Introduction to Reliability Engineering
Professor. Neeraj Kumar Goyal
Subir Chowdhury School of Quality and Reliability
Indian Institute of Technology, Kharagpur
Lecture 17
System Reliability Modelling (Contd.)

Hello everyone. So, now we are moving to lecture number 17. Today we will continue our discussion about the system reliability modelling. In previous lectures, we discussed how a series system is modelled for relative evaluation and how can we evaluate reliability for a parallel system. In series system, all parts of the system should work for the system to work and in parallel system, there is a redundancy, if one part does not work but another one is working, then system will still be continue functioning. Today, we will discuss some more configurations in which system can be there and for a liability evaluation purpose.

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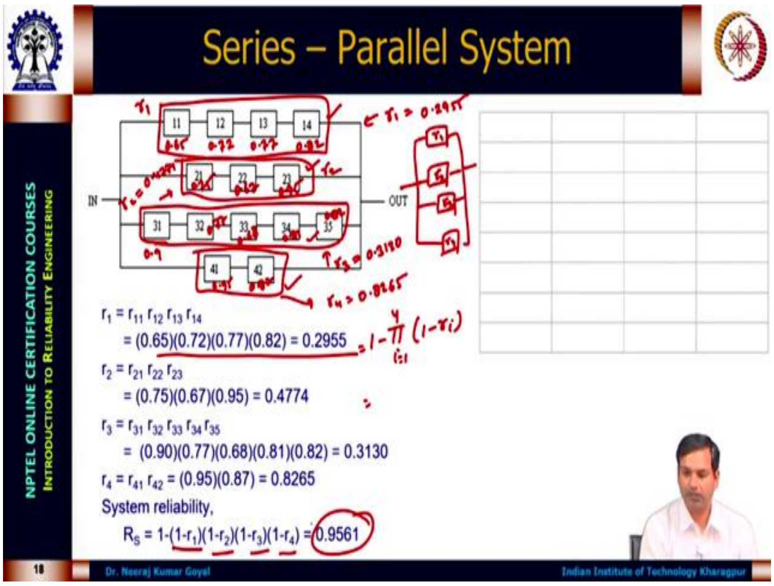

One is series parallel system. So, it is series combinations which are connected in parallel. If we see here, so here we have series of 4 components here, series of 3 here, series of 5 here, series of 2 components here, these components are connected in parallel. Now we want to know the reliability for this. So, to solve this, first generally we solve the series components.

$$r_1 = r_{11}r_{12}r_{13}r_{14}$$
$$= (0.65)(0.72)(0.77)(0.82) = 0.2955$$
$$r_2 = r_{21}r_{22}r_{23}$$
$$= (0.75)(0.67)(0.95) = 0.4774$$
$$r_3 = r_{31}r_{32}r_{33}r_{34}r_{35}$$
$$= (0.90)(0.77)(0.68)(0.81)(0.82) = 0.3130$$
$$r_4 = r_{41}r_{43} = (0.95)(0.87) = 0.8265$$

So, here let us say relative values I have here like this component has 0.65, this is 0.72, this is 0.77, this is 0.82. So, reliability as we know reliability for series system is the multiplication of reliability values. So, this reliability r1 will be multiplication of 0.65, 0.72, 0.77 and 0.82, this turns out to be 0.2955 as we have evaluated it here.

Similarly, r2, so we have now converted this into 1 block r1. Similarly, these 3 blocks can be converted into a single block that is we can call it r2. So, this is r1, then r2, as we calculate this is multiplication of (())(2:33) of 21 22 23. 21 is 0.75, 22 is 0.67 and 23 is 0.95. So, when we multiply the velocity of these 3, I get the r2 value, r2 value comes out to be 0.4774. So, I can say this is my r2, similarly we can calculate r3; r3 will be a combined block of this and r3 is equal to, as we have calculated similarly by multiplying, this is 0.9, this is 0.77, this is 0.68, this is 0.81, and this is 0.82. When we multiply, our reliability turns out to be 0.3130.

Similarly, for these two components, reliability is 0.95 and 0.87, 0.95, 0.87, then we multiply the 2, this reliability turns out to be r4 equal to 0.8265. As we see here, our system is now converted into a parallel system of 4 components, each one is having a series of the components and these are connected in parallel. So, now reliability if I want to calculate, reliability will be parallel connection of this, and for parallel we know 1 minus Pi of 1 minus ri, we have 4 components, so i equal to 1 to 4.

