Introduction to Reliability Engineering
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Lecture 10
Weibull Distribution (2 Parameter)

In lecture 10, we will delve into the Weibull distribution, which is another widely used probability distribution. Unlike the exponential distribution, which assumes a constant failure rate, the Weibull distribution can accommodate increasing, decreasing, or constant hazard rates, making it a versatile model. The 2-parameter Weibull distribution is more commonly used than the 3-parameter version.

The Weibull distribution is highly popular in data analysis for reliability estimation. In fact, it is so ubiquitous that many refer to reliability analysis as "Weibull analysis." Its versatility and applicability have contributed to its widespread adoption.

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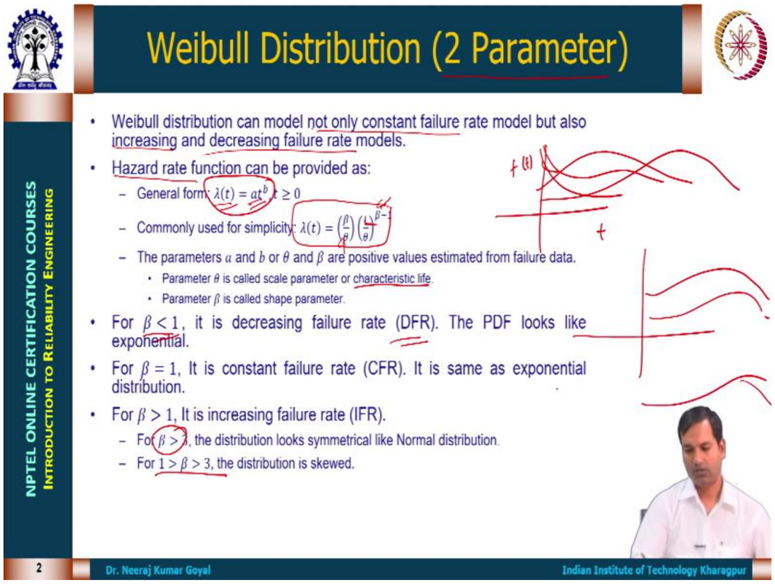

So, today, we are going to start discussing about the Weibull distribution. Weibull distribution the first one which we will be discussing is 2 parameter Weibull distribution, which is the most common one. So, similar to exponential distribution, which is 1 parameter this becomes 2 parameters, similar to 2 parameter exponential distribution, Weibull distribution has a 3 parameter Weibull distribution.

$$\lambda(t) = at^b, t \geq 0$$

For 2 parameter Weibull distribution, this can not only consider constant failure rate model, but it can also model increasing hazard rate and decreasing hazard rate model. So, we can say that hazard rate or failure rate instantaneous failure rate lambda t is a function of time as well as some constant a.

$$\lambda(t) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}$$

So, time raise to the power beta now, when for different values of beta it will have the different forms. So, when beta is 0, b0 and b0 then lambda t will be equal to a t raise to the power 0 will be 1. So, this becomes time independent that is constant failure rate. If b is less than 1 less than 0 negative quantity in that case, what you will see lambda t will be a t to the negative power. So, that means it is some it is a inversely proportional or in it is a inverse function of time with some power. When b is greater than 0, that means b is a positive quantity in that case, with time lambda t will increase.

So, that becomes a increasing function of time. This is a form which is sometimes used, but more commonly form which is used for representing failure rate for Weibull distribution is beta upon theta t upon theta raise to the beta minus 1 where theta is called the scale parameter beta is called the shape parameter. Generally the parameter which comes as the division or multiplication for the random variable, here random variable of interest is t.

So, since theta comes in division, so, it modifies or changes the scale of t that is why this is called scale parameter. But the power changes the behavior or the pattern or the curve. So, beta is called the shape parameter with the changing of beta the shape of lambda t shape of rt functions will change, but if you change the theta, then this will result in only contraction or expansion of the scale or the axis.

So, it will be like stretching the distribution or it will be like contracting the distribution. Theta is also called the characteristic life. Because the as we will see later when t is equal to theta then reliability is same. Which is same as the when t is equal to MTTF for the exponential distribution. Like some cases like when beta is less than 1 then this is called decreasing failure rate and decreasing failure rate will look like exponential only but it will be sharp, beta equal to 1 is a constant failure rate which is also an exponential distribution. And for beta greater than 1, it may have any shape it may be your Ft versus t may have a different shape. It can be like this can be like this can be like this.

So, different kinds of distributions shape it can take depending on the value of beta. When beta is equal to beta is somewhere greater than 3, then it is taking a quite similar shape as normal distribution something like this. As it becomes more and more it becomes more looking like a normal distribution when beta is some value between 1 and 3 then distribution is skewed it may be looked like looking like something like this.

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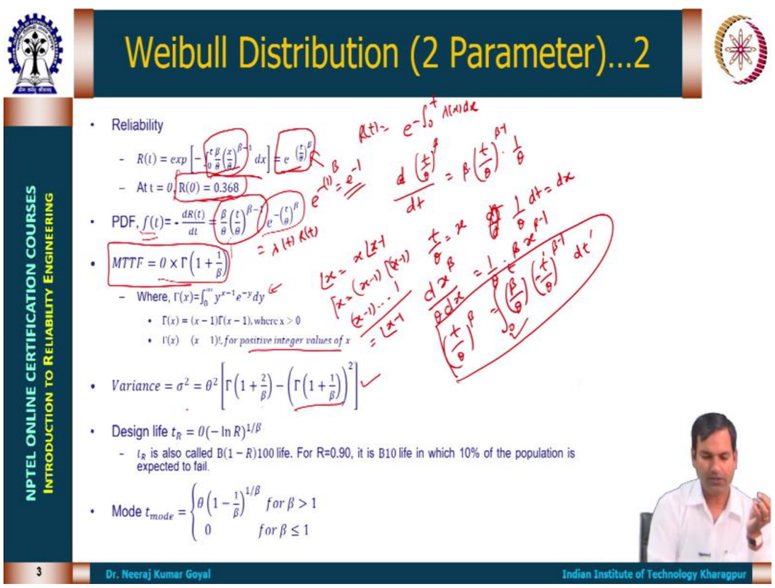

Or another way, we for if we ignore the Weibull distribution, we know this is our lambda t. So, we know reliability can be given as e to the power minus integration from 0 to t lambda x dx. Here lambda is. So, if we integrate this beta upon theta x upon theta raise to the power beta minus 1 this becomes t upon theta raise to the power beta. In another way in a simpler way if we see, if you differentiate this with respect to t what we will get we will get t upon theta raise to the power beta minus 1 beta into 1 upon theta.

$$R(t) = \exp\left[-\int_0^t \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} dx\right] = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

This is same like if we assume t upon theta as x then dt over sorry so, we will have 1 upon theta dt is equal to dx. So, this will be t upon theta is x so the x raise to the power beta divided by dt, dt we will replace as theta dx. So, that will be 1 upon theta then we differentiate this this will become beta x to the power beta minus 1 and so, this will become beta upon theta x if we convert into t x is equal to t upon theta.

So, this is t upon theta raise to the power beta minus 1, which is same as this beta upon theta x upon theta raise to the power beta minus 1. So, if we integrate this 0 to t dt dash what we will get is t upon theta raise to the power beta. So, here once we integrate this what we get is e to the power minus t upon theta raise to the power beta. So, that is why we write failure rate as a differentiation of, this is much easier to process. So, that is why more commonly use expression for reliabilities e to the power minus t upon theta raise to the power beta where theta is the characteristic like beta is called the shape parameter.

Now, if we take t equal to theta, then in all cases irrespective of beta what will happen t will be theta. So, this will become e to the power minus 1 raise to the power beta, which is equal to e to the power minus 1. So, irrespective of value of beta, what if at time t equal to theta the reliability is always equal to 0.368 which is same as when t is equal to MTTF of a exponential distribution. We can calculate ft, ft is the PDF, we know PDF is minus dR t over dt or we can say it is equal to lambda t dt lambda t Rt.

$$f(t) = -\frac{dR(t)}{dt} = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

So, this is my lambda t and this is my Rt. Or if we differentiate this differentiation of this will produce the same this value the exponential term when we differentiate it will produce this beta upon theta t upon theta raise to the power beta minus 1 which we have already got it here multiplied by e to the power minus t upon theta raise to the power beta.

$$\text{MTTF} = \theta \times \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Where, } \Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy$$

$$\Gamma(x) = (x-1)\Gamma(x-1), \text{ where } x > 0$$

$$\Gamma(x) = (x-1)!, \text{ for positive integer values of } x$$

Now, if we want to calculate MTTF then MTTF because this is little complex evaluation, we are not going into the derivation the integration of reliability, if we do we are able to get MTTF function is theta into gamma function of 1 plus 1 upon beta, where gamma function is mathematically defined as integration from 0 to infinity of y to the power x minus 1 e to the power minus y dy.

Gamma function has certain properties like we know factorial x is equal to x factorial x minus 1, but for gamma function gamma function of x is x minus gamma function of x minus 1. This is the property of gamma function. And similarly, gamma function of x as we see if we continue to do this if x is a integer quantity positive integer quantity then the same we can continue and finally, we will reach up to let us say 1.

So, 1 into till x minus. So, we can say this is equal to factorial x minus 1. So, gamma x becomes factorial x minus 1, if x is a positive integer. For non-integer values we have to refer certain tables there are gamma tables available in which we can find out the values of the known values from 0 to 1, when x value is somewhere from 0 to 1. Various for this distribution is calculated as theta square gamma of 1 plus 2 upon beta minus gamma of 1 plus 1 upon beta whole square this is the square of this gamma function. Design life we can achieve from again from same from here. So, I will erase this.

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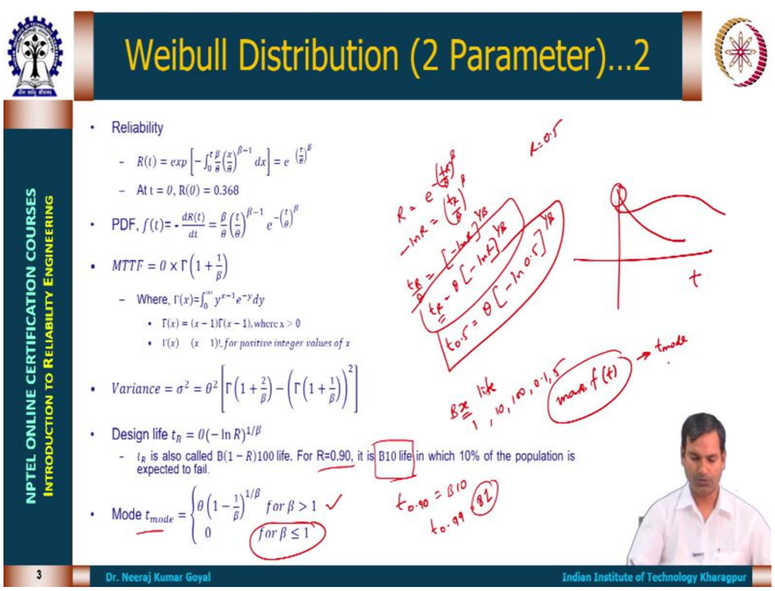

Design that we know that R is equal to e to the power minus t upon theta raise to the power beta. So, from here at when we want to know tR that time at which reliability is equal to R. So, this will be equal to if I take minus ln of R this will be equal to tr divided by theta raise to the power beta or if I take n tR upon theta will be equal to minus ln of r raise to the power 1 upon beta. So, tR will be equal to theta into minus ln of R raise to the power 1 upon beta.

So, if I am interested to know median time to failure or the reliability is. So, for median time to failure my reliability will be equal to 0.5. So, I can get t 0.5 that is median time that will be equal to theta minus ln of 0.5 whole raise to the power 1 upon beta. So, we can use this formula to get the median life.

