Introduction to Reliability Engineering
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Lecture 14
Lognormal Distribution

Hello everyone, in previous lecture, we discussed about normal distribution. Now, we will be discussing about lognormal distribution in this lecture. So, we have a brief discussion about lognormal distribution here.

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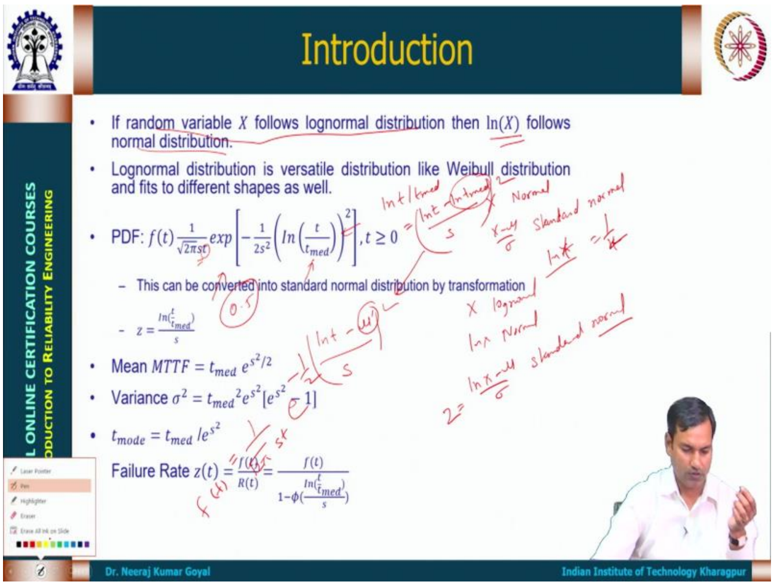

So, lognormal distribution is kind of special form of normal distribution we can say that, if we say that variable X the random variable X if it is following lognormal distribution then ln of x will follow the normal distribution. Like we said that, if X follows normal distribution, then X minus mu divided by sigma follows the standard normal distribution. Same way here if X follows the lognormal distribution, then ln of x will follow the normal distribution or we can say that ln of X minus mu divided by sigma will follow the standard normal distribution.

So, we are able to do same thing what we are able to do with the normal distribution. Here since ln of X is coming so, because of ln of X then X is changed to an ln of X. In earlier case, we had our sigma under root 2 pi sigma here we have additional parameter t here, because of the when we differentiate X ln of X then this comes 1 upon X or when we differentiate ln of t this will become 1 upon t. So, this 1 upon t also comes here and regarding this exponential part which is that is minus half of here if you see this, this is nothing but ln of t divided by t median. This can also be written as ln of t minus ln of t median divided by S whole square.

This is square and this is also square. So, this will become ln of t minus ln of t median divided by S whole square. If I want I can also write it in another form that is ln of t minus mu dash divided by S whole square minus half. So, this can sometimes you will find that this distribution is represented in this format that is e to the power minus half 1 upon under root 2 pi S into t. This form is also many times used, many times we use the t median directly. As we see either we see mu dash or we say ln of t median both are same.

So, why we use T median? Because it is as we know when if I put t equal to t median here then this will become ln of 1, ln of 1 is 0 that means the f t at 0 which is we know from normal distribution comes out to be 0.5. So, since the value of t at which it becomes CDF becomes 0.5 that value is of t is t median. So, that is why this is called the median value.

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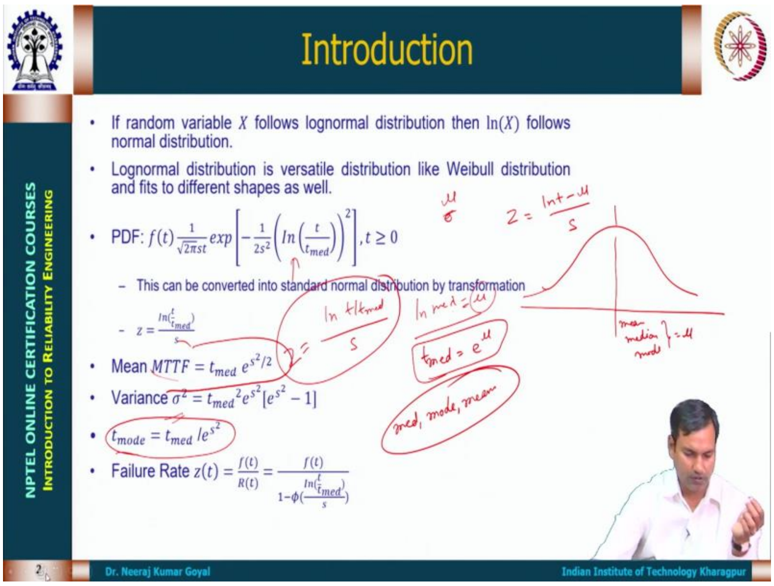

