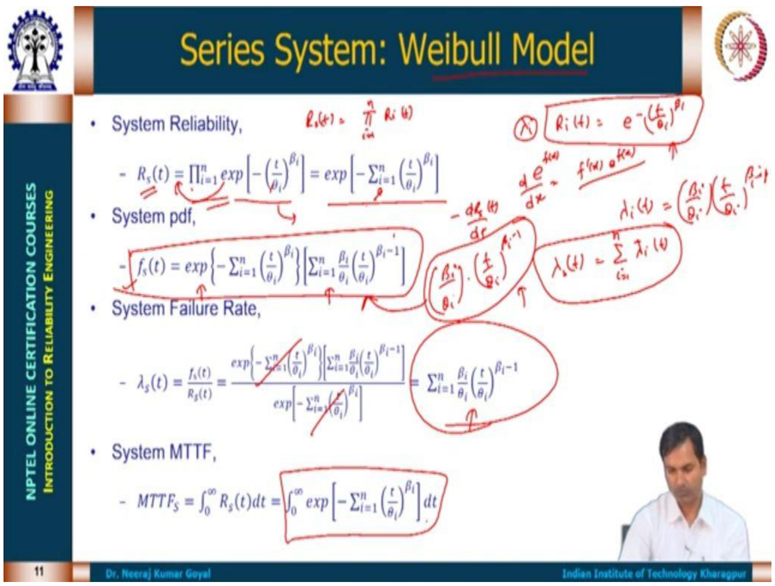
Introduction to Reliability Engineering
Professor. Neeraj Kumar Goyal
Indian Institute of Technology, Kharagpur
Lecture No. 16
System Reliability Modelling (Contd.)

Hello everyone. So, welcome back to lecture number 16. We will continue our discussion from where we left in lecture number 15. In lecture number 15, we discussed about series model and we also discussed about the exponential distribution.

(Refer Slide Time: 00:44)



Continue our discussion. Today we will discuss about the Weibull model. That means when distribution for component is Weibull. So, if each component is following Weibull distribution, let us say I said lambda i or we can say Ri; Ri t is equal to e to the power minus t upon, we generally use theta and beta. Weibull distribution two parameter Weibull distribution has theta and beta as a parameter. So, component reliability is we can express as Ri t is equal to e to the power minus t upon theta i, to raise to the power beta i. So, different components will have different values of theta and different values of beta i.

$$R_s(t) = \prod_{i=1}^{n} \exp\left[-\left(\frac{t}{\theta_i}\right)^{\beta_i}\right] = \exp\left[-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}\right]$$

Now, I want to know the system reliability. For system reliability, the formula is same; that is multiplication of reliability values irrespective of the distribution. So, Rs t here is equal to multiplication of i equal to 1 to n, Ri t. Ri t values, I can take from here that is exponential of minus t upon theta raised to the power, theta i raised to the power beta i. This if we see as this we have already discussed in case of multiple failure mode that if we multiply all this, effectively, exponential terms will get added up. So, if I take exponential outside this, then because of this power feature, the pie multiplication will become summation.

And this will become summation of i equal to 1 to n, t upon theta i raised to the power beta i. This becomes my integration of failure rate for the all components at the system level. So, if I want to know the system PDF; system PDF is nothing but the exponential dR t over dt, minus dR t over dt. So, this if we differentiate, effectively, this exponential we know exponential of some function of x. If we differentiate it with respect to x, I get f dash x, then e to the power of fx. Same formula if you use, then this remains same; differentiation of this gives you the beta i divided by theta i.

$$f_s(t) = \exp\left\{-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}\right\}\left[\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta_i-1}\right]$$

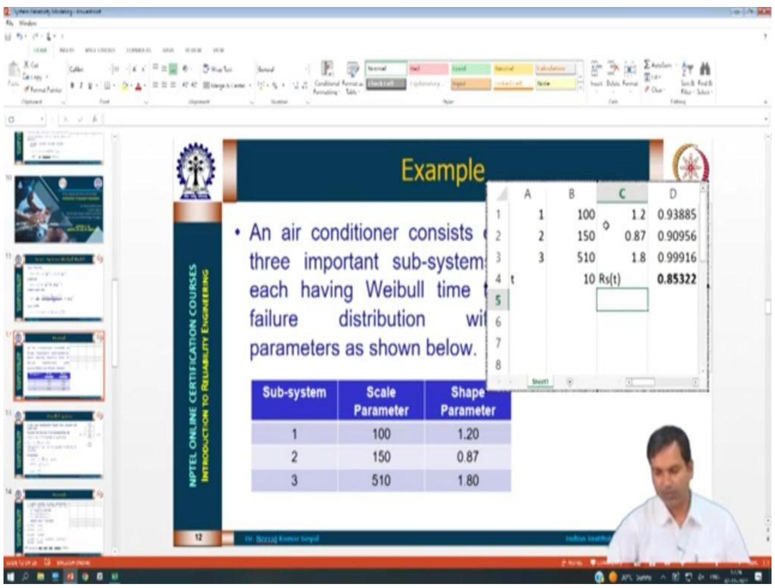
Beta i is coming because of the differentiation x raise to the power n is equal to n x raise to power n minus 1. So, theta is coming because it is coming as a multiplying factor to t. So, 1 upon theta i, beta i; and this raise to the power beta i minus 1. The same thing comes here and this value overall this gives the PDF. If system failure rate I am interested, then fs upon Rt. So, this divided by this if you do, then exponential term will get cancelled; only this term which we have calculated summation that will be remaining. So, effectively it is nothing but this, this is if we differentiate this, then and take the same process if you follow; then lambda i t is nothing but the differentiation of this exponential term.