If I am interested to know 90 percent reliability life which we are calling as a design life then R will be equal to 0.95. The 0.5 will replace with 0.90. There is another indication another parameter which is used this is for exponential or any distribution that is we are calling as Bx life. This x can be 1, can be 10, can be 100, can be 0.1 any value, can be 5, what does it mean B10 or Bx life means that we are taking x percent of failure that means, we want to know the life at which we will observe x percent of failure.

So, this similar to tR R when I am saying t 0.90 that means, time at each reliability is 90 percent or we can say this is equal to B 10 that means time average unreliability is 10 percent. So, t 0.9 is equal to B10. Similarly, t is 0.09 t 0.99 will be equal to B1, 1 percent failure. So, t 0 99 percent reliability means 1 percent failure. So, Bx is a common notation many times it is referred by many people that be B1 life B1 life means the time at which we are expecting that 1 percent of the or x percent of the population will fail.

Similarly, we can get the mode as we have seen that when beta is less than equal to 1 for exponential et cetera, the maximum value comes at t equal to 0 only. So, that is a mode for beta less than 1 or beta equal to 1 is at t equal to 0. But for beta greater than 1 this model is changing this shape changes.

So, because of the change in shape the beta the mode value comes out to be at theta into 1 minus 1 upon beta whole raise to the power 1 upon beta. So, this formula can be used to calculate the mode of the distribution. To get the mode actually you once you have this mode means you have the maximum value of ft. So, if you maximize ft the value of t at which ft is maximum that is your t mode. So, if you want you can get it by differentiation of ft and then setting it equal to 0 to get the value of t mode.

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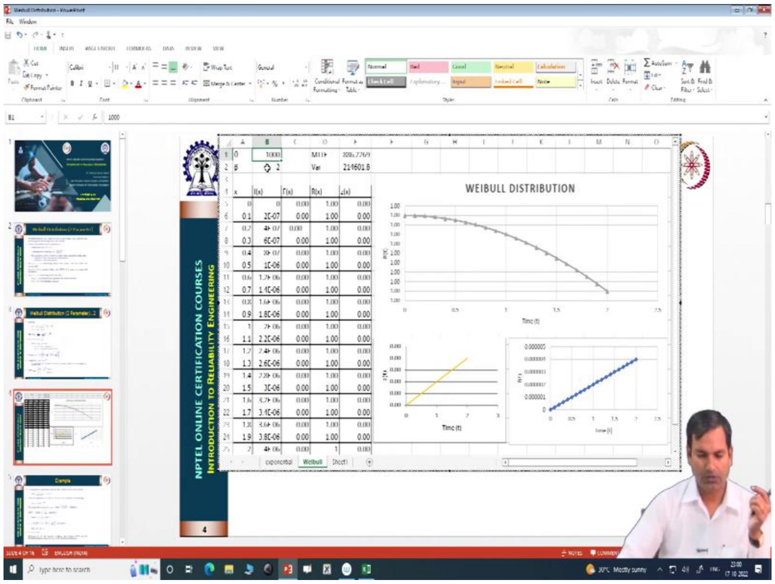

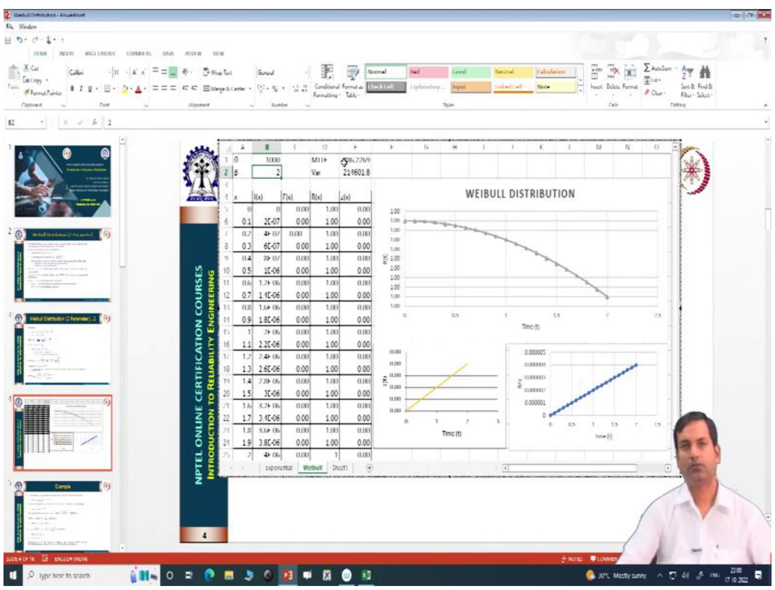

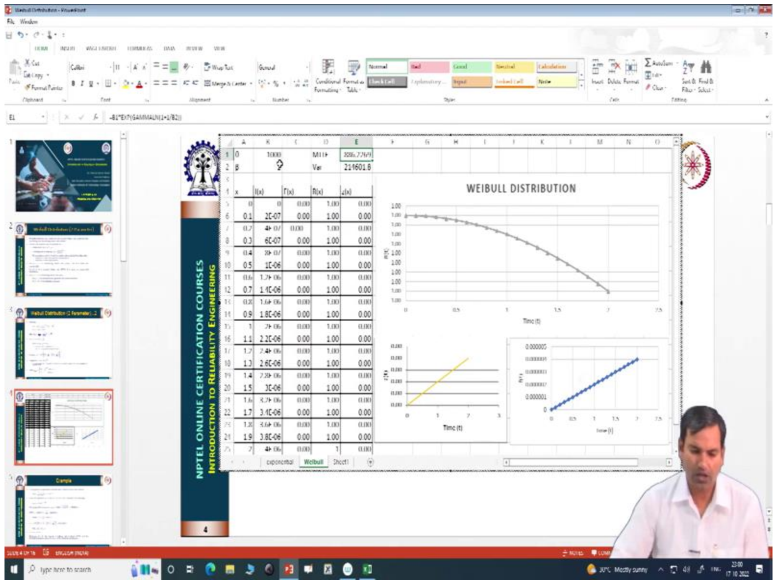
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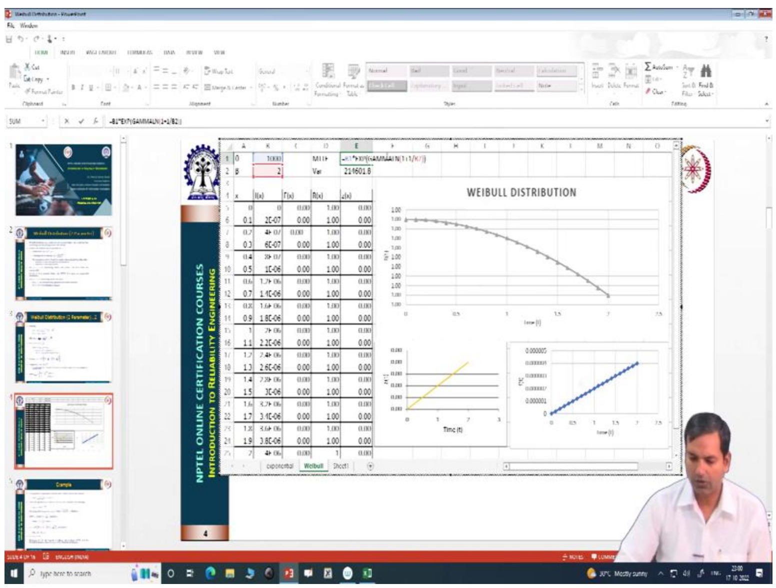
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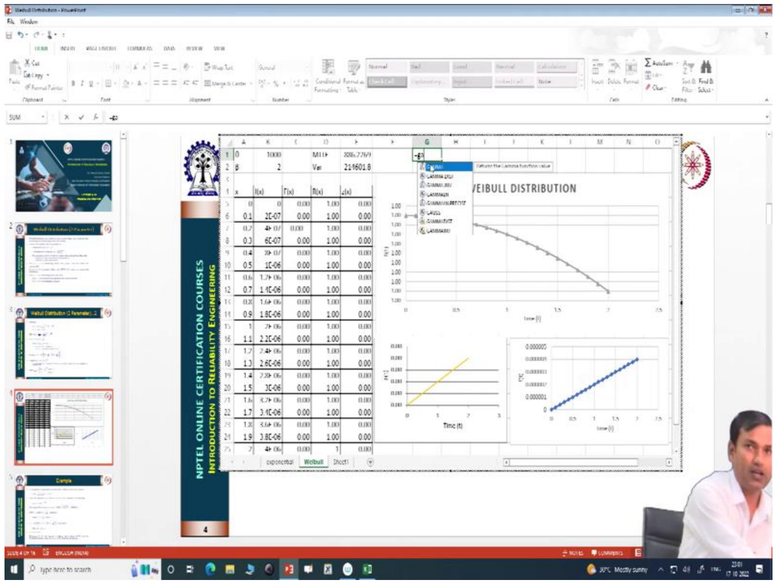

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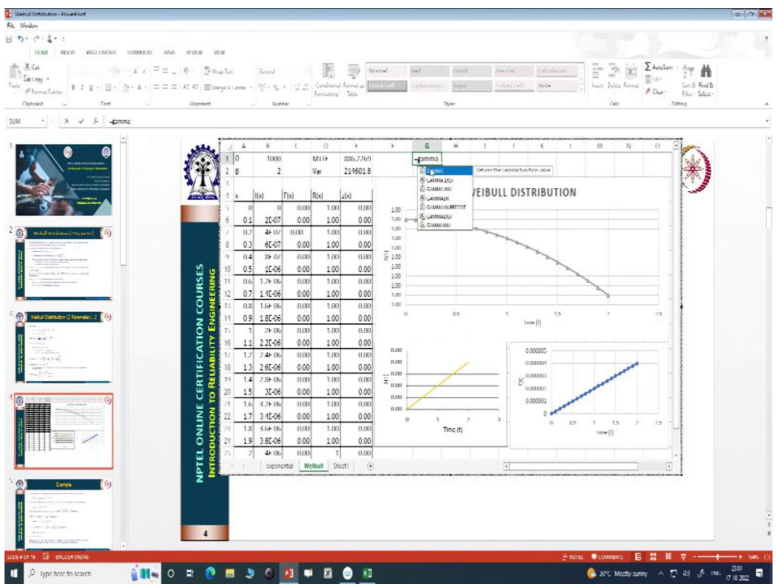

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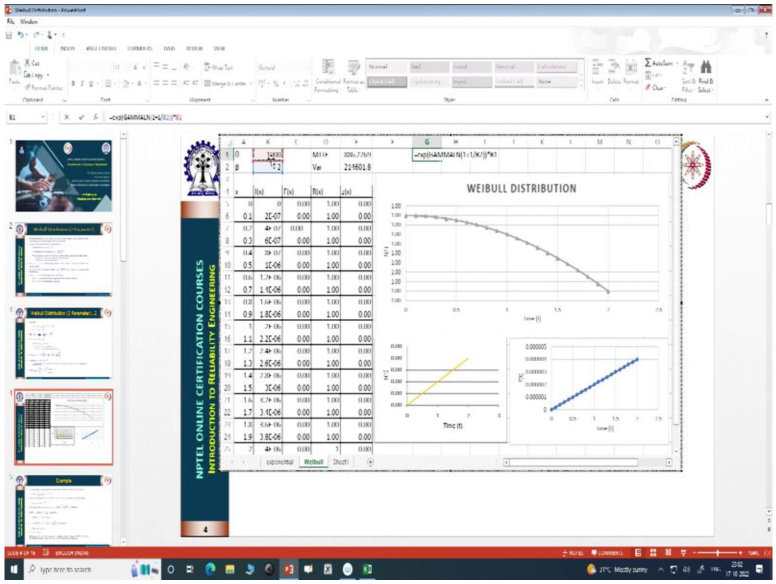

