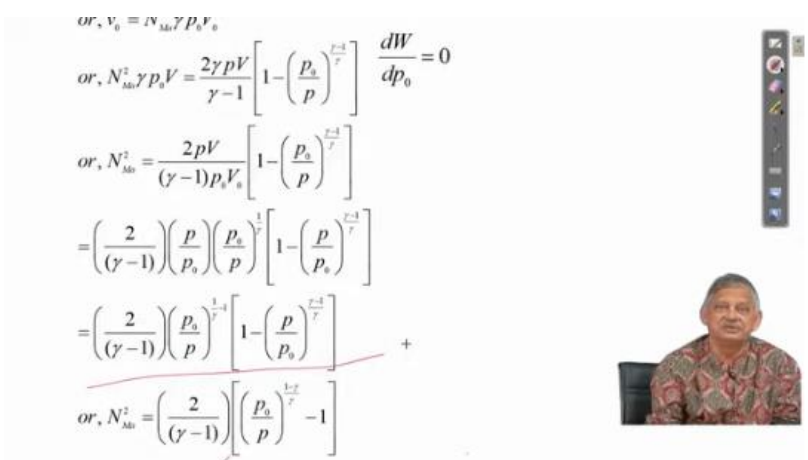
Earlier, Here you see that 2 P V, capital V by gamma minus 1 into Po Vo into 1 minus P_o by P to the power gamma minus 1 by gamma. So, that we have taken as 2 by gamma minus 1 remains Now P by P_o, that is identical and Po by P, we have P_o by P equals to V by V_o

or V to the power gamma by V_o to the power gamma is P_o by P. So, P_o by P is V by V_o to the power 1 by gamma, yeah, right.

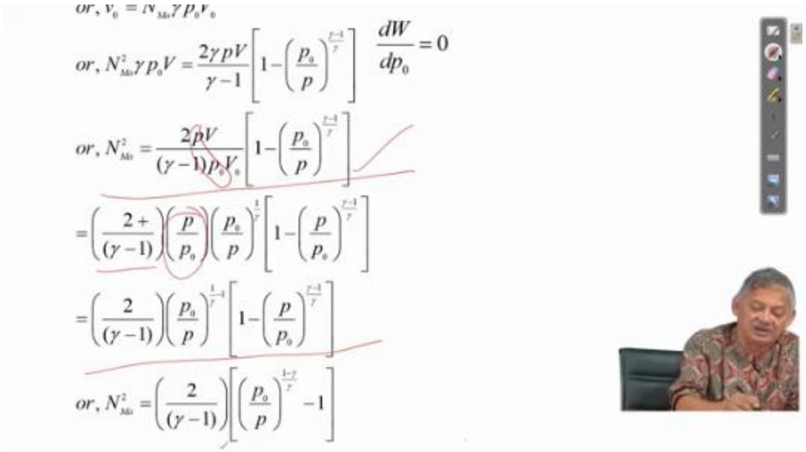


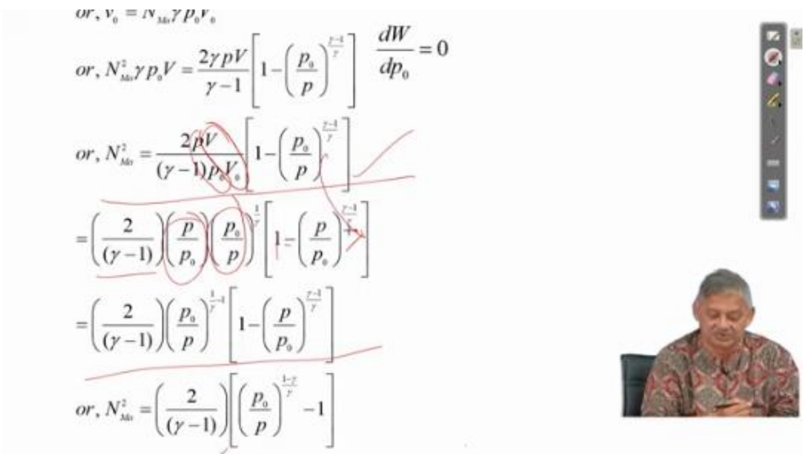
So, that is 1 minus P by P_o to the power gamma minus 1 by gamma or 2 by gamma minus 1 is P_o by P to the power. 1 by gamma minus 1, because P_o by P is inverted as P by P_o is inverted as P_o by P, right. So, if we take it inside, then it becomes 1 plus, it becomes 1 plus 1 by gamma, that is gamma 1 plus gamma 1 by gamma rather. Now, again taking that here, the difference here is that the problem is solved. I do not know. I told you right from the beginning that in my slides, there could be some errors, which particularly I have kept and not cleared or corrected.