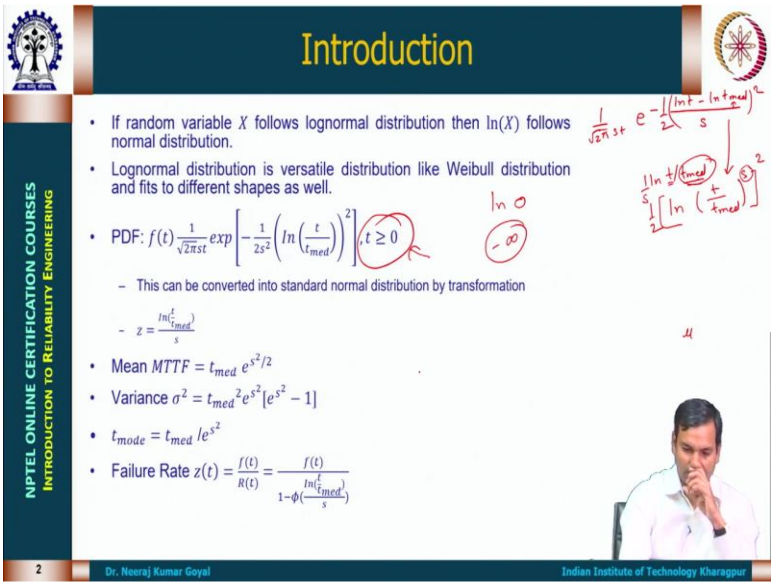

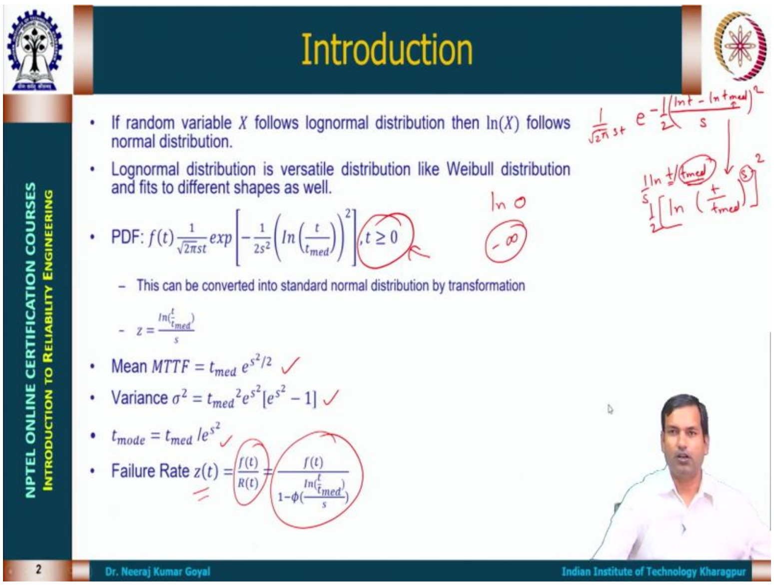

So, we can use median value directly here or we can use the mu function also there. As we discussed earlier, I can use this part that is ln of t divided by t median whole divided by S this I can call it as Z. Once I do this, then I can convert lognormal distribution into the standard normal distribution. And I can use the standard normal tables to get the CDF values or vice versa that I can get the values of Z for given CDF values. Unlike normal distribution, in normal distribution, we had the mean as mu and standard deviation as sigma. But here this relationship is not X minus mu this relation is ln of t minus ln of t median.

Because of that or ln of t median here we can say is the mu. So, here this mu is not the exactly the mean here, mean comes out for this distribution the mean is given using this formula that is t median into e to the power S square by 2. If I am using mu here, then median will be equal to t median will be equal to exponential of mu. So, this t median, if I use this earlier form, then this will become exponential of mu. So, I can say t median is e to the power mu if I am using this another format here, I am representing Z is equal to ln of t minus mu divided by S.

Similarly, variance for this is given us t median square into e to the power S square multiplied by e to the power S square minus 1. And more value for this is like for a normal distribution median, mode and mean all three are same, because there is no tail it is a completely it is having all the reflective and left side right side at all property symmetric distribution. So, because of symmetry, and there is no tail like left side right side there is no tail. Around the mean both sides, it is having a similar shape and distance.