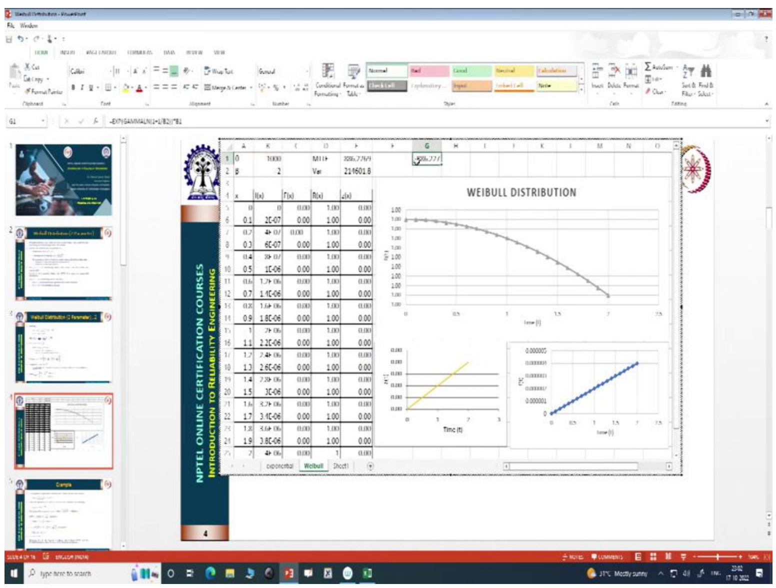

Now, let us see how does this Weibull distributions look like. So, I have made again use the Excel sheet here. So, if you look at here my theta value I have put it as 1000 here. If my theta is 1000 and beta is 2. In that case I can calculate everything here. My MTTF, as you see here, MTTF is theta into gamma function of 1 plus 1 upon beta. If you see this is equal to theta 1000 into actually this Excel function did not have earlier Excel versions did not have the gamma function directly they have given the gamma ln function gamma ln function gives the log gamma values of the value.

So, we have to take the exponential of that to get the gamma function value. New Excel features have this gamma function directly. So, you can use them directly gamma of 1 plus 1 upon beta.

So, beta is B2 here B is the column number 2 is the row number B2 means 2 here to have a better explanation, what I can do this is equal to let me check if gamma is here or not. See, here gamma function is not there. So I will use the gamma ln, gamma ln of 1 plus 1 upon beta. So, 1 plus 1 divided by beta, beta is 2 and this value I have to take the exponential again.

So, I will use the exponential function here and this whole value has to be multiplied with the characteristic life theta, theta is 1000 here once I multiply I get the MTTF here which is seem like shown here.

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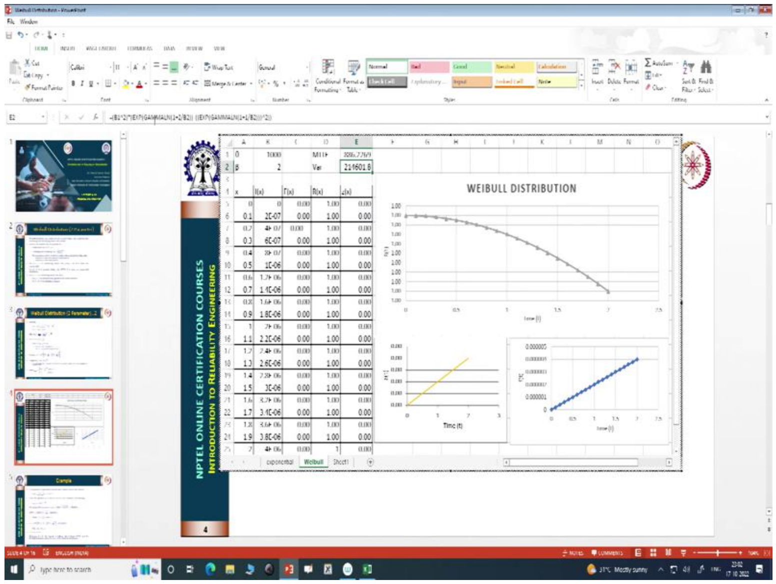

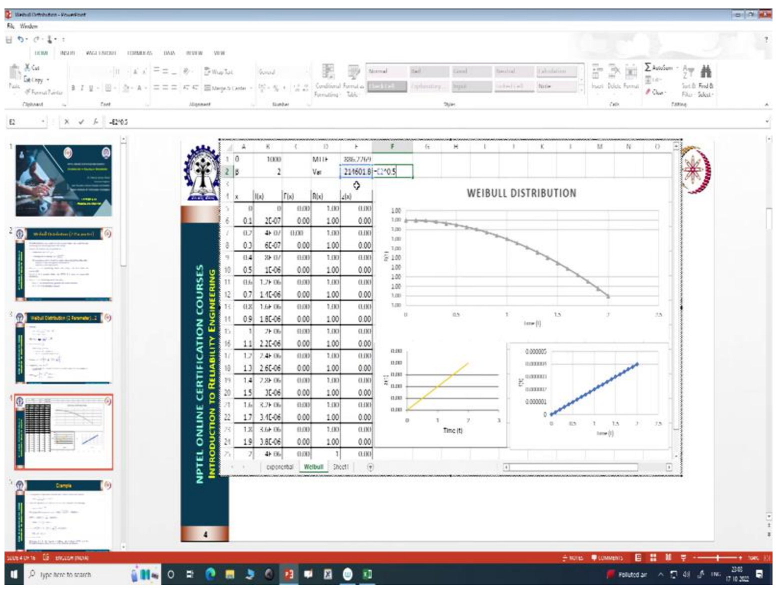

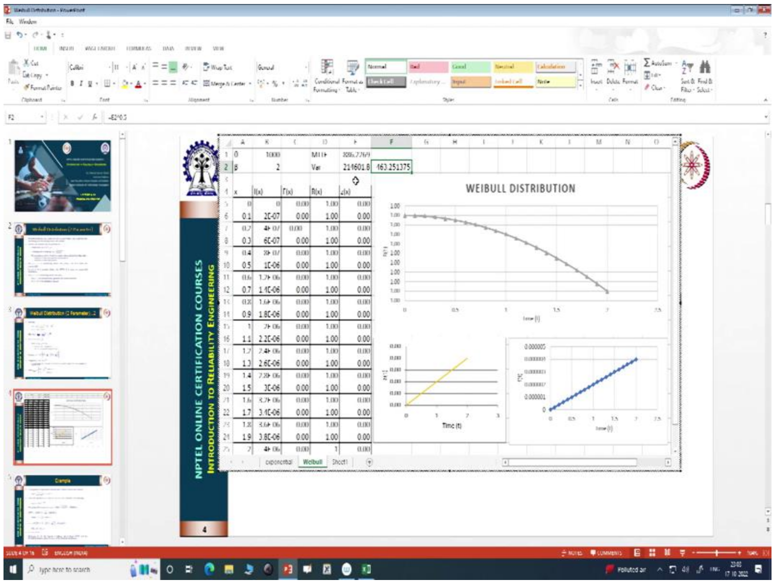

Variance again calculated in a similar way. So, we take the gamma value of 1 plus 2 upon beta and we take gamma value of 1 plus 1 upon beta, but when we take the gamma value of 1 plus 1 upon beta that is quiet and subtracted from gamma value 1 plus 1 upon beta and this whole whatever comes is multiplied with a squire of the theta value B1 is theta, that is 1000. So, we can get the variance here. If I want to know the standard deviation standard deviation will be this power 0.5 square root of this value that comes out to be 463.

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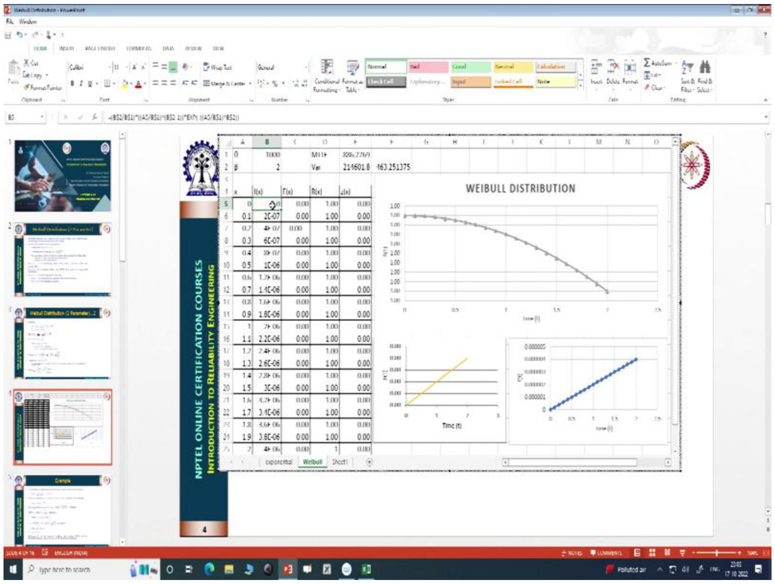