This when we apply same formula, we use 1 minus r1, 1 minus r2, 1 minus r3, 1 minus r4 and we evaluate the reliability we get this final reliability. This we can, let us see and do it by the Excel sheet.

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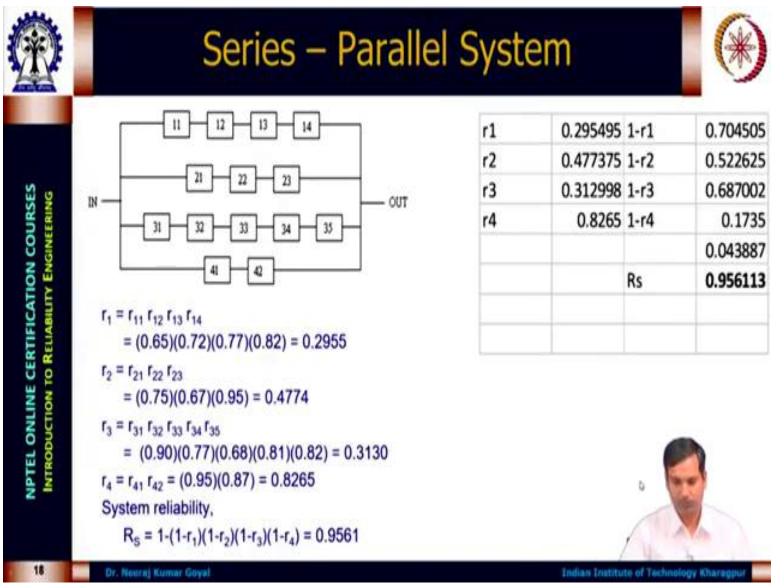

If we have to do this calculation, so just for, so what is r1? r1 is equal to 0.65 multiplied by 0.72, multiplied by 0.77, multiplied by 0.82, so this becomes my r1. Now, what will be my r2? r2 is equal to 0.75 multiplied by 0.67, multiply by 0.95. And how much is r3? r3 is equal to 0.9 multiply by 0.77, multiplied by 0.68, multiplied by 0.81, multiply by 0.82. And r4 is multiplication of 2 components, that is 41 and 4 2, which is 0.95 and 0.87.

Now, if this is the series, so we have taken the series, now these are in parallel. So, for parallel what we need to do? first we will take 1 minus of r1, so we will get 1 minus r1, similarly we will get 1 minus r2, then we will get 1 minus r3, 1 minus r4. So, this I can easily get by 1 minus of r1, and similarly if I copy and paste, I get the 1 minus r2, 1 minus r3, and 1 minus r4. Then, what I need to do? I need to multiply them. So, I will take the product of these terms.

So, when I take product of these terms, what I get is the system unreliability, because these, when I have taken 1 minus r1 that probability has become unreliability. So, as we discussed earlier for parallel system, the system will be unreliable where all parallel components are unreliable. So, that means all becomes multiplication. So, this product when I take, it will give me unreliability. So, this is my unreliability, and if I want to know reliability, reliability will be 1 minus of this.

Here, as you see that, as we discussed earlier, and as you see now that for series component, reliability gets multiplied to get the system reliability, and for parallel systems we first convert relative values into the unreliability, like this was the reliability we converted this into unreliability, and unreliability values are multiplied to get the system reliability, unreliability and from this reliability we can get again subtracting from 1. So, this becomes our system reliability.

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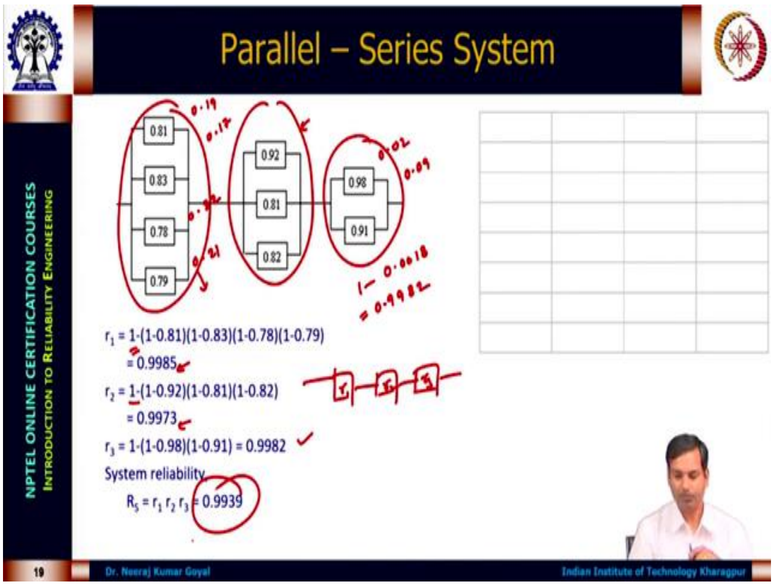