Why? Because if you are looking at it, then it is your duty to check whether it was correct or not. This means you are checking. Here, our thing was N_{Ma} square is equal to 2 gamma PV capital V by. gamma minus 1 P_o V_o into 1 minus P_o by P to the power gamma minus 1 by gamma. Now, suddenly in the next line, we wrote 2 by gamma minus 1 into this P by P_o that remains there.

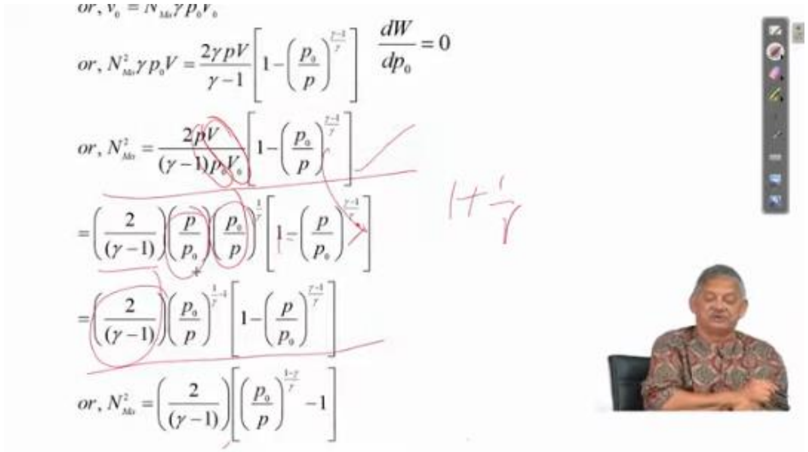


So, P by P_o. And V by V_o that is P_o by P to the power 1 by gamma, that is also right. And we have changed suddenly here, 1 minus P by P_o to the power gamma minus 1 by gamma. That means this we have inverted; there is a negative sign because gamma minus 1 by gamma, that there should have been a negative sign. Which we have not shown, right.

