Evolutionary Dynamics

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Week 04

Lecture 16

All right, welcome back to the next video. As we saw in the previous one, the Bacterial growth can be compartmentalized into several phases. And when we compare different species of bacteria and ask the question of which one is fitter in a particular context—a particular environmental context—it is not so easy to comment because we might have situations where species A is better in some of those compartments and species B is better than A in the other compartments. So how does this sum total...

How do we compare bacterial species growing in an environment and actually be able to say which one is fitter? We need some sort of quantitative understanding to be able to comment on these. And that's what we'll be focusing on in this particular video. So let's imagine that we have species A. The most popular model of growth, which models exponential growth, is represented by

So this is a little bit of calculus, which we will take our time going over and trying to understand what this actually means. So this model is used for modeling growth in the exponential phase. And let's try to understand what this means. Before we go there, let's try to understand what we mean by dn by dt. Let's imagine we have a scenario where we have variables x and y. So y is some function of x, and we have some sort of relationship between these two variables.

Let's imagine this. Now, if I choose a particular time x1 and another particular time called x2, then at these two times, I can ask the following question. And the question I am asking is: what is the rate of change? What is the rate of change in y with respect to x? We can answer that question by observing that when the value of x was x1, the value of y was y1.

And when the value of x was x2, the value of y was y2. So, over this period of time going from x2 to x1, the rate of change can be measured as delta Y—how much change took place in Y—divided by delta X—how much change took place in X. So, this is change in Y divided by change in X. This can be estimated as: change in y is simply y2 minus y1, and change in x is x2 minus x1. This quantity is measured over a finite range of x, when x changes from x1 to x2. The next thing we do is ask the same question.

However, we bring x2 a little closer to x1. We change the location of x2 and bring it here. Now, our answer changes slightly because the new y2 is here. So, The formula remains the same; everything remains the same except for the window of x over which we are asking this question has shrunk from this larger window to this smaller window now.

We can keep doing this process and keep bringing X2 closer to X1 and so on and so forth. When we bring X2 extremely closely to X1, so now we keep Estimating the above quantity, we keep estimating the above quantity but by bringing X2 closer to X1. They will come at time when x2 is really really close to x1 and is only infinitely larger than x1. When x2 is equal to x1 plus h where h is infinitely small.

So in this graph, this is X1, and we keep bringing X2 closer. Eventually, we'll reach a stage where X2 is very, very close to X1. I mean, what I've drawn X2, you can still see something, but we can zoom in and keep bringing X2 closer to X1. And what you will see is that as a result, y2 is also becoming closer to y1 and x2 is very close to x1. In such a scenario when x2 is infinitely close to x1 but never quite equal to x1, in that case,

Delta Y by delta X is simply referred to as written as T Y by DX and that is sort of the beginning of calculus understanding what this means. in this geometric context. And I hope this is clear to everyone that when we are talking over finite differences between this range of x in this finite window x1 to x2, delta y by delta x is simply the change in y divided by change in x. And dy by dx is exactly the same except for a special case that x2 is infinitely close to x1. And that's the only difference. OK, if I hopefully that makes sense to everybody.

And now let's go back to our growth experiment. Modeling this growth process. So what we have here is... When we write dN/dt, we are asking how N is changing. With time.

That is what the left-hand side means. The right-hand side tells us the answer to the question we are asking on the left-hand side. It says that N is changing via multiplication of these two factors. One is simply N. So the more N there is, the faster the change. This makes sense in the context of binary divisions, because if we have only one bacterium, then after one round of growth, this goes to two.

So what is the ΔN that took place? That's only one. But if we had two bacteria, after the same amount of time—if there are sufficient resources—we now have four. So what was the ΔN that we now have? It's two.

And same idea that if we repeat this, now delta N is equal to four. So the rate at which here transition happened from 1 to 2, here it happened from 2 to 4, then it happened from 4 to 8 and so on and so forth. So the rate is proportional to how many bacteria did you have at that time when you are measuring that how is this rate changing with time. The more bacteria you have, the faster the rate of change it's going to be. All of this is assuming that this is exponential growth and there are sufficient resources available for everybody to grow.

R is simply a measure of growth rate in these conditions. That is just a measure of how long does each one of these take. Is it 20 minutes or is it 2 hours? And that depends on the environmental context in which this growth is being measured. So I hope that this makes sense to everybody, but what this tells us is that growth in exponential phase can be measured as DN by DT is equal to RN.