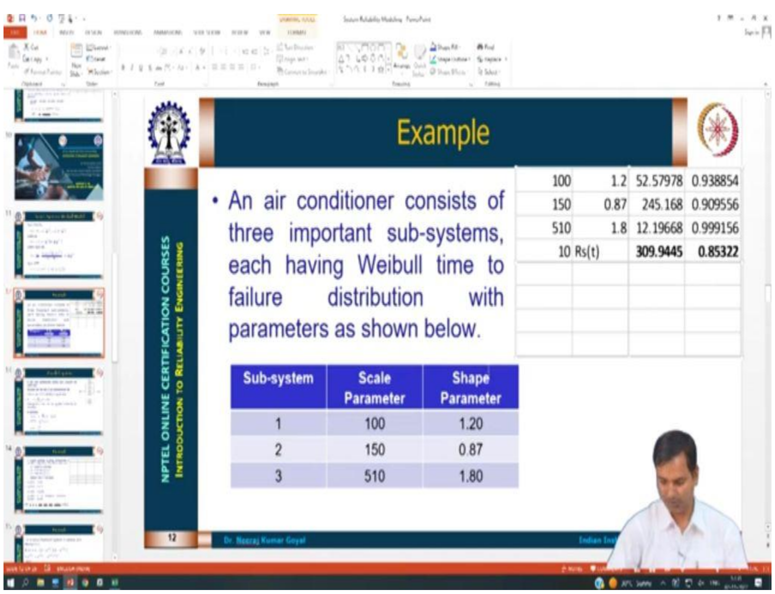
$$\lambda_s(t) = \frac{f_s(t)}{R_s(t)} = \frac{\exp\left\{-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}\right\}\left[\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta_i-1}\right]}{\exp\left[-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}\right]} = \sum_{i=1}^{n}\frac{\beta_i}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta_i-1}$$

So, that is equal to beta i upon theta i, t upon theta i raised to the power beta i minus 1. And same thing comes here, if we sum it up we get the system; so, system failure rate is always; for series system, system failure rate is summation of individual failure rates, lambda i t. Whether it is exponential distribution that is Weibull distribution, this will be the term. Now, here if you look at it, if I want to know the system MTTF here; that is little tricky here. Unlike, exponential distribution, which was easily integratable and we were able to get the system MTTF. Same is not the case here. Here we have too many time numerically solve this.

$$MTTF_S = \int_0^{\infty} R_s(t)dt = \int_0^{\infty} \exp\left[-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}\right]dt$$

(Refer Slide Time: 05:09)





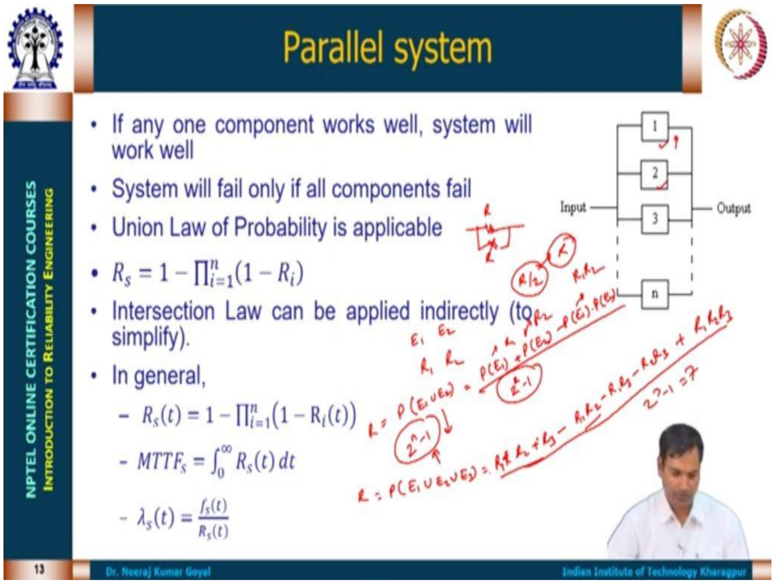
Let us take an example. An air conditioner consists of three sub-systems, each having Weibull time to failure distribution with parameters as shown below. So, let us put the same thing here; I will put same thing here; I will use text only. So, this is my system number, this is my scale parameter, this is my beta. Now, if I want to know the reliability, let us say time is given to me. Time is let us say I have the 10, I can have 10. Then, how much will be the reliability for this? I know the reliability formula is Rt is equal to exponential minus of t; t divided by theta, divided by theta.