So, because of that, all mean, median, mode all are equal to mu here, but same is not true for the lognormal distribution. If we look at this lognormal distribution a little bit more deeper, then you see that I am seeing that here the parameter is this is coming 1 upon under root 2 pi S t e to the power minus ln of t minus ln of t median divided by S square half as we see here. Since this is ln, this parameter which I am taking here, this becomes ln of t divided by t median.

$$\text{PDF: } f(t)\frac{1}{\sqrt{2\pi}st}\exp\left[-\frac{1}{2s^2}\left(In\left(\frac{t}{t_{med}}\right)\right)^2\right], t \geq 0$$

So, this t medium is not the location parameter here, this t median becomes the scale parameter here that means, if I change the t median, I have to change the t with the similar amount like if I made the t median as 10 times, then I have to multiply t with the 10 times to have the same values. So, here and this is divided by S. So, effectively, if I take it in ln function, this becomes t divided by t median raised to the power S squared, this value I can write it like this. If you see if I write it like this, then S becomes the shape parameter.

So, if I change the S, then the shape of the distribution will change, because this is becoming a power of t. That is why the lognormal distribution also represent a family of distributions. Because it can take different shapes unlike normal distribution, normal distribution shape is same that is bell-shaped, bell-shaped curve, but this can have various shapes like the Weibull distribution. So, because of that, this lognormal distribution also fits to a variety of the data.

Further, if you see that lognormal distribution is defined for values of random variable, which are greater than equal to 0, because we know ln of negative values does not exist. So, minimum value of X can be 0 and for that, it will be minus infinity. So, we cannot take log of negative values. So, this distribution is not valid for the random variables which are having a negative value. So, it is very well suited for the reliability applications. And it is very highly used for the repair distributions repair time TTR, time to repair many times follow the lognormal distribution.

Even time to fill your data when you try to fit with lognormal distribution many times it will feel better and this is very well usable because it follows the time properties also that time cannot be negative. So, here this becomes very useful distribution for the reliability studies where maintainability and reliability are studied.

So, here we can use these formulas to get the interesting quantities in which we are interested and we can also get the failure rate for this lognormal distribution using same ft upon Rt. So, let us say if we do this by let us see how this looks like by taking the I am again use the Excel sheet for the same thing.

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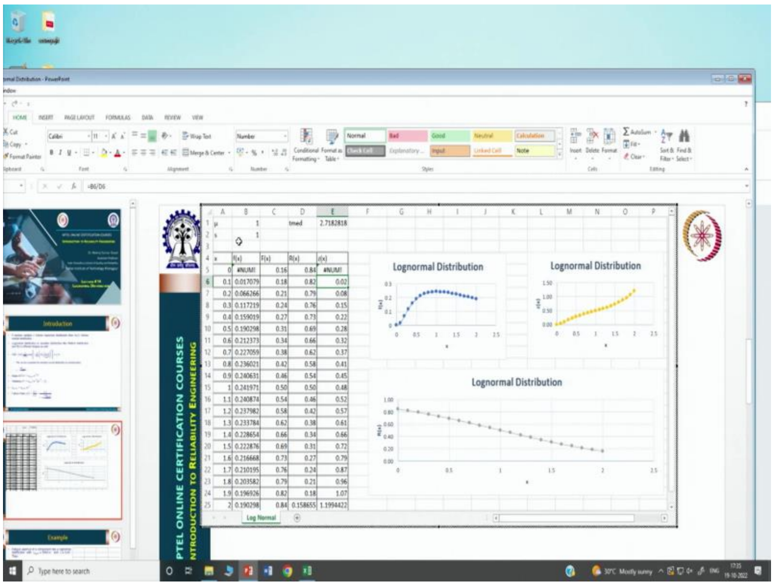

