Introduction to Reliability Engineering
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Lecture 18
System Reliability Modelling (Contd.)

Hello everyone. So, we will continue our discussion on system reliability modelling. We have discussed about series, configuration, parallel configuration, parallel series, series parallel, we also discussed k out of m models. So, now we will discuss, today we will discuss further or more configurations, which is little bit complex configuration.

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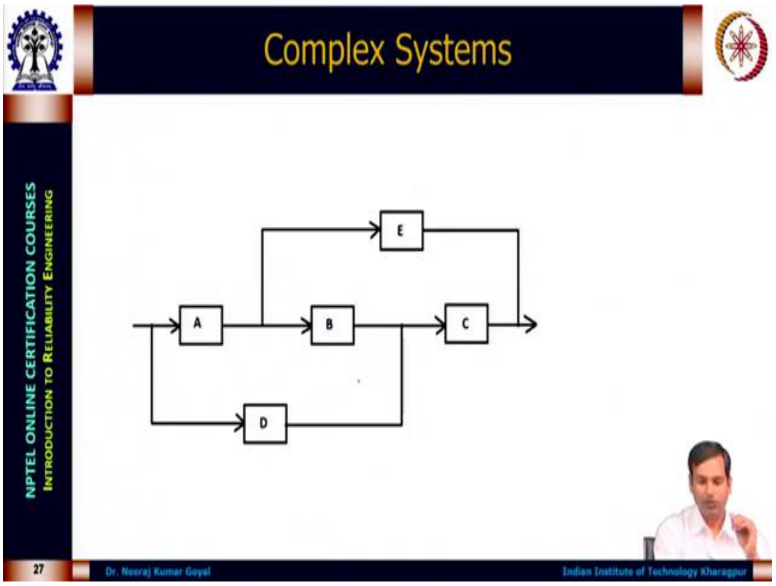

So, like if we look at this kind of system, this can be a network here, communication network also called a bridge network. So, here we look at it here this component is not series parallel, like if I want to reduce this using series parallel configuration, I am not able to do, that that E is not in parallel with B. So, neither it is in series with B, neither B is C Series in with B because there is another connection coming up.

So, because of this configuration, the series parallel reduction methods which we discuss in which we try to reduce each series element of parallel series, parallel element by step-by-step manner is not possible here. So, to solve these kinds of problems, we have to use different methods.

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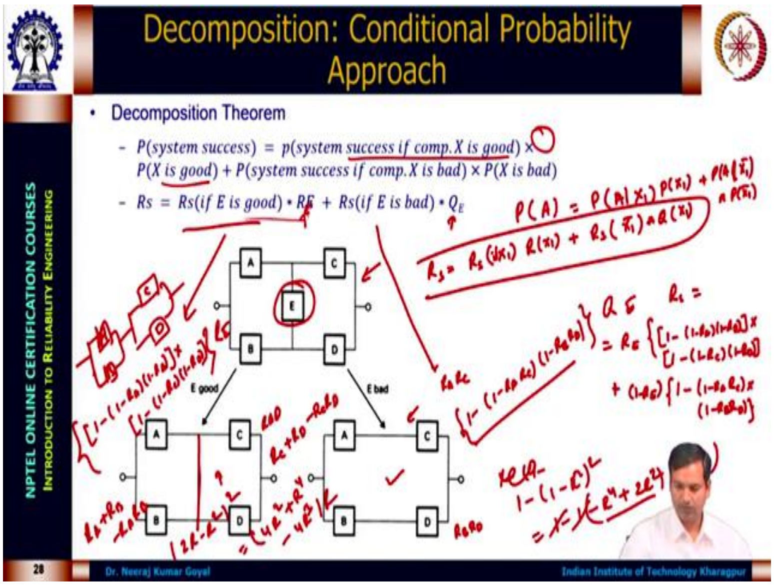

First method, let us discuss is the decomposition method. Decomposition method which is also the conditional probability approach. It says that I can evaluate reliability for this by performing a decomposition, that means, I can make this network into two conditions. So, I can take any element of this and I can provide the two conditions; one condition is calculating the reliability, then a component X is good, multiply the reliability of the component, that X is good. So, this is as we discussed that is total probability theorem.

We know that total probability theorem that probability of A is equal to probability of A given, let us say X1 multiply by probability of X1 plus probability of A given X1 bar multiply by probability of X1 bar. So, here what is X1 let us say the component is working. So, here generally we can choose any element, this is applicable to any element, but as we see this element is the one which is because of, which we are having trouble, because of which we are not able to convert it into series and parallel. So, we will apply this theorem to this component, component A.