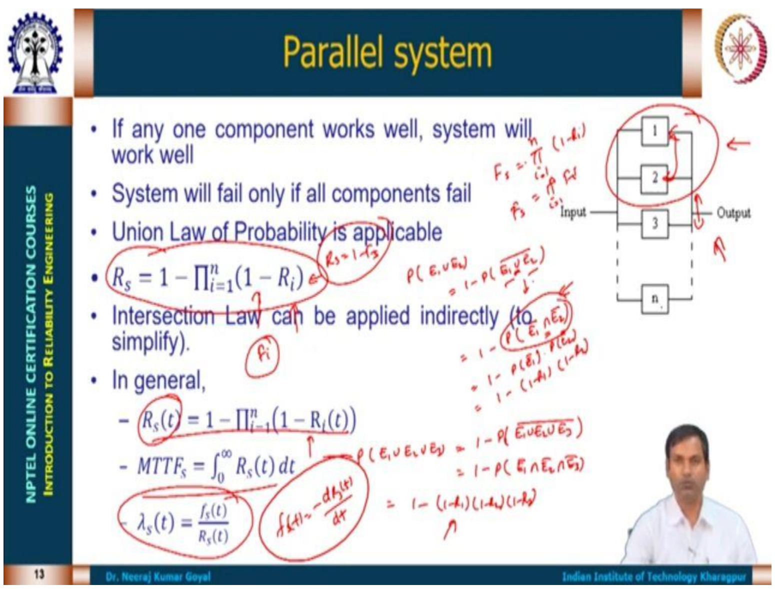
This is my scale parameter theta, whole raised to the power beta; this becomes my reliability for this component. Similarly, I can get the reliability for these components. As you see, I can get the reliability for the each component. From here if I am interested to know the system reliability Rs t, that would be nothing but the product or multiplication of above terms. Because it is a series system; for series system, the system reliability is multiplication of component reliabilities. So, this value turns out to be my system reliability. If I am interested in any other parameter, if I am interested in system failure rate; now system failure rate will be a function of time. But, for a given time, I can calculate.

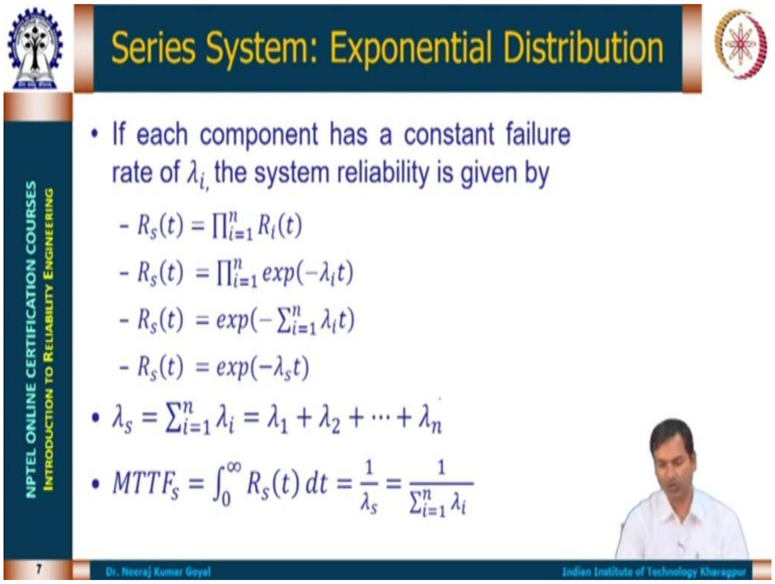
For a given time, if I want to calculate I have to, I can calculate individually. Let us say if I insert here. If I calculate individual failure rate, the failure rate is equal to as we discussed earlier that is theta divided by beta, multiply by t. T, I will again use dollar signs; t divided by theta, whole raised to the power beta minus 1. So, this becomes my failure rate for each sub-system. If I want to know the failure rate for system that will be sum of these failure rate. If I want to calculate reliability as you know for system reliability that is equal to exponential; here I have to integrate, so that will be problematic.

I may unlike, I could get it get the reliability from the, for the exponential distribution by simply taking exponential and multiplying by t; that is not feasible here, because here it is the function of time t. So, I have to integrate it with respect to time t, then only I will be able to get. Because, it is a function of t, so I cannot do it directly. If I do, I will not get the same answer. So, to calculate reliability, I have to calculate individually and then get this; but this is giving me the system level failure rate. This is my system failure rate for 10 days. So, here like this whatever parameter I am interested, I can calculate and use it.