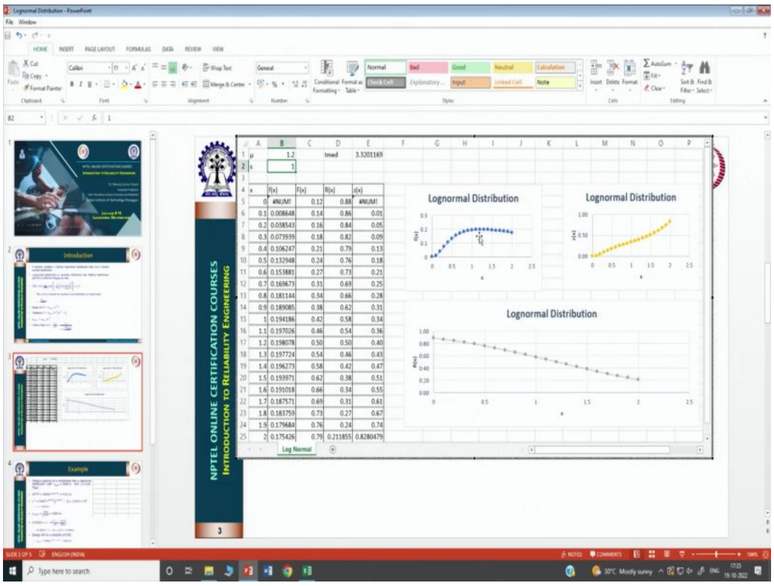

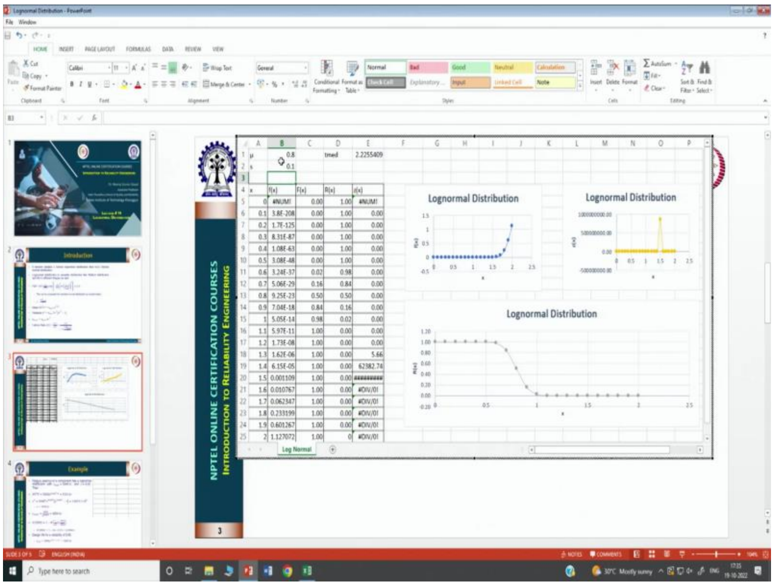
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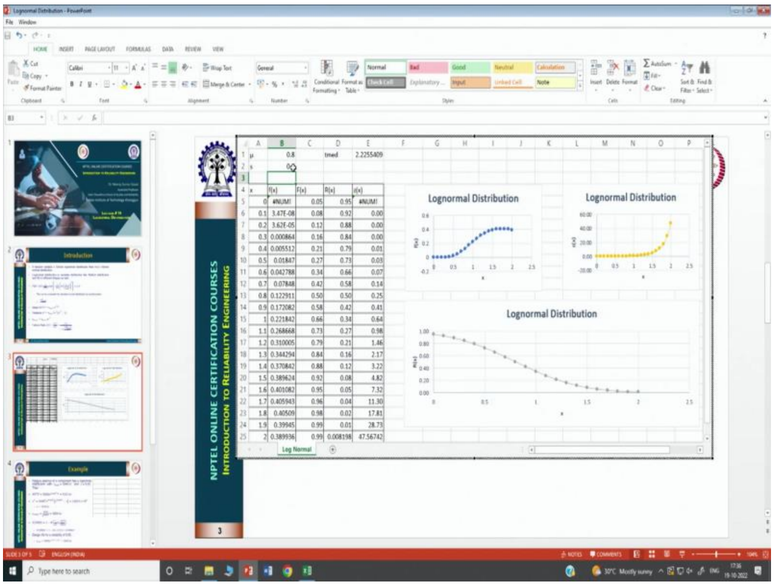
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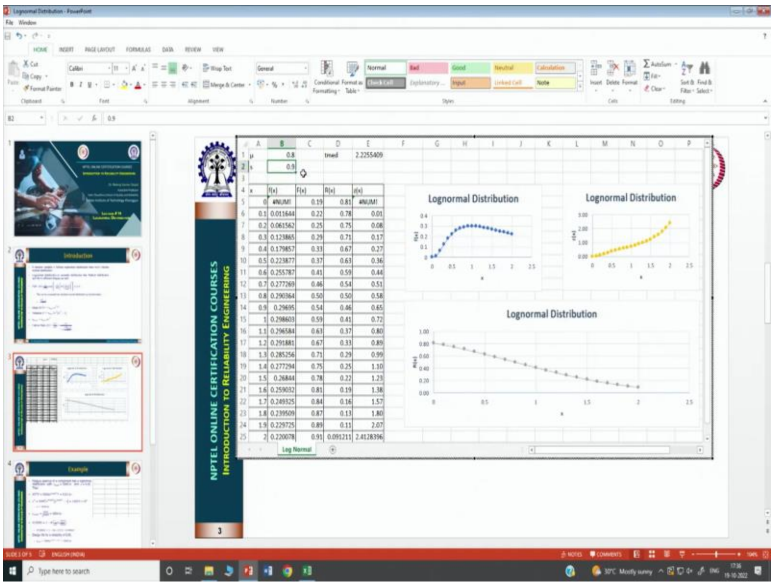
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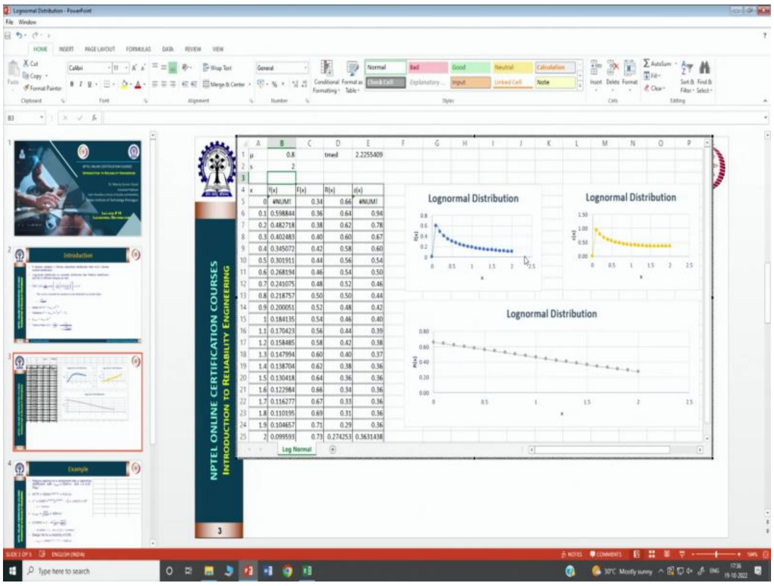