We will say that we want to know the reliability, so reliability of system, so I want to know reliability of system given X1 is working. If X1 is working, that means probability that X1 is working is reliability of X1, I can say R1, plus reliability of system, if X1 is not working multiply by probability that X1 is not working is nothing but unreliability Q of X1. So, here we are able to calculate system reliability in two parts.

First, we calculate reliability that component is working condition, that means it is perfectly reliable, it cannot fail and multiply by the reliability of the component. So, this is R small RE another case is that component is not working that means component is in field condition and multiply by the probability of failure. So, if we say component is working, component working means this is perfectly reliable, reliability is 1, reliability is 1 that means it is always working. So, there is no block here, this is the sorting link. So, this component is converted into a sorting link in the diagram.

Another case is when E is not working, E is not working means it is not at all available for the working. So, this is not existing, so this is removed from the system, so this becomes open. This circuit when we solve or this reliability block diagram, when we solve, we will get the reliability of the system, then component E is working, and this is giving the reliability, this is giving the reliability while E is not working.

So, how can we get the reliability is working? This now this is solved, so we can easily solve what will be the reliability for this that means we can say this is A and B in parallel and again C and D in parallel. So, we can get reliability easily here, that is 1 minus, 1 minus Ra, 1 minus Rb, this is in series with this, that is 1 minus 1 minus Rc and 1 minus rd, this gives me the reliability for this.

Similarly, if I want to calculate reliability for this, then this is series, so Ra Rc and this is series Rb Rd, both are in parallel, so that will be equal to 1 minus Ra Rc into 1 minus Rb Rd. So, this gives me this reliability, and this gives me this reliability, whatever I get I will multiply this with Re, I will multiply this with QE, and then take the summation of both, and that will give me the system reliability.

So, my system reliability here will be equal to this that is Re into 1 minus Ra, 1 minus Rb into 1 minus 1 minus Rc 1 minus Rd plus 1 minus Re, Q is 1 minus Re, into 1 minus 1 minus Ra Rc into 1 minus Rb Rd. This if we solve, I get the system reliability.

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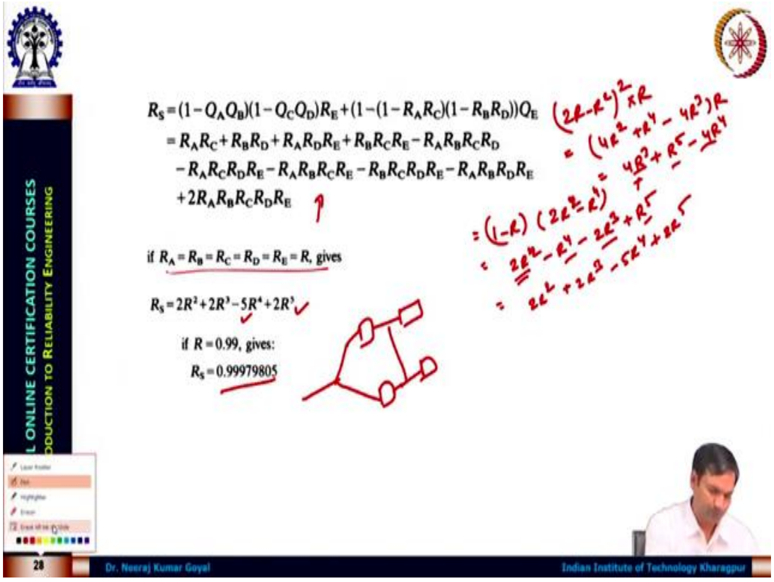

$$
\begin{aligned}
R_S &= (1-Q_A Q_B)(1-Q_C Q_D)R_E + (1-(1-R_A R_C)(1-R_B R_D))Q_E \\
&= R_A R_C + R_B R_D + R_A R_D R_E + R_B R_C R_E - R_A R_B R_C R_D \\
&\quad - R_A R_C R_D R_E - R_A R_B R_C R_E - R_B R_C R_D R_E - R_A R_B R_D R_E \\
&\quad + 2 R_A R_B R_C R_D R_E
\end{aligned}
$$

The same thing is shown here, when we solve, then, and if you open this now and if we assume that all are equal to R, then R, when we solve this then this actually becomes Ra plus Rb minus Ra Rb, and this will become, second term will become Rc plus Rd minus Rc Rd or in a way if all are equal we can say this is 2R minus R square and multiply by again whole square, this will be as we discussed this is R square R square, this is 1 minus or we can say enough way we can say 1 minus Ra Rc, so 1 minus 1 minus R square whole square. So, that will be 1 minus 1 minus R to the power 4 plus 2 R square.