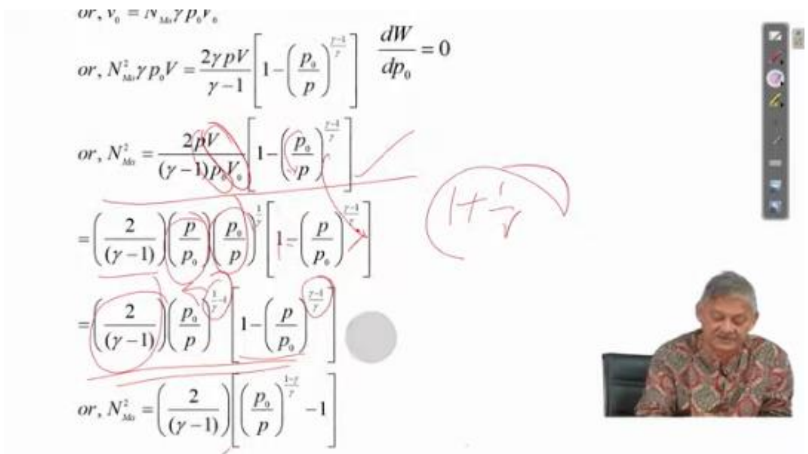




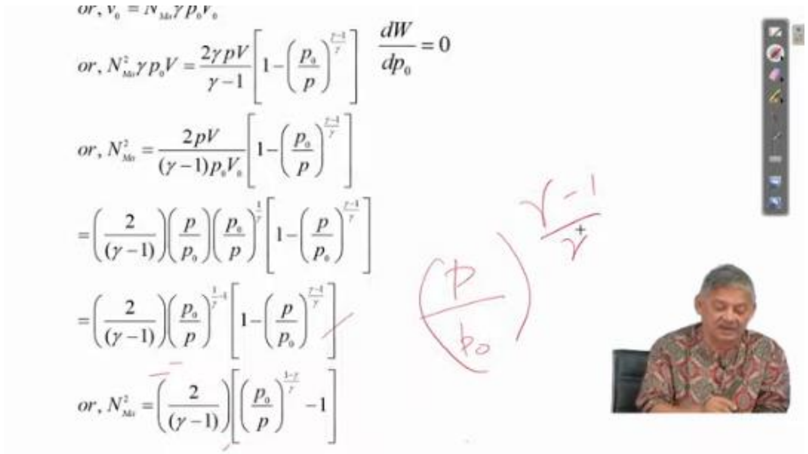
So, that means, in the next line, 2 by gamma minus 1 remains identical: P_o by P into P_o by P. Meaning 1 plus 1 by gamma because they are adding, they are multiplying, so adding. So, it became 1 plus 1 by gamma, but we have taken 1 by gamma minus 1. That means a negative 1 was there. So, this negative has taken care of this P by P_o, suddenly inversion of this, right?



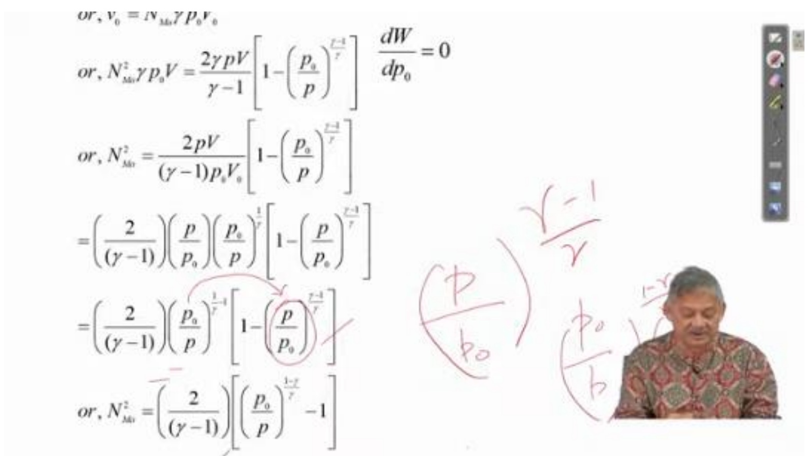
So, it is 2 by gamma minus 1 into P_o by P to the power 1 by gamma minus 1, that into 1 minus P by P_o to the power gamma minus 1. So, this means This means that the negative sign is duly taken care of, right? The negative sign is duly taken care of. So, we can write N_{Ma} squared is equal to 2 by gamma minus 1.

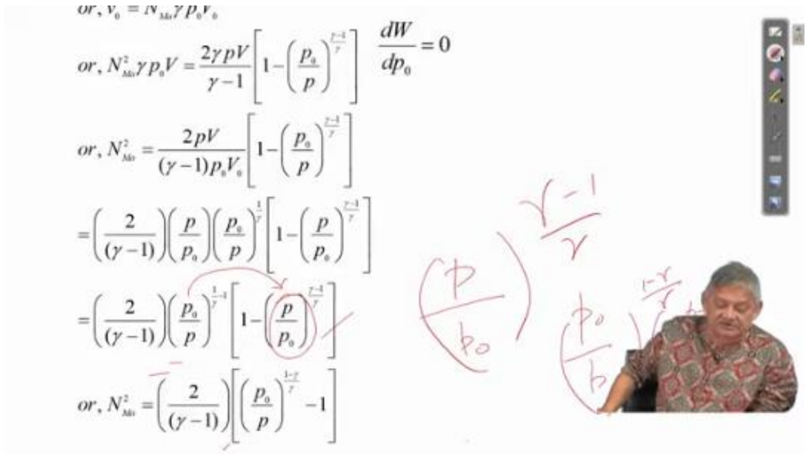


into P_o by P to the power 1 minus gamma by gamma minus 1. Why again? You see here, it was P by P_o to the power gamma minus 1 by gamma. Here, P_o by P to the power 1 by gamma minus 1, that means 1 minus gamma by gamma, and this is adding with or multiplying with this P_o by P, that means it will be changed to P by P_o. OK.