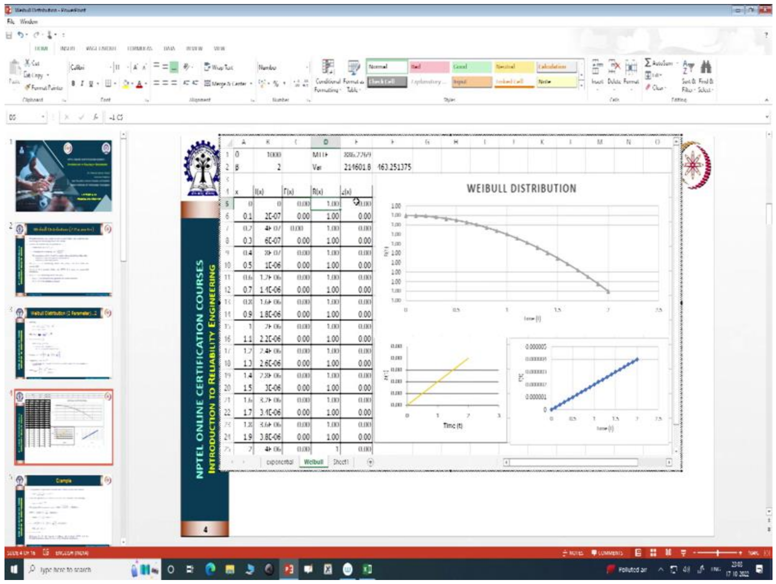
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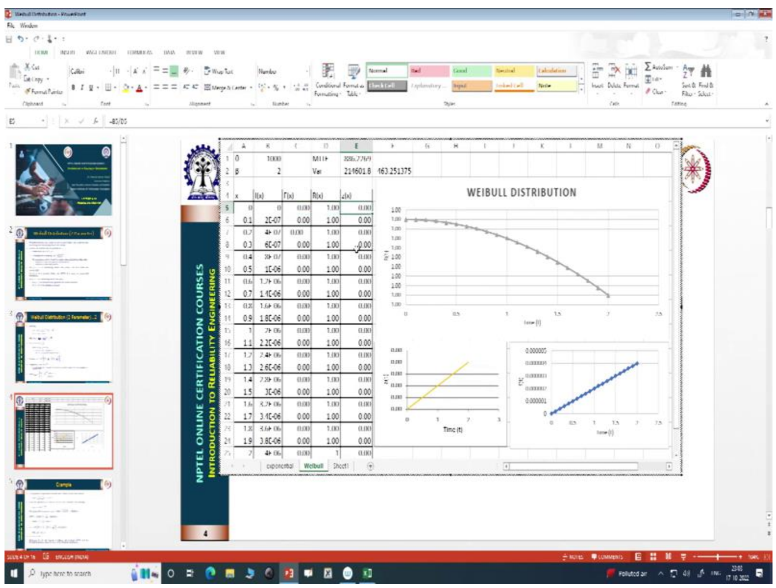
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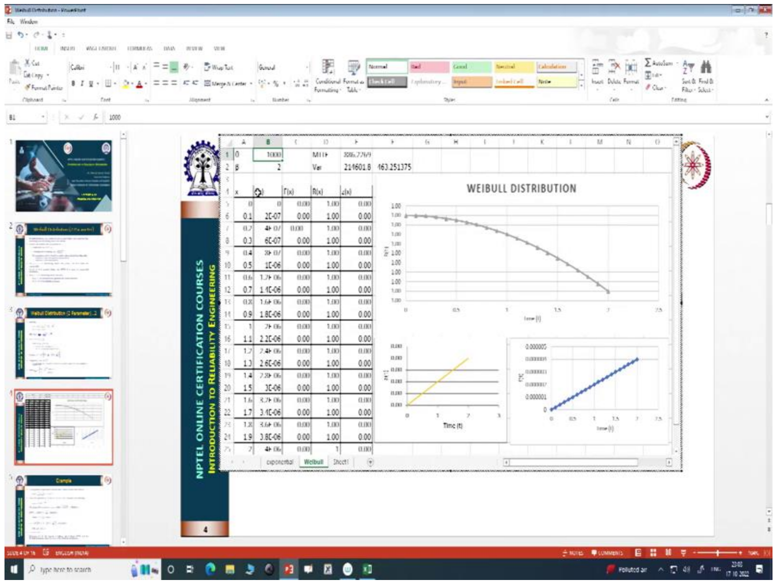
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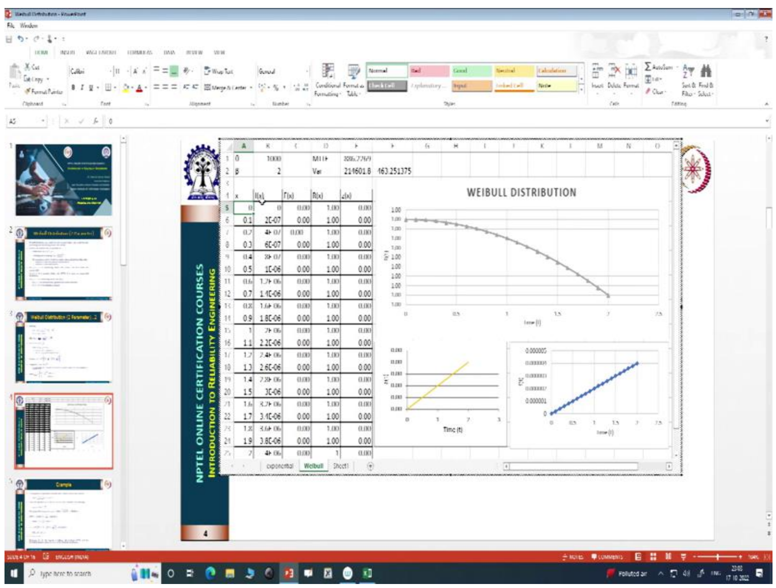
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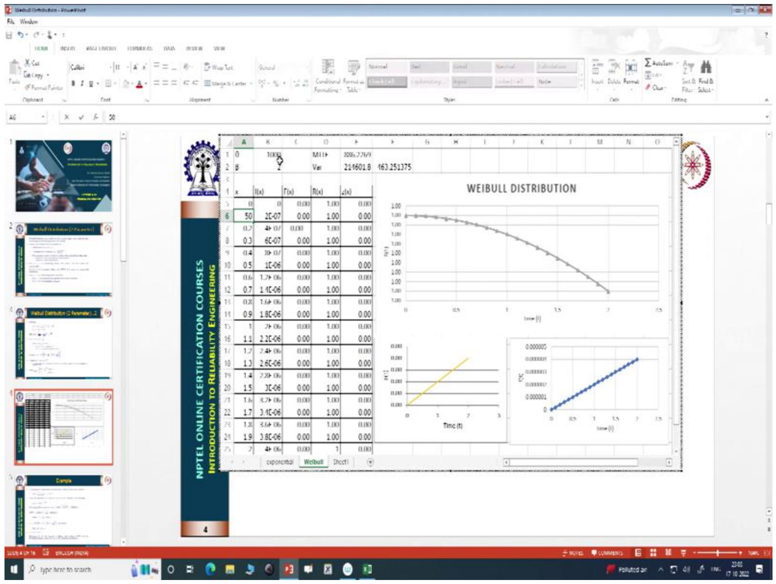
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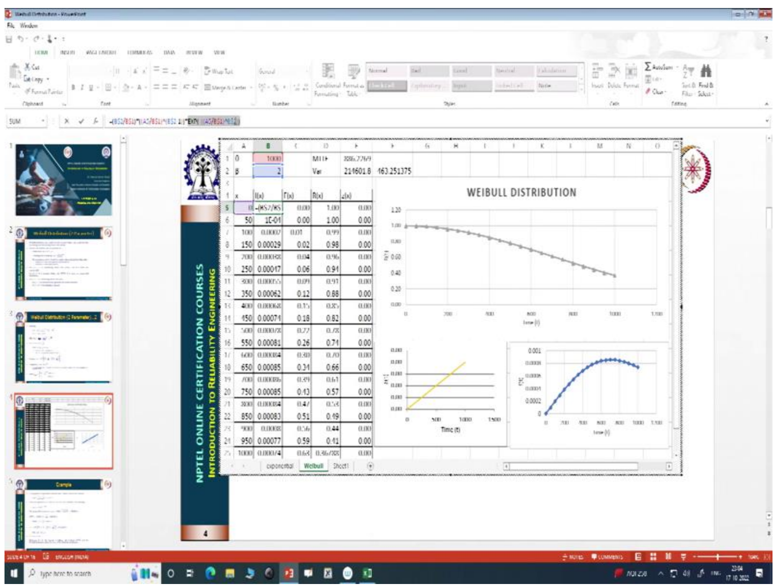
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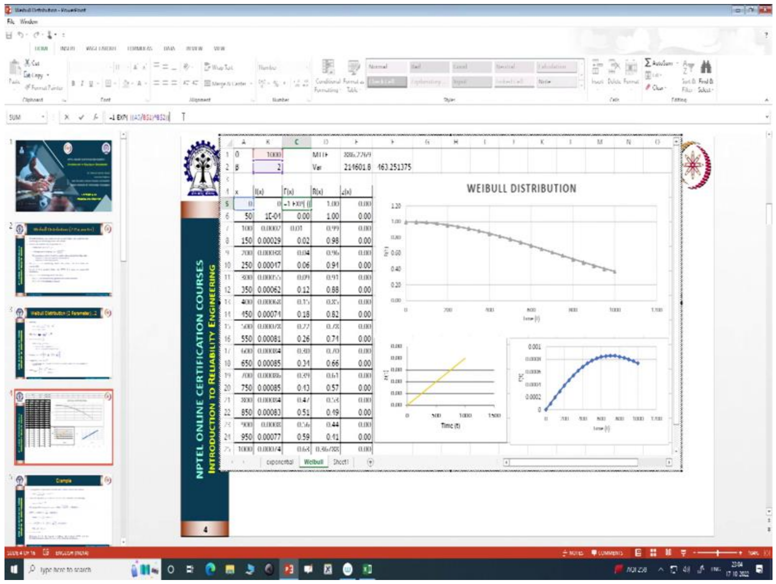
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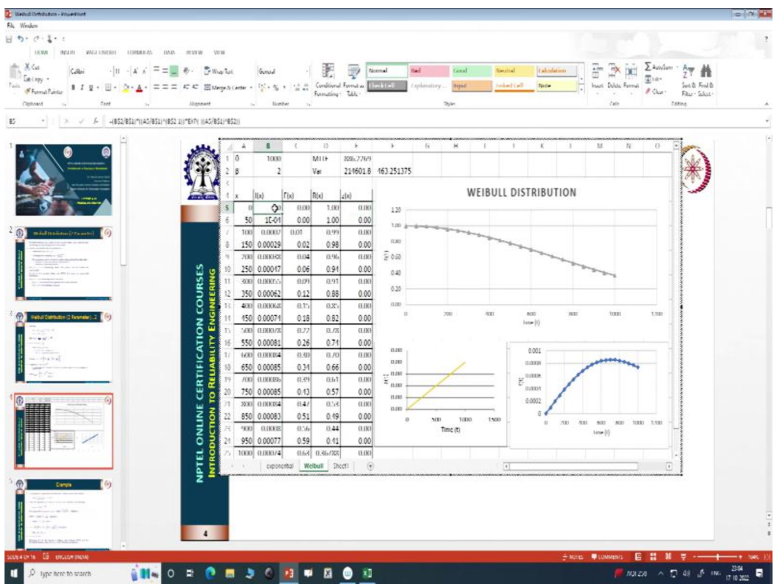
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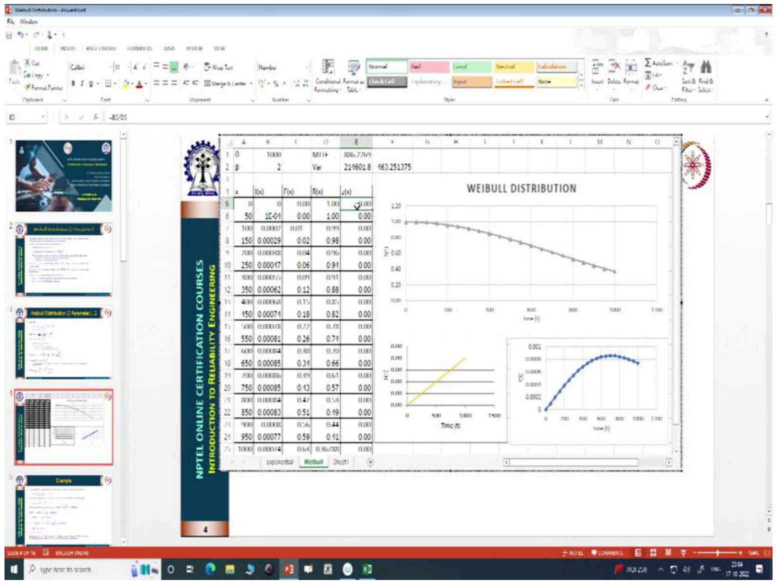
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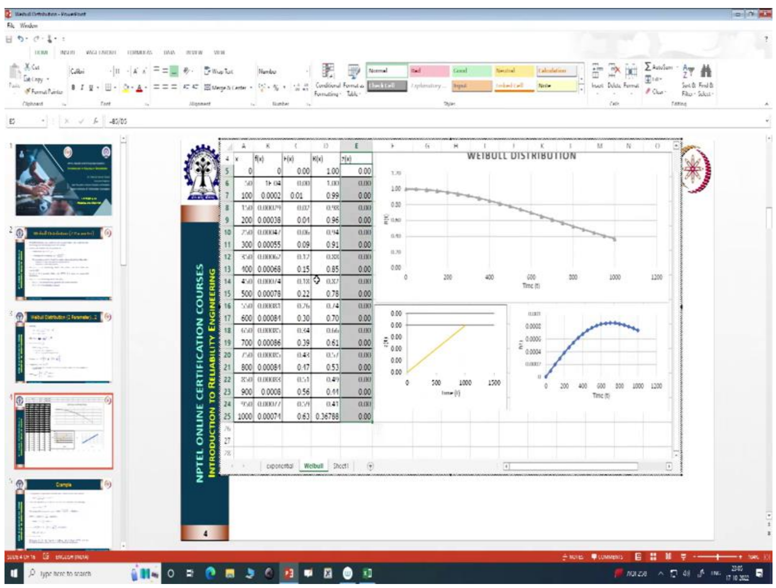
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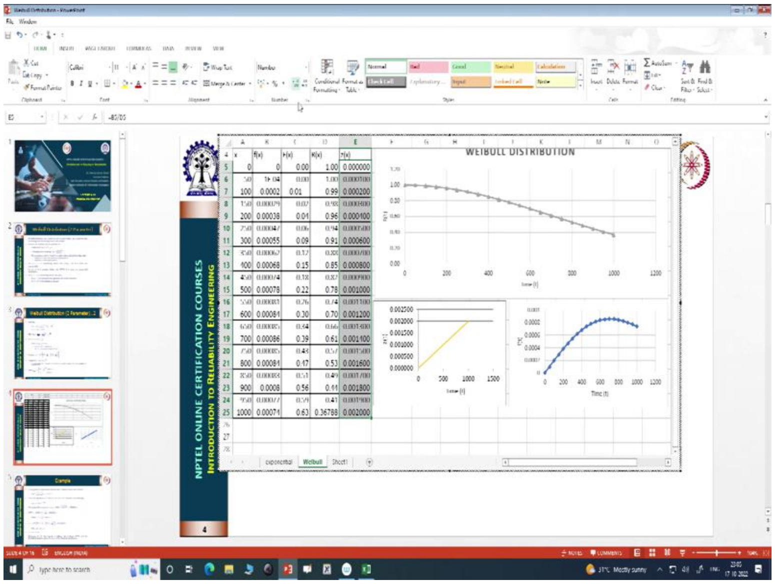