(Refer Slide Time: 09:47)







Now, let us discuss about parallel system. Before going for parallel system, let us discuss a little bit more. We have already discussed actually, that is why I did not include it here. But, just as a refresher that let us say two components of exponential model if we discuss about, exponential distribution.

$$R_S(t) = 1 - \prod_{i=1}^{n}\left(1 - \mathrm{R}_i(t)\right)$$
$$MTTF_S = \int_0^\infty R_S(t)dt$$
$$\lambda_S(t) = \frac{f_S(t)}{R_S(t)}$$

Then, let us say if you are talking about two components here, then reliability will be, failure rate will be summation of this. Now, let us go forward with the parallel system. For parallel system, we know that this is in reliability block diagram, it is in parallel. But, actually functionally or the in the circuit, it is not necessary they should be in parallel. Like even though, let us say if we put two resistance in parallel.

Let us say this value is R, this value is also R. But, my reliability, my resistance requirement is R by 2. So, in that case even if one fails, my job is not done; because in that case also, my reliability resistance becomes R, and R is not acceptable; because in that case, my circuit will fail. So, in that case, though functionally they are in parallel, but from reliability purpose they are not in parallel; because for reliability purpose if one of the resistance fails, my system fails, the system does not work. So, we do not have to look at the functional layout. We have to look from the purpose that whether the systems are required to work, compulsory to work, or they can replace the functional requirement.

When they are able to replace the functional requirement, then only we use them in the parallel. In parallel, this component and this component will be doing generally the same function, but in parallel; that means both are doing the same thing. But, so if one fails to do, then another will take care of it. In this case, the reliability, system reliability if I have to calculate, I have to use the Union Law. So, let us say if I am having two components R1 and R2 reliabilities R1 and R2. Then, I have to get the probability; let us say as we discussed earlier, this is E1t E1, this is E1t E2.

So, here I am interested to know the reliability that will be equal to probability of union of E1t E1 and E2. So, that function as we know from the Union Law, this will be equal to probability of E1 plus probability of E2 minus probability of E1 into probability of E2, given that E1 and E2 are independent. If they are not independent, then this will become E1 given E2, or multiply by E2 like that. Now, here, this formula then we use, this becomes a trouble because it is increasing. Now, if I am having n component, I, here the number of terms which are generated is 2 raised to the power 2 minus 1.

If I am having n terms, then I will be having 2 to the power n minus 1, like for let us take an example of 3 terms R equal to probability of E1 union E2 union E3. If I am going to do this, this will be R1 plus R2 plus R3; this is R1, this is R2 and this is R1R2. Same way when we write it here for 3 events, this will become minus combination of 2 events. So, that is R1R2 minus R1R3 minusR2R3; three C two, out of the 3 events, I choose the 2 events as a time, and that will become minus. Here I am choosing one out of the three single combinations, here double combinations, single (combi) all number of combinations are preceded by the plus sign wherever, or even number terms are there that is preceded by the negative sign.

And three out of three, when we use plus R1R2R3. If you see here, this is 2 to the power 3 minus 1; 7 terms are there 1, 2, 3, 4, 5, 6, 7. If we use more, then similarly the number of terms becomes very very high; so this formula is little large to solve. In place of solving this, we can do this in a little easier way. And that easier way is using the reverse formula. What is the reverse formula? This also we have discussed briefly earlier; I will again discuss it in detail here. If I want to know the probability of E1 union E2; I can write it as this is 1 minus probability of E1 union E2 whole bar.

That means complimentary of E1 union E2. Now, we can use the De Morgan's theorem. De Morgan's theorem says that E1 union E bar E2 whole bar is equal to E1 bar intersection with E2 bar; that means individual from the whole bar or complete event, I get the individual event inversion and this sign is also inverted; from Union it becomes intersection. If it was intersection, it would have become union. So, by changing the sign and by, the individual event become; so this is the De Morgan's theorem we are using. Now, what happens? This becomes a simpler one; because now I can calculate probability of E1 bar and probability of E2 bar.

And what is probability of E1 bar? That is 1 minus R1; and what is the probability of E2 bar? That is 1 minus of R2. Similarly, if I am having 3 event, I am interested in probability of E1 union E2 union E3. Then, I can write it in same way 1 minus probability of E1 union E2 union E3 whole bar; that if I solve again that will become 1 minus probability of E1 bar intersection E2 bar intersection E3 bar. So that same thing I can write it as 1 minus 1 minus R1 into 1 minus R2 into 1 minus R3. In general, if I am having n elements, I can simply write this formula Rs t or Rs is equal to 1 minus multiplication of 1 minus Ri's.