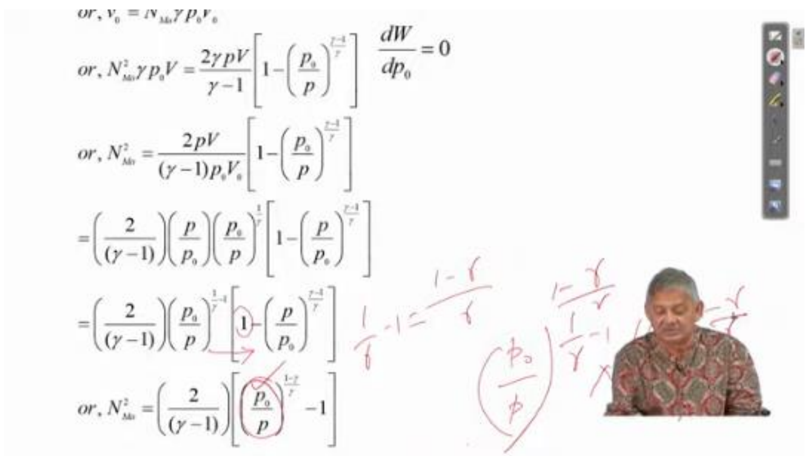


So, if we make Po by P that means, P_o by P if we make this. then its power gets changed that is minus 1. So, 1 minus gamma minus gamma by gamma that means gamma 1 minus gamma by gamma is the solution right and that minus 1, this we are multiplying with this P_o by P to the power 1 minus gamma, right and this and that cancels out making a 1, only 1 is there right. So, this on multiplication with 1 has become P_o by p to the power 1 minus gamma by gamma because this is 1 minus gamma by gamma only because 1 by gamma minus 1 is equal to 1 minus gamma by gamma only right.

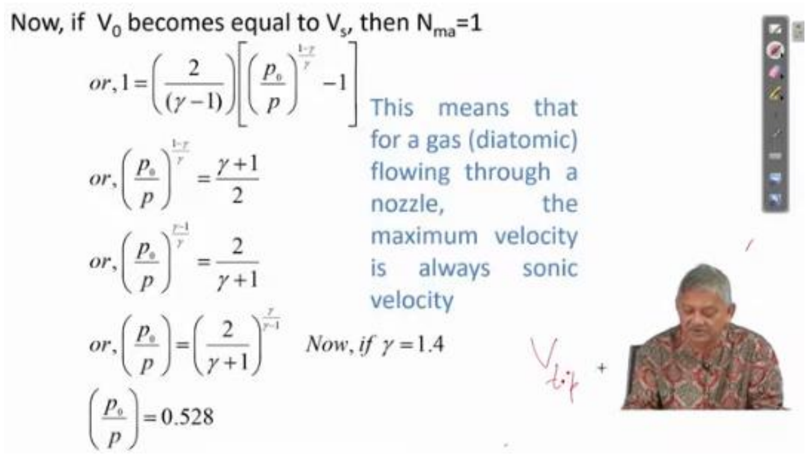




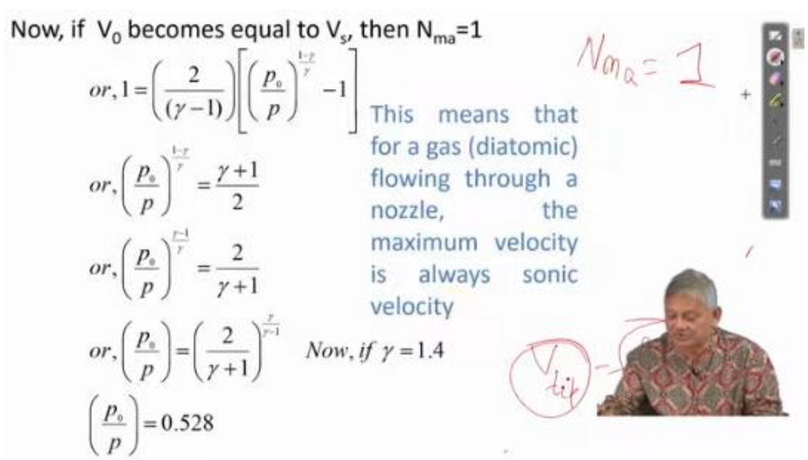
So, this is ok. Now, P_o by P to the power 1 by gamma minus 1 or 1 minus gamma by gamma right into P by P_o to the power 1 minus gamma by gamma. right. So, 1 minus gamma by gamma remains same and this is P_o by P and P by P_o. So, they are cancelling out, right.