And we can integrate this, N is a function of time, so we get DN upon N equal to RdT. And we integrate this from t equal to 0 to some future time t. At t equal to 0, the number of bacteria were n naught and at some future time t, the number of bacteria were n t. And when we integrate this, we get ln of n and we apply the limits n naught and n t. And this is simply r t because integral of a constant is just 1 here is just t and we apply the limits 0 to t. And we get ln of nT minus ln of n0 equal to RT minus 0, which is just RT. And this can be written as ln of nT upon n0 is equal to RT.

$$\frac{dN}{dt} = rN$$

which is simply just N_t upon N_naught equal to e to the power r t, which leads me to the final expression N_t is equal to N_naught e to the power r t. And that is what that is where we get the equation for growth in the exponential phase. So if at the start of the exponential phase, the number of bacteria is N_naught and growth has taken place for T amount of time. Then after this T amount of time, the number of bacteria that's present in the culture is N_T. So suppose we have the value of N_naught, R is given to us, and T is something that we want to find out—how many bacteria are there after, let's say, four hours or six hours or something.

$$N_t = N_o e^{rt}$$

Then if these three quantities are given to us, we can estimate the number of bacteria that's present in the culture by simply applying this formula. All of this is assuming that exponential growth is valid for the duration and the conditions in which we are trying to model. Now, the problem with the exponential phase is that it assumes that numbers are going to increase like this. And as we will see, that exponential growth—if I assume that E. coli is dividing every 20 minutes—and so let's say, 20 minutes means that r is equal to 1 upon 20.

And I start a culture in a flask where I seed, let's say, just one E. coli. So N_naught is just equal to one. My starting population size at the beginning of the exponential phase is just equal to one. And I let this culture grow for three days. which means that it's 24 into 3, that's 72 hours.

And the way it works out is that since E. coli divides every 20 minutes, there are three divisions taking place in one hour. So if we start with one bacterium, after 20 minutes, I have two. After another 20 minutes, I have 4. And after another 20 minutes, that's 1 hour now, I have 8. I have eight now.

So what that means is that if these are the conditions, then in one hour, I have three generations, as we just saw here. So in 72 hours, I have 72 multiplied by three, that's 216 generations. If N0 was equal to 1, then NT is simply equal to N0 multiplied by 2 to the power of 216. That is the number of bacteria that I have after exponential growth for 3 days. Here is an interesting exercise for you to try.

E. coli has the following dimensions. Let's say this is roughly 2 micrometers and this is 1 micron. Assuming that the density of E. coli As an order of magnitude estimate, is not too different from the density of water. So let's just say this is 10 to the power of 3 kg per meter cubed.

As we all know, our bodies are primarily composed of water. So this is not, as an order of magnitude estimate, bad at all. So the density of E. coli is this. These are the dimensions. So what is the mass of one E. coli?

And we can simply, as an order of magnitude estimate, compute the volume and multiply by the density. So we can assume that this is a cylinder for $\pi r^2 h$, where π radius is 10 to the power of minus 6, squared by 4 because radius is half of this, and h is 2 microns, which is 2×10 to the power of minus 6. That's the volume of E. coli times the density, which is 10 to the power of 3. So this can be worked out.

We have π, 2, 4. This becomes 10 to the power of minus 12. This is another 10 to the power of minus 6. So that is 10 to the power of minus 18 times 10 to the power of 3. This is 1.

This is 2. This is $\pi/2 \times 10^{-15}$ kilograms. We are interested in only an order-of-magnitude estimate, so let us just assume that this is roughly equal to 1. So, an E. coli roughly weighs 10^{-15} kg. If that is the case, as an exercise, I would like you to compute how much 2^{216} E. coli weigh.

And if you do this exercise, what you will find is that the combined mass of these 2^{216} E. coli is greater than the mass of the planet we are on. And now, that obviously poses a problem. What that tells you is that somewhere along the line, our assumptions were wrong because clearly the mass of E. coli cannot exceed the mass of that flask. So somewhere along the line, our assumptions went wrong. And where our assumptions went wrong was in assuming that E. coli was going to grow exponentially for three days.

That environment that we provided E. coli—that small flask with 100 milliliters of culture—is not sufficient to support exponential growth for three days. And hence, we need models that are different compared to simply exponential phase, because no environmental niche that we are talking about can support exponential phase for any extended amount of time. And that is what we are going to continue with in our next video. Thank you.