So, let us look at it so, here if you see that this is t median, I am having an I am having the S value, just give me 1 second. So, actually lognormal distribution uses mu. So, it is using that. So, if I say this actually uses the formula that is Z is equal to ln of X minus mu divided by S. So, this value is S and for mu I have to get the value of mu from the t median or I can get the t median from the mu.

So, the parameter here are the mu and S not the t median and the S. So, in Excel sheet we have to supply the value of mu and S that is why it is taken like this here. Now, let us look at it. So, as we discussed earlier ln of t median is mu. So, we can calculate t median as the value exponential of mu. So, here if we see that what we saw that if I get t median, t median is exponential of mu. So, I can calculate that exponential of mu will give me the t median. This sigma is nothing but the S I can write it S for better clarity.

Now, here I have taken few values of X and I have got the value of fx, fx is as we see that is the lognormal distribution value of with respect to X and mu values, I have taken B 1 and S value is given us B 2 and false means I want to take the PDF value. And if I take CDF, I will make it true same thing I will make it true. So, I will get the CDF value and RX is nothing but the 1 minus CDF and ZX is small fx divided by RX that I get it from here, same thing I have done. So, let us see if I change the mu then what happens and if I change the S then what happens? So, let us change mu.

So, with mu what we expect mu is the scale parameter. So, it will stretch or it will become if I increase this will be stretched more. So, this same distribution, what will happen it will stretch it will be spread over a more values. If I make it little S it will contract in a smaller region you will have these values. Similarly, if I take different values of S like for S equal to 1 these values are there, If I take S equal to 0.1 then you will see that shape is changed completely. If I make it 0.5 it gets different shape, it is kind of curved shape, if I make it or 0.9 it will be in this shape. If I make it let us say 2 that is another shape.

So, here by changing the value of S, we are able to change the, we are able to capture different failure patterns. Here shape means, since I am changing the shape of PDF means, PDF is telling that how failures and where the failures are more where the failures are less. So, when we are able to change the shape that means, we are able to change the how failures are distributed over the time. So, but in normal distribution most of the failures are distributed in the central region, but if you have the tails like in this case, if we see here then we see that most of the distribution are in the initial, most of the failures are in the initial region only.

And later on number of failures are reducing that is coming and failure rate is also same like it is a decreasing failure rate. So, we can here also we can have decreasing failure rate increasing failure rate if you see 0.5 then we have the increasing failure rate here the failure rate is increasing here Zx is increasing here. And if you see here the density function that means failures are more (())(16:22) failures are less that is also behavior is change. So, because of this what is happening?