Similarly, we may have a parallel series system. Parallel series systems as we see here this is first systems components are in parallel and then they are then connected in series. So, this is one parallel, this is another parallel, this is another parallel and these are connected in series. Now, for parallel system reliability if you want to calculate as we discussed earlier, first we will convert into unreliability, so this will become 0.19, and point, unreliability will be 0.17, and reliability will be 0.22, and this is 0.21. And then we will multiply this unreliability and what the values which we get then we will subtract from 1, this gives me the unreliability for this block or reliability for this block as 0.9985.

Similarly, I will take 1 minus of this, multiply, then whatever comes I will subtract from 1, this gives me 0.9973. Similarly, for this, I will take 0.02 and 0.09, when we multiply, it will give me 0.0018, and when I subtract from 1, I will get 0.9982. And now these 3 are connected in series, but I got r1, r 2, r3, they are connected in series. So, I will multiply this whatever relative values I have got 0.9985 into 0.9973 into 0.9982, which gives me final relative as 0.9939. This again, so for practice purpose, I am showing this once again using Excel sheet.

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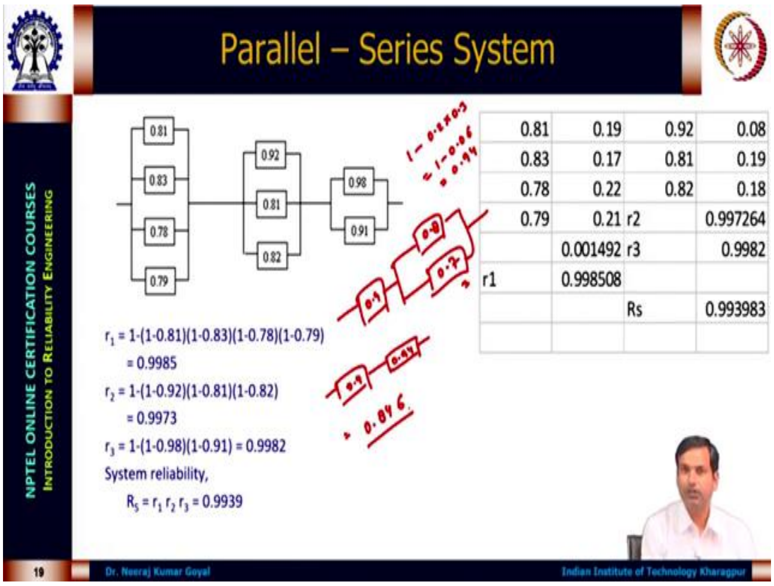

So, for Excel sheet, as we discussed, our, if I am going to solve for first component, let us, I will just first put 0.81, 0.83 or 0.78, 0.79. so, how do I get unreliability of each component will be 1 minus of this. So, this gives me unreliability, from this I get the product term. I will use product, all these terms. So, this gives me the unreliability of first block or 1 minus r1, and r1 will be equal to 1 minus of this value.

Similarly, I can get the second block reliability that is 0.92 0.81 0.82, I will take 1 minus of this and then I will take the 1 minus product of these terms and this gives me this r2, this is my r1, and similarly I can get r3, r3 we already shown by the calculation that is this is equal to 1 minus 0.98 up into 1 minus 0.91, this is my r3. So, my system will have ths series of r1, r2, r3. So, RS would be equal to r1 multiply by r2 multiplied by r3 which comes out to be 0.9939. So, here, as we see that when components are in series parallel combination we can solve them, whatever is the combination, we try to solve that according to that.

You may take some more configurations, let us say if we have a system like this where we have 0.9 here and we have 0.8 reliability here and we have 0 point let us say 7 reliability here and I want to know how much is the relative for this. So, we try to reduce this as we see this is parallel combination. So, first I can reduce this, so this will become 0.9 and 0.8 into 0.8 and 0.7, so what I will do 0.2 into 0.3 1 minus that will be 1 minus 0.06 that will turn out to be 0.94. So, this block will become 0.94.

Now, I can get the reliability easily 0.9 into 0.94 that will be 0.846. Similarly, any other combination if it is available if that is in series and parallel, we can always solve using these methods, we try just try to solve them.