So, R 2 R square minus R to the power 4 and this is 2R minus R square whole square, and when we solve them, some, both when we put summation of both like this will become 4 R square plus R to the power 4 minus 4R to the power 3. So, if we sum it up, we will become R square 4 here 2 here 6 R square, R4 will get cancelled minus, I have to multiply here with another R and here I have to multiply with 1 minus R. So, when we solve this, we will get these values, we will get the same answers or let me do this here.

4R square minus 4 2R minus R square and whole square multiply by R this will give me 4 R square plus R to the power of 4 minus 4 R Cube into R this will give me 4 R cube plus R to the power of 5 minus 4R to the power 4. Similarly, second one, when I solve this portion that is 1 minus R into 2 R square minus R to the power 4, this will be equal to R to the power 4 2 R square, sorry, 2 R square minus R to the power 4 minus 2 R cube Plus R to the power 5.

Once we sum it up both, what we will get? we let us go with the small 2 high. So, R square term if you see, only 1, that is 2 R square, then R cube term if you look 4 R cube here minus 2 R cube here, so that will become 2 R cube. Now, if you look at the R to the power 4 term minus 4 here minus 1 here, so minus 5.

Then R to the power 5 term, only 1 1 here, that is 2R to the power 5, same term we are getting here. So, same Solutions when we apply, we are able to if let us say each component left is 0.99, I can get the system to actually using this same formula. This problem, let us say we have another network which is more complex. Let us say if we have another system one here like this then we have another system here.

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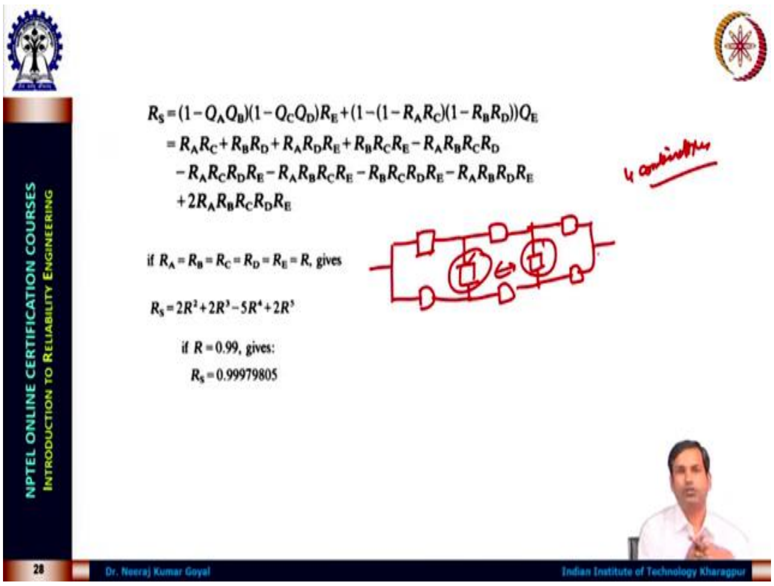

Let us make another system here like we have made one system like this, now this system can be, there may be another system like this, if you see many complexities may be higher, now here when we are applying then with this when I solve again this component will become will be there which is non series parallel. So, this component solution will again require me to solve using the decomposition theorem. So, here if I take these two components, I will actually have 4 combinations.

So, similarly if I have more complex Network where more such elements are there, number of combinations will multiply every time by 2. If I have 3 such components it, will become 8 combinations. So, as we see that more complexity and more irreducibility is there due to the non-parallel series combinations, the combinations will rise and because of that it will become difficult and time consuming and to solve the problem using this decomposition theorem. However, this can be solved using the decomposition term, only thing is that we may require more time and effort.