We are able to capture a variety of failure patterns and those failure patterns we can represent by the same lognormal distribution by having the different parameter values of mu and S. So, lognormal distribution reliability function also if you see this is reflective of how fx and Zx will be there. And as usual reliability is a decreasing function.

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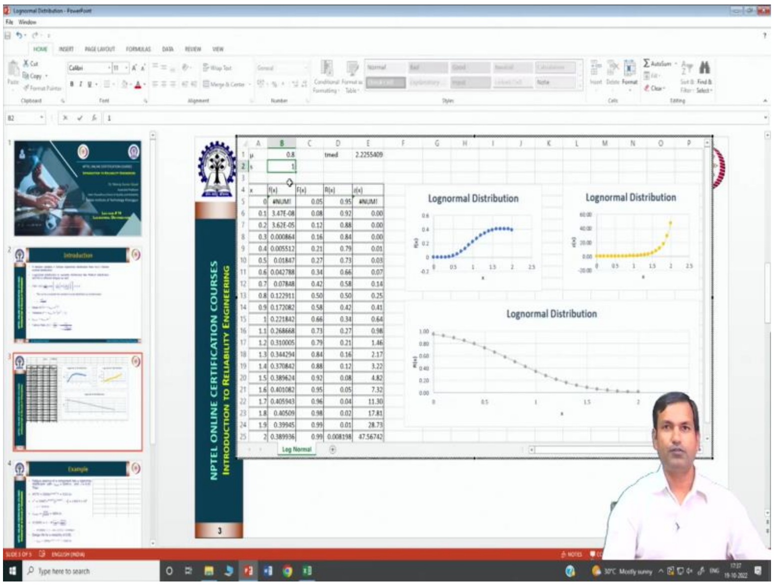

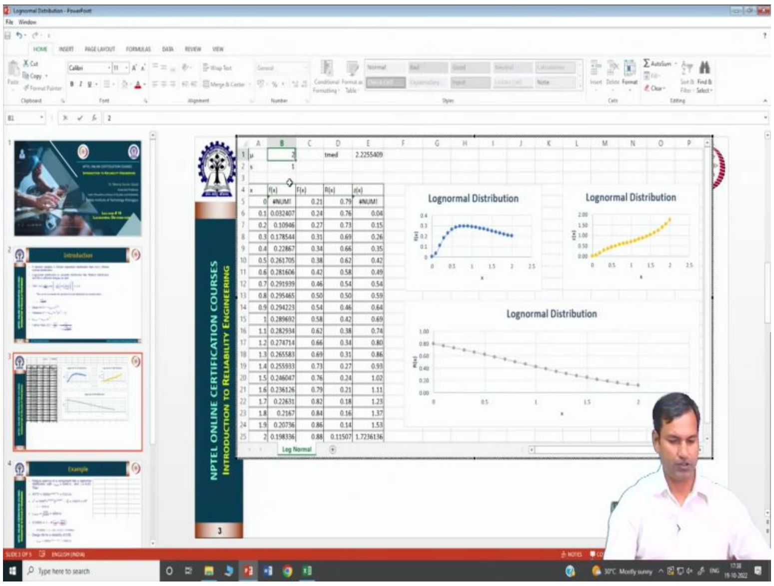

So, in different cases like for I have taken 0.5 if I take 1 then it will be looking like this and let us say this is 2 so, in as it will take more time. If I increase the mu it will take more time to deteriorate. So, mu will be reflecting a if mu is high that means, it will take more time to fail or life will be more. Similar to what we have seen in case of exponential or VFC, in exponential 1 upon lambda does the same thing and or MTBF does the same thing. And in case of lognormal mu will also have the same impact the scale parameters, scale parameters will have the direct implication of on the mean. So, here we can use this.

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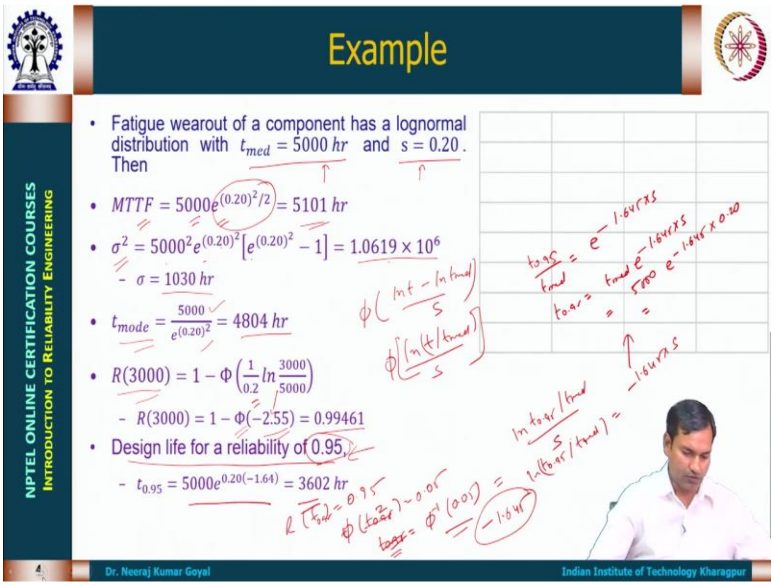