That means here essentially what we are saying 1 minus Ri is the failure probability Fi, failure probability of component. So, system will fail. In this case, let us say we are talking about two elements. So, any one of the elements work, system will work; so in a way we can say the system will fail when both the elements fail. That is, E1 bar intersection E2 bar; this one. So, when both the element fails, then only the system fails; so, we are trying. So, if n elements are there, then all n elements should be failed, then only system will fail. So, what is 1 minus R1 that is the failure probability of component Ri, component i.

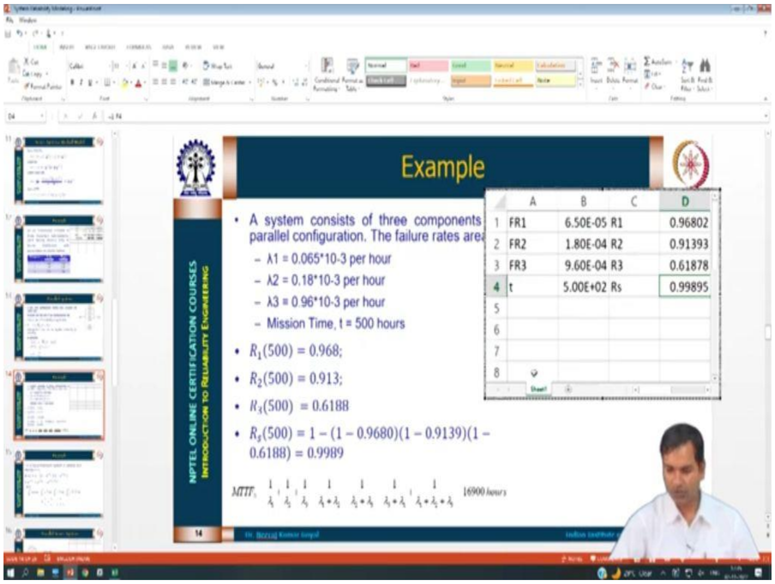
So, how can I get the system failure probability? That is the multiplication of failure probabilities of each component; by multiplying this, I get the unreliability for this system. So, unreliability for the system, Fs is equal to multiplication of i equal to 1 to n, 1 minus Ri. What is 1 minus Ri? 1 minus R is actually the Fi. And if I am interested to know Rs, Rs is equal to 1 minus Fs; so that same formula when I subtract from 1, I get the reliability. So, as we know here, we are using two things that whenever series is there, reliability is multipliable to get the system reliability.

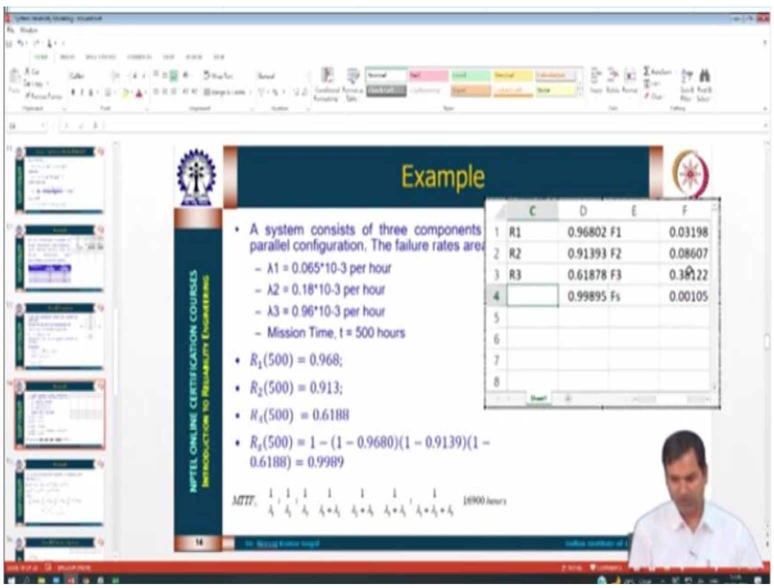
Whenever parallel system is there, then we can multiply the failure probabilities to get the system failure probability. Because for series systems, system will be reliable; then all components are going to work. And similarly for parallel system, the system will be unreliable only when all components will fail. So, we use that analogy and this analogy when we use our computation becomes simpler. And we do not have to use the formula which we were using for inclusion-exclusion formula, where we will need to calculate so many values. So, we can calculate the reliability by using this formula.