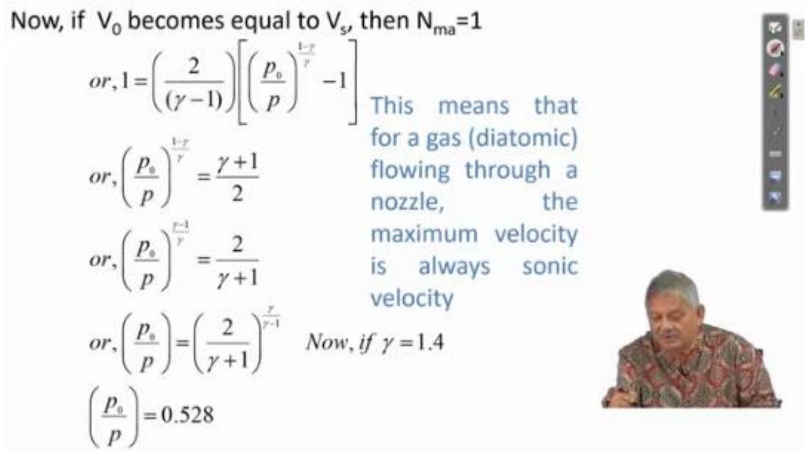


So, this way it has become P_o by P to the power 1 minus gamma by gamma that is there and N_{Ma} square, we have written to be 2 by gamma minus 1 into P_o by P to the power gamma minus 1, 1 minus gamma by gamma minus 1. right. Then N_{Ma} square, we have seen this, and we can further write if v_o, v_o becomes equal to v_s that is v_{tip} is equal to v_s, v_{tip}. is equal to v_s right, tip velocity is the sound velocity.



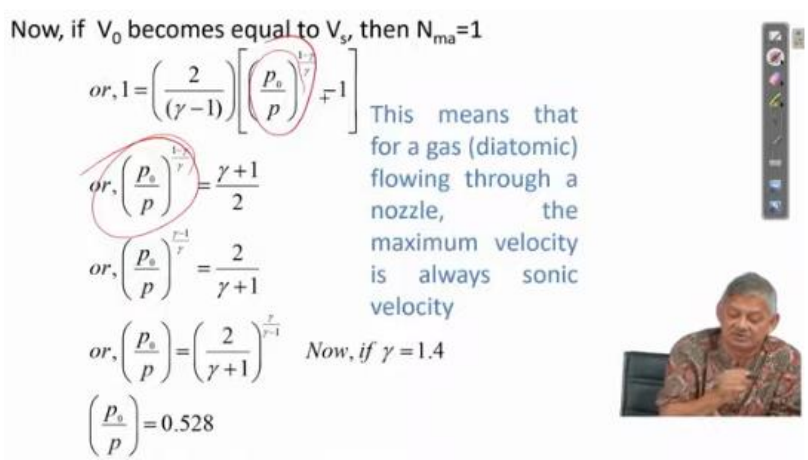
Then N_{Ma}, that is the Mach number that becomes equal to 1, right. So, in the previous slide, we have shown in the previous slide, we have shown that N_{Ma} squared, N_{Ma} squared into 2 by gamma minus 1 into P_o by p, 2 to the power 1 minus gamma by gamma minus 1. Right. So now, N_{Ma} has become, because when we said, when V_{naught} is equal to Vs, that is, the tip velocity becomes the velocity of sound, then N_{Ma} becomes equal to 1, which means this is now 1, okay.



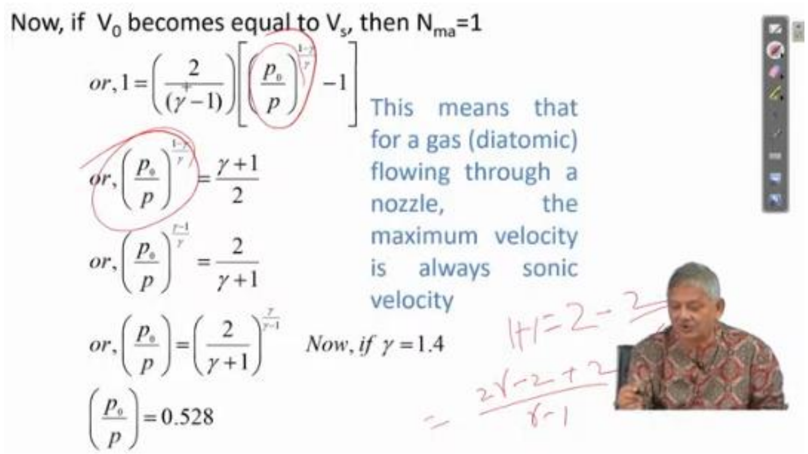


Let us look into If we make N_{Ma} equal to 1, then where do we go? We go here that 1 is equal to 2 by gamma minus 1 into P_o by P to the power 1 minus gamma by gamma minus 1. Therefore, we can write by changing the side, the P_o by P to the power 1 minus gamma by gamma.