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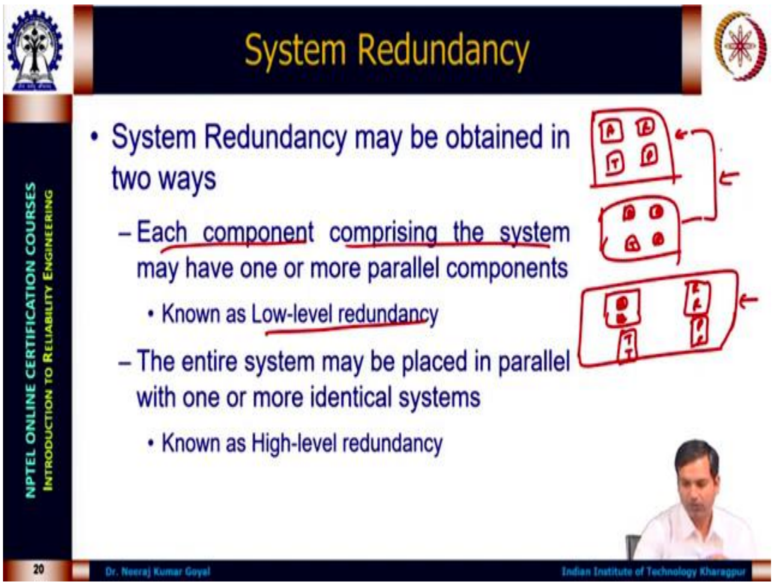

Generally, system may be having a different way of redundancies, like you have multiple components with available video. So, for to get the system function you can connect them as in parallel system.

So, if components you are putting in parallel that means, for a particular system failure multiple components has to fail, then only system will fail. So, in that case we are doing the lower-level redundancy, because let us say if we have a design here where we have systems here, let us say if we talk about a general Communication System, you may have an antenna here, you may have a receiver here, you may have a transmitter here, you may have a power supply here.

Now, if you want redundancy there are 2 possible ways to do the redundancy that this complete system you want to have a redundancy. So, what you can do you can use 2 such systems separately, A R T P or another case is that you use a system where each component is having redundancy that means both A can be twice, similarly 2 R are there in 1 system, 2 T are there in 1 system to power system are there.

Here, this is a low-level redundancy compared to the overall scenario because here these major systems you have duplicated. This is high level redundancy because here at the system level itself you have duplicated. So, when we investigate the reliability for such kind of systems, this is called Low, high-level redundancy, this is called low level redundancy.

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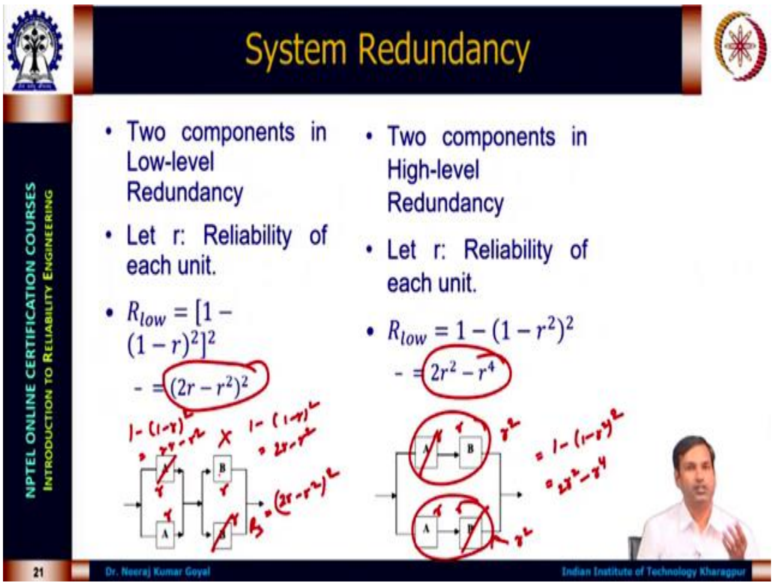

Let us say, if we have the low-level redundancy the same thing, we have discussed 4 here, let us see if there are only 2 if we talk about transceiver system, we may have a transmitter here and receiver only, let us say we talk about only 2. Then let us say a means transmitter B means receiver and our transmit transceiver system will work when both transmitter and receiving is working. So, in that case, in low level redundancy what you mean?

We are connecting 2 transmitters in parallel. So, if any of the transmitter work, we will be able to transmit similarly. So, we at the design level, in the circuit level, we will have 2 transmitter and we will have 2 receivers. Transmitter function whenever it is to be done, either of the unit can do, similarly receiver function need to do either of the unit can do, we can say A1 A2 B1 B2 to differentiate between them.