Now, here I have taken few values of x like x equal to 0 to 2 I have taken and then we have tried to calculate the fx capital FX or Rx and zx. So, since theta is 1000 these values looks very smaller. Now, I can take values something like. So, if I take values of x something like 1 then since, these are some where 10,000 let us says 50 difference.

If I take a difference of 50 you will see that how small fx is coming now, for small fx calculation, we know the formula small fx is t upon theta raise to the power beta minus 1. So, if you see here this is t upon theta this is raise to the power beta minus 1 multiplied by beta upon theta beta is this and theta is this, multiply by exponential of minus t upon theta raise to the power beta.

Once we get this we are able to take the fx. Similarly, we can get the capital Fx which is 1 minus exponential of t divided by theta whole raise to the power beta and negative value of the same. Similarly R we can take 1 minus of Fx and zx is nothing but Fx upon Rx so, same thing we are able to get here maybe the resolution is poor zx is not visible, I will increase the resolution here to see we able to see. So, as you can see here that we are able to.

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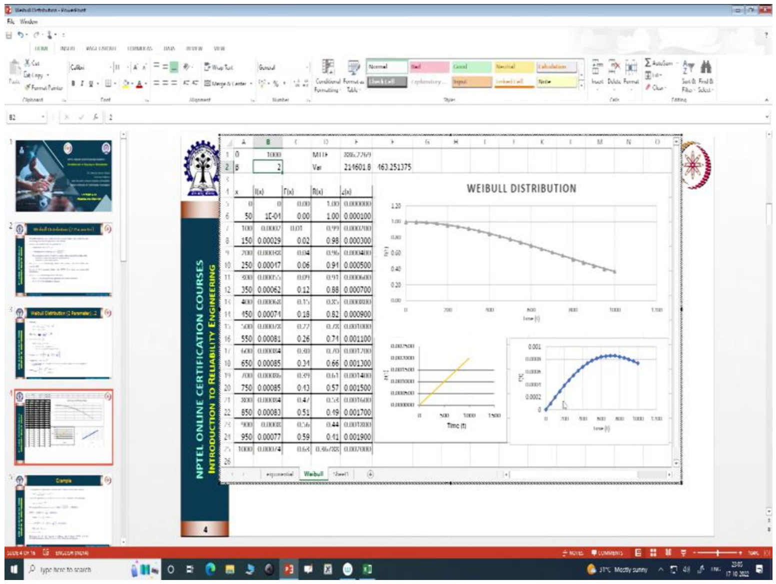

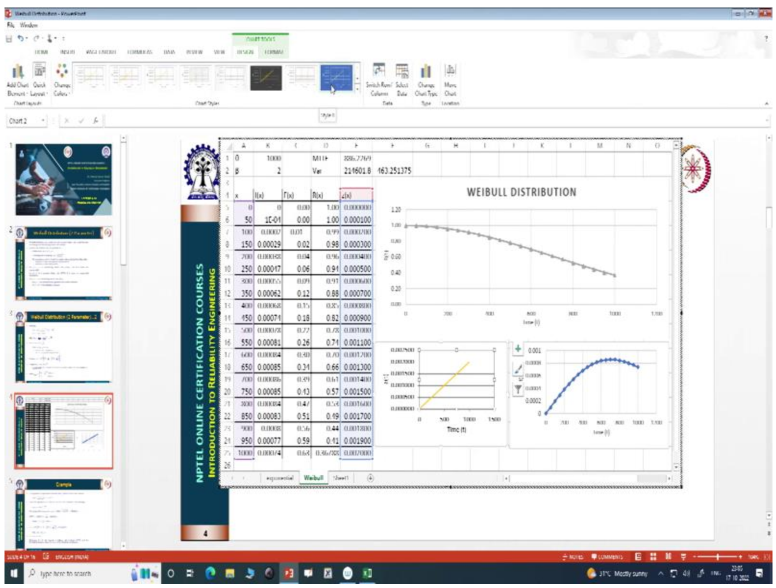

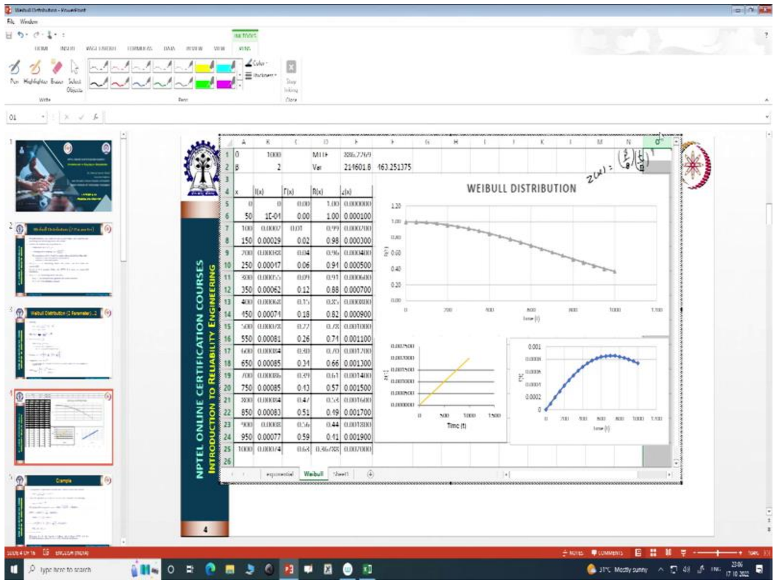

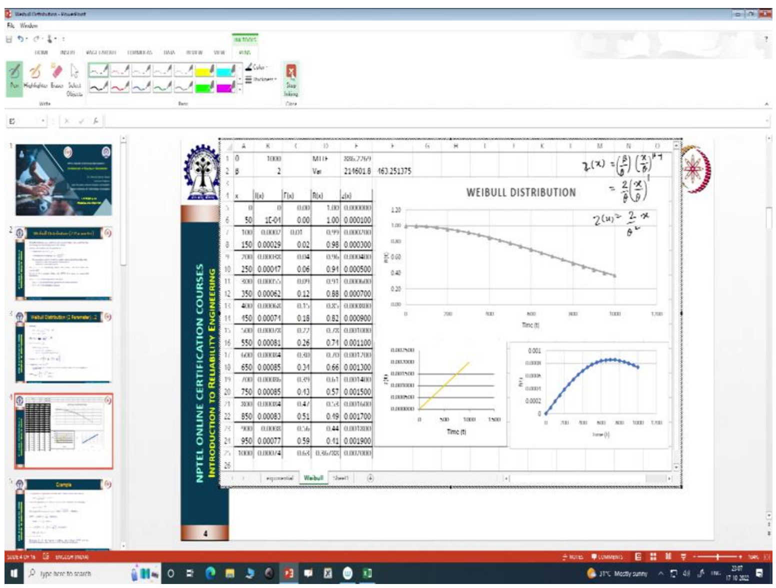

Now, let us investigate that how this Weibull distribution looks like that, when I am taking beta equal to 2, then if you see zt is a zt becomes linear because beta minus 1 is 1. Since beta minus 1 is 1 my use of pen here. We know that zx or lambda x is equal to beta upon theta then t upon theta raised to the power beta minus 1. This is getting disturbed I will use rubber here. We know that zx here zx is equal to beta upon theta into x upon theta raise to the power beta minus 1. Now, when theta is equal to 2 what will happen this will be equal to 2 upon theta into x upon theta raise to the power 1. So, that if we see that is 2 upon theta square x.

So, we say that zx is proportional to x. So, it is a linear function. So, but when beta is equal to 1 this will be x to the power 0 that will be constant value. If x beta is equal to 3 then this will become x square so, zx will be square function of this same we can see.

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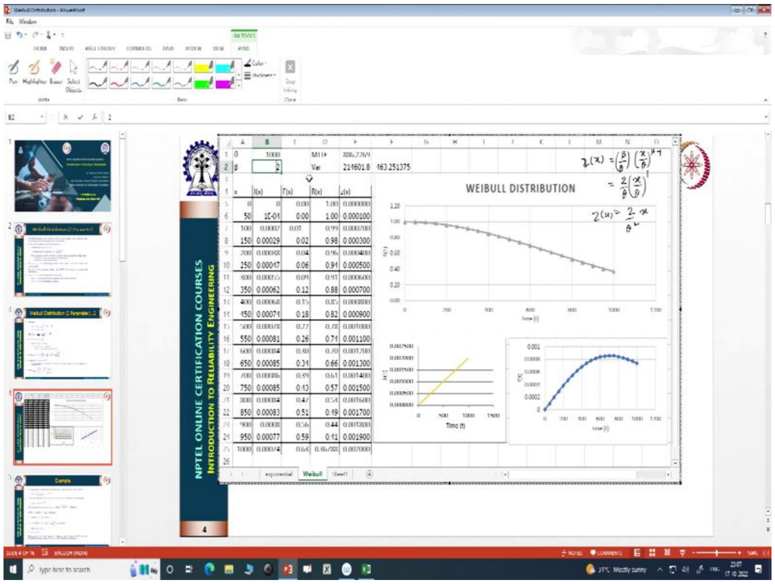