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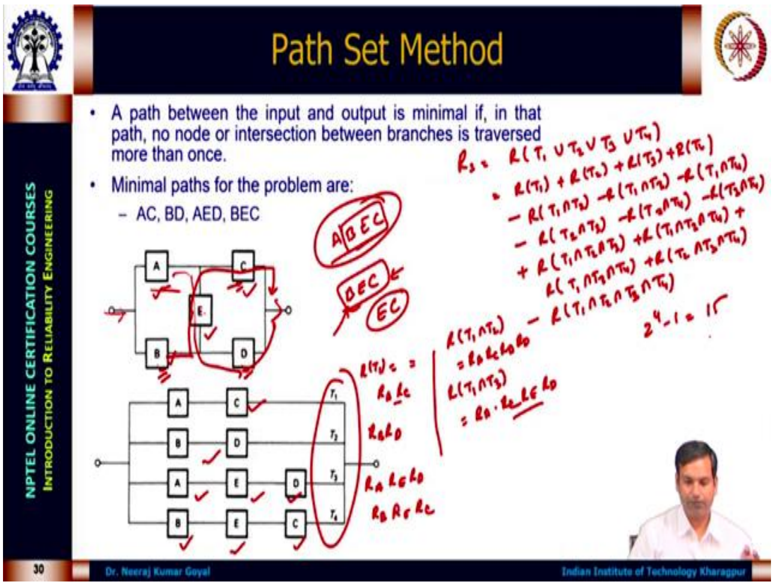

To solve these problems, another method which can be used is the path set method. So, path set is a set of components which if they are working like here A and C, if they work my system will work, I will be able to transfer the signal from here to here. So, in RBD also this is going to be working. So, or if my B and E D is working, then also my system will work. So, these we have to make the combination of the components when they work, then the system is working.

So, I can have multiple combinations here like A and C is working, my system will work, B and D is working my system will work. Another path can be from A we have E and then from we have D, that means A is working, E is working, and D is working, A is working, E is working, D is working, then also my system will work or B is working, E is working, and C is working, then my system will also about B E D, there is no other combination here.

Though I can say that if ABE, A is working, B is working, E is working, C is working, ABEC, then also system is going to work, but this is a path, this is also called a path, but this is not a minimal path, because even if B E and C is working that is sufficient for the system to work.

So, a minimal path set is the one for which if I take a subset of any subset of that will not become a path. Like, BEC is a minimal path, because if B does not work, then only E and C cannot ensure, like if only E and C Works B does not work, then I cannot ensure that my system will be working. But, if B E and C, all 3 are working, then I will be sure that my system is going to work. So, subset of this is not possible to make sure that my system is going to work.

So, this becomes my path now any of the path is working my system will work if combination of A and C works, my system will work. Combination of B and T work, my system will work. A E and D work, B E and C work, my system will work. So, now I have the 4 components, my system reliability is that reliability that T1 is working or T2 is working or T3 is working or T4, any path working my system will be reliable.

To solve this, I can get it by reliability of T1. As we know, we can use the inclusion exclusion formula as we discussed earlier, that is individual element I will take the individual reliabilities, I will sum it up, plus R T3 plus R T4. Then second order combinations will be subtracted minus RT1 intersection T2 minus R T1 intersection T3 minus reliability of or probability we can say of T1 intersection T4, this is 4, 4C2 means 4 into 3 by 2, 6 combinations will be there.

Then minus R or T2 T3 minus R T2 T4 minus R T3 T4, then I will take the 3 level combination there will be plus. So, 4 C 3 means 4 combinations will be there that is R T1 working T2 working and T3 working, plus R T1 working T2 working and T4 working plus R T1 working T3 working T for working plus R T2 working T3 working T4 working, we have 4 combinations here then I will take the negative of 5 or fourth level combination, like even combinations are negative and odd combinations are positive, that is R of T1 intersection T2 intersection T3 intersection T4.

Now what is my T1? T1 is A and C and what is relative value when I am calculating individually? R of T1 means Ra Rc, R of T 1 and R of T2 will be equal to Rb Rd, this is Ra Re Rd, this is Rb Re RC. But when I take combination when I take combination that is T1 and T2 are T1 intersection T2. Now, T1 is A and C, T 2 is B and D. So, that will be equal to Ra RC Rb RD when I say R of T1 T3, then if we say Ra Rc and we have 3 here we see that Ra is coming twice.

So, when it is coming Ra intersection with Ra as we know from the atom potential Ra intersection with Ra will give only Ra it will not become Ra square. So, that will become Ra into Rc. Now, this Ra will not be becoming Square, it will become Re Rd. So, same way, we have to solve all these combinations and we will be getting the probabilities.