Another case is that we have one system where A1 B1 is there and another system we have A2 and B2 is there, that means in this case we have one PCB where or we have a complete system of transceiver 1 and we have another transceiver here, either of the transfer transceiver working the system will work. Now, how this makes a difference, let us see this.

If we want to know the reliability for this. So, let us say all reliabilities are same r r r r, then as per our principles as we discussed, here this is parallel, so this will become 1 minus 1 minus r square, similarly, this will also become 1 minus 1 minus r square, which if we solve this will become 2r minus r square, this is also 2r minus r square. Now, these are in series, so we have to multiply them. So, 2r minus r square has to be multiplied, so this will become our reliability.

Now, here we have the high-level redundancy that means this pair should work. In this case, this is also r this is also our this is r, so first we will they are connected in series, so reliability will be r square, this reliability will also be r square, then 1 minus 1 minus r square, whole Square, so this will become, just give me a second, so 1 minus r square, whole Square. This if we know this will become r square 2r square minus r to the power 4. So, we have low level redundancy, we have this reliability and high-level redundancy we have this reliability. So, natural question arises which 1 is higher, whether this reliability is higher or this reliability is higher.

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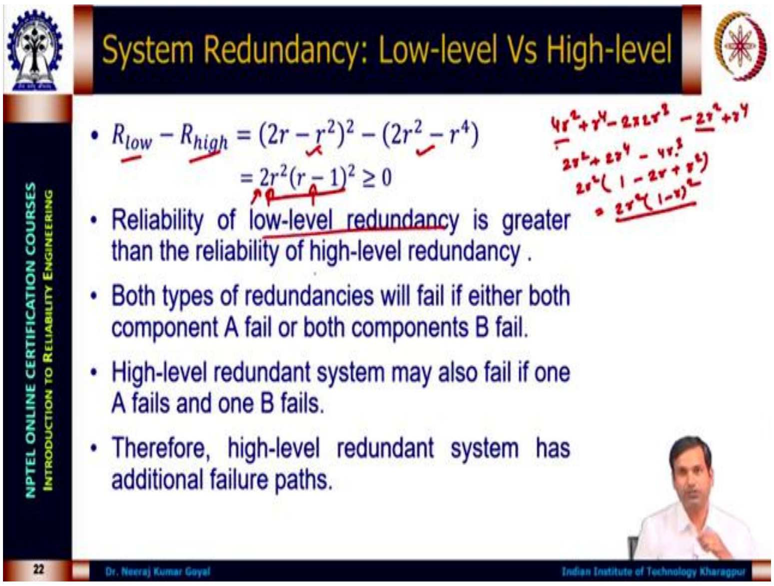

$$R_{\text{low}} - R_{\text{high}} = \left(2r - r^2\right)^2 - \left(2r^2 - r^4\right)$$
$$= 2r^2(r-1)^2 \geq 0$$

Generally, as we see here, if I can take reliability of low redundancy and reliability of high redundancy, this is my low, this is my high, if I solve this, then this will become 4 r square plus r to the power of 4 minus 2 into 2 r cube, this is the expansion of this minus 2 r square plus r to the power 4. This if we solve this will become 2 r square r to the power 4 plus r to the power 4 is 2 r to the power 4 minus 4 r to the power Q.

If I take 2 r square s common, then this will become 1 minus 2r for this and plus r square, this if we know we know this is 2 r square into 1 minus r to the power 2. Now, this value as we see this is a square, this is also square and this is positive 2. So, this value, for any value of r this value can never be negative value, that means, this is always going to be greater than 0 that means r low is always going to be higher than r high. So, in a way we can say that that lower-level redundancy gives a better reliability compared to the high-level redundancy, which is obvious also.

If we look at here that if this A fails and this B fails, then still my system can continue to work with this A and this B. But here, if this A fails and this B fails, my system will not work because this system will also feel this system will also fail. So, here we have a more combination, here the system will work in more combination, here the system will work in less combinations. So, there are more possibilities in which system will work, there are less possibilities here in which system will work. So, our lower-level redundancy generally gives better reality, but as a designer, implementing lower-level redundancy is challenging.

How the two transmitters will connect together, so that and the signal will go to the receiver here and then how two receivers will again be able to do the same thing, that becomes challenging from the design, while it is much easier to put two transceivers in parallel and just see that and show that they are working. So, this is easier to design high level redundancy, but it is giving you little lesser relative but low-level redundancy though it gives a higher liability, but it may pose the designing challenges.