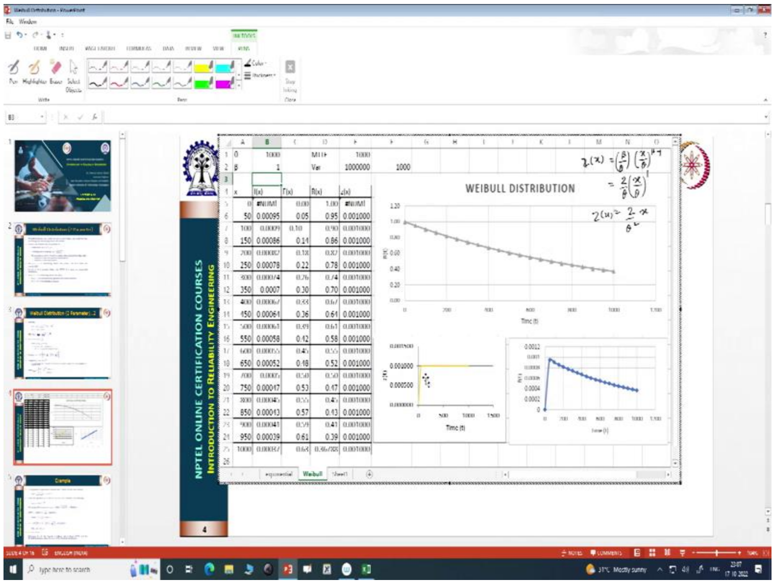

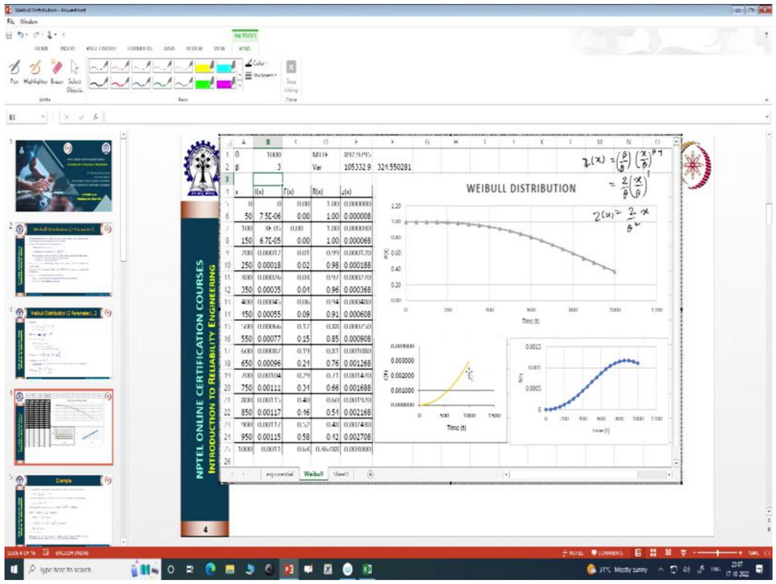
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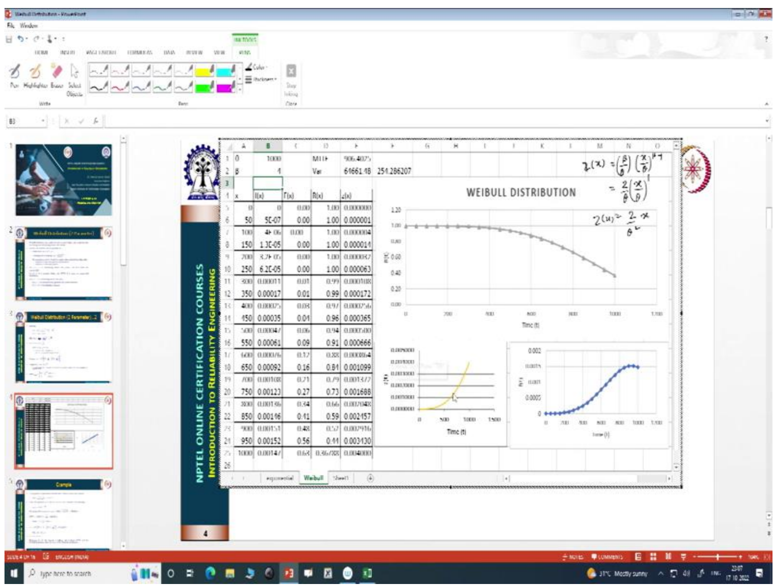
850 0.00117 0.46 0.54 0.002168
950 0.00115 0.58 0.42 0.002708
Weibull
Time (t)
4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WEIBULL DISTRIBUTION
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x f(x) F(x) R(x) z(x)
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150 1.3E-05 0.00 1.00 0.000014
250 6.2E-05 0.00 1.00 0.000063
350 0.00017 0.01 0.99 0.000172
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550 0.00061 0.09 0.91 0.000666
650 0.00092 0.16 0.84 0.001099
750 0.00123 0.27 0.73 0.001688
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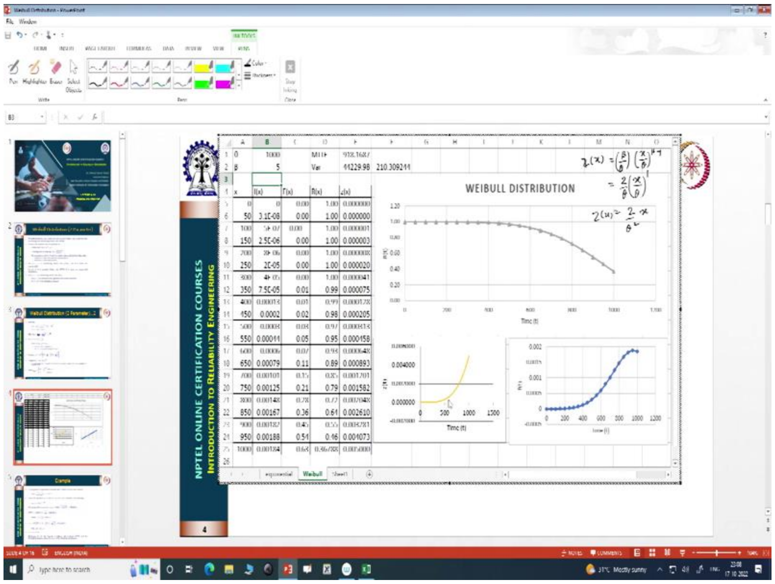
Weibull
Time (t)
4



So, if we change the value of beta let us say if we make it 1 then this zt will become constant same constant value. And when we make beta equal to 3 you see that this will become kind of parabolic function. Now, if we look make beta equal to 4 this will be much sharper increase if I make it let us say 5 much sharper going and then increasing faster.

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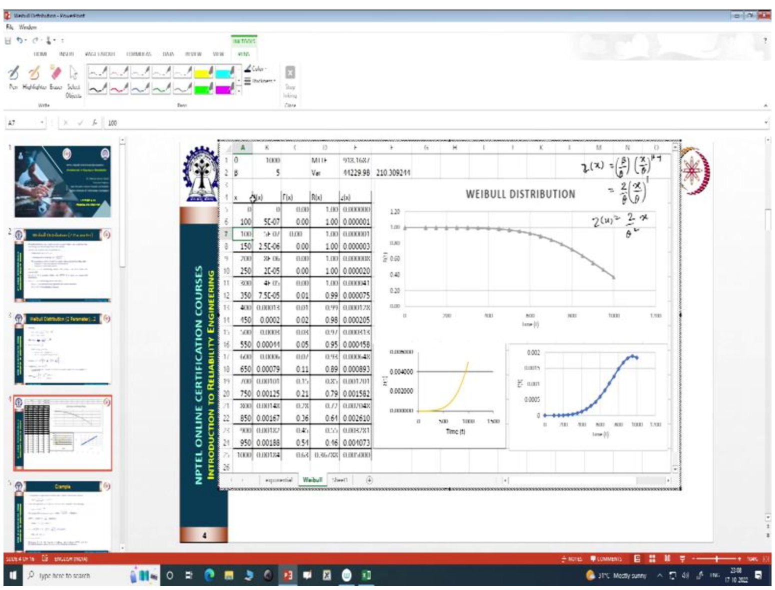

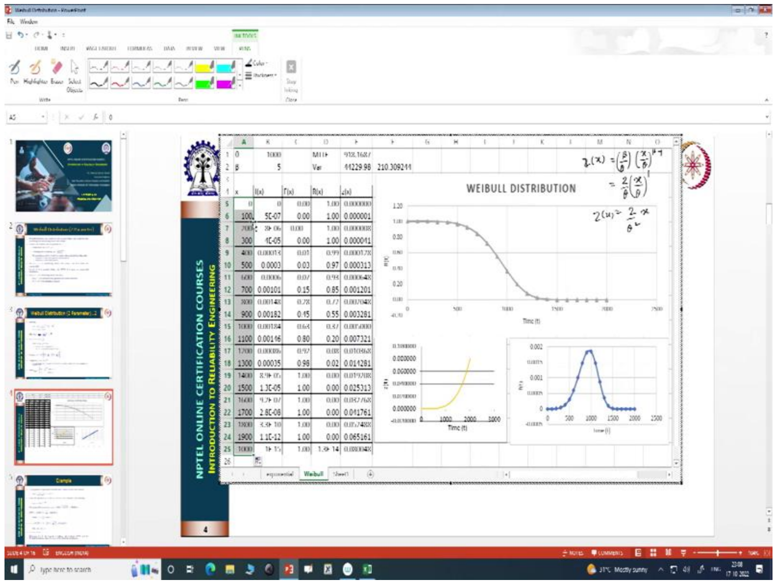
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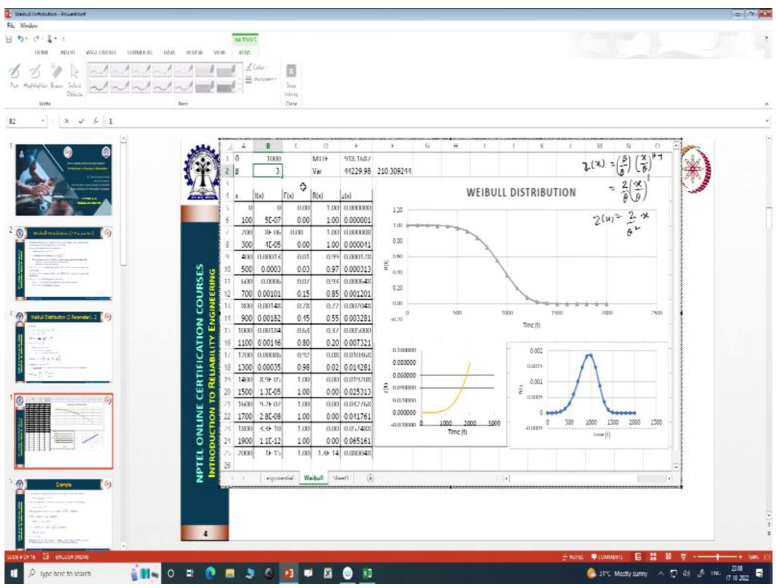
Time (t)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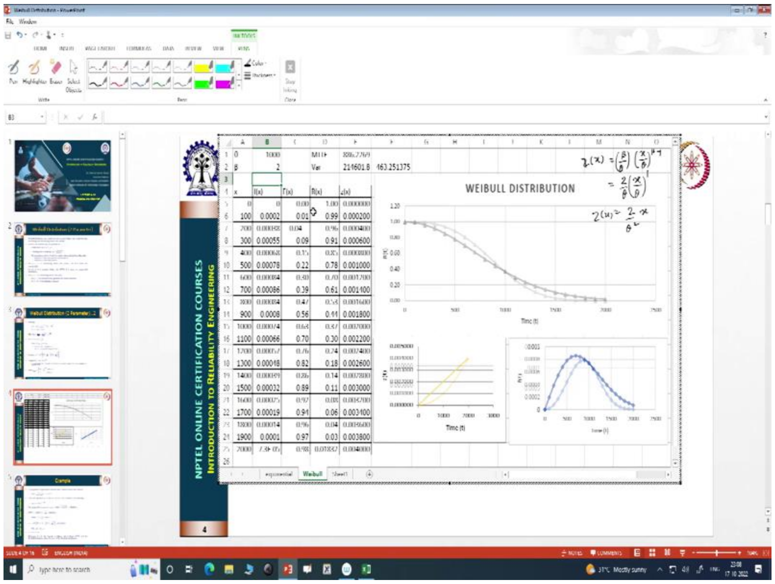

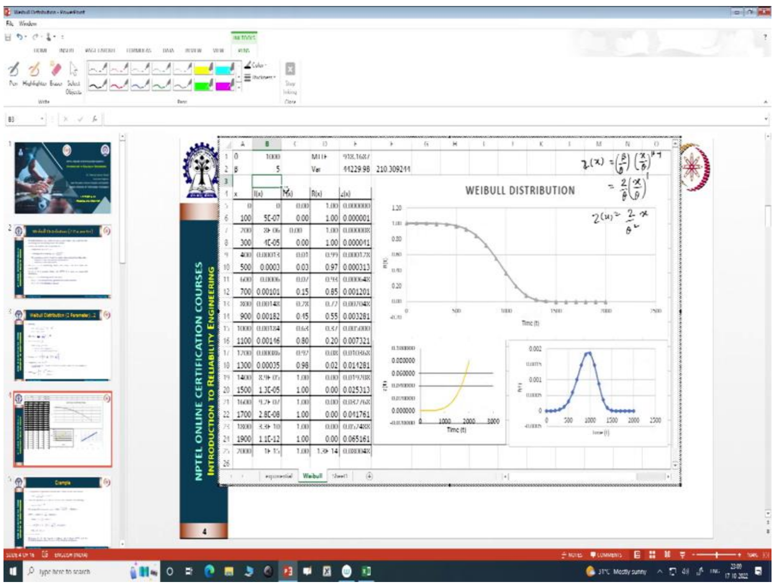

Now, let us investigate how the distribution is changing. Now, let us look into the PDF function. So, PDF functions as we as you see here though it is all plotted only up to 1000. So, let us do little more let us try it 100 gap. So, if you see that when beta is equal to 5 it is almost shape is almost looking like the normal distribution. If you see beta equal to 3 almost looking like but when beta is equal to 2 this is called Rayleigh distribution you have a little skewness is there. But when beta becomes larger this is almost similar to normal distribution.