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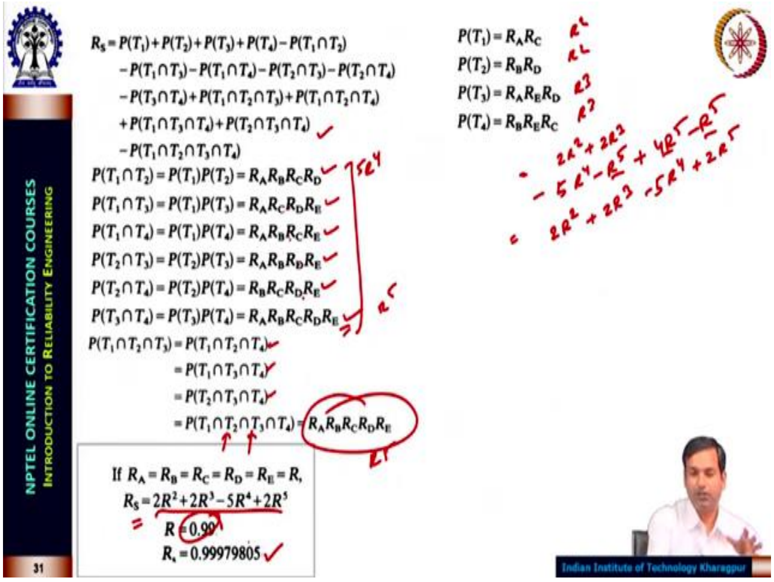

$$
\begin{aligned}
R_S &= P(T_1) + P(T_2) + P(T_3) + P(T_4) - P(T_1 \cap T_2) \\
&\quad - P(T_1 \cap T_3) - P(T_1 \cap T_4) - P(T_2 \cap T_3) - P(T_2 \cap T_4) \\
&\quad - P(T_3 \cap T_4) + P(T_1 \cap T_2 \cap T_3) + P(T_1 \cap T_2 \cap T_4) \\
&\quad + P(T_1 \cap T_3 \cap T_4) + P(T_2 \cap T_3 \cap T_4) \\
&\quad - P(T_1 \cap T_2 \cap T_3 \cap T_4)
\end{aligned}
$$

$$P(T_1) = R_A R_C$$
$$P(T_2) = R_B R_D$$
$$P(T_3) = R_A R_E R_D$$
$$P(T_4) = R_B R_E R_C$$

$$P(T_1 \cap T_2) = P(T_1)P(T_2) = R_A R_B R_C R_D$$
$$P(T_1 \cap T_3) = P(T_1)P(T_3) = R_A R_C R_D R_E$$
$$P(T_1 \cap T_4) = P(T_1)P(T_4) = R_A R_B R_C R_E$$
$$P(T_2 \cap T_3) = P(T_2)P(T_3) = R_A R_B R_D R_E$$
$$P(T_2 \cap T_4) = P(T_2)P(T_4) = R_B R_C R_D R_E$$
$$P(T_3 \cap T_4) = P(T_3)P(T_4) = R_A R_B R_C R_D R_E$$

$$
\begin{aligned}
P(T_1 \cap T_2 \cap T_3) &= P(T_1 \cap T_2 \cap T_4) \\
&= P(T_1 \cap T_3 \cap T_4) \\
&= P(T_2 \cap T_3 \cap T_4) \\
&= P(T_1 \cap T_2 \cap T_3 \cap T_4) = R_A R_B R_C R_D R_E
\end{aligned}
$$

$$R_A = R_B = R_C = R_D = R_E = R,$$
$$R_S = 2R^2 + 2R^3 - 5R^4 + 2R^5$$
$$R = 0.99$$
$$R_c = 0.99979805$$

So, here we have shown here this formula T1 T2 gives this T1 T3 gives this, T1 T4 T2 T3, T3 T2 T4 T3 T4 we get all these combinations and T1 T2 T3 which is in the all these cases T1 T2 T3 T1 T2 T4 T1 T3 T4, whenever we combine 3 path sets, we always get all elements all 5 elements need to work. Similarly, whenever we take all the parts set then also it becomes same.

Now, when we apply the formula, if we assume all are equal, then if and we apply the formula, then our reliability of the system turned out to be, like here if we see this is Ra square this is Ra square this is R cube this is R cube, we get 2 Ra square plus 2 R cube here. Now, these terms will be negative, this is R to the power 4, 1 2 3 4 5 times and RT to the power 5 will be coming 1 time. So, minus 5 R to the power 4 minus R to the power 5, because of this.