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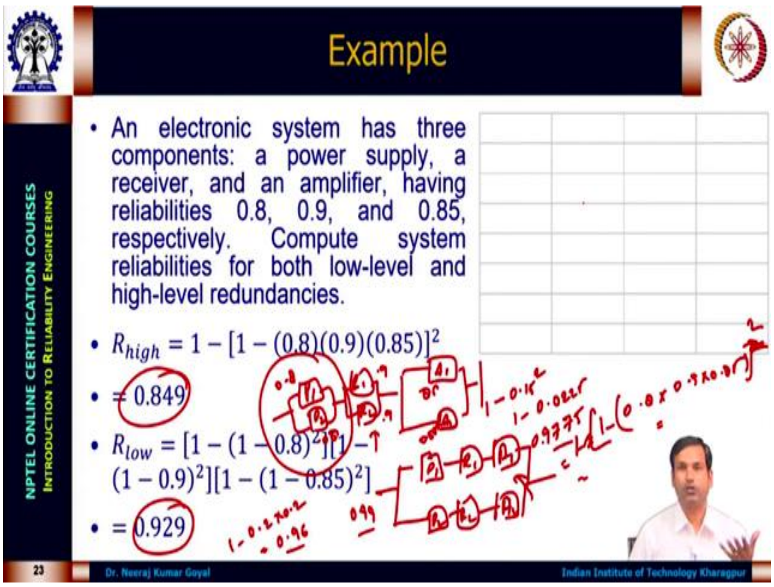

Let us say if we have the 3 level systems. So, here what happens in low level and high level, in both cases both we have to see in both the combinations, we have two transmitter, two receiver, same amount of components are being used but that design is little different.

Now, if we take the same example, let us say one electronic system has power supply receiver and amplifier, three parts are there and their reliabilities are 0.8, 0.9 and 0.85, then what will be the reliability for low level and high-level redundancies? So, for low level redundancies we know that reliability has to be, for each component we have the reliability, so we have the 3 three here, we have 3 components and both. So, that means we have the two-level redundancy here, we have, so for amplifier we have 2 amplifiers A1, A2.

$$R_{\text{high}} = 1 - [1 - (0.8)(0.9)(0.85)]^2$$
$$= 0.849$$
$$R_{\text{low}} = \left[1 - (1 - 0.8)^2\right][1 -$$
$$(1 - 0.9)^2\Big]\left[1 - (1 - 0.85)^2\right]$$
$$= 0.929$$

Then we have or we can say power supply, we can take first P1, P2, which are in PC parallel, then we have the receiver r1 r2, both are same but two different, same make same design. So, this is my low-level redundancy and high-level redundancy would be P1, then r1, and A1, all are in one set and another set of P2, r2 and A2. So, here we can get the low-level redundancy, now we know this is 0.8, both 0.8 0.8, this is 0.9, 0.9 reliability, this is 0.85 0.85 reliability.

So, unreliability, I want to calculate that reliability for this block, so that will be 0.2 into 0.2 1 minus, so that will be 0.96. Similarly, for this 0.1 0.1, so this reliability will be 1 minus 0.01, that would be 0.99, for this 1 minus 0.15 whole square, that means 1 minus 0.0225, that will be 757, 0.9775.

Once we multiply the 3, this, this and this, we get reliability for, sorry, we get this reliability 0.929. For this, we have to take the multiplication of 3, that is 0.8 into 0.9 into 0.85, this, once we multiply and then we take the square of this and subtract from 1. We take subtract from 1, then we take a square and then we subtract from 1 again, that will give this reliability for high which is 0.849. So, we can solve this and as we have solved earlier this can also be solved in the same way without much difficulty.

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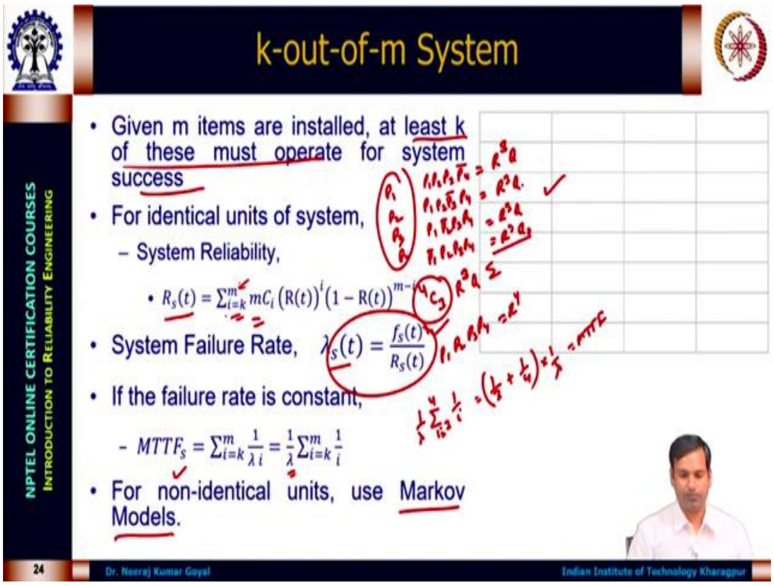