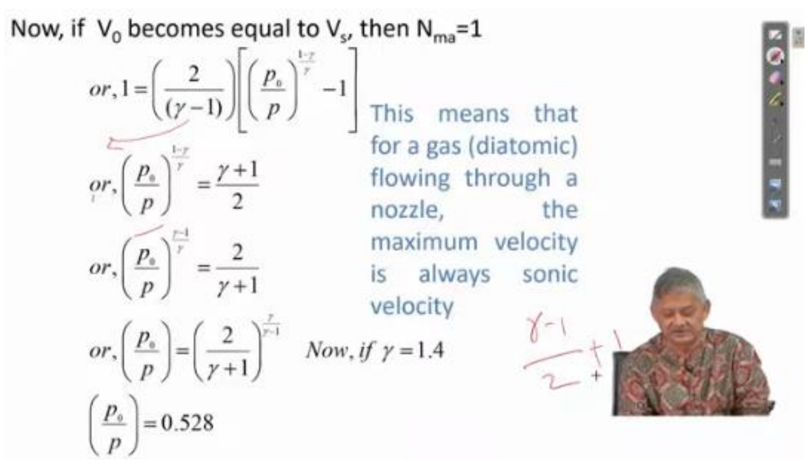
This is equal to 1 minus 2 by gamma minus 1, right. Here you see that, this P_o by P to the power 1 by gamma, 1 minus gamma by gamma, that remains and this we are shifting to that. So, it is becoming 1, that one, and this one becoming Right, and this becomes minus 2 by gamma minus 1.



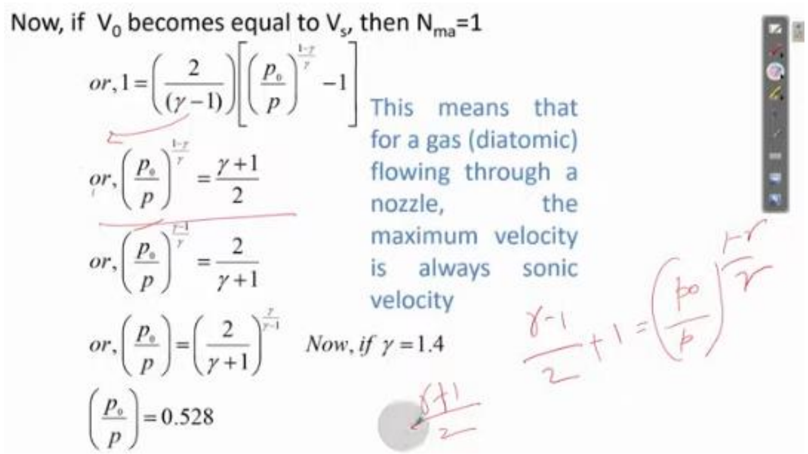
So, that means if it is gamma minus 1, then it is 2 gamma minus 2 plus 2, right. 2 gamma minus 2 plus 2, that is gamma plus 1 by 2, right? OK. No, the other way round. The other way round means this way. What is the other way round? The other way round is that this is divided here first, right?



Then gamma minus 1 by 2. OK, and this minus 1 becomes plus 1, right. This minus 1 becomes plus 1, and this is equal to P_o by P to the power 1 minus gamma by gamma. Right. So, 2 minus 1 that is gamma minus 1 by 2 is equal to P_o by P to the power 2.