Now, these 4 combinations will be summed up and in every case the probabilities R to the power 5, Plus 4 times R to the power 5 and 1 combination for 5 is subtracted R to the power 5, this turns out to be 2 Ra square plus 2 R cube, then R to the power 3 is there R to the power 4 is only 1 term minus 5 R to the power 4 and R to the power 5 if we see 5, 3 terms are there 2 negative and 4 positive. So, that will become 2R to the power 5. Same formula we are getting here and once we apply efficient reliability is 0.99 for each component, then my system relative will be nothing but applying this formula, we will get this value.

So, as we see here using path sets, we can invert the reliability, but here when we are calculating the reliability, it is a little lengthy process, because if we see here, we have to find the reliability of each combination and these number of combinations as we see here, if we have 4 paths, combination is 2 to the power 4 minus 1, that means 15. We have 15 combination here, like 4 here 5 6 7 8 9 10 11 12 13 14 and 15. So, we have to evaluate 15 combinations reliability, and then we have to apply this plus minus formula to get the system reliability.

There are other methods, a lot of research has been done and which gives other methods, which helps to calculate this reliability shorter formula versions. That is out of the scope for this lecture, we will not be covering that but you can study more in the literature and you will you will be able to find those methods.

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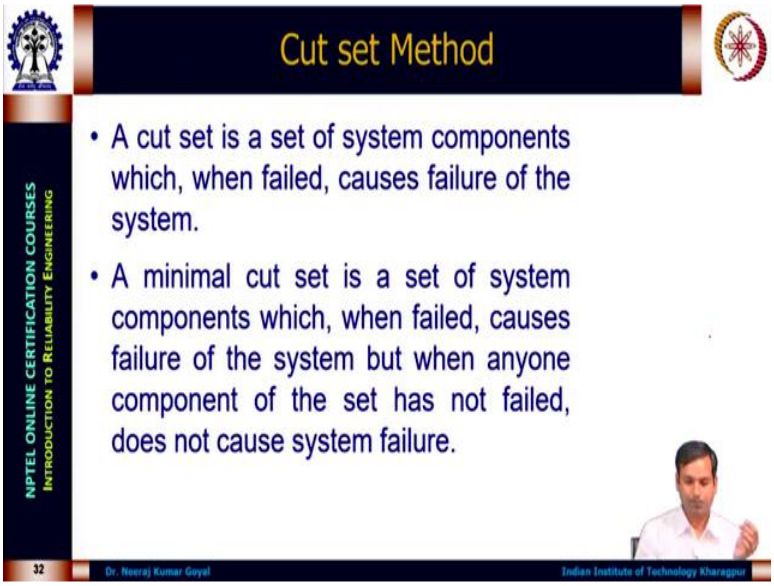

Similar to path set method, we have the cut set method. In path set method, we are trying to find the ways the system will work that means, when a set of components are working the system will work. A \ cut set is a opposite concept; in a cut set concept we try to find out the set of elements when they fail, the system will fail.

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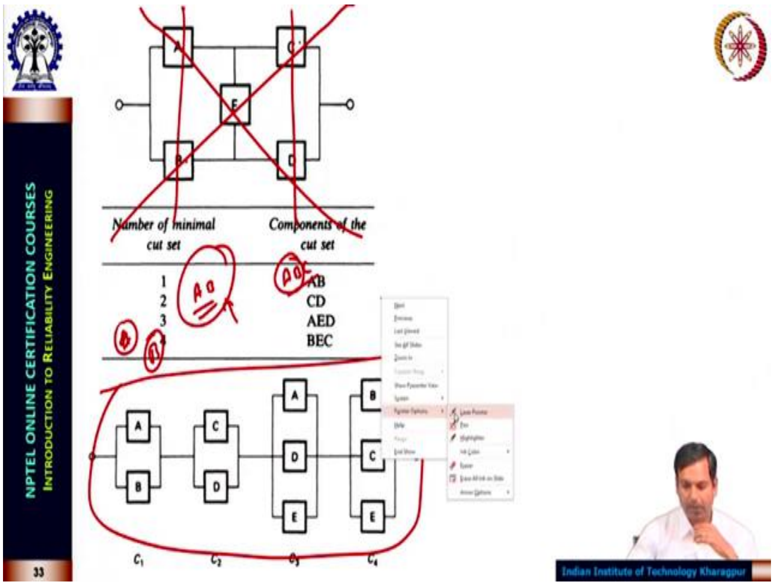

So, like if we see here, in the same diagram, same RBD, relative block diagram, what are the cases where we see that system will not work? Generally, it is called cut set because it is kind of cut, it cuts the system into two parts. So, if element A and B fails together, then in under no other whether it is C E and D is working or not working makes no difference, my system will not work.