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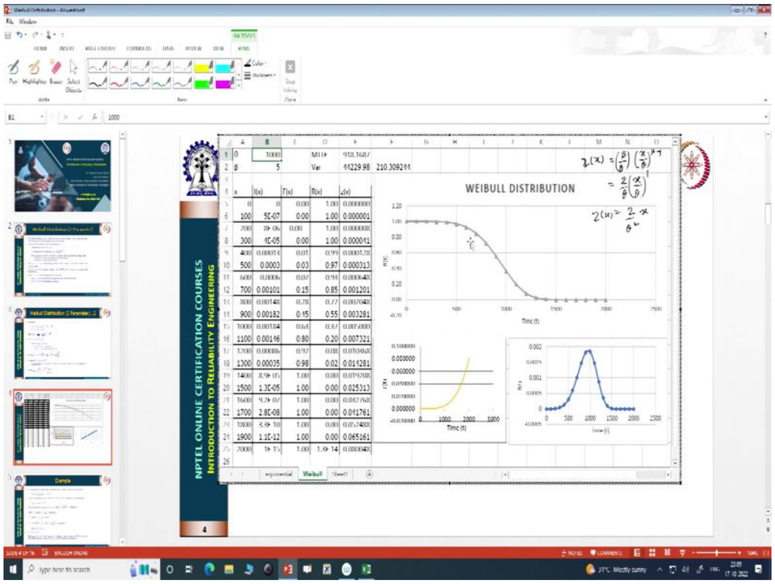

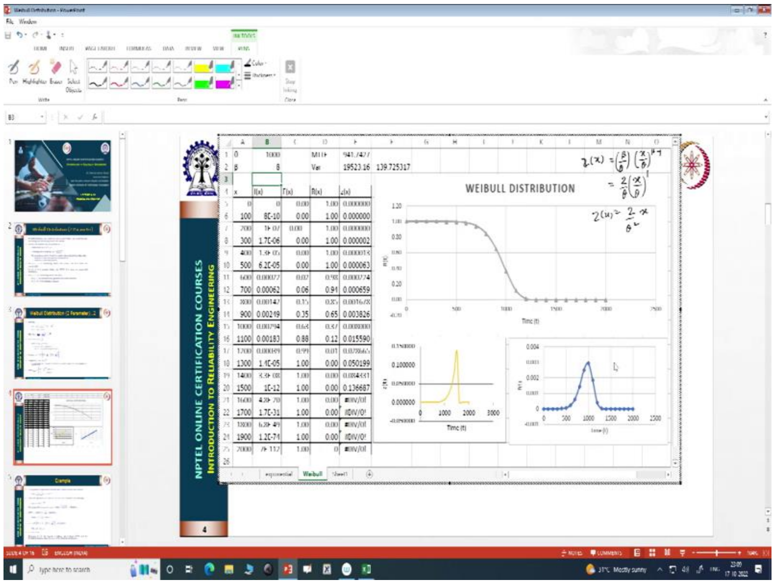

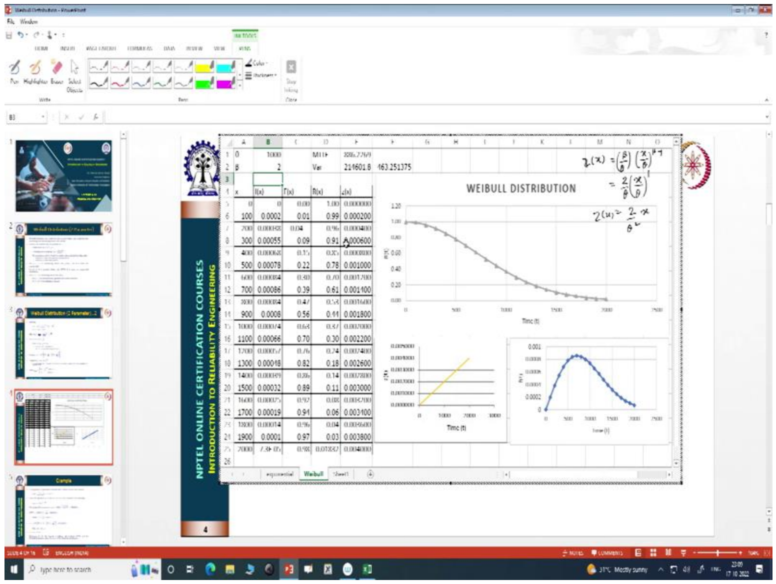

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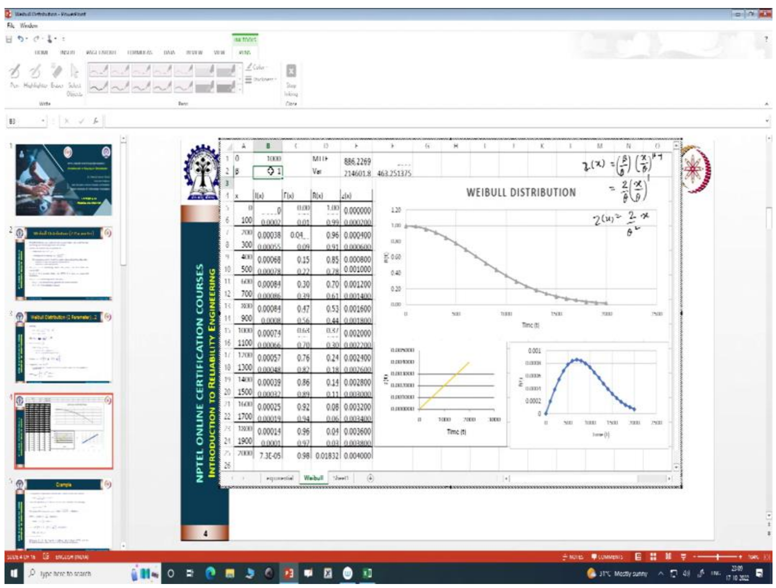
Time (t)



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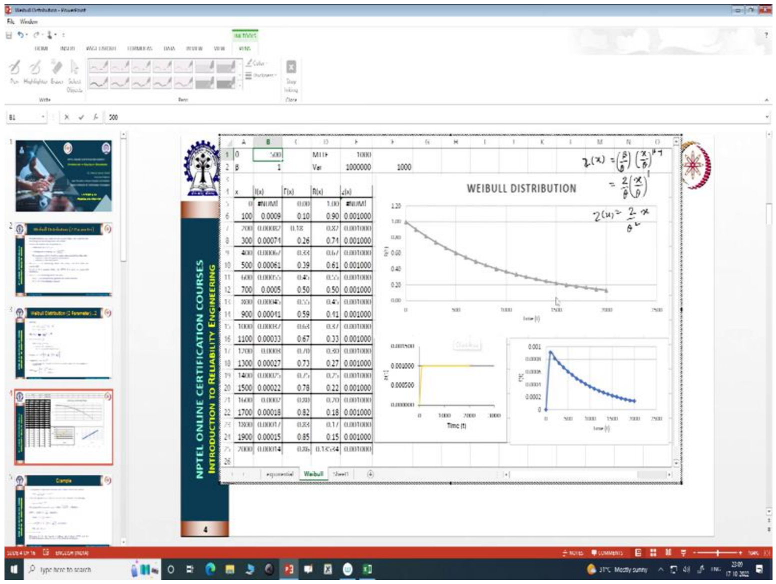
Time (t)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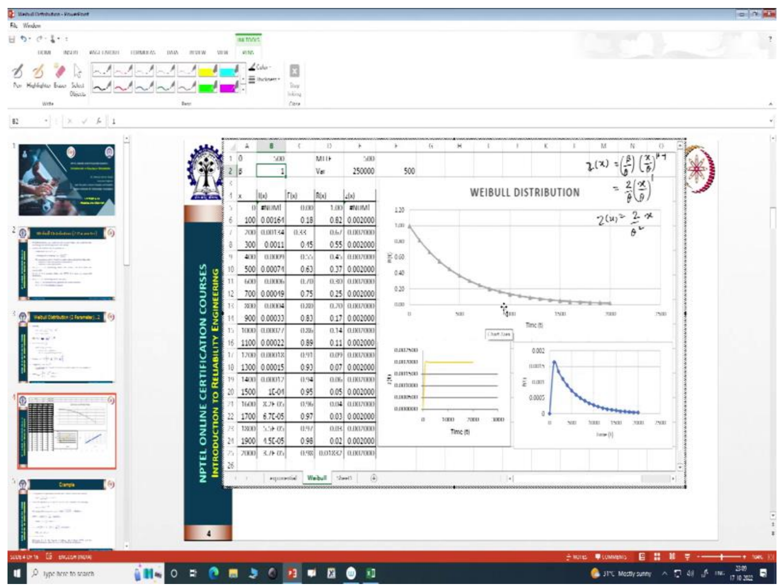
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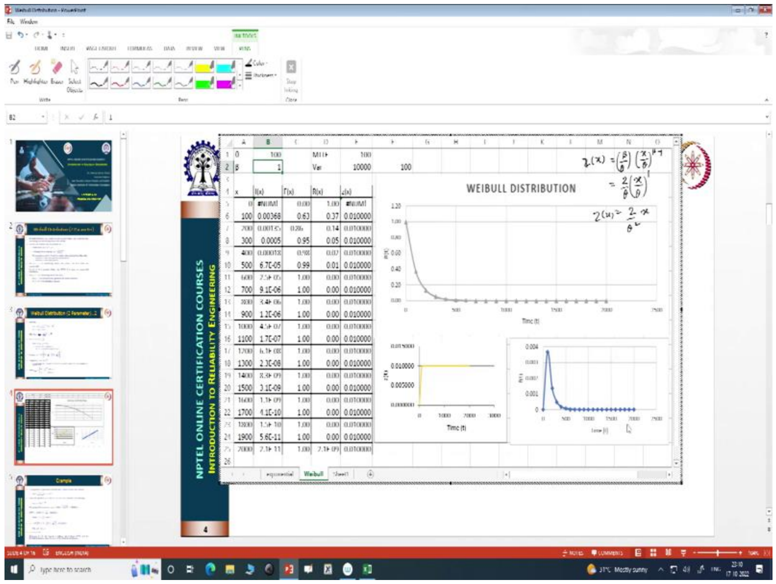

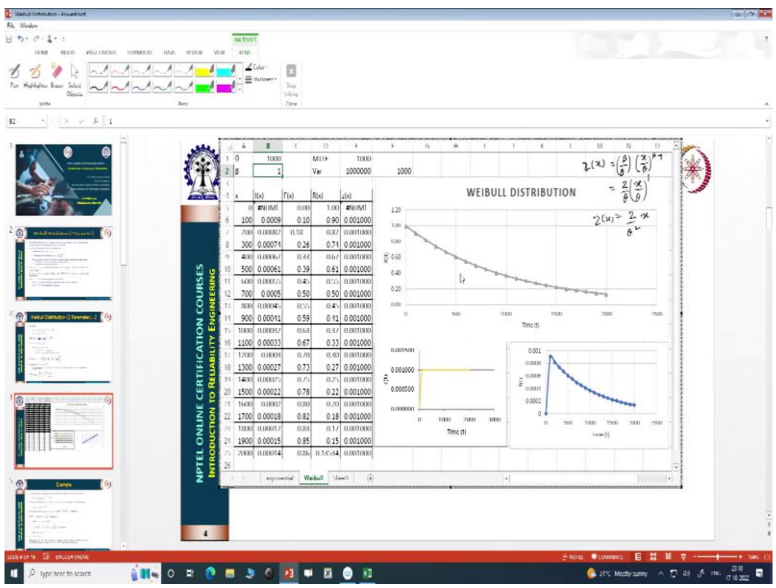

Similarly, if you see that shape, so, since PDF shape is changing with the beta, whenever I change beta a shape is having a change, then that is why this is called scale parameter, let beta is equal to 1 this will become exponential. All the values beta theta has to be greater than 0 they cannot be less than equal to 0. Now, let us see what is the impact of theta.