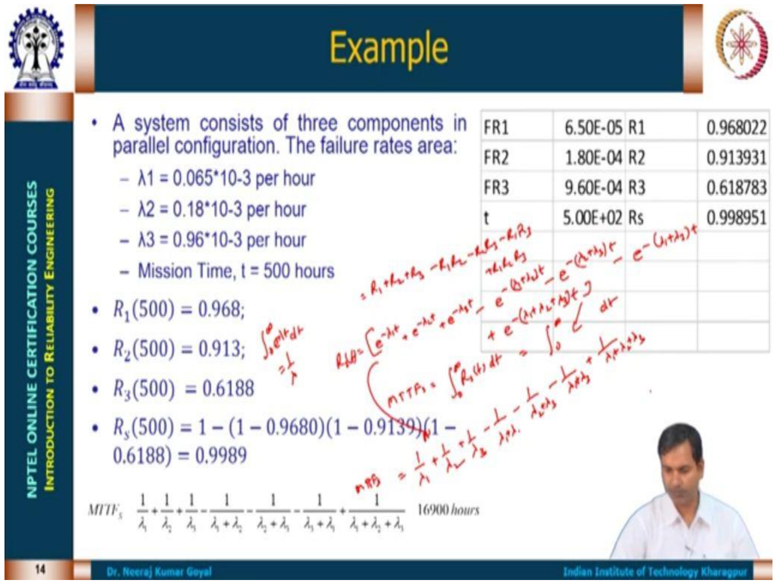
And once we get this reliability equation, if we do the integration from 0 to infinity, I can get the system MTTF. And if I want to know the system failure rate, that is nothing but the fs t divided by Rs t; and where Fs t is nothing but minus dR t over, Rs t over dt. So, same formula which we discussed, we have to apply it here to get the values. So, in a simple way, we simply remember that for series system, reliability gets multiplied to get the system reliability; and for parallel system, the unreliability will get multiplied to give the system unreliability. So, for parallel system, system unreliability decreases. For series system, system reliability decreases; because by each multiplication, the probability will become lower.

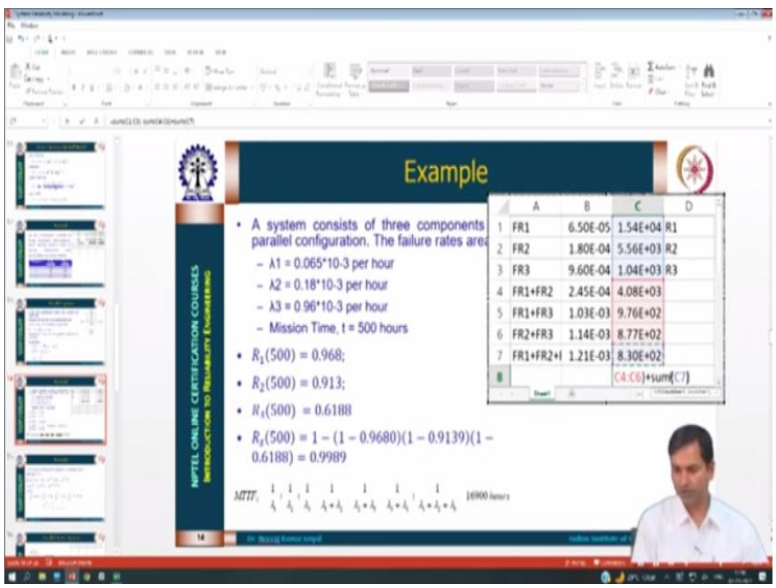
So, in a way, we can say we were saying for series system; for series system, where reliability is determined by the least reliable component. Similarly, for parallel system, the reliability will be determined by the most reliable component or the least unreliable component. Because, the most, the system reliability will be higher than the reliability of the highest component here; because here, the when we use the formula the failure rate will failure probability is going to decrease. Failure probability will always be lesser than that. So, here the reliability of system would be higher than the strongest system or strongest component which we have.

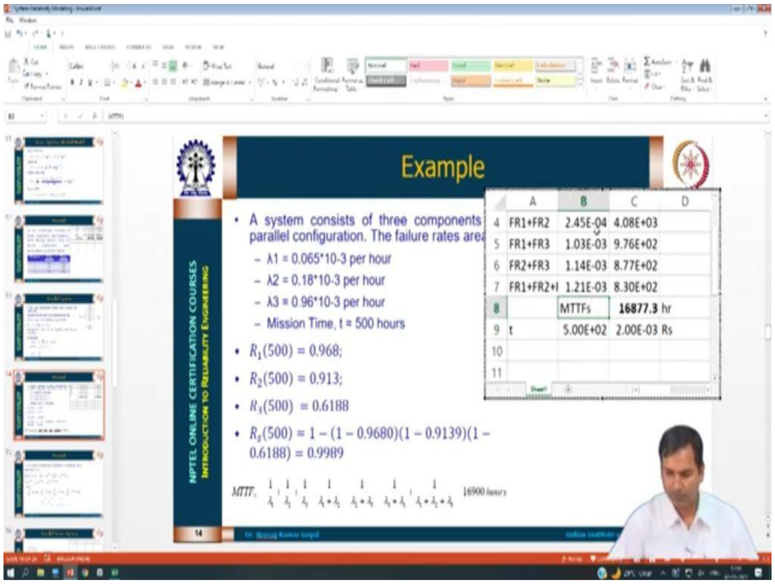
(Refer Slide Time: 20:58)