Same way, as we discussed earlier, for path set, if a and C works, then whether E B and D working or not working makes no difference my system will work because that is a path set. Similarly, if A and B fails, my system is going to fail irrespective of the status of C D and E. Another possibility is my C and D fails, then also my system will fill ice my system will not work; I will not be able to transfer the signal from here to here.

Another case is that my components A E and D fails, because that is also bisecting if these 3 components fail, then also my system will not work alone B and C Works my system will not work. Similarly, if B E and C fails, then also my system will fail. So, I have 4 cut sets here, 4 ways, I can cut it and 4 ways, 4 (com), 4 sets of the components will be there which will be giving me the minimal cut set.

Similar to minimal part set concept, we have the minimal cut set concept. In minimal cut set that means, we have A B as the minimal cut set ABC is a cut set but it is not minimal because A B alone is enough to make the system failure. So, if C fails along with that, the system will be in failed condition, but subset of ABC that is A and B is enough for the system to fail, since this is enough to system to fail this becomes minimum, like if I want to, if I take only A, my system will not be failed this is not sufficient condition for the system to fail.

If I take only B failure then also my system will keep on working a further element are working. So, that is also not sufficient condition to failure. This in RBD can be represented like this, like A B fails CD fails, A D E fails BCE fails. So, to calculate this we can calculate the reliability for this to calculate the relative for this we first calculate the unreliability for this.

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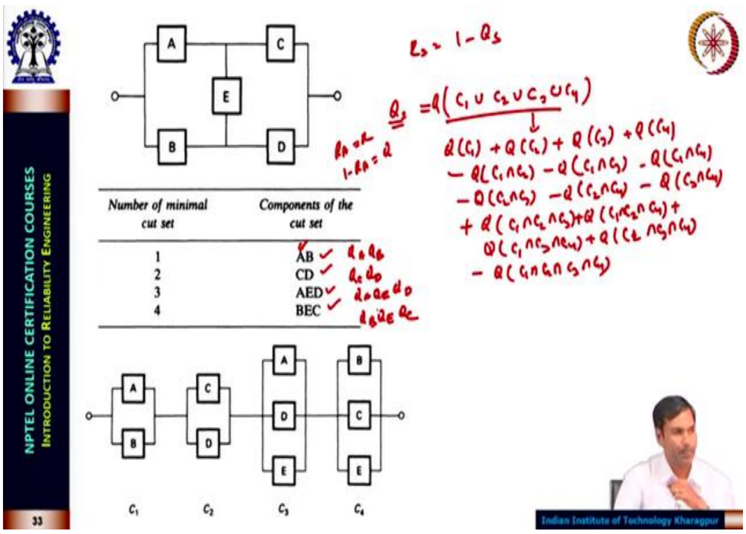

We have AB CD AD. So, our relation becomes same my system will be unreliable if any one of the cut is unreliable. We have 4 cut sets here, if any of the cut set is unreliable any of the cut set is resulting in failure my system will be in failed state.

So, similar concept will be applicable as we discussed earlier, this will generate 15 combinations that is probability failure, probability of cut set 1 plus failure probability of cut set 2 plus QC 3 plus QC 4 minus Q, I will write C1 C2, Q C1 C3, Q C1 C4, Q C2 C3, Q C2 C4, Q C3 C4, then plus terms of 3 combinations 4 terms will be there, Q of C1 C2 C3 plus Q of C1 C2 C4 plus Q of C1 C3 C4 plus Q of C2 C3 C4 and minus combination of 5, 4 terms that is Q of C1 C2 C3 C4. So, this formula will give us the unreliability failure probability of system I want to know the reliability. So, gravity will be 1 minus QS. So, whatever value I get here I will subtract from 1, I will be able to get the envelope T.

So, if what will be the let us say the reliability of A is Ra or R and let us say then 1 minus Ra will be unreliability let us say that is Q. So, probability of A B will be QA QB, this is QC QD, this is QA QE QD, this is QB QE QC. Similarly, we calculate the like we solve for path set, we can solve for cut set also, we can get the C1 C2; C1 C2 is QA QB QC QD QC, 1 C3 is QA QB QA is again coming, we will not count it. So, QA QB QE QD, C1 C4 is QA QB QB is not counted again QE QC C2 C3 means QC QD into QA QE QD is again coming. So, we will not count it again because it is the intersection same term will not repeat it then we have C2 C4. So, QC QD QC is again coming QB QE QC QD QB QE, C3 C4 QA QE QD QB QC because Q is duplicated again.