If I make theta equal to 500 half of if I make theta half. So, what will happen? The distribution is supposed to stretch, what I am getting from here to whatever values I am getting from 0 to 2000. Same value I will get from the scale from 0 to 1000. You see, it will have the same pattern for 0 to 1000.

So, because it is contracting, because of the if you see shape is same here also shape is same, the only change happened is that scale changed. It was happening on this is 100. So, is this shorter further can I make 1000 and this will be going to the larger in.

So, when theta is change the shape is not change, shape of the distribution remains in the only thing either when theta becomes higher this is a stretched out and when theta becomes lesser than it is contracted, this becomes the reliability function is always starting at 1 at t equal to 0 and it is a non-increasing function it is a decreasing function. So, as time progresses the reliability is decreasing.

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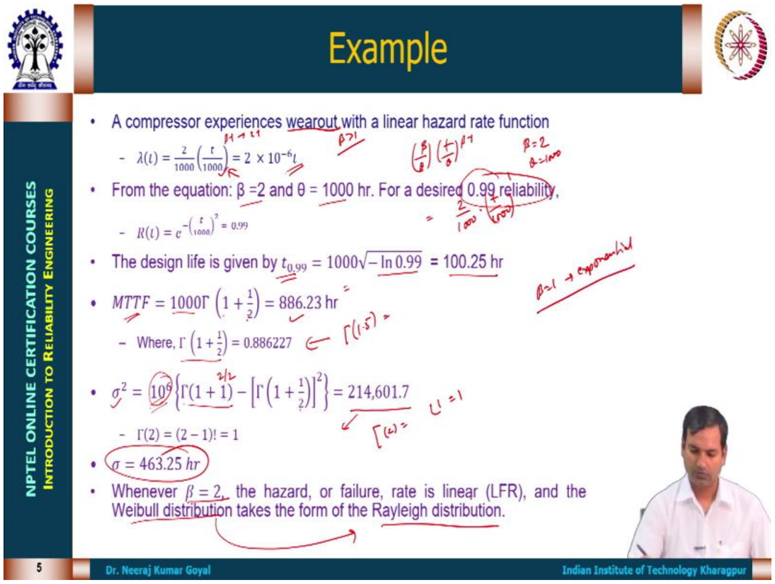

Now let us take one example here let us say that there is a compression which is experiencing wear out. So, wear out means whenever we say wear out this is the increasing hazard rate or beta is greater than 1. So, here it is given we are given that lambda t is following this pattern that lambda t is equal to 2 upon 1000 multiplied with t divided by 1000.

$$\lambda(t) = \frac{2}{1000}\left(\frac{t}{1000}\right) = 2 \times 10^{-6} t$$

If we compare this, if we compare this with our function beta upon theta than t upon theta raise to the point beta minus 1. So, if you see t to the power 1 so, that definitely beta is equal to 2. So, when theta is equal to 2, then t to the power beta minus 1, so, this will become 1 and here if you see compared with this then for beta equal to 2 this will become beta minus 1. So, 2 minus 1 and theta is equal to 1000.

So, if you put beta equal to 2 and theta equal to 1000 here, we will get 2 upon 1000 multiply by t upon 1000 raise to the power beta minus 1 which is same as this. So, from here by comparison we are able to get that beta is equal to 2 and theta is equal to 1000. Now, from this equation we get beta to equal to 2 theta equal to 1000.

Now, we want to have a let us say we want our desire is that we want to know the design life at which reliability is 0.99. So, Rt is equal to 0.99 I want to know what the value of t. So, that is t 0.99 I want to know. As we discussed earlier, this is equal to I can take log of both sides that will give me minus ln of 0.99 and that will be equal to t upon 1000 square.

So, t upon 1000 will be equal to square root of minus log of 0.99 and t will be equal to 1000 into square root of minus ln of 0.99. This gives me 100.25 hours. For MTTF calculation again theta gamma function of 1 plus 1 upon beta, beta is 2 theta 1000. Once you put this, we will be able to get the MTTF value. Gamma function of 1 plus 1 point 1 by 2 that is gamma function of 1.5 is equal to this value 0.886227. Similarly, we can calculate sigma square sigma squared is theta square. So, 1000 square will become 10 to the power 6, gamma of 1 plus 2 by 2, gamma 1 plus 2 upon beta.

So, 2 upon 2 will be 1 minus gamma of 1 plus 1 upon beta whole square. So, gamma of 1 plus 1 by 2 whole square. This if we solve we get the 214601.7 gamma value of 2 as we know it is a positive integers. So, this is equal to factorial of 1 that is equal to 1. If we take a square root of this my sigma value comes out to be 463.25 hour.

So, when beta is equal to 2 then our lambda t is a proportional function of time t. So, hazard of failure it is linear it is a linearly increasing function and the Weibull distribution becomes Rayleigh distribution. So, as we know that Weibull distribution for different values of beta it represents a different kind of distribution when beta is equal to 1 it becomes exponential distribution.

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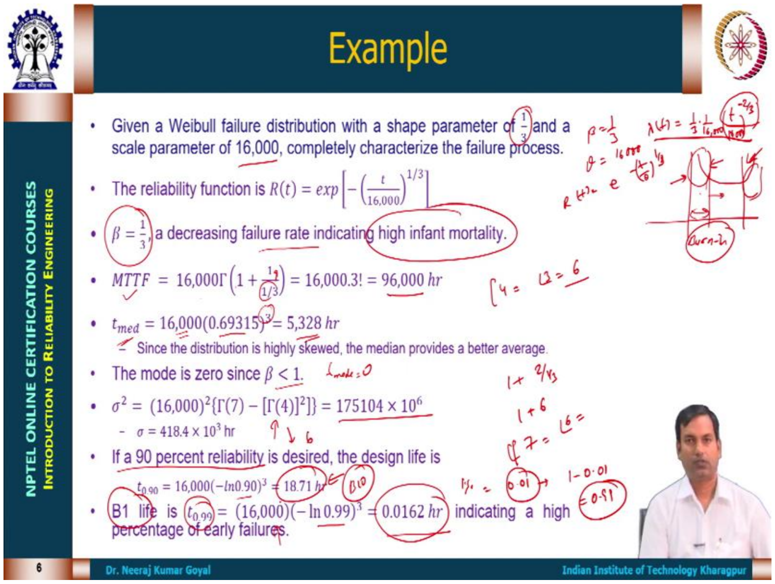

Let us take 1 more example that of given failure rate Weibull distribution shape parameter is 1 by 3. So, my beta is equal to 1 by 3 and scale parameter that is theta, theta is equal to 16,000. In that case, we can find out all the values, but is it reliability, reliability Rt will be equal to e to the power minus t upon theta raise to the power beta, beta is less than 1. So, it is a decreasing failure rate.

So, with time like, if I take failure rate, then failure rate will be equal to 1 by 3 into 1 upon theta that is 1 by 16,000 t raise to the power theta minus 1 so, that will be minus 2 by 3 divided by 16,000, t upon theta is raise to the power 1 minus beta. So, this if you see it is a negative function negative power of t, since t has a negative power that means, with time the failure rate is going to decrease. So, decreasing failure rate.

Decreasing failure rate reason in bathtub curve we are also calling as infant mortality period, because, here in this case this curve infant mortality here the failure rate is decreasing. Now, this is undesired because what will happen in this case, many failures will happen at the start of the life which is undesired.

Because any person who is using this product the moment he purchase and started to use, it fails, so, it is the worst customer experience which is highly undesirable. This reason, when it is failing that means, in this reason the equipment has already worked for the desired life whatever is desired life maybe 10 years, 5 years depending on the equipment.

So, here if it is failing it is expected and customer has already used the equipment for the certain period of the time. So, he is satisfied, but these failures are going to lead to the higher and very high dissatisfaction among the customer. And also company reputation will be affected because new product launch whenever they do and they start having this kind of failure the company image will be severely affected. So, this reason we have to address this reason we have to make sure that customer is not seeing these fillers.

So, the one way of doing that is we do the Burning. In Burning we are keeping this product within the company and providing at the highest stress and checking. So, make sure that these do not fail during the initial some period. So, this period we passed it and those components which do not fail, and those products are only sent to the customer. So customers start seeing the life from where which is near to the constant failure it reason.

Now, how can we calculate MTTF for this we know the formula for MTTF. MTTF is theta gamma function of 1 plus 1 upon beta. So, that will become theta gamma function of 1 plus 1 by 3. So, 1 by 2 if we take this will become 3. So, gamma function of 4 comma functional 4 is factorial 3 that will be 6.

So, 6 into 16,000 gives me the 96,000, t median as we have discussed earlier t median is theta minus ln of 0.5 that comes out to be 0.693 raised to the power of 1 upon beta 1 upon beta becomes 3 because beta is 1 by 3 this gives me 5328 hours. Since beta is less than 1 mode will be equal to 0. So, t mode is equal to 0 and sigma square is theta square gamma functional 1 plus 2 upon beta. So, 1 plus 2 divided by 1 by 3.

So, that will be 1 plus 6 that will be equal to 7. So, gamma of 7 means you have the factorial 6 and similarly, 1 plus 1 upon beta is gamma of 4, gamma of 4 means factorial 3 that is 6. Once we solve this we get this value of sigma square and from here we can get the standard deviation sigma equal to 418.4 into 10 to the power 3 hours.

Now, if I am interested to know the design life for 90 percent reliability that means, I want to sell the product when and I want to promise the life as the life at which reliability is not falling below 90 percent. So, I will promise the product for around 18 hours. So, for 18 hours when the product is used, there is 90 percent chance that system it will continue to work for 18 hours. So, my reliability is 90 percent here or this also we can say this is the B10 life. Similarly, if I want to calculate B1 life B1 life means 1 percent is equal to 0.01 failure probability.

So, my reliability will be equal to 1 minus 0.01 that will be equal to 0.99. So, I want to know the life at which reliabilities 0.99 that is t 0.99 this will be same formula we will use that will be 16,000 square minus ln of 0.9 whole cube, this comes out to be 0.0162 hours. So, if you see that initially number of failures are very high 1 percent is happening like 10 percent failures are happening within 18 hours. So, we will stop it here. We will continue this discussion in our next lecture. Thank you.