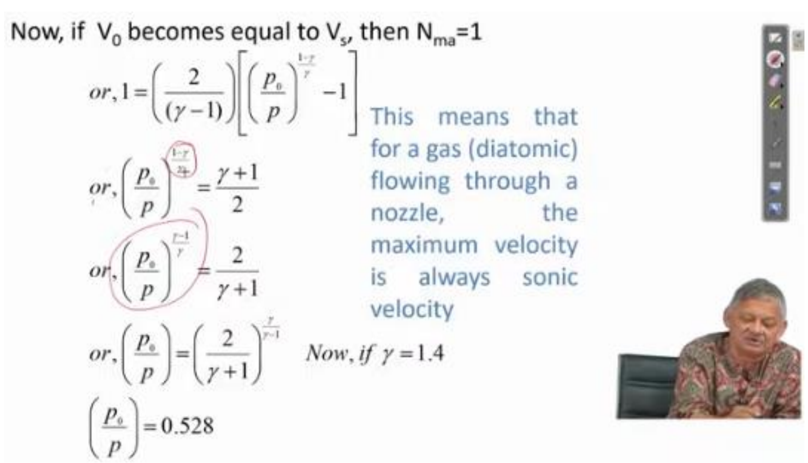


Gamma 2 plus 1, right? Sorry, this is 2 minus 1, this is gamma plus 1 by 2, is equal to P_0 by P to the power 1 minus gamma by gamma. So, that is exactly what we are getting, OK. Then that is exactly what we are getting. Then we have found out that what is P_0 by P to the power 1 by gamma minus 1.

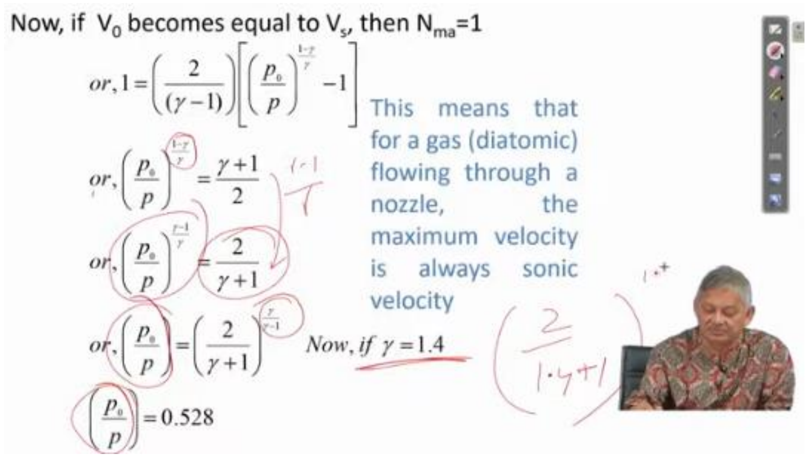


So, rather what is P_o by P to the power 1 minus gamma by gamma that is equal to gamma plus 1 by 2, right? So, this we can rewrite as P_o by P to the power gamma minus 1 by gamma. It is the inverse of that, which is 2 by gamma plus 1, right? So, here only we have changed the sign, that is 1 minus gamma by gamma to gamma minus 1 by gamma, right? Sorry. We write that, hence we write that.

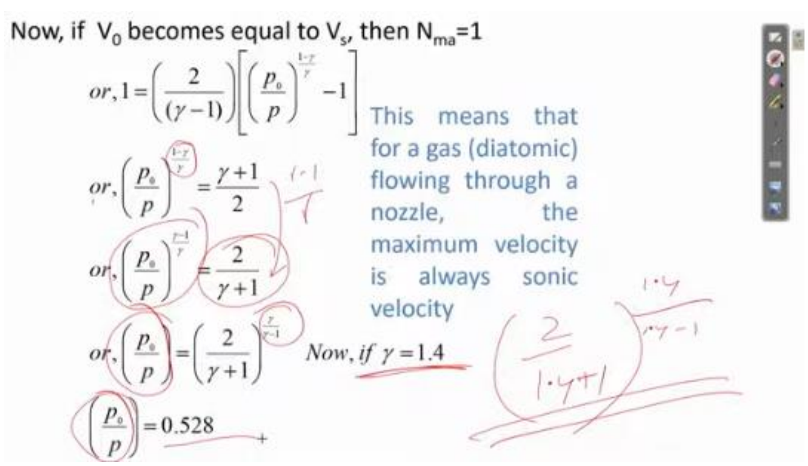
P_o by P to the power gamma minus 1 because this was 1 minus gamma by gamma. So, we have made 1 minus that, which is gamma minus 1 by gamma with negative, right? So, that we have made this the inverse, 2 by gamma plus 1. Right, therefore, P_o by P can be written as 2 by gamma plus 1 to the power gamma by gamma minus 1, right? So, P_o by P, that 2 by gamma plus 1, if gamma is 1.4, so that means 2 by 1.4 plus 1 to the power



1.4 by 1.4 minus 1, this becomes equal to 0.528, earlier also we have seen. This becomes equal to 0.528. This is a unique thing, that is for maximum discharge for a medium whose gamma is 1.4, the tip velocity to the tip pressure to the rather interior that pressure ratio becomes 0.528. Now, if it is 0.528, we have seen earlier that means the velocity is the sonic velocity rather, the other way around, the velocity gives maximum discharge and maximum velocity.