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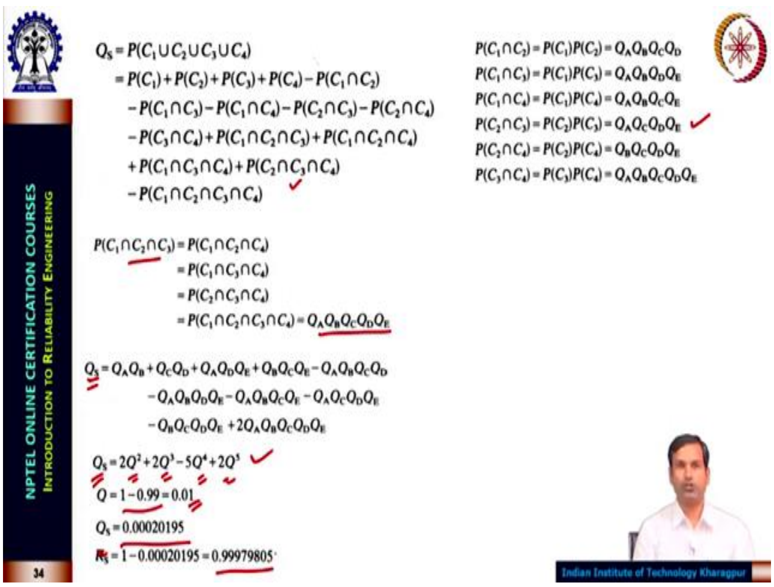

$$Q_S = P(C_1 \cup C_2 \cup C_3 \cup C_4)$$
$$= P(C_1) + P(C_2) + P(C_3) + P(C_4) - P(C_1 \cap C_2)$$
$$-P(C_1 \cap C_3) - P(C_1 \cap C_4) - P(C_2 \cap C_3) - P(C_2 \cap C_4)$$
$$-P(C_3 \cap C_4) + P(C_1 \cap C_2 \cap C_3) + P(C_1 \cap C_2 \cap C_4)$$
$$+P(C_1 \cap C_3 \cap C_4) + P(C_2 \cap C_3 \cap C_4)$$
$$-P(C_1 \cap C_2 \cap C_3 \cap C_4)$$
$$P(C_1 \cap C_2 \cap C_3) = P(C_1 \cap C_2 \cap C_4)$$
$$= P(C_1 \cap C_3 \cap C_4)$$
$$= P(C_2 \cap C_3 \cap C_4)$$
$$= P(C_1 \cap C_2 \cap C_3 \cap C_4) = Q_A Q_B Q_C Q_D Q_E$$
$$Q_S = Q_A Q_B + Q_C Q_D + Q_A Q_D Q_E + Q_B Q_C Q_E - Q_A Q_B Q_C Q_D$$
$$-Q_A Q_B Q_D Q_E - Q_A Q_B Q_C Q_E - Q_A Q_C Q_D Q_E$$
$$-Q_B Q_C Q_D Q_E + 2Q_A Q_B Q_C Q_D Q_E$$
$$Q_S = 2Q^2 + 2Q^3 - 5Q^4 + 2Q^5$$
$$Q = 1 - 0.99 = 0.01$$
$$Q_S = 0.00020195$$
$$R_S = 1 - 0.00020195 = 0.99979805$$

So, this way we get this we take this we get these values and similarly for third level also, third level all 5 terms will come, as we have seen for C1 C2 C3 QA QB QC QD, then QA will not repeat but QE will come from here, that will become QA QB QC QD QE, same way when we take other 3 combinations always, we will have the all 5 terms in this case, that is QA QB QC QD QE and reliability of system will be summation of all this.

If we assume, the similar expression will come what has come for the reliability, but the difference is here we have unreliability system and this is unreliability of each element and if we, unreliability of each element let us say reliability was 0.99 as we have taken earlier, then unreliability of each element will be point 01 and reliability system will be calculated using this formula which we have taken here and reliability of the system will be 1 minus of this which turns out to be this which is same as what we have evaluated earlier.

So, we stop it here and we will continue our discussion for more common configurations for system relative evaluation. Thank you.