So, this gives us reliability for low-level redundancy, high level redundancy. There can be other type of systems where we have multiple items available to us which is working like, generally let us say production plant where we have m number, let us say we have 10, let us say there, but generally we require around 7 Lids, 2 Lids and that will be enough for our 2-meter production requirement.

So, that means if 7 lets out of the 10 systems are working, then our production requirements are fulfilled. So, we have the redundancy here, that means if we had only 7 Lids, we, our job would continue, but this extra 3 Lids which we have, they help us to provide the redundancy and improve the reliability, because in case any of 1 of the Lids filled or more Lids are filled, we are able to use them.

So, and or they are continuously used. So, the load can be transferred to those Lids. So, here we need k of these to operate for the system to success, this same can also be there for, let us say, if we are talking about the industry where we are having the gas supply or having the let us say some liquid supply. So, multiple pumps may be there, and all the pumps may be continuously working, but for meeting the critical requirement you may not require all pumps to work, even if 1 or 2 fails, the system will continue to function or requirement will be fulfilled.

So, in this case, we only need k systems to work. So, let us say if you talk about, we have 4 pumps let us say we have P1, P2, P3, P4 and all are same. So, if all are same, that means their failure probabilities or reliabilities are same and we need only 3 pumps. So, in that case we have combinations like P1, P2, P3 is working and P4 does not work, then also my system will work, if P1, P2, P3 is not working, then my system will work if P1, P2 is not working P3, P4 working my system will work. If P1 does not work, but P2, P3, P4 works, then also my system will work.

So, here if we see that we have 4 items out of the 4 if 3 works, so I have 4 C3 combinations, in all these combinations the system will work and for each combination the reliability is same. How much is the reliability here? that is the 3 system should work. So, r to the power q and 1 system can, is not working that is Q. Similarly, 3 same reliability, 3 should work, 1 is not working, 3 should work, 1 is not working, 3 should work and 1 is not working. So, 4C3 r to the power 3 and Q, because if we sum it 4C 3 times, we get this reliability which is equal to this.

So, similarly, but another case where system will work that is all 4 are working, P1, P2, P3, P4, that is r to the power 4. So, in all cases that either k or more than k systems are working, our reliability, our system will be reliable. So, we will take all such combinations where i is equal to k to m. So, k working or more than k up to all are working, then MCI will be the number of such combinations and each combination for i r t raised to the power i as we have taken for 3 r to the power 3 and remaining may not be working. So, 1 minus r t raised to the power m minus i, this then we take summation, we get the system reliability.

Once we get the system reliability, we can get other things, we want to calculate system failure rate, we can get it by, first taking negative differentiation of this with respect to T and then divide by the RST. And, if failure rate is constant, then MTTF is calculated as 1 upon summation of i equal to 1 to k, 1 upon Lambda i or we can say 1 upon Lambda summation of i equal to 1 to k because Lambda is independent of i, 1 upon i.

So, like for 4C 3 system we can say this is i equal to 3 to 4, 1 upon Lambda, 1 upon i. So, that will be equal to 1 upon 1 by 3 plus 1 by 4 into 1 upon Lambda, this will give me the MTTF. For non-identical units, we may have to either evaluate combinations, we have to make all the combinations, then all combinations need to be evaluated or we will discuss little later in next week about the Markov model. So, Markov models can also be used for the determination of reliability for such systems. So, here for identical units this follows the binomial distribution or…