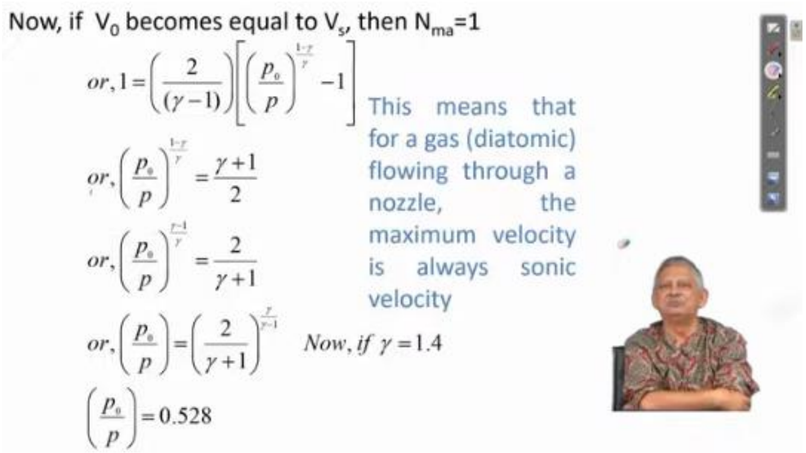


Here, we have said earlier that vo becomes equal to vs, that is, the tip velocity has become the velocity of sound. When the Mach number is 1, and when the Mach number is 1, then the velocity of sound, the pressure ratio becomes critical. That is, the discharge is maximum, and the velocity is also maximum. So, the maximum velocity attains the sonic velocity, that is vs, right? So, that is what we have said, that this means for a diatomic flowing through—for a diatomic



flowing through a nozzle, the maximum velocity is always the sonic velocity, right? That is why we get the sound, right? The sonic velocity—unless it is sonic velocity, we cannot get sound. In this respect, I tell you one thing that I said sometime back in one of the classes: those who stay near the airways—the airways, I mean, the defense airways—those who are there, they have definitely come across that at some point, it is becoming a sound suddenly coming, right? And that sound, when it is coming, it is like the velocity of the airplane that is either crossing the velocity of sound—that is called supersonic—or it is under the velocity of sound—that is subsonic. So, whenever this change is occurring, right? I do not know how many of you have seen very Very recent cars also have this facility that in those cars, what is there? In those cars, there is a number of kilometers selected, like if it is going above 80. Then there is a beep sound, and when it is coming from above 80 to below, then also there is a beep, right?

So, here also, if the velocity of sound is equal to the velocity of the plane, then fine, but when it is crossing that, then it becomes. And when it is again coming below the velocity of sound, then it is called subsonic. In both cases, there will be a booming. A very loud sound, and yeah, it is to the extent that your windows, if they are made of glass or have loose glass, may crack or get broken. It gets.



We, being at IIT Kharagpur, have faced this or we face it very, very commonly, right? So, they are the play of the velocity of sound, right? So, we have shown that when the Mach number is 1, then the velocity of sound is the sonic velocity, which is the maximum velocity, and the discharge is also maximum, right? If the tip velocity becomes the velocity of sound, then the velocity is maximum, and we are getting the discharge maximum, right?

Getting the discharge maximum, right. With this, our time for this class is over. So, thank you for carefully listening. Thank you all.

Now, if V_0 becomes equal to V_s, then N_{ma}=1

$$or, 1 = \left(\frac{2}{(\gamma-1)}\right)\left[\left(\frac{p_0}{p}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]$$

$$or, \left(\frac{p_0}{p}\right)^{\frac{1-\gamma}{\gamma}} = \frac{\gamma+1}{2}$$

$$or, \left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} = \frac{2}{\gamma+1}$$

$$or, \left(\frac{p_0}{p}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad Now, if\ \gamma = 1.4$$

$$\left(\frac{p_0}{p}\right) = 0.528$$

This means that for a gas (diatomic) flowing through a nozzle, the maximum velocity is always sonic velocity