Example
NPTEL ONLINE CERTIFICATION COURSES
INTRODUCTION TO RELIABILITY ENGINEERING
A system consists of three components in parallel configuration. The failure rates area:
λ1 = 0.065*10-3 per hour
λ2 = 0.18*10-3 per hour
λ3 = 0.96*10-3 per hour
Mission Time, t = 500 hours
$R_1(500) = 0.968;$
$R_2(500) = 0.913;$
$R_3(500) = 0.6188$
$R_s(500) = 1-(1-0.9680)(1-0.9139)(1-0.6188) = 0.9989$
$MTTF_s = \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3}-\frac{1}{\lambda_1+\lambda_2}-\frac{1}{\lambda_2+\lambda_3}-\frac{1}{\lambda_1+\lambda_3}+\frac{1}{\lambda_1+\lambda_2+\lambda_3}$ 16900 hours
FR1 6.50E-05 R1 0.968022
FR2 1.80E-04 R2 0.913931
FR3 9.60E-04 R3 0.618783
t 5.00E+02 Rs 0.998951
14
Dr. Neeraj Kumar Goyal
Indian Institute of Technology Kharagpur


Example
A system consists of three components in parallel configuration. The failure rates area:
λ1 = 0.065*10-3 per hour
λ2 = 0.18*10-3 per hour
λ3 = 0.96*10-3 per hour
Mission Time, t = 500 hours
$R_1(500) = 0.968;$
$R_2(500) = 0.913;$
$R_3(500) = 0.6188$
$R_s(500) = 1-(1-0.9680)(1-0.9139)(1-0.6188) = 0.9989$
16900 hours
A B C D
1 FR1 6.50E-05 1.54E+04 R1
2 FR2 1.80E-04 5.56E+03 R2
3 FR3 9.60E-04 1.04E+03 R3
4 FR1+FR2 2.45E-04 4.08E+03
5 FR1+FR3 1.03E-03 9.76E+02
6 FR2+FR3 1.14E-03 8.77E+02
7 FR1+FR2+I 1.21E-03 8.30E+02
8 C4:C6)+sum(C7)
14



Let us take one example, that of. A system is consists of three components in parallel configuration. These are the failure rates, so I can put them here again. I think these are similar to what we did earlier; I will again type it. So, FR1 is 0.065e minus 3, then FR2 is 0.18e minus 3, and this is per hour; so, my results would be in hours, FR3 is 0.96e minus 3, and my time is 500 hours. Remember the unit should be in same; we should use the same unit. Now, if I want to know, I can get the R1, R2, R3 like we have got earlier. So, R1 is equal to exponential minus lambda t, lambda into t; because I am going to use the same t for all.

So, that is why I am going to put; I have got the R1, R2, R3. Now to solve this, what I will do? I will take 1 minus R1, R2, R3 here; rather than reliability, I will calculate the unreliabilities here F1, F2, F3. What is F1? F1 is 1 minus of R1; similarly, I will get. So, as we discussed earlier, for parallel system, I have to take the multiplication of unreliabilities. In series system, I had taken the multiplication of reliabilities; for parallel, I have to take the multiplication of unreliability; and my reliability will turn out to be 1 minus of this. So, my system failure probability is this and system reliability is this. Again, same thing is done here point 99895 becomes my reliability.

And how much will be the system failure rate here? System failure rate turns out to be. For system failure rate we let us see how do we get the system failure rate. To calculate system failure rate, let us do this calculation. For that we have to use either I can do it here, but that will be a little difficult to understand; I will use the formula which we discussed earlier. We have the three elements. So, that will be R1 plus R2 plus R3, minus R1R2, minus R2R3, minus R1R3, plus R1R2R3; we know the R1. R1 is e to the power minus lambda 1 t, plus e to the power minus lambda 2 t, plus e to the power minus lambda 3 t, minus e to the power minus R1R2.

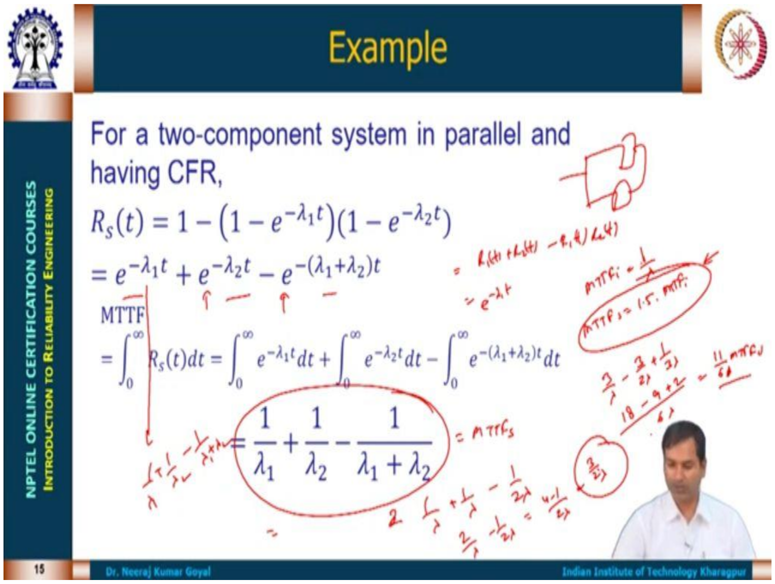
So, lambda 1 plus lambda 2 into t, minus e to the power minus R2R3 means lambda 2 plus lambda 3 t, minus e to the power minus R1R3 means lambda 1 plus lambda 3 t, plus e to the power minus lambda 1 plus lambda 2 plus lambda 3 t. I want to know this is my Rs t; I want to know MTTF. So, MTTF of system is integration of Rs t from 0 to infinity dt. So, this what I can do? I can take 0 to infinity of integration of whole these terms which I have got it here and dt. Now, integrating this with respect to time t, what do I get? We know that integration of 0 to infinity of e to the power minus lambda t dt is equal to 1 upon lambda.