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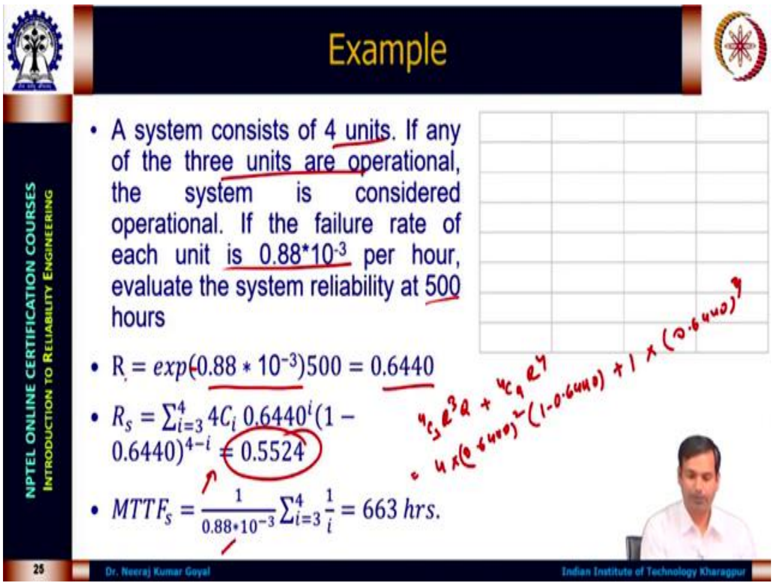

$$R = \exp\left(-0.88 * 10^{-3}\right)500 = 0.6440$$

$$R_s = \sum_{i=3}^{4} 4C_i\, 0.6440^i (1 -$$

$$0.6440)^{4-i} = 0.5524$$

$$MTTF_s = \frac{1}{0.88 * 10^{-3}} \sum_{i=3}^{4} \frac{1}{i} = 663\text{hrs.}$$

So, here let us take one example that a system is consists of 4 units, if any 3 units are operational like we discussed earlier, if 3 are working our system is considered to be working. So, 3 working, 4 working both are the working scenario. The failure rate of the each unit is 0.88 into 10 to the power minus 3. So, reliability will become e to the power minus, this minus sign should also come minus of Lambda into T, T is 500, the reliability turns out to be 0.66440 for each unit, so this becomes my r. Now, for system reliability I have 4C 3 r cube Q plus 4 C4 to the power 4. So, 4 into 0.6440 Q and 1 minus 0.6440 plus 4C 4 is 1 multiplied by 0.6440 raised to the power of 4.

Once we calculate this, we get this system reliability, that is 0.5524, and as we discussed earlier system MTTF can be calculated as 1 upon Lambda, 1 divided by Lambda multiplied by 1 by 3 plus 1 by 4, this comes out to be 663 hours.

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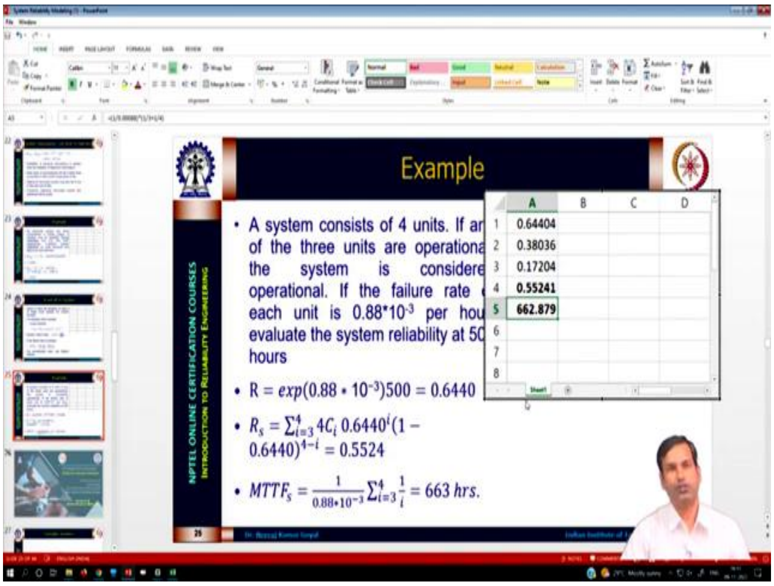

We can do this quickly in Excel sheet, just to explain you. So, we have, this is equal to exponential minus Lambda T, reliability is exponential minus Lambda T, Lambda is 0.88 e minus 3 into T, T is 500, this becomes my per unit reliability. Now, I want to calculate the first that is 4 into 4C 34 that is 4 into reliability raised to the power 3 multiply by unreliability, that is 1 minus of reliability raised to the power 1. This becomes my first combination.

Second combination is this power 4 and total relative will become that that means either 3 working or 4 working, that becomes my system relative reliability. And MTTF is equal to 1 divided by Lambda, Lambda is 0.88 e minus 3, multiply by 1 by 3, plus 1 by 4, you get this. So, thank you for listening, we will continue our discussion on system reliability, other configurations will discuss and we will stop it here for the today. Thank you.