Now, let us take an example here to understand that how we can calculate this? So, for calculation purpose, let us see, we have taken an example, that fatigue wearout of a component follows the lognormal distribution and the parameters given a t median that is equal to 5000 hours and S is equal to 0.20. So, if we have these 2 parameters, we know full about the lognormal distribution now. So, we can calculate everything what is MTTF?

$$MTTF = 5000e^{(0.20)^2/2} = 5101\text{hr}$$

$$\sigma^2 = 5000^2 e^{(0.20)^2}\left[e^{(0.20)^2} - 1\right] = 1.0619 \times 10^6$$

$$-\sigma = 1030\text{hr}$$

$$t_{\text{mode}} = \frac{5000}{e^{(0.20)^2}} = 4804\text{hr}$$

$$R(3000) = 1 - \Phi\left(\frac{1}{0.2}\ln\frac{3000}{5000}\right)$$

$$-R(3000) = 1 - \Phi(-2.55) = 0.99461$$

MTTF is t medium e to the power S square by 2. You can calculate this; this comes out to be 5101 hours. Similarly, we can calculate sigma square, sigma square was t median is called e to the power S squared multiplied by e to the power S squared minus 1 we apply this we get this value that is for sigma square.

So, standard deviation would be the square root of this that is 1030-hour. Mode value was t median divided by e to the power S squared. So, 5000 divided by S square this case mode is 4804. Now, if I am interested to calculate the reliability for 3000 hours, so as we discuss the reliability 1 minus CDF and what is CDF here? CDF is from a standard normal distribution we can get CDF that is phi of ln of t minus ln of t median divided by S. So, S is 0.2. And this I can also write it as phi of ln of t divided by t median whole divided by S. So, this will be ln of 3000. Because I want to calculate for 3000 divided by mean is 5000, median is 5000. And ln of this divided by 0.2, this value comes out to be minus 2.55.

And if I take the standard normal CDF value for this, that and if I subtract that from 1 that turns out to be 0.99461. Now, let us say my design target for reliability is 0.95 I want to know that how much life is there or how much design life can be offered for the reliability target of 0.95. So, what does it mean? It means my reliability at time t is 0.95 or t 0.95 is 0.95. So, I can see that means my phi at t 0.95 is equal to 1 minus 0.95, 0.05. So, t 0.95 will be nothing but phi inverse of 0.05, phi inverse of 0.05 I can calculate and then I can do take the inverse from the that was minus 1.645 as we remember from it.

Now, this minus 1.645 is for Z. Now, this Z value is nothing but Z value is ln of t 0.95 divided by t median divided by S. So, my t 0.95 ln of t 0.95 divided by t median will be equal to minus 1.645 into S from here if I want to calculate then have to take exponential here. So, t 0.95 divided by t median will be equal to e to the power minus 1.645 into S. So, t 0.95 will be equal to t median e to the power minus 1.645 into S I know the S value that is 0.20 t median is 5000. So, that will become 5000 e to the power minus 1.645 into 0.20 this if I solve the same thing is coming here this comes out to be 3602 hours. Let us solve this using the Excel sheet also.

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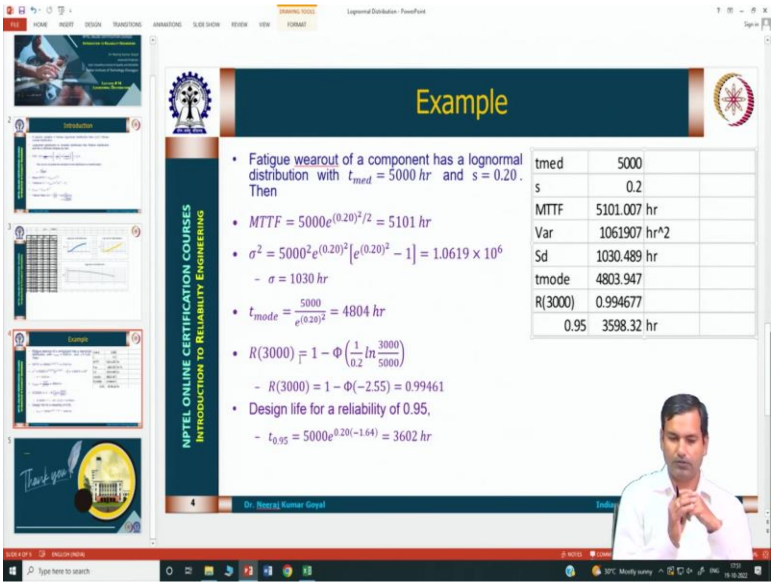