So, same thing when we integrate from here what we get? This will give 1 upon lambda 1, plus this will give 1 upon lambda 2, this will give 1 upon lambda 3. Minus sign will come, minus of 1 upon 1 plus, 1 upon lambda 1 plus lambda 2; minus, again this is minus 1 upon lambda 2 plus lambd3. This is again minus that is minus 1 upon lambda 1 plus lambda 3; and this is plus, plus 1 upon lambda 1 plus lambda 2 plus lambda 3; this becomes my system MTTF. And when I use this formula, I can get the MTTF of system. I can do this in Excel sheet. So, here if I use this formula, I have got FR1, FR2; I can take FR1 plus FR2 that will be equal to this plus this.

Similarly, FR1 plus FR3 will be equal to FR1 plus FR3, FR2 plus FR3 is equal to 2 plus 3. Similarly, I can get FR1 plus FR2 plus FR3; that will be equal to this plus this plus this. Now, if I want to know the MTTF, for of MTTF system in an easier way if I want to calculate, I will insert here. I will take 1 upon FR1 that is equal to 1 divided by this; this I will take here. I am not interested in 1 upon t; so this value I am going to move to little lower. I will delete this row. Now, I know what is the formula? Formula is equal to sum of individual FR, 1 upon FR, minus some of these individual or double combinations, plus sum of 1 upon FR which is coming from combination of three.

If you see this becomes my around 16877 or 16,900 hours becomes by MTTF. So, for MTTF calculation in parallel case what I have to do? I have to take a single combination, double combination; depending on if number of combinations, this will be 2 to the power n minus 1 like we have seven combinations here. So, accordingly we use and then we can get the system or MTTF.

(Refer Slide Time: 29:19)



See one more example, simpler one. Let us say, if we have two components in parallel which is following the constant failure rate; both are constant failure rate. So, we know this is Rs t will be equal to 1 minus this, or we can say that is equal to R1 t plus R2 t, minus R1 t into R2 t. So, R1 t is e to the power minus lambda 1 t, R2 is e to the power minus lambda 2 t, and R1R2 t, R1 t into R2 t is e to the power minus lambda 1 plus lambda 2 t. From this, if I want to get the MTTF, I will integrate this; 0 to infinity. The moment I do integration, this will become 1 upon lambda; this will give 1 upon lambda 1, this will give 1 upon lambda 2. And this will become minus 1 upon lambda 1 plus lambda 2. Same thing we get it here. So, as you see here this becomes my MTTF of system.

Now, let us see if both lambda are same. Then, this will become 1 upon lambda plus 1 upon lambda minus 1 upon 2 lambda; or we can say 2 upon lambda, minus 1 upon 2 lambda. If I take 2 lambda, then this will become 4 minus 1; that is equal to 3 divided by 2 lambda. So, as we discussed earlier here, 1 upon lambda is MTTF of component; that is equal to 1 upon lambda. So, MTTF of system is equal to 1.5 because 3 by two is 1.5 into MTTF of individual component. So, as we see that if we use two component in parallel, MTTF is not doubled; but when we use two component in series, MTTF became half.

So, improvement is diminishing return whenever we use three components, then we will have a different value. That will not be like three lambda or even not that value will become 3 upon lambda, plus minus three cases of 1 upon 2 lambda. So, that will become 3 upon lambda minus 3 by 2 lambda, plus one case of 3 lambda. So, if I take 6 lambda is common here and that will become 18 Minus 9 plus 2; 20 minus 9 is 11 by 6 or lambda, or 1 lambda I can write it as MTTF of i. So, it is around it is less than 2.

So, for two component, it is became 1.5 MTTF; for three components, it is not even 2 MTTF. It is a little bit less than the 2 MTTF. So, we will continue our discussion; we will stop it this lecture here. And we will continue our discussion in the next lecture. Thank you.