So, if I solve this using Excel sheet then let us take t median, t median is given as 5000. I have the value of S, S is given as 0.2. Now I can get the value of all the values whichever I am interested I can calculate MTTF, MTTF is equal to t median multiply by e to the power x square by 2. So, exponential of S square, S is 0.2 square whole divided by 2, 5101 which is similar to what we have calculated. Then variance if I want to calculate, variance is equal to or t median square multiply by e to the power S square exponential of square multiply by same e to the power S square minus 1.

Once we apply, we get the 1061119 and standard deviation same value that will be equal to square root of this power 0.5, 1030 hour and this will hour square variance will be the square of the unit and this will be hour, this will be hour. Similarly, we can get t mode, t mode is equal to t median divided by e to the power S square exponential of S square 4803.95 that is 4804 approximately. Then I want to calculate reliability at 3000 hours, reliability 3000 hours I can directly do in Excel because Excel also has the lognormal distribution.

So, 1 minus lognormal distribution, lognormal distribution value for X is 3000. I want to know this value for 3000. And what is the mu? Here, since I am taking mean as we remember, mean is mu is ln of t median. So, I will take ln of t median, this is not t median this is ln of t median and standard deviation is value of S. And I want to know cumulative values I will put true; this gives me the hour 3000 if you see that same 0.99468. And I want to know the design reliability.

So, my reliability target is let us say 0.95. So, that means I will take the norm inverse, norm inverse sorry, log norm inverse. So, inverse what is the probability I want to calculate that is 1 minus of 0.95 because this is reliability. So, unreliability would be 1 minus of this or CDF would be 1 minus of this and mean is as we discussed earlier mean is ln of t median and S is 0.2. Once we calculate this, I will get the I will be able to hear actually in calculation, there is a why our calculations are not exactly matching with the Excel sheet?

Because, in these calculations, we have done the we have dropped the higher digits we have taken like here we have taken only minus 1.64. But that value is minus 1.645 something like that. So, once we take the accurate values, because Excel is giving the all-accurate values. So, because of point values, there is a slight change here. But, as we see here, we are able to gather design life here. So, we can use Excel sheet to solve our problems, and we can get all the probabilities at sector values, which we have been using for various purposes. So, as we can also do the same calculation by hand, we can use a calculator and then also we can do.

For normal and lognormal distribution, we may have to refer the normal distribution standard normal tables, which is giving the for different values of Z, what is the cumulative value of Z. So, if I want to know the probability, I will search on the row and column find out the value of Z and corresponding probability value picked up from the values given. If I want to know the Z value, then I will first search for the probability which is matching in the table. And corresponding to that value, I will look into the row and column value and based on that, I will find out the what is the value of Z?

Once I find out the value of Z, then if it is normal distribution, then I will multiply the sigma and I will sum up the mu, that will give me the random variable X value. And if I am interested in taking the value for lognormal distribution, then whatever inverse value I have got for the Z, then Z value will be multiplied with S. And that will be then either, if I am having t median, then what I will do? I will take the exponential of that, and then I will multiply by the t median. If I am having mu, then I will first add the mu then take the exponential that will give me the value of t.

So, this way, we are able to with this like most of the major distributions, there are other distributions like gamma distribution, there are beta distribution, there is so many distributions. But these are very commonly used distribution for the purpose of the capturing the failure patterns or capturing or repair patterns.

And these are quite as we discussed, we can use them it is not very difficult to use, we can use them for calculating the reliability, unreliability, failure rate, and PDF whatever value we require. These are the four major parameters, then there is MTTF, then there is a variance. All these things we can calculate once we know that parameters of the distribution.

So, here, we will stop discussing about these major distributions in next time, we will start discussing about the system reliability models. System reliability models in those we will be discussing that system is made up of components. So, if we know the reliability of components, how can we get the reliability of the systems? So, I will stop it here and we will continue in next lecture